MATH237 Lecture 08

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1 Clicker Review!

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Does $L(x,y)$ exist at $(0,0)$? Yes, $L(x,y) = 0$.

2 Rigourous Definition of Differentiability

Definition 2.1. A function $f : B(r, \underline{a}) \subset \mathbb{R}^2 \to \mathbb{R}, r > 0$ is **differentiable** at $\underline{a} = (a,b)$ if he partial derviatives exist at $\underline{a}$ and the linear approximation

$$L_{\underline{a}}(x) = f(\underline{a}) + f_x(\underline{a})(x-a) + f_y(\underline{a})(y-b)$$

satisifies

$$\lim_{\underline{x}\to\underline{a}} \frac{|f(x) - L_{\underline{a}}(\underline{x})|}{||\underline{a} - \underline{a}||}$$

2.1 Example

Let

$$f(x,y) = x^2 + y^2$$

and

$$\underline{a} = (1,2)$$

Is f differentiable at $(1,2)$? Yes.

Solution:
Calculate $L(x,y)$ at $(1,2)$.

$$f(1,2) = 5$$

$$\begin{aligned} f_x(x,y) &= 2x \implies f_x(1,2) = 2 \\ f_y(x,y) &= 2y \implies f_y(1,2) = 4 \end{aligned}$$

$$L(x,y) = -5 + 2x + 4y$$

Now we need to show
$$\lim_{\underline{x} \to \underline{a}} \frac{|f(x,y) - L(x,y)|}{||(x,y) - (1,2)||} = 0$$
so
$$\begin{aligned} \frac{|f(x,y) - L(x,y)|}{||(x,y) - (1,2)||} &= \frac{|x^2 + y^2 - 5 - 2(x-1) - 4(y-2)|}{\sqrt{(x-1)^2 + (y-2)^2}} \\ &= \sqrt{(x-1)^2} \end{aligned}$$