MATH237 Lecture 02

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1 Scalar functions and graphs continued

1.1 Example of graphing: $z = 2x + y$

Recall that we should sketch the level curves and cross-sections, then consider the symmetry.

- If $z = c$ for some constant c, we have the line $y = -2x + c$. So our level curves will look like lines with negative slope.
- Consider the cross sections $z = 2x + c$ and $z = 2c + y$. The cross sections are also lines (on the $z - x$ and $z - y$ plane). They are also lines, both with positive slope, with the first one being steeper than the other.
- Now we sketch the entire graph.

1.2 Example 2: $z = x^2 + y^2$

- Level curves: $c = x^2 + y^2$ - circle
- Cross Sections: $z = c^2 + y^2$ and $z = x^2 + c^2$ - parabola
- Now we sketch the entire graph - it's a parabolic cone, infinite paraboloid

1.3 Example 3: $z = \sqrt{x^2 + y^2}$

- Level curves: $c = \sqrt{x^2 + y^2} = c^2 + x^2 + y^2$ - circle again
- Cross Sections: $z^2 = x^2 + c^2$ - what shape is this? If $c = 0$ then the graph looks like $z = |x|$ on the $z - x$ plane.
- The shape is a cone - hyperbola

1.4 Example 4: $z = xy$

- Level curves: $c = xy$ - shape of the reciprocal function
- Cross Sections: $z = cx$ and $z = cy$ - lines with different slopes

1.5 Matching with images from the slide

- Figure 1 corresponds to example 4
- Figure 2 corresponds to example 1
- Figure 3 corresponds to example 3
- Figure 4 corresponds to example 2

2 Limits

Definition: A function $f : \mathbb{R} \to \mathbb{R}$ is **continuous** at a if $\lim\limits_{x \to a} f(x) = f(a)$.
Definition: A function $f : \mathbb{R}^2 \to \mathbb{R}$ is **continuous** at (a, b) if $\lim\limits_{(x,y) \to (a,b)} f(x, y) = f(a, b)$.
What does this mean? For a function $f : \mathbb{R} \to R$, $\lim\limits_{x \to a} f(x) = L$ means as x gets close to a, $f(x)$ gets close to L. More precisely, for every interval around L, we can construct an interval around a such that f maps every element in that interval to $L's$ interval.

How about for $f : \mathbb{R}^2 \to \mathbb{R}$?.
Definition: $\lim_{(x,y)\to(a,b)} f(x, y) = L$ means that for any $\varepsilon > 0$, there is a $\delta > 0$ such that if $||\underline{x} - \underline{a}|| < \delta$ then $|f(\underline{x}) - L| < \varepsilon$.

2.1 Example

$$f(x, y) = \begin{cases} \frac{\sin xy}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$