MATH237 Lecture 01

Peter He

January 6 2020

1 Overview of the Course

We covered functions of 1 variable, in general $f : \mathbb{R} \to \mathbb{R}$. We covered topics such as

- Limits
- Continuity
- Linear approximation
- Differentiation
- Integration

We will now cover funcctions of > 1 variables, in general $f : \mathbb{R}^n \to \mathbb{R}^m$. Examples of multivariable functions:

- Ocean temp
- Temp of Canada
- wind in Canada
- factory production

Scalar functions ($f : \mathbb{R}^n \to \mathbb{R}$)

- Scalar functions are the focus of this course
- most of the discussion is for $n = 2$
- Generalization to $n =$ and arbitrary n

2 Graphs

We will discuss **terminology** and **visualization**

Review:
A function $f : A \to B$ is a rule that associates each $a \in A$ to a unique element of B, $f(a)$. The domain of f is A, $D(f)$. The range of f is $R(f) = \{b \in B | b = f(a), a \in A\}$. $f(x, y)$ can mean the value of f at the point (x, y) or more usually that f is a function of 2 variables.

Example: $f(x, y) = x^2 + y^2$. We can see that $f : \mathbb{R}^2 \to \mathbb{R}, D(f) = \mathbb{R}^2$, and $R(f) = \{z \in \mathbb{R} | z \geq 0\} \subset \mathbb{R}$

2.1 Visualization of $f : \mathbb{R} \to \mathbb{R}$

Methods:

- Level curves
- Cross-sections
- Symmetry
- Analysis
- Computer plots

Example: $f(x, y) = z$.

- Level curves: $z = c$: $x^2 + y^2 = c$ for some constant c (we would get some sort of circle, analogous to looking "down" on the shape)
- Cross-sections: $y = c$ or $x = c$: $x^2 + c^2 = z$ (we would get some sort of parabola)

We might guess that the shape is a parabolic cone thing.

Let's look at $f(x, y) = \sin(x^2 + y^2)$. Level curves are of the form $c = \sin(x^2 + y^2)$ or $\arcsin c = x^2 + y^2$. We expect to see a bunch of circles when graphed.
Cross sections: $z = \sin(x^2 + c^2)$. We expect to see a distorted sine curve shifted left.
Symmetry: