Image Segmentation

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Abstract

*Image segmentation problem asks us to classify all pixels in a general image into two groups: Pixels which belong to the background and the pixels which belong to the foreground, given the **tendency** of each pixel to go into foreground and background. We define 2- pixels as neighbours if they are in **4- Neighbourhood** connectivity of each other. Decission of a pixel whether to go into foreground or background is also dependent on the neighbour pixels since neighbouring pixels are more likely to go into the same category. So for each pair of neighbour pixels, we have a **penalty** such that if they are put in separate groups, penalty is paid. So, the aim of the image segmentaion problem is to divide set of pixels into 2 groups such that maximum number of tendencies are met while incurring minimum penalty.*

I. Problem Statement

Let S be the set of all pixels. Then the image segmentation problem can be formally stated us under:
Input:
1. Two values $a_i \geq 0$ and $b_i \geq 0$, $\forall i \in S$ indicating forward and backward tendencies respectively.
2. Penalty $p_{ij} \geq 0$ $\forall i, j \in S$ s.t. i and j are neighbours.
Output: Sets A and B s.t. $A \cup B = S$ and $A \cap B = \phi$.
Goal: To maximize $q(A, B) = \sum\limits_{i \in A} a_i + \sum\limits_{j \in B} b_j - \sum\limits_{i \in A, j \in B} p_{ij}$.

Since $q(A, B) = Q - q^{'}(A, B)$
where $Q = \sum\limits_{i \in S} (a_i + b_i)$
and $q^{'}(A, B) = \sum\limits_{i \in A} b_i + \sum\limits_{j \in B} a_j + \sum\limits_{i \in A, j \in B} p_{ij}$
Since Q is a constant, goal can be rewritten as:
Goal: To minimize $q^{'}(A, B) = \sum\limits_{i \in A} b_i + \sum\limits_{j \in B} a_j + \sum\limits_{i \in A, j \in B} p_{ij}$

II. Conversion to Minimum-Cut Problem

Image segmentation problem can be polynomially converted to minimum-cut problem.For that we need to make a directed graph $G^{'} = (V^{'}, E^{'})$ as under:

$V^{'} = s \cup V \cup t$, $V = \{v_i, \forall i \in S\}$

$E^{'} = E_s \cup E \cup E_t$ where

$E_s = \{(s, j), \forall j \in B\}$, $\quad w((s, j)) = a_j$
$E = \{(i, j), \forall i \in A, j \in B$ s.t. i and j are neighbours$\}$, $\quad w((i, j)) = p_{ij}$
$E_t = \{(i, t), \forall i \in A\}$, $\quad w((i, t)) = b_i$

Clearly cost of the $s - t$ cut = $\sum\limits_{i \in A} b_i + \sum\limits_{j \in B} a_j + \sum\limits_{i \in A, j \in B} p_{ij} = q^{'}(A, B)$.
Thus, Solving image segmentation problem on the given pixel data is equivalent to solving minimum $s - t$ cut problem on graph $G^{'}$.

III. Solving Minimum Cut

Max- flow min-cut theorem states that value of minimum $s - t$ cut on a directed graph is equal to value of maximum $s - t$ flow on the same graph. Thus, value of minimum cut can be obtained by solving maximum flow using **Ford- Fulkerson** algorithm.

To obtain the sets A and B, start from vertex s and do a DFS on the residual graph left after obtaining maximum flow on $G^{'}$. Set of vertices which are reachable from s, other than s itself, constitute A and remaining vertices (other than t) consitute B.