Fundamentals of Artificial Neural Networks

ECE 657
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Introduction

- Artificial Neural Networks (ANNs) are physical cellular systems, which can acquire, store and utilize experiential knowledge.
- ANNs are a set of parallel and distributed computational elements classified according to topologies, learning paradigms and at the way information flows within the network.
- ANNs are generally characterized by their:
  - Architecture
  - Learning paradigm
  - Activation functions

A Brief History

- ANNs have been originally designed in the early forties for pattern classification purposes.
- ANNs are now used in almost every discipline of science and technology:
    - from Stock Market Prediction to the design of Space Station frame,
    - from medical diagnosis to data mining and knowledge discovery,
    - from chaos prediction to control of nuclear plants.

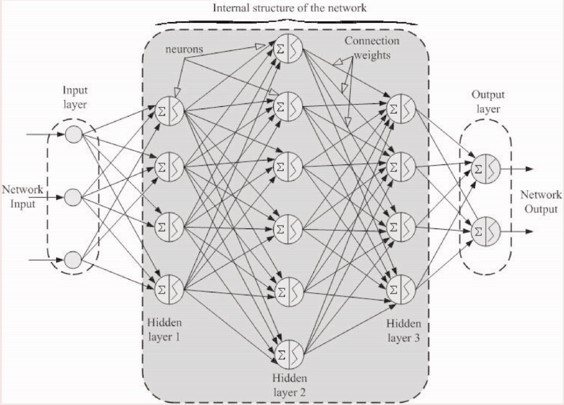
Features of ANNs

ANN are classified according to the following:

- Architecture: Feedforward and Recurrent
- Activation Functions
- Learning Paradigms: Supervised, Unsupervised, and Hybrid

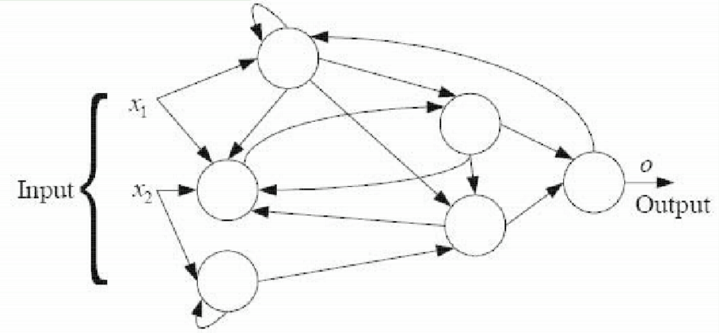
Neural Network Topologies

Feedforward Flow of Information



Neural Network Topologies

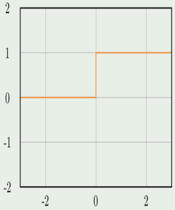
Recurrent Flow of Information



Neural Network Topologies

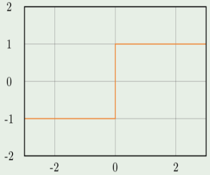
Step Function

$$\text{step}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$



Signum Function

$$\text{sigum}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{otherwise} \end{cases}$$

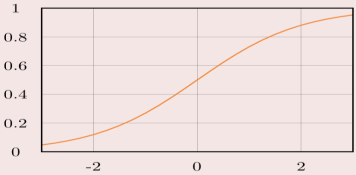


Neural Network Topologies

Differentiable functions

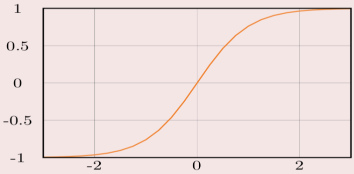
Sigmoid function

$$\text{sigmoid}(x) = \frac{1}{1+e^{-x}}$$



Hyperbolic tangent

$$\tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

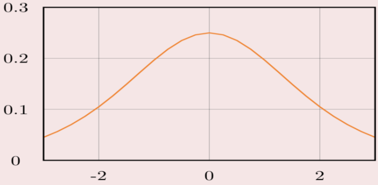


Neural Network Topologies

Differentiable functions

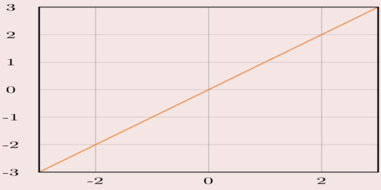
Sigmoid derivative

$$\text{sigderiv}(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$



Linear function

$$\text{lin}(x) = x$$



Learning Paradigms

Supervised Learning

- Multilayer perceptrons
- Radial basis function networks
- Modular neural networks
- LVQ (learning vector quantization)

Unsupervised Learning

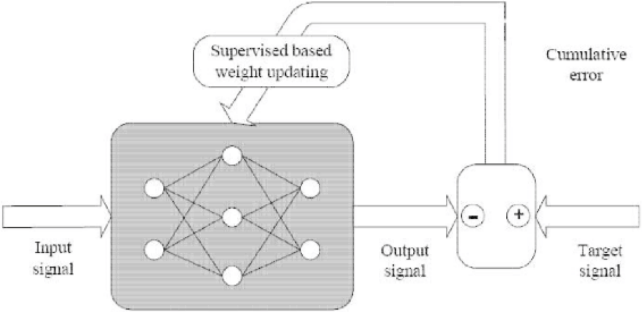
- Competitive learning networks
- Kohonen self-organizing networks
- ART (adaptive resonant theory)

Unsupervised Learning

- Autoassociative memories (Hopfield networks)

Supervised Learning

- Training by example; i.e., priori known desired output for each input pattern.
- Particularly useful for feedforward networks.



Supervised Learning (cont.)

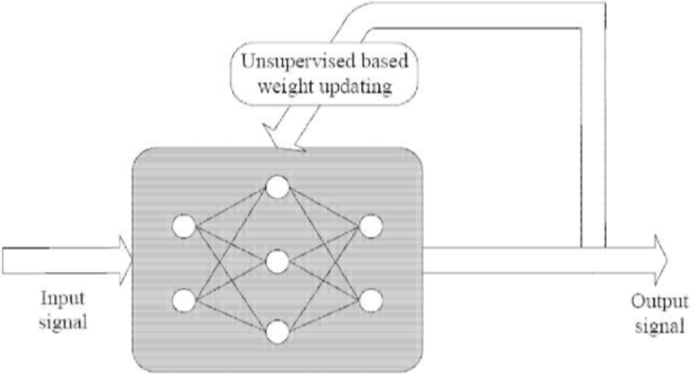
Training Algorithm

1. Compute error between desired and actual outputs
2. Use the error through a learning rule (e.g., gradient descent) to adjust the network's connection weights
3. Repeat steps 1 and 2 for input/output patterns to complete one epoch
4. Repeat steps 1 to 3 until maximum number of epochs is reached or an acceptable training error is reached

Unsupervised Learning

- No priori known desired output.
- In other words, training data composed of input patterns only.
- Network uses training patterns to discover emerging collective properties and organizes the data into clusters.

Unsupervised Learning: Graphical Illustration



Unsupervised Learning

Unsupervised Training

1. Training data set is presented at the input layer
2. Output nodes are evaluated through a specific criterion
3. Only weights connected to the winner node are adjusted
4. Repeat steps 1 to 3 until maximum number of epochs is reached or the connection weights reach steady state

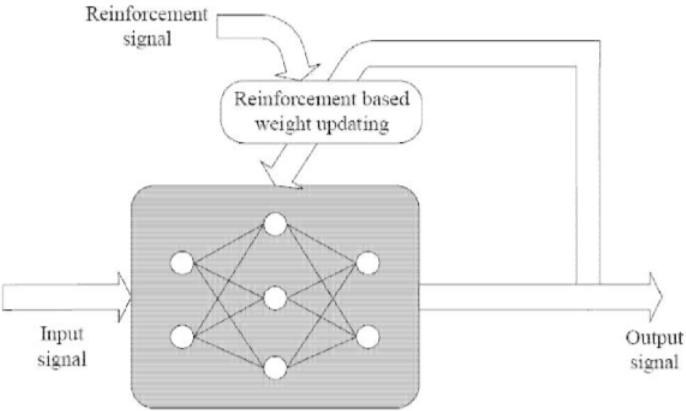
Rationale

1. Competitive learning strengthens the connection between the incoming pattern at the input layer and the winning output node.
2. The weights connected to each output node can be regarded as the center of the cluster associated to that node.

Reinforcement Learning

- Reinforcement learning mimics the way humans adjust their behaviour when interacting with physical systems (e.g., learning to ride a bike).
- Network's connection weights are adjusted according to a qualitative and not quantitative feedback information as a result of the network's interaction with the environment or system.
- The qualitative feedback signal simply informs the network whether or not the system reacted "well" to the output generated by the network.

Reinforcement Learning



Reinforcement Training

Reinforcement Training Algorithm

- Present training input pattern network.
- Qualitatively evaluate system's reaction to network's calculated output
  1. If response is "Good", the corresponding weights led to that output are strengthened
  2. If response is "Bad", the corresponding weights are weakened.

Fundamentals of ANNs

Late 1940's : McCulloch Pitt Model (by McCulloch and Pitt)

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Mid 1960's : Adaline (by Widrow)

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Mid 1960's : Adaline (by Widrow)

Mid 1970's : Back Propagation Algorithm - BPL I (by Werbos)

Fundamentals of ANNs

Late 1940's : McCulloch Pitt Model (by McCulloch and Pitt)

Late 1950's – early 1960's : Perceptron (by Roseblatt)

Mid 1960's : Adaline (by Widrow)

Mid 1970's : Back Propagation Algorithm - BPL I (by Werbos)

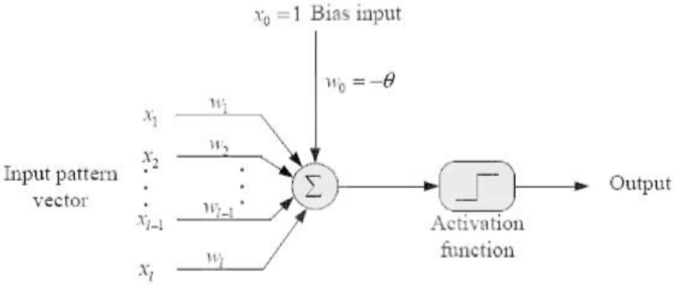
Mid 1980's : BPL II and Multi Layer Perceptron (by Rumelhart and Hinton)

McCulloch-Pitts Model

Overview

- First serious attempt to model the computing process of the biological neuron.
- The model is composed of one neuron only.
- Limited computing capability.
- No learning capability.

McCulloch-Pitts Model: Architecture



Perceptron

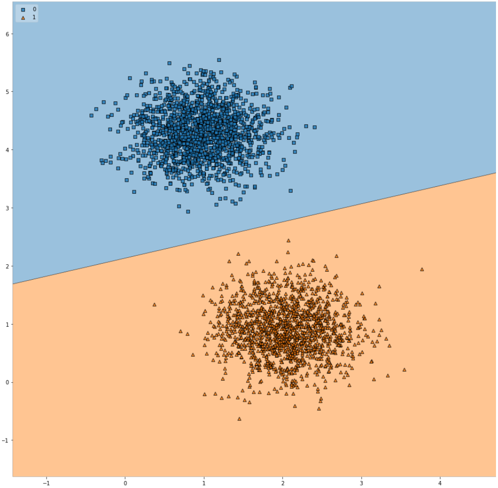
Overview

- Uses supervised learning to adjust its weights in response to a comparative signal between the network's actual output and the target output.
- Mainly designed to classify linearly separable patterns.

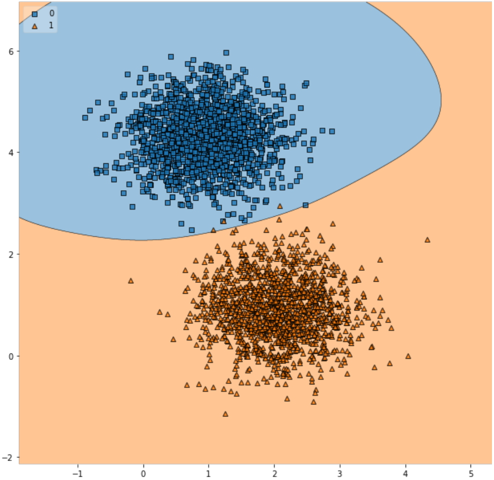
Definition: Linear Separation

Patterns are linearly separable means that there exists a hyperplanar multidimensional decision boundary that classifies the patterns into two classes.

Linearly Separable Patterns



Non-Linearly Separable Patterns

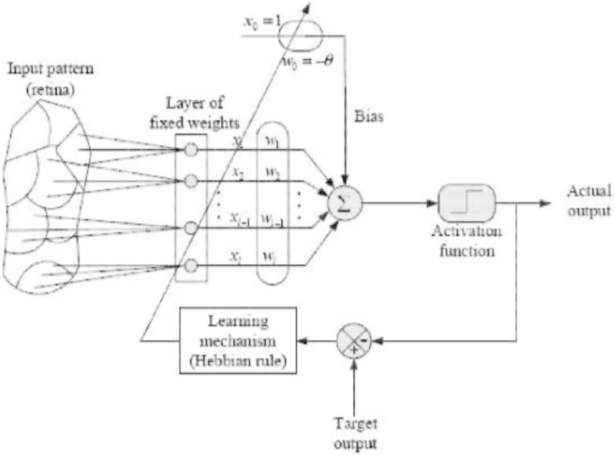


Perceptron

Remarks

- One neuron (one output)
- I input signals: $x_1, x_2, \ldots, x_I$.
- Adjustable weights : $w_1, w_2, ..., w_I$ and bias θ.
- Binary activation function; i.e., step or hard limiter function

Perceptron: Architecture

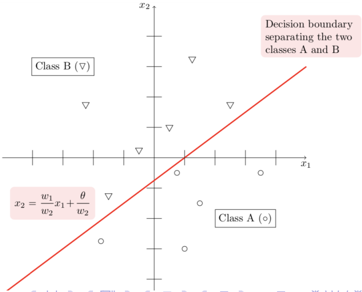


Perceptron (Cont.)

Perceptron Convergence Theorem

If the training set is linearly separable, there exists a set of weights for which the training of the Perceptron will converge in a finite time and the training patterns are correctly classified.

In the two-dimensional case, the theorem translates into finding the line defined by $w_1x_1 + w_2x_2 - \theta = 0$, which adequately classifies the training patterns.



Training Algorithm

1. Initialize weights and thresholds to small random values.:
2. Choose an input-output pattern $(x^{(k)}, t^{(k)})$ from the training data.
3. Compute the network's actual output $c^{(k)} = f(\sum_{i=1}^{l} w_i x_i^{(k)} - \theta)$
4. Adjust the weights and bias according to the perceptron learning rule: $\Delta w_i = \eta[t^{(k)} - o^{(k)}]$, and $\Delta\theta = -\eta[t^{(k)} - o^{(k)}]$. where $\eta \in [0, 1]$ is the perceptron's learning rule. If f is the signum function, this becomes equivalent to

$$\Delta w_i = \begin{cases} 2\eta t^{(k)} x_i^{(k)} & \text{if } t^{(k)} \neq o^{(k)} \\ 0 & \text{otherwise} \end{cases}$$

Training Algorithm

$$\Delta\theta = \begin{cases} -2\eta t^{(k)} & \text{if } t^{(k)} \neq o^{(k)} \\ 0 & \text{otherwise} \end{cases}$$

- If a whole epoch is complete, then pass to the following step; otherwise go to Step 2.
- If the weights (and bias) reached steady state ($\Delta w_i \approx 0$)through the whole epoch, then stop the learning; otherwise go through one more epoch starting from Step 2.

Example

Problem Statement

- Classify the following patterns using $\eta = 0.5$:
  - Class 1 with target value (-1):
    $T = [2, 0]^T, U = [2, 2]^T, V = [1, 3]^T$
  - Class 2 with target value $(+1)$:
    $X = [-1, 0]^T, Y = [-2, 0]^T, Z = [-1, 2]^T$
- Let the initial weights be $w_1 = -1, w_2 = 1, \theta = -1$
- Thus, initial boundary is defined by $x_2 = x_1 - 1$

Example

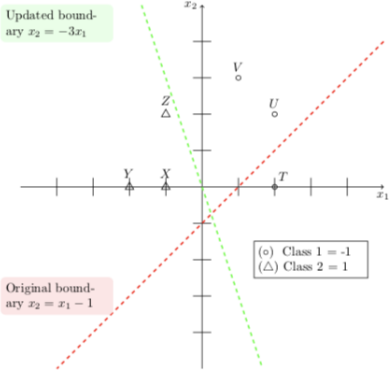
Solution

- T properly classified, but not U and V.
- Hence, training is needed.

$$\begin{aligned} \mathrm{sgn}(2 \times (-1) + 2(1) + 1) &= 1 \\ \Delta w_1 = \Delta w_2 = -1 \times (2) &= -2 \\ \Delta \theta &= +1 \end{aligned}$$

- Updated boundary is defined by $x_2 = -3x_1$
- All patterns are now properly classified.

Example: Graphical Solution



Case Study: Binary Classification Using Perceptron

- We need to train the network using the following set of input and desired output training vectors

$$(x^{(1)} = [1, -2, 0, -1]^T; t^{(1)} = -1),$$
$$(x^{(2)} = [0, 1.5, -0.5, -1]^T; t^{(1)} = -1),$$
$$(x^{(3)} = [-1, 1, 0.5, -1]^T; t^{(1)} = +1)$$

- Initial weight vector $w^{(1)} = [1, -1, 0, 0.5]^T$
- Learning rate $\eta = 0.1$

Epoch1

Introducing the first input vector $x^{(1)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(1)} &= sgn(w^{(1)^T} x^{(1)}) \\ &= sgn([1, -1, 0, 0.5][1, -2, 0, -1]^T) \\ &= +1 \neq t^{(1)}, \end{aligned}$$

- Updating weight vector

$$\begin{aligned} w^{(2)} &= w^{(1)} + \eta[t^{(1)} - o^{(1)}]x^{(1)} \\ &= w^{(1)} + 0.1(-2)x^{(1)} \\ &= [0.8, -0.6, 0, 0.7]^T \end{aligned}$$

Epoch1

Introducing the first input vector $x^{(2)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(2)} &= sgn(w^{(2)^T} x^{(2)}) \\ &= sgn([0.8, -0.6, 0, 0.7][0, 1.5, -0.5, -1]^T) \\ &= -1 = t^{(2)}, \end{aligned}$$

- Updating weight vector

$$w^{(3)} = w^{(2)}$$

Epoch1

Introducing the first input vector $x^{(3)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(3)} &= sgn(w^{(3)^T} x^{(3)}) \\ &= sgn([0.8, -0.6, 0, 0.7][-1, 1, 0.5, -1]^T) \\ &= -1 \neq t^{(3)}, \end{aligned}$$

- Updating weight vector

$$\begin{aligned} w^{(4)} &= w^{(3)} + \eta[t^{(3)} - o^{(3)}]x^{(3)} \\ &= w^{(3)} + 0.1(2)x^{(3)} \\ &= [0.6, -0.4, 0.1, 0.5]^T \end{aligned}$$

Epoch2

We reuse the training set $(x^{(1)}, t^{(1)}), (x^{(2)}, t^{(2)}) and (x^{(3)}, t^{(3)})$ as $(x^{(4)}, t^{(4)}), (x^{(5)}, t^{(5)}) and (x^{(6)}, t^{(6)})$, respectively.

Introducing the first input vector $x^{(4)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(4)} &= sgn(w^{(4)^T} x^{(4)}) \\ &= sgn([0.6, -0.4, 0.1, 0.5][1, -2, 0, -1]^T) \\ &= +1 \neq t^{(4)}, \end{aligned}$$

- Updating weight vector

$$\begin{aligned} w^{(5)} &= w^{(4)} + \eta[t^{(4)} - o^{(4)}]x^{(4)} \\ &= w^{(4)} + 0.1(-2)x^{(4)} \\ &= [0.4, 0, 0.1, 0.7]^T \end{aligned}$$

Epoch2

Introducing the first input vector $x^{(5)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(5)} &= sgn(w^{(5)^T} x^{(5)}) \\ &= sgn([0.4, 0, 0.1, 0.7][0, 1.5, -0.5, -1]^T) \\ &= -1 = t^{(5)}, \end{aligned}$$

- Updating weight vector

$$w^{(6)} = w^{(5)}$$

Epoch2

Introducing the first input vector $x^{(6)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(6)} &= sgn(w^{(6)^T} x^{(6)}) \\ &= sgn([0.4, 0, 0.1, 0.7][-1, 1, 0.5, -1]^T) \\ &= -1 \neq t^{(6)}, \end{aligned}$$

- Updating weight vector

$$\begin{aligned} w^{(7)} &= w^{(6)} + \eta[t^{(6)} - o^{(6)}]x^{(6)} \\ &= w^{(6)} + 0.1(2)x^{(6)} \\ &= [0.2, 0.2, 0.2, 0.5]^T \end{aligned}$$

Epoch 3

We reuse the training set $(x^{(1)}, t^{(1)}), (x^{(2)}, t^{(2)}) and (x^{(3)}, t^{(3)})$ as $(x^{(7)}, t^{(7)}), (x^{(8)}, t^{(8)}) and (x^{(9)}, t^{(9)})$, respectively.

Introducing the first input vector $x^{(7)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(7)} &= sgn(w^{(7)^T} x^{(7)}) \\ &= sgn([0.2, 0.2, 0.2, 0.5][1, -2, 0, -1]^T) \\ &= -1 = t^{(7)}, \end{aligned}$$

- Updating weight vector

$$w^{(8)} = w^{(7)}$$

Epoch 3

Introducing the first input vector $x^{(8)}$ to the network

- Computing the output of the network

$$\begin{aligned} o^{(8)} &= sgn(w^{(8)^T} x^{(8)}) \\ &= sgn([0.2, 0.2, 0.2, 0.5][0, 1.5, -0.5, -1]^T) \\ &= -1 = t^{(8)}, \end{aligned}$$

- Updating weight vector

$$w^{(9)} = w^{(8)}$$

Epoch 3

Introducing the first input vector $x^{(9)}$ to the network

- Computing the output of the network

$$
\begin{aligned}
o^{(9)} &= sgn(w^{(9)^T} x^{(9)}) \\
&= sgn([0.2, 0.2, 0.2, 0.5][-1, 1, 0.5, -1]^T) \\
&= -1 \neq t^{(9)},
\end{aligned}
$$

- Updating weight vector

$$
\begin{aligned}
w^{(10)} &= w^{(9)} + \eta[t^{(9)} - o^{(9)}]x^{(9)} \\
&= w^{(9)} + 0.1(2)x^{(9)} \\
&= [0, 0.4, 0.3, 0.3]^T
\end{aligned}
$$

Epoch 4

We reuse the training set $(x^{(1)}, t^{(1)}), (x^{(2)}, t^{(2)}) and (x^{(3)}, t^{(3)})$ as $(x^{(10)}, t^{(10)}), (x^{(11)}, t^{(11)}) and (x^{(12)}, t^{(12)})$, respectively.

Introducing the first input vector $x^{(10)}$ to the network

- Computing the output of the network

$$
\begin{aligned}
o^{(10)} &= sgn(w^{(10)^T} x^{(10)}) \\
&= sgn([0, 0.4, 0.3, 0.3][1, -2, 0, -1]^T) \\
&= -1 = t^{(10)},
\end{aligned}
$$

- Updating weight vector

$$w^{(11)} = w^{(10)}$$

Epoch 4

Introducing the first input vector $x^{(11)}$ to the network

- Computing the output of the network

$$
\begin{aligned}
o^{(11)} &= sgn(w^{(11)^T} x^{(11)}) \\
&= sgn([0, 0.4, 0.3, 0.3][0, 1.5, -0.5, -1]^T) \\
&= +1 \neq t^{(11)},
\end{aligned}
$$

- Updating weight vector

$$
\begin{aligned}
w^{(12)} &= w^{(11)} + \eta[t^{(11)} - o^{(11)}]x^{(11)} \\
&= w^{(11)} + 0.1(-2)x^{(11)} \\
&= [0, 0.1, 0.4, 0.5]^T
\end{aligned}
$$

Epoch 4

Introducing the first input vector $x^{(12)}$ to the network

- Computing the output of the network

$$
\begin{aligned}
o^{(12)} &= sgn(w^{(12)^T} x^{(12)}) \\
&= sgn([0, 0.1, 0.4, 0.5][-1, 1, 0.5, -1]^T) \\
&= -1 \neq t^{(12)},
\end{aligned}
$$

- Updating weight vector

$$
\begin{aligned}
w^{(13)} &= w^{(12)} + \eta[t^{(12)} - o^{(12)}]x^{(12)} \\
&= w^{(12)} + 0.1(2)x^{(12)} \\
&= [-0.2, 0.3, 0.5, 0.3]^T
\end{aligned}
$$

Final Weight Vector

- Introducing the input vectors for another epoch will result in no change to the weights which indicates that w(13) is the solution for this problem;
- Final weight vector:
  $w = [w_1, w_2, w_3, w_4] = [-0.2, 0.3, 0.5, 0.3]$.