MATH237 Lecture 09

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1 Clicker!

Consider

$$f(x,y) = |xy|$$

. You are given $f(0,0) = f_x(0,0) = f_y(0,0) = 0$. Is f differentiable at $(0,0)$?

Yes!

2 Relationship between Continiuity and Differntiability

- Continuity doesn't imply existence of partial derivatives or differentiability
- Existence of partial derivatives does not imply diff or continuity
- However, differentiability does imply continuity

Theorem 1. *If $f : B(r, \underline{a}) \subset \mathbb{R}^n \to \mathbb{R}^m, R > 0$ is differentiable at $\underline{a}$ then it is continuous at $\underline{a}$.*

Proof. Differentiability implies f is defined at $\underline{a} = (a,b)$. It must be shown that $\lim_{\underline{x}\to\underline{a}} f(\underline{x}) = f(\underline{a})$ or $\lim_{\underline{x}\to\underline{a}} |f(\underline{x}) - f(\underline{a})| = 0$.

$$\begin{aligned} 0 \leq |f(\underline{x}) - f(\underline{a})| &\leq |f(\underline{x}) - f(\underline{a}) - f_x(\underline{a})(x-a) - f_y(\underline{a})(y-b)| + |f_x(\underline{a})(x-a) + f_y(\underline{a})(y-b)| \\ &= |f(\underline{x}) - L(\underline{x})| + |f_x(\underline{a})(x-a) + f_y(\underline{a})(y-b)| \\ &= \frac{|f(\underline{x}) - L(\underline{x})}{||\underline{x} - \underline{a}||} ||\underline{x} - \underline{a}|| + |f_x(\underline{a})(x-a)| + |f_y(\underline{a})(y-b)| \end{aligned}$$

Since f is differentiable, $\lim_{\underline{x}\to\underline{a}} \frac{|f(\underline{x}) - L(\underline{x})|}{||\underline{x}-\underline{a}||} = 0$ by definition of differentiability. The Product Limit Theorem and Sum Limit theorem imply the limit of the right hand side is 0. Thus, by the Squeeze theorem $\lim_{\underline{x}\to\underline{a}} |f(\underline{x}) - f(\underline{a})| = 0$. By definition, f is continuous at $\underline{a}$. □