Report on Binary Classification and SLT

1. Introduction to Binary Classification

Binary classification is one of the core tasks in supervised learning, where the objective is to classify an input X into one of two possible classes. In formal mathematical terms, given an input space X and an output space Y, binary classification aims to find a function $f : X \to Y$, where Y belongs to $\{-1, +1\}$. The classifier f attempts to predict the label Y for any given input X, minimizing the classification errors on unseen data points drawn from the same underlying distribution.

2. Mathematical Framework of Binary Classification

In the standard SLT framework for binary classification, the problem is as follows:

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n) \in X \times Y$ represent the training examples, where X_i are inputs and Y_i are their corresponding labels. The goal is to find a classifier $f : X \to Y$ that minimizes the risk, or the expected loss:

$$R(f) = E[\ell(Y, f(X))]$$

where ℓ is a loss function, commonly the 0-1 loss in classification:

$$\ell(Y, f(X)) = \begin{cases} 0, & \text{if } f(X) = Y \\ 1, & \text{if } f(X) \neq Y \end{cases}$$

3. Statistical Learning Theory (SLT) and Generalization

SLT offers a mathematical framework for understanding the relationship between minimizing empirical risk and generalizing to unseen data. It balances the complexity of the classifier and its performance on the training set.

4. SLT Framework for Binary Classification

Key concepts of SLT for binary classification include:

- **Function Space $\mathcal{F}$:** The space of candidate classifiers f needs to be restricted.
- **Generalization Bounds:** SLT provides probabilistic bounds on the true risk $R(f)$, based on the empirical risk $R_{\text{emp}}(f)$ and model complexity (VC dimension).

5. Conclusion

SLT offers a mathematical foundation for binary classification, guiding algorithm development and balancing classifier complexity and training performance.