A Tutorial Manual for

Mathematics I
(3110014)

B.E. Semester1 (All Branches)

Directorate of Technical Education, Gandhinagar, Gujarat

Mathematics I (3110014)

Tutorial Manual is prepared by

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Mathematics-01 (3110014)			
CO-PO Matrices			
Table			
Sr. No.	COs	Statement	
1	1	**Evaluate limits of indeterminate forms, test the convergence of improper integrals and compute length, area and volume using definite integrals**	**15**
2	2	**Test the convergence of series of numbers and power series. Establish the Taylor series and Fourier series expansions of appropriate functions and understand its applications.**	**30**
3	3	**Compute partial derivatives and make its use for finding the maxima or minima of functions of several variables**	**20**
4	4	**Compute double and triple integrals and use them for finding area and volume of bounded regions**	**20**
5	5	**Perform various matrix operations effectively, use matrices for solving system of linear equations and compute the eigen values and eigen vectors of a square matrix.**	**15**

Table CO – PO Matrix

Sr. No.		PO1 Engineering knowledge	PO2 Problem analysis	PO3 Design/ development of solutions	PO4 Conduct investigations of complex problems	PO5 Modern tool usage	PO6 The engineer and society	PO7 Environment and sustainability	PO8 Ethics	PO9 Individual and team work	PO10 Communication	PO11 Project management and finance	PO12 Life-long learning
1	CO1	3	2										
2	CO2	3	2										
3	CO3	3	3										
4	CO4	3	3										
5	CO5	3	3										

Guidelines for Faculty members

1. Teacher should provide the guideline with demonstration some problems to the students.
2. Teacher shall explain in shortly, basic concepts/theory related to the Assignment or Tutorial to the students before starting of each Assignment or Tutorial
3. Involve all the students in Assignment or Tutorial.
4. Teacher is expected to share the skills and competencies to be developed in the students and ensure that the respective skills and competencies are developed in the students after the completion of the Assignment or Tutorial.
5. Teachers should give opportunity to students for hands-on experience after the demonstration.
6. Teacher may provide additional knowledge and skills to the students even though not covered in the manual but are expected from the students.
7. Give assignment and assess the performance of students based on task assigned to check whether it is as per the instructions or not.
8. Teacher is expected to refer complete curriculum of the course and follow the guidelines for implementation.

Instructions for Students

1. Students are expected to carefully listen to all the theory classes delivered by the faculty members and understand the COs, content of the course, teaching and examination scheme, skill set to be developed etc.
2. Students shall organize the work in the group and make record of all work.
3. Students shall develop maintenance skill as expected by course.
4. Student shall attempt to develop related hand-on skills and build confidence.
5. Student shall develop the habits of evolving more ideas, innovations, skills etc. apart from those included in scope of manual.
6. Student shall refer book and resources.
7. Student should develop a habit of submitting the Assignment or Tutorial work as per the schedule and s/he should be well prepared for the same.
8. * refer to challenging problem in the tutorial.

Date:	
Tutorial No: 1	
Topic:	Linear Algebra
Sub Topics:	Elementary row operations in a matrix, Row echelon and reduced row echelon form, Rank by row echelon form, Inverse by Gauss- Jordan method, Solution of system of linear equation by Gauss Elimination method
Relevant CO:	5
Objectives:	1. find Row echelon and reduced row echelon form 2. inverse of matrix using elementary row operations 3. solution of system of linear equations using Gauss Elimination method

Sr. No.	Question	CO	PI	B.T. level
1	Find row echelon and reduced row echelon form of the given matrix $(i)\begin{bmatrix}1 & 5 & 4\\ 0 & 3 & 2\\ 2 & 13 & 10\end{bmatrix}$ $(ii)\begin{bmatrix}1 & 2 & 3 & 4\\ -2 & 0 & 5 & 7\end{bmatrix}$	5	1.1.1	U
2	Find the rank of the given matrices by reducing it into row echelon form (i) $\begin{bmatrix}1 & 5 & 3 & -2\\ 2 & 0 & 4 & 1\\ 4 & 8 & 9 & -1\end{bmatrix}$ (ii) $\begin{bmatrix}0 & 6 & 7\\ -5 & 4 & 2\\ 1 & -2 & 0\end{bmatrix}$ (iii) $\begin{bmatrix}1 & 3 & 4 & 5\\ 1 & 2 & 6 & 7\\ 1 & 5 & 0 & 10\end{bmatrix}$	5	1.1.1	U
3	Find inverse of the following matrices by Gauss Jordan method $(i)\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$ (ii) $\begin{bmatrix}2 & 6 & 6\\ 2 & 7 & 6\\ 2 & 7 & 7\end{bmatrix}$	5	1.1.1	U
4	Discuss the consistency of system of linear equations using concept of rank of matrix		1.1.1	R
5	Examine consistency of following equations using Gauss elimination method and find the solution of following if exists (1) $\begin{aligned}x+y+2z&=8\\ -x-2y+3z&=1\\ 3x-7y+4z&=10.\end{aligned}$ (2) $\begin{aligned}2x+2y+2z&=0\\ -2x+5y+2z&=1\\ 8x+y+4z&=-1\end{aligned}$ (3) $\begin{aligned}x-2y+z&=4\\ 3x+5y+z&=6\\ 6x-y+4z&=2\end{aligned}$		1.1.1	U, A
*6	Solve the following homogenous linear system using Gauss elimination method. $v+3w-2x=0$ $2u+v-4w+3x=0$ $2u+3v+2w-x=0$ $-4u-3v+5w-4x=0$		1.1.1	U, A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 02	
Topic:	Limit of Indeterminate forms, Improper integrals
Sub Topics:	Indeterminate forms and L'Hospital's rule, Improper integrals, Convergence and divergence of the integrals
Relevant CO:	1
Objectives:	1. find limit of indeterminate forms using L'Hospital's Rule 2. find Convergence and divergence of the Improper integrals

Sr. No.	**Question**	**CO**	**PI**	**B.T. level**
1	State L'Hospital's Rule and use it to evaluate following limit. 1. $\lim_{x \to 0} \frac{1-\cos x}{x^2}$ 2. $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$ 3. $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$ 4. $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$ 5. $\lim_{x\to \pi/2} \frac{\sec x}{1+\tan x}$ 6. $\lim_{x\to 0+} \sqrt{x} \ln x$	1	1.1.1	R, U
2	Determine indeterminate forms and evaluate the limit 1. $\lim_{x\to 0}\left[\frac{1}{x^2} - \frac{1}{sin^2 x}\right]$ 2. $\lim_{x\to 1}\left[\frac{x}{x-1} - \frac{1}{\ln x}\right]$ *3. $\lim_{x\to 0}\left(\frac{1^x+2^x+3^x}{3}\right)^{\frac{1}{x}}$	1	1.1.1	A
3	Define Improper integral of First kind and evaluate $\int_0^\infty \frac{dx}{x^2+1}$	1	1.1.1	R, U
4	Evaluate 1) $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ 2) $\int_{-\infty}^{0} e^{2x}\, dx$	1	1.1.1	U
5	Find 1) $\int_{-\infty}^{\infty} \frac{1}{e^x+e^{-x}} dx$ 2) $\int_0^\infty \frac{dx}{(1+x^2)(1+\tan^{-1} x)}$	1	1.1.1	U
6	Prove that p -integral $\int_1^\infty \frac{1}{x^p}\, dx$ converges when $p > 1$ and diverges when $p \leq 1$.	1	1.1.1	U
7	Define Improper integral of Second kind and evaluate $\int_0^3 \frac{1}{\sqrt{3-x}}\, dx$.	1	1.1.1	R,U
8	Test the convergence of $\int_0^1 \frac{dx}{1-x}$. If convergent, then evaluate the same.	1	1.1.1	U
9	Check the convergence of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$.	1	1.1.1	U
*10	Determine whether $\int_4^\infty \frac{\sin^2 x}{\sqrt{x}(x-1)}$ is convergent?	1	1.1.1	U

B. T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing ,E: Evaluating, C: Creating

Date:	
Tutorial No: 3	
Topic:	Linear Algebra
Sub Topics:	Solution of system of linear equation by Gauss Jordan method, Eigen values and Eigen vectors, Caley-Hamilton theorem, Diagonalization of a matrix
Relevant CO:	5
Objectives:	1. solve linear system using Gass Jordan method 2. find eigen values and eigen vectors of matrix 3. Inverse of matrix by Caley-Hamilton theorem

Sr. No.	**Question**	**CO**	**PI**	**B.T. level**
1	Solve the following linear system using Gauss-Jordan method $x+2y-z=-1$, $3x+8y+2z=28$, $4x+9y-z=14$, $x+y+z=6$, $x+2y+3z=14$, $2x+4y+7z=30$	5	1.1.1	U, A
2	Investigate for what values of *a* and *b* the following system of linear equations have (1) No solution (2) Infinite solutions (3) Unique solution $x+y+z=6$ $x+2y+3z=10$ $x+2y+az=b$	5	1.1.1	U, A
3	(i) Find eigen values of A and A^{-1}; $A=\begin{bmatrix}1&2\\0&4\end{bmatrix}$ (ii) if eigen values of an 3x3 matrix are 1, 2 and 0 then what is det(A)?	5	1.1.1	U
4	Find the eigenvalues and corresponding eigen vectors of the following matrices (i) $A=\begin{bmatrix}4&0&1\\-2&1&0\\-2&0&1\end{bmatrix}$ (ii) $A=\begin{bmatrix}1&2&2\\0&2&1\\-1&2&2\end{bmatrix}$	5	1.1.1	U
5	Find $\det(A)$ given that A has $P(\lambda)$ as its characteristic polynomial $i)P(\lambda)=-\lambda^3+2\lambda^2-\lambda-5$ $ii)P(\lambda)=\lambda^4-\lambda^3+7$	5	1.1.1	U
6	If 1 is an eigen value of the matrix $\begin{bmatrix}2&1&0\\-1&0&1\\0&0&1\end{bmatrix}$ then find its corresponding eigen vectors.	5	1.1.1	U
7	State Cayley -Hamilton theorem. Find A^{-1} using Caley-Hamilton theorem;	5	1.1.1	U, A

	(i) $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ (ii) $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$			
*8	Find a 3×3 matrix A that has eigen values 1, -1 and 0 and for which $\begin{bmatrix}1\\ -1\\ 1\end{bmatrix},\begin{bmatrix}1\\ 1\\ 0\end{bmatrix},\begin{bmatrix}1\\ -1\\ 0\end{bmatrix}$ are their corresponding eigen vectors.	5	1.1.1	A
9	Find A^3 using Cayley -Hamilton theorem if $A=\begin{bmatrix}1 & 0 & 1\\ 1 & -1 & 1\\ 0 & 1 & 0\end{bmatrix}$	5	1.1.1	U, A
*10	Diagonalize the matrix A= $\begin{bmatrix}0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}$. Hence find A^{25} .	5	1.1.1	U, A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 04	
Topic:	Fourier Series
Sub Topics:	Fourier Series
Relevant CO:	2
Objectives:	1. Understand Derichlet Condition 2. Derive Euler's formula for Fourier Series 3. Expand periodic function in Fourier series

Sr. No.	Question	CO	PI	B.T. level
1	Explain Dirichlet condition for Fourier series. Give examples of functions whose are not fit to expand as Fourier Series	2	2.1.3	U
2	$\int_0^{c+2\pi} sinmx\ cosnx\ dx =$ ____ for n and m are positive integers	2	1.1.1	U
3	Derive Euler's formulae for Fourier series expansion.	2	1.1.1	U
4	Draw the graph of following Function where f(x +2π) = f(x) (1) $f(x) = x^2$ (2) $f(x) = x$ (3) $f(x) = 0\ for -\pi < x < 0$ $= x\ for\ 0 < x < \pi$ (4) $f(x) = x \quad for\ 0 < x < \pi$ $= 2\pi - x\ for\ \pi < x < 2\pi$	2	1.1.1	U
5	Find Fourier Series expansion for $f(x) = \frac{1}{4}(\pi - x)^2 \quad 0 < x < 2\pi\ and\ f(x + 2\pi)$	2	1.1.2	A
6	Obtain Fourier Series f(x) = e^{ax} $a \neq 0$ in - π < x < π hence find series for $\frac{\pi}{\sinh(\pi)}$	2	1.1.2	A
7	Expand f(x) = xcosx ,0 < x < 2π and f(x +2π) = f(x)	2	1.1.2	A
*8	Find Fourier Series for f(x) = 2x – x^2 in the interval (0,3) and f(x + 3) = f(x)	2	1.1.2	A
9	Find Fourier Series for the function f(x), where $f(x) = 1 + \frac{2x}{\pi} - \pi \leq x \leq 0$ $= 1 - \frac{2x}{\pi}\ 0 \leq x \leq \pi$	2	1.1.2	A
10	Find Half range Cosine series for f(x) = sinx define in (0,π)	2	1.1.2	A
11	Find half range sine series for $f(x) = mx, 0 < x < \frac{\pi}{2}$ $= m(\pi - x), \frac{\pi}{2} < x < \pi$	2	1.1.2	A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 05	
Topic:	Partial Derivative
Sub Topics:	Functions of several variables, Limits and continuity, Test for non existence of a limit, Partial differentiation,
Relevant CO:	3
Objectives:	1. Find Domain and range of function of several variables. 2. Evaluate Limit function of several variables. 3. Discuss continuity of function of several variables. 4. Calculate Partial derivative

Sr. No.	Question	CO	PI	B.T. level
1	Find the domain and range and sketch the graph of domain of following function(1)$f(x) = \sqrt{x^2 - y}$ (2)$f(x) = \frac{x^2}{\sqrt{x^2+y^2-4}}$	3	1.1.1	U
2	Sketch the following surfaces by using level curves (1) z = x^2 + y^2(2) z^2 = x^2 + y^2(3) z = x + y (4) y = x^2 (5) $\frac{x^2}{4} - \frac{y^2}{9} = 1$	3	1.1.1	U
3	Evaluate following limits (1) $\lim_{(x,y)\to(0,0)} \frac{2x^2-2xy}{\sqrt{x}-\sqrt{y}}$ (2)$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{(x^2+y^2)}$ (3)$\lim_{(x,y)\to(0,0)} \frac{4xy^2}{(x^2+y^2)}$	3	1.1.1	U
*4	Discuss the continuity of f(x,y) = $\frac{xy}{\sqrt{x^2+y^2}}$ $if\ (x,y) \neq (0,0)$ $0\ \ if\ (x,y) = (0,0)$	3	1.1.1	A
5	If f$(x,y) =$ $\frac{2xy(x^2+y^2)}{x^2+y^2}$ $if\ (x,y) \neq (0,0)$ find $0\ \ \ if\ (x,y) = (0,0)$ $\left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(0,0)}$ and$\left(\frac{\partial^2 f}{\partial y \partial x}\right)_{(0,0)}$	3	1.1.1	A
6	Explain a geometric interpretation of partial derivatives.	3	2.1.3	U
7	For following function show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (1) $u = tan^{-1}\left(\frac{y}{x}\right)$ (2) $u = log\sqrt{x^2+y^2}$ (3) $u = e^x cosy$ (4) $u = x^3 - 3xy^2$	3	1.1.1	A
8	If $u = e^{xyz}$find $\frac{\partial^3 u}{\partial x \partial y \partial z}$	3	1.1.1	A
9	If $u = \frac{x^2}{a^2+v} + \frac{y^2}{b^2+v} + \frac{z^2}{c^2+v} = 1$ where v is function of x,y,z then prove that $\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 = 2\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\right)$	3	1.2.1	A
10	If $x = rcos\theta\ and\ y = rsin\theta$ prove that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left(\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right)$	3	1.2.1	A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 06	
Topic:	Partial Derivative
Sub Topics:	Mixed derivative theorem, differentiability, Chain rule, Implicit differentiation
Relevant CO:	3
Objectives:	1. Use of chain rule 2. Using partial derivative to find total derivative 3. Understanding of partial derivative of composite function

Sr. No.	**Question**	**CO**	**PI**	**B.T. level**
1	Explain differentiability of function of two variables.	3	1.1.1	U
2	Explain Chain Rule for composite functions.	3	1.1.1	U
3	Find $\frac{du}{dt}$ if $u = x^2 + y^2 + z^2$, $x = e^{2t}$, $y = e^{2t}cos3t, z = e^{2t}sin3t$	3	1.1.1	U
*4	The hight of a right circular cone is 15cm and is increasing at the rates 0.4cm/s. The radius of the base is 10cm and is decreasing at the rate of 0.6cm/s. Find the rate of change of volume.	3	1.2.1	A
5	If $u = f(x - y, y - x, z - x)$ $prove$ $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	3	1.1.1	U
6	If u = f(x,y) where x = rcosθ and y = rsinθ prove that (1) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$ (2) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$	3	1.1.2	A
7	Using partial derivative find the value of $\frac{dy}{dx}$ for $xe^y + \sin(xy) + y - \log 2 = 0$ at (0, log2)	3	1.1.1	A
8	If $y^{x^y} = sinx$ then find $\frac{dy}{dx}$.	3	1.1.1	A
9	If $\sin(xyz) + x^3y^2z^2 + \log(x^2 + y^2 + x^2) = 0$ find $\frac{\partial z}{\partial x}, \frac{\partial y}{\partial z}, \frac{\partial y}{\partial x}$	3	1.1.1	A
10	If $u^2 - v - 6x - y = 0$ and $u - v^2 - 3x + 2y = 0$ then find u_x, u_y	3	1.2.1	A
11	If $u = 2xy$, $v = x^2 - y^2$ and $x = r\cos\theta$, $y = r\sin\theta$ evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$.	3	2.1.3	U

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 07	
Topic:	Infinite Sequence & Series
Sub Topics:	Converges and divergence of Sequences, The Sandwich theorem, The Continuous function theorem, bounded monotonic sequence, Convergence and divergence of infinite series, Geometric series, telescoping series
Relevant CO:	2
Objectives:	1. Understand convergence and divergence of the Sequence and series 2. Find limit of Convergent sequence 3. Find sum of Convergent series

Sr. No.	Question	CO	PI	B.T. level
1	Explain Convergence and Divergence of sequence. Examine whether the following sequences are convergent or divergent. (1) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ (2) $\{k\}_{n=1}^{\infty}$ (3) $\left\{(-1)^{n+1}\right\}_{n=1}^{\infty}$	2	1.1.1	R, U
2	Describe the sandwich theorem for sequences. Use it to check convergence of following sequences. (1) $\left\{\frac{\cos n}{n}\right\}_{n=1}^{\infty}$ (2) $\left\{\frac{(-1)^{n+1}}{2n-1}\right\}_{n=1}^{\infty}$	2	1.1.1	R, U
3	Write down continuous function theorem for sequences. Use it to examine convergence of following sequences. (1) $\left\{2^{\frac{1}{n}}\right\}_{n=1}^{\infty}$ (2) $\left\{\left(1+\frac{1}{n}\right)^{n}\right\}_{n=1}^{\infty}$	2	1.1.1	R, U
4	Define bounded and monotonic sequences. Classify following sequences in terms of bounded and monotonic and discuss their convergence. (1) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ (2) $\left\{\frac{3n+1}{n+1}\right\}_{n=1}^{\infty}$ (3) $\left\{\left((-1)^{n}+1\right)\left(\frac{n+1}{n}\right)\right\}_{n=1}^{\infty}$	2	1.1.1	R, U
*5	Find limit of sequence $(an)=\left(\frac{n^{n}}{n!}\right)$	2	1.1.1	A
6	Define infinite series and discuss their convergence using sequence of the partial sum of the series.	2	1.1.1	R, U
7	Explain telescoping series and use it to check convergence of following series. (1) $\sum_{n=1}^{\infty}\frac{4}{(4n-3)(4n+1)}$ (2) $\sum_{n=1}^{\infty}\frac{2n+1}{(n)^{2}(n+1)^{2}}$ (3) $\sum_{n=1}^{\infty}\left(\tan^{-1}(n)-\tan^{-1}(n+1)\right)$	2	1.1.1	R, U
8	Define geometric series and explain its convergence. Examine convergence of following series and find sum of series if it is convergent. (1) $\sum_{n=1}^{\infty}\frac{(-3)^{n}}{2^{n+2}}$ (2) $\sum_{n=1}^{\infty}\frac{3^{n-1}-1}{6^{n-1}}$ (3) $\sum_{n=1}^{\infty}\frac{\cos n\pi}{5^{n}}$	2	1.1.1	R, U

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 08	
Topic:	Partial Derivative
Sub Topics:	Gradient, Directional derivative, tangent plane and normal line, total differentiation, Local extreme values, Method of Lagrange Multipliers
Relevant CO:	3
Objectives:	1. Evaluate gradient of scalar function 2. Calculate directional derivative 3. Find error using partial derivative 4. Find maximum and minimum value

Sr. No.	Question	CO	PI	B.T. level
1	"The flow of the heat in a temperature field take place in the direction of maximum decrease of temperature." If T is a temperature field then find the direction of maximum change of temperature at given point. $(1) T(x,y,z) = \frac{x}{x^2+y^2}\ at\ point\ (1,-1,2)$ $(2) T(x,y) = e^{x^2+y^2}\sin(2xy)\ at\ point\ P\left(\frac{\pi}{2},0\right)$	3	1.1.2	A
2	If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $\lvert\vec{r}\rvert = \sqrt{x^2+y^2+z^2}$ then find grad(($\lvert\vec{r}\rvert^n$).	3	1.1.2	A
3	If $f(x,y,z) = e^{xyz} + tan^{-1}\left(\frac{x}{y}\right)$ then find grad(f) at point (1,1,1)	3	1.1.1	U
4	Explain directional derivative and find the directional derivative of f(x,y,z) = $3e^x\cos(yz)$ at point p(0,0,0) in the direction of $\vec{a} = 2 + 2\hat{\jmath} - 2\hat{k}$.	3	1.2.1	A
5	Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at point (2, - 1 , 2).	3	1.1.1	A
6	Find the equation of tangent plane and normal line to the surface $2xz^2 - 3xy - 4x = 7$ at point (1, - 1, 2)	3	1.1.1	A
*7	For simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$. Find the maximum error in T due to possible error 2.4% in l and 1% in g.	3	1.1.1	A
8	Find the maximum and minimum value of $2(x^2 - y^2) - x^4 + y^4$.	3	1.1.1	A
9	Find the shortest distance from origine to the surface $xyz^2 = 2$.	3	1.2.1	A
10	Prove that the rectangular solid of maximum volume that can be inscribe in a sphere is a cube	3	1.2.1	A
11	Find the numbers x,y and z such that xyz = 8 and xy + yz +zx is maximum using Langrage Multipliers method	3	1.2.1	A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 09	
Topic:	Multiple Integration
Sub Topics:	Multiple integral, Double integral over Rectangles and general regions, double integrals as volumes,
Relevant CO:	4
Objectives:	1. Evaluate multiple Integral 2. compute area and volume by using Multiple integral

Sr. No.	Question	CO	PI	B.T. level
1	Explain double integration with its geometric meaning.	4	1.1.2	U
2	Evaluate following Multiple integral $$(1)\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}\ (2)\int_1^4\int_{2x^2}^{3x^2} xe^{x^2+y}dydx$$ $$(3)\int_0^1\int_0^{\sqrt{1-y^2}} x^2+y^2dxdy\ (4)\int_o^{\pi}\int_{y=0}^{x}\frac{sinx}{x}dydx$$	4	1.1.1	U
3	Evaluate $\int\int_R (x+y)^2$dxdy where R is a region enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	4	1.1.1	U
4	Evaluate $\iint_R\ x^2\,dA$ where R is a region in the first quadrant bounded by the hyperbola xy = 16 and line y = x , x = 4 and x = 8.	4	1.1.1	U
5	Evaluate $\iint_R\ r\sqrt{a^2-r^2}drd\theta$ over the upper half of the circle r = a cosθ	4	1.1.1	U
*6	Find the volume of prism where base is triangle in XY plane bounded by y axis y= x and y = 1 and whose top lies in the plane z = 2- x – y .	4	1.1.1	A
*7	A thin plate covers the triangular region bounded by x axix and line x = 1 and y = 2x in the first octant. The plate density at point (x,y) is $\rho(x,y)=6x+6y+6.$ Find the plate mass, first moments and center of mass about the coordinate axis.	4	1.1.1	A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 10	
Topic:	Infinite series
Sub Topics:	Integral test, The p-series, The comparison test, The limit comparison test, Ratio test, Raabe's test, Root test,
Relevant CO:	2
Objectives:	1. Discuss convergence of infinite series using various test

Sr. No.	Question	**CO**	**PI**	**B.T. level**
1	State nth term test for divergence of an infinite series. Examine that following series are divergent. (1) $\sum_{n=1}^{\infty}\frac{2n}{3n-1}$ (2) $\sum_{n=1}^{\infty} n\sin\frac{1}{n}$	2	1.1.1	R, U
2	Examine following series for their convergence using Integral test. (1) $\sum_{n=1}^{\infty}\frac{8\tan^{-1}(n)}{1+n^2}$ (2) $\sum_{n=1}^{\infty}\frac{e^n}{1+e^{2n}}$ (3) $\sum_{n=2}^{\infty}\frac{\ln n}{n}$	2	1.1.1	U, A
3	Prove that the P- series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if $p>1$ and divergent if $p\le 1$ also discuss convergence of harmonic series.	2	1.1.1	U
4	Apply comparison test to find convergence of following series. (1) $\sum_{n=1}^{\infty}\frac{1}{n^2+30}$ (2) $\sum_{n=1}^{\infty}\frac{\cos^2 n}{n^{\frac{3}{2}}}$ (3) $\sum_{n=1}^{\infty}\frac{3n^2-3n}{n^2(n-1)(n^2+5)}$ (4) $\sum_{n=1}^{\infty}\frac{1}{n3^n}$ (*5) $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$ (6) $\sum_{n=2}^{\infty}\ln\left(1+\frac{1}{n^2}\right)$	2	1.1.1	U, A
5	Examine convergence of following series by using Ratio Test (1) $\sum_{n=1}^{\infty}\frac{(n!)^2}{2^{n^2}}$ (2) $\sum_{n=1}^{\infty}\frac{4^{n(}(n+1)!}{n^{n+1}}$ (3) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$	2	1.1.1	U, A
6	Examine convergence of following series by using Raabe's Test (1) $\sum_{n=1}^{\infty}\frac{1.4.7...(3n+1)}{n^2 3^n n!}$ (2) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}-1}$	2	1.1.1	U, A
7	Examine convergence of following series by using Cauchy Root Test. (1) $\sum_{n=1}^{\infty} ne^{-n^2}$ (2) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$ (3) $\sum_{n=2}^{\infty}\frac{n}{(\log n)^n}$	2	1.1.1	U, A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 11	
Topic:	Multiple Integration
Sub Topics:	Change of order of integration, double integration in polar coordinates, Area by double integration,
Relevant CO:	4
Objectives:	1. To change order of integration 2. To convert cartesian coordinate to another coordinate for integral using Jacobian.

Sr. No.	Question	CO	PI	B.T. level
1	Describe change of order of multiple integration with figures.	4	2.1.3	U
2	Establish relation between cartesian coordinate and polar coordinate. Also explain role of Jacobian in multiple integral	4	2.1.3	U
3	Evaluate following multiple integration (1) $\int_0^\infty \int_x^\infty e^{-y^2} dydx$ (2) $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$ (3) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dydx$ (4) $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}} dydx$	4	1.1.1	A
4	$\iint_R \; x^2 + y^2 \, dA$ by charging variables where R is the region lying in the first quadrant and bounded by the hyperbolas $x^2 - y^2 = 1$, $xy = 2$, $x^2 - y^2 = 9$, $xy = 4$.	4	1.1.2	A
5	Evaluate following by change variable in polar coordinate (1) $\int_0^a \int_o^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\,dxdy$ (2) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} x^2+y^2 dydx$	4	1.1.2	A
*6	If $\operatorname{erf}(x) = \int_0^x \frac{2e^{-t^2}}{\sqrt{\pi}} dt$ then find $\lim_{x\to\infty} \operatorname{erf}(x)$.	4	1.1.2	A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 12	
Topic:	Infinite series, Power series
Sub Topics:	Alternating series test, Absolute and conditionally convergent, Power series, Radius of convergence of power series, Taylor and Maclurin series
Relevant CO:	2
Objectives:	1. Understand Alternating series and its convergence 2. Examine convergence of power series 3. Expansion of a function as a Taylor and Maclurin series

Sr. No.	**Question**	**CO**	**PI**	**B.T. level**
1	Define Alternating series. Using Leibnitz test discuss convergence of the series. (1) $\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+...$ (2) $\sum_{n=2}^{\infty}\frac{(-1)^n}{\log n}$	2	1.1.1	R, U, A
*2	Examine convergence of the series: $1-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+...\quad where\quad p>1$	2	1.1.1	U
3	Define Absolutely convergent series and Conditionally convergent series. Give an example of conditionally convergent series which is not absolutely convergent.	2	1.1.1	R, U
4	Which of the following series is conditionally convergent but not absolutely convergent? 1) $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^3+1}$ (2) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[3]{n}}$ (3) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$	2	1.1.1	R, U
5	Discuss convergence of following power series. Also find radius and interval of convergence of these power series. (1) $\sum_{n=1}^{\infty}\frac{n+1}{n}x^{n-1}$ (2) $\sum_{n=1}^{\infty}\frac{(n+1)^n}{n^{n+1}}x^n$ (3) $\sum_{n=1}^{\infty}\frac{(-3)^n}{\sqrt{n+1}}x^n$ (4) $1-\frac{1}{2}(x-2)+\frac{1}{4}(x-2)^2-\frac{1}{8}(x-2)^3+...$	2	1.1.1	R, U
6	Express following function in power series using formula of Maclaurin series (1) e^x (2) $\sin x$ (3) $\log(1+x)$ (4) $\tan^{-1}x$	2	1.1.1	U
7	Expand following functions in powers of $(x-a)$ using Taylor's series. (1) $f(x)=x^3-2x+4,\quad a=2.$ (2) $f(x)=x^4-11x^3+43x^2-60x+14,\quad a=3.$	2	1.1.1	U
8	Expand $\sin\left(\frac{\pi}{4}+x\right)$ in power of x. Find approximate value of $\sin 46^o$ and $\sin 44^o$	2	1.1.1	U, A
9	Using Taylor's series find approximate value of $\sqrt{36.12}$ and $\sqrt{9.12}$	2	1.1.1	U, A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial: 13	
Topic:	Multiple Integration
Sub Topics:	Triple integrals in rectangular, cylindrical and spherical coordinates, Jacobian, multiple integral by substitution.
Relevant CO:	4
Objectives:	1. Evaluate multiple Integral 2. compute area and volume by using Multiple integral

Sr. No.	**Question**	**CO**	**PI**	**B.T. level**
1	Derive relation between cartesian coordinate and spherical coordinate and cylindrical coordinate. Explain important of Jacobian in multiple integral	4	2.1.3	U
2	Evaluate following triple integration (1) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ (2) $\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{asin\theta} \int_{z=0}^{\frac{a^2-r^2}{a}} rdzdrd\theta$	4	1.1.1	A
3	Evaluate $\iiint_D \frac{dv}{(x^2+y^2+z^2)^{\frac{3}{2}}}$ where D is the region bounded by the sphere $x^2+y^2+z^2=a^2$ and $x^2+y^2+z^2=b^2$	4	1.1.1	A
4	Find the volume of the region B bounded by paraboloid $z = 4 – x^2 – y^2$ and XY plane.	4	1.1.2	A
5	$\iiint_B \sqrt{x^2+y^2+z^2}dv$ where B is the region bounded by the plane z = 3 and the cone $z = \sqrt{x^2+y^2}$	4	1.1.2	A
*6	Find the volume of “ice cream cone” cut from the solid sphere $\rho \leq 1$by the cone $\emptyset = \frac{\pi}{3}$	4	2.1.3	A
*7	Show that the volume of sphere of radius r is 4/3 πr^3 by using (i) single integral (ii) double integral (iii) triple integral	4	1.1.2	A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating

Date:	
Tutorial No: 14	
Topic:	Beta & Gamma function, Application of definite integral
Sub Topics:	Beta & Gamma function, Volume using cross section, Length of plane curves, Areas of surfaces of revolution
Relevant CO:	1
Objectives:	1. Evaluate integrals using Beta & Gamma function 2. Compute area&volume as application of definite integral

Sr. No.	**Question**	**CO**	**PI**	**B.T. level**
1	Define Gamma function. Show that $i)\ \overline{n+1} = n\overline{n} \quad ii)\ \overline{n} = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$	1	1.1.2	R, U
2	Evaluate: i) $\overline{7/2}$ ii) $\int_0^\infty \frac{x^5}{5^x} dx$ iii) $ii)\int_0^1 x^4 (\log x)^4 dx$	1	1.1.2	U
3	Define Beta function. Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$	1	1.1.2	R, U
4	Evaluate: $i)\int_{-1}^{1} (1+x)^4 (1-x)^3 dx \quad ii)\int_0^{\frac{\pi}{2}} \sin^6\theta \cos^7\theta d\theta$	1	1.1.2	U
5	The solid lies between planes perpendicular to the x-axis at x = -1 and x = 1. The cross-sections perpendicular to the x-axis between these planes are squares whose diagonals run from the semicircle $y = -\sqrt{1-x^2}$ to the semicircle $y = \sqrt{1-x^2}$.	1	1.1.1	A
6	Find the volumes of the solids generated by revolving the regions bounded by $y = x^2, y = 0, x = 2$ about the x-axis by the disk method.	1	1.1.1	A
7	Find the volumes of the solids generated by revolving the regions bounded by $y = x^2 + 1, y = x + 3$ about the x-axis by the washer method.	1	1.1.1	A
8	Find the length of the curve $y = \frac{1}{2}\left(e^x + e^{-x}\right); 0 \le x \le 2$	1	1.1.1	A
9	Find the length of asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.	1	1.1.1	A
10	Find the area of the surface of revolution of the solid generated by revolving the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ about the x-axis.	1	1.1.1	A
*11	Find the surface area generated by revolving the loop of the curve $9ay^2 = x(3a-x)^2$about the x-axis.	1	1.1.1	A
12	Find the surface area of the solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$about x- axis.	1	1.1.1	A

B.T. Level: R: Remembering, U: Understanding, A: Applying, N: Analyzing, E: Evaluating, C: Creating