MATH237 Lecture 12

Peter He

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1 Last Class

I was not in class lol.

2 This Class: Proof of the chain rule: look at slides

3 Examples of applying the chain rule

Let $f(x, y) = (xy)^{1/3}$. We have that $x(t) = t$, $y(t) = t^2$. Define $F(t) = f(x(t), y(t))$. Find $F'(0)(\frac{DF}{dt})$.
Solution:

Formally, $\frac{\partial f}{\partial x} = (1/3)(xy)^{-2/3}y$. When $x(0) = 0, y(0) = 0$, it's not defined, so we must used the defn.

$$\lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

Thus, $f_x(0,0) = 0$. Similarly, $f_y(0,0) = 0$.

1. Using the chain rule,

$$\begin{aligned}\frac{\partial F}{\partial t}(0) &= \frac{\partial f}{\partial x}(0,0)\frac{dx}{dt}(0) + \frac{\partial f}{\partial y}(0,0)\frac{dy}{dt} \\ &= 0 + 0 = 0\end{aligned}$$

2. Directly,

$$\begin{aligned}F(t) &= (tt^2)^{1/3} = t \\ \frac{dF}{dt} &= 1\end{aligned}$$

   Which one is right? Using the chain rule, we assumed f is differentiable at $(0,0)$, which is false.

Lesson: Must make sure everything is differentiable.

4 Example 2

$$f(x, y, z) = e^x y z^2$$
$$x(t) = \cos(t), y = t^2, z = t$$

Define

$$F(t) = f(x(t), y(t), z(t))$$

We find that

$$f_x = e^x y z^2, f_y = e^x z^2, f_z = 2e^x y z$$

The Continuity Theorems imply that all the partial derivatives are continuous on $\mathbb{R}^2$. Let's find $F'(t)$.

$$\begin{aligned}
\frac{dx}{dt} &= -\sin t \\
\frac{dy}{dt} &= 2t \\
\frac{dz}{dt} &= 1
\end{aligned}$$

$$\begin{aligned}
\frac{dF}{dt} &= \frac{\partial f}{\partial x}(x, y, z)\frac{dx}{dt} + \frac{\partial f}{\partial y}(x, y, z)\frac{dy}{dt} + \frac{\partial f}{\partial z}(x, y, z)\frac{dz}{dt} \\
&= (e^x y z^2)(-\sin t) + (e^x z^2)(2t) + (2e^x y z)(1) \\
&= e^{\cos t} t^3 (4 - t \sin t)
\end{aligned}$$

5 Extension 1 of the Basic Chain Rule - More than 1 independent variable