

Ex.

For the given dataset calculate skewness using Karl Pearson's coeff-coefficient

class	0-10	10-20	20-30	30-40	40-50
frequency	5	7	20	12	6

Soln:- We know that $KPS = \frac{\text{Mean} - \text{Mode}}{S.D}$

class	Frequency	x_i	$F_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
0-10	5	5	25	$5 - 26.4 = -21.4$	457.96
10-20	7	15	105	$15 - 26.4 = -11.4$	129.96
20-30	20	25	500	$25 - 26.4 = -1.4$	1.96
30-40	12	35	420	$35 - 26.4 = 8.6$	73.96
40-50	<u>6</u>	45	<u>270</u>	$45 - 26.4 = 18.6$	<u>345.96</u>
	<u>50</u>		<u>1320</u>		<u>1009.08</u>

$$\sum f_i(x_i - \bar{x})^2$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1320}{50} = 26.4$$

$$39.2$$

$$\text{Mode} = l_1 + \left(\frac{F_1 - F_0}{2F_1 - F_0 - F_2} \right) \times h$$

$$62.01.99$$

Highest frequency class = 20-30

$$\therefore l_1 = 20, F_1 = 20, F_0 = 7, F_2 = 12, h = 10$$

$$\text{Mode} = 20 + \left(\frac{20-7}{2(20)-7-12} \right) \times 10$$

$$= 20 + \left(\frac{13}{21} \right) \times 10$$

$$\text{Mode} = 20 + (0.619) \times 10$$

$$= 20 + 6.19$$

$$= 26.19$$

~~$$S.D = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}}$$~~

$$= \sqrt{\frac{6201.99}{50}}$$

$$= \sqrt{124.0398}$$

$$= 11.137$$

~~$$= 11.14$$~~

$$KPS = \frac{\text{mean} - \text{mode}}{S.D} = \frac{26.4 - 26.19}{11.14}$$

$$= \frac{0.21}{11.14}$$

$$= 0.018$$

Q1

Practice problem

- Q1. A company send following dataset to statistical Analysis Department and asked to generate skewness report

worker policies	5-10	10-15	15-20	20-25	25-30	30-35
	2	10	30	50	6	2

worker	Policies(Fi)	(xi)	(xi)F _i	x _i - \bar{x}	(x _i - \bar{x}) ²	F _i (x _i - \bar{x}) ²
5-10	2	7.5	15	-12.7	161.29	322.58
10-15	10	12.5	125	-7.7	59.29	592.9
15-20	30	17.5	525	-2.7	7.29	218.7
20-25	50	22.5	1125	2.3	5.29	264.5
25-30	6	27.5	165	7.3	53.29	319.74
30-35	2	32.5	65	12.3	151.29	302.58
	100	2020				2021

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2020}{100} = 20.2$$

$$\text{Mode} = l_1 + \left(\frac{F_1 - F_0}{2F_1 - F_0 - F_2} \right) \times h$$

$$= 20 + \left(\frac{50 - 30}{2 \times 50 - 30 - 6} \right) \times 5$$

$$= 20 + \left(\frac{20}{64} \right) \times 5$$

$$= 20 + (0.132) \times 5$$

$$= 20 + 1.562$$

$$= 21.562$$

$$SD = \sqrt{\frac{2072.3}{100}} = \sqrt{\frac{2021}{100}}$$

$$= \sqrt{20.723} \sim 20.21$$

$$= 4.55 \quad \underline{\underline{4.49}}$$

$$g'kps = \frac{\text{mean} - \text{mode}}{SD}$$

$$\frac{20.2 - 21.56}{4.49}$$

$$= -1.36$$

$$\underline{\underline{4.49}}$$

$$= -0.298 = -0.302$$

$$= -0.3$$

Worker	sf	Worker	fi	x_i	$f_i x_i$	$x_i - \bar{x}$
5-10	6	5-10	6	7.5	45	$7.5 - 22 = -14.5$
10-20	8	10-15	8	12.5	100	$12.5 - 22 = -9.5$
20-25	17	15-20	17	17.5	297.5	$17.5 - 22 = -4.5$
25-30	21	20-25	21	22.5	472.5	$22.5 - 22 = 0.5$
30-35	15	25-30	15	27.5	412.5	$27.5 - 22 = 5.5$
35-40	11	30-35	11	32.5	357.5	$32.5 - 22 = 10.5$
		35-40	2	37.5	75	$37.5 - 22 = 15.5$
			80		1760	

$$\begin{aligned}
 & (x_i - \bar{x})^2 & f(x_i - \bar{x})^2 \\
 210.25 & & 1261.5 \\
 90.25 & & 722 \\
 20.25 & & 344.25 \\
 0.25 & & 5.25 \\
 30.25 & & 453.75 \\
 110.25 & & 1212.75 \\
 240.25 & & 480.5 \\
 & \hline & 4480
 \end{aligned}$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1760}{80} = 22$$

$$\text{Mode} = l_1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) x_h$$

$$= 15 + \frac{17 - 8}{2 \times 17 - 8 - 2} = 20 + \left(\frac{21 - 17}{2 \times 21 - 17 - 15} \right) \times 5$$

$$= 20 + \left(\frac{21-17}{2 \times 21 - 17 - 15} \right) \times 5$$

$$= 20 + \left(\frac{4}{10} \right) \times 5$$

$$= 20 + 0.4 \times 5$$

$$= 20 + 2$$

$$= \underline{\underline{22}}$$

$$S.D = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{4480}{80}} = \sqrt{56} = 7.483$$

$$KPS = \frac{\text{mean} - \text{mode}}{SD}$$

$$= \frac{22 - 22}{7.483}$$

$$= \frac{0}{7.483}$$

$$= \underline{\underline{0}}$$

Bowley's coefficient of skewness

$$= \frac{(Q_3 + Q_1 - 2 \times \text{median})}{Q_3 - Q_1}$$

Q. Calculate the Bowley's coefficient of skewness

e

class

Frequency

C.F

5 - 10	7	7
10 - 15	9	16
15 - 20	16	32
20 - 25	22	54
25 - 30	14	68
30 - 35	12	80
35 - 40	3	83

$$Q_1 = l_1 + \frac{(l_2 - l_1)}{f} \left(\frac{N}{4} - F \right)$$

$$\therefore N/4 = 83/4 = 20.75$$

Hence Q.D class = 15 - 20

$l_1 = 15$, $f = 16$, $F = 16$

$$Q_1 = 15 + \left(\frac{20 - 15}{16} \right) (20.75 - 16)$$

$$= 15 + (0.3125) (4.75)$$

$$= 15 + 1.482$$

$$= 16.482$$

Q3 =

$$\therefore 3N/4 = \frac{3 \times 83}{4} = \frac{249}{4} = 62.25$$

0.D class = 25-30

$$Q_3 = l_1 = 25, l_2 = 30, F = 14, F' = 54$$

$$Q_3 = 25 + \left(\frac{30 - 25}{14} \right) (62.25 - 54)$$

$$= 25 + (0.357) (8.25)$$

$$= 27.95$$

$$\text{Median} = l_1 + \left(\frac{n/2 - CF}{F} \right) \times h = 20$$

$$= 10 + \left(\frac{41.5 - 7}{9} \right) \times 5 = 20 + \left(\frac{41.5 - 32}{22} \right) \times 5$$

$$= 10 + \left(\frac{34.5}{9} \right) \times 5 = 20 + (0.4318) \times 5$$

$$= 20 + 2.159$$

$$= 10 + 3.833$$

$$= 22.16$$

$$= 10 + 19.16$$

$$= 29.16$$

$$BCS = \frac{Q_3 + Q_1}{Q_3 - Q_1} \div 2 \times \text{median}$$

$$= \frac{27.95 + 16.48}{27.95 - 16.48} - 2 \times 22.16$$

$$= \frac{0.11}{11.47}$$

$$= 0.0096$$

Q.

x f CF

10-15 2 2

15-20 5 7

20-25 7 14

Q₁ 25-30 13 27

Q₂ 30-35 21 48

Q₃ 35-40 18 64

40-45 8 72

45-50 3 75

~~Q₂~~ =

$$Q_1 = l_1 + \frac{(l_2 - l_1)}{f} \left(\frac{N}{4} - CF \right)$$

$$\therefore N/4 = 75/4 = 18.75$$

$$Q_1 = 25 + \frac{30-25}{13} (18.75 - 14)$$

$$= 25 + \frac{5}{13} (4.75)$$

$$= 25 + 0.3846 (4.75)$$

$$= 25 + 1.8268$$

$$= \underline{\underline{26.82}}$$

$$Q_3 = l_1 + \frac{(l_2 - l_1)}{f} \left(\frac{3N}{4} - f \right)$$

$$Q_3 = \frac{3 \times 75}{4} - \frac{225}{4} = 56.25$$

$$Q_3 = 35 + \frac{(40 - 35)}{16} (56.25 - 48)$$

$$= 35 + (0.3125) (8.25)$$

$$= 35 + 2.578$$

$$= 37.578$$

Median

$$N/2 = \frac{75}{2} = 37.5$$

$$Q_2 = 30 + \frac{37.5 - 30}{21}$$

$$Q_2 = l_1 + \frac{(N/2 - CF)}{f} \times h$$

$$Q_2 = 30 + \left(\frac{37.5 - 27}{21} \right) \times 5$$

$$= 30 + \left(\frac{10.5}{21} \right) \times 5$$

$$= 30 + 0.5 \times 5$$

$$= 30 + 2.5$$

$$= 32.5$$

$$BGS = \frac{Q_3 + Q_1 - 2 \times \text{Median}}{Q_3 - Q_1}$$

$$= \frac{37.578 + 26.82 - 2 \times 32.5}{37.578 - 26.82}$$

$$= \frac{-6.61}{10.75}$$

$$= -0.056$$

class	F	CF
30-35	5	5
35-40	10	15
Q1 40-45	30	45
Q3 - 45-50	35	80
50-55	15	95
55-60	5	100

$$\therefore N/2 = 100/2 = 50$$

$$\therefore N/4 = 100/4 = 25$$

$$\therefore 3N/4 = \frac{3 \times 100}{4} = 75$$

$$Q_1 = l_1 + \frac{(l_2 - l_1)}{f} (N/4 - F) \quad Q_3 = l_1 + \frac{(l_2 - l_1)}{f} \left(\frac{3N}{4} - F \right)$$

$$= 40 + \frac{(45-40)}{30} (25-15)$$

$$= 45 + \frac{(50-45)}{35} (75-45)$$

$$= 40 + (0.1666) (10)$$

$$= 45 + (0.1428) (30)$$

$$= 40 + 1.66$$

$$= 45 + 4.2857$$

$$= 41.66$$

$$= 49.285$$

- 0.061

$$\text{Median} = l_1 + \left(\frac{n/2 - CF}{F} \right) \times h$$

$$= 45 + \left(\frac{100/2 - 45}{35} \right) \times 5$$

$$= 45 + (0.1428) \times 5$$

$$= 45 + 0.7142$$

$$= 45.71$$

$$BCS = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{45.28 + 41.66 - 2 \times 45.71}{45.28 - 41.66}$$

$$= -0.48$$

$$= -0.062$$

skewness

Karl Pearson's Method of skewness = 258

i) $KPS = \frac{\text{Mean} - \text{Mode}}{S.D}$

$$\therefore \text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore \text{Mode} = l + \left(\frac{F_1 - F_0}{2F_1 - F_0 - F_2} \right) \times h$$

$$\therefore S.D = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}}$$

ii) $KPS = \frac{3(\text{Mean} - \text{Median})}{S.D}$

$$\therefore \text{Median} = l + \left(\frac{n/2 - CF}{f} \right) \times h$$

Left edge
Date:

Bowley's coefficient of Skewness

$$BCS = \frac{Q_3 + Q_1 - 2 \times \text{median}}{Q_3 - Q_1}$$

$$\therefore Q_3 = l_1 + \frac{(l_2 - l_1)}{f} \left(\frac{3N}{4} - F \right)$$

$$\therefore Q_1 = l_1 + \frac{(l_2 - l_1)}{f} \left(\frac{N}{4} - F \right)$$

$$\therefore \text{Median} = l_1 + \frac{\left(\frac{n}{2} - CF \right) \times h}{F}$$