# More Spatial Filtering

CS 450: Introduction to Digital Signal and Image Processing

Slides created by Dr. Bryan Morse
BYU Computer Science



## Sharpening

The goal of sharpening is to *enhance differences*, so all sharpening kernels involve differences—some positive and some negative weights.

- Unsharp masking
- 1st-derivative operators
- 2nd-derivative operators



## Unsharp Masking

- Idea: "subtract out the blur"
- Can be done in analog (originated in dark rooms)
- Procedure:
  1. Blur the image
  2. Subtract from the original
  3. Multiply by some weighting fraction
  4. Add back to the original



## Unsharp Masking

Mathematically:

$$g = f + \alpha \left( f - \bar{f} \right)$$

where

- $f$ is the original image
- $\bar{f}$ is the blurred image
- $g$ is the sharpened result
- $\alpha$ controls how much sharpening is added



## Unsharp Masking - Alternative Formulation

- Unsharp Masking:

$$g = f + \alpha \left( f - \bar{f} \right)$$

- Rather than working with fractional weights $\alpha$, we can write this as

$$g = Af + \left( f - \bar{f} \right)$$

- In other words, what really matters is the ratio of the original image to the "unsharp masked" (sharpening) part, not the absolute values used.
  - Larger A: more original image, less removal of the blur
  - Smaller A: less original image, more removal of the blur



## Implementing Unsharp Masking

Can directly blur, weight, and subtract, OR

$$\frac{1}{A}\left[ \underbrace{\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & A & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}}_{\text{Original } (\times A)} + \left( \underbrace{\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 5 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}}_{\text{Original}} - \underbrace{\begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}}_{\text{Blur}} \right) \right]$$

$$= \frac{1}{A} \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & A+4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

## Derivative Operators

- Local derivatives are a great way of measuring local image geometry
- *Differential Geometry*: a branch of mathematics built around using derivatives to describe local geometry
- Derivatives can be measured (or at least approximated) using convolution

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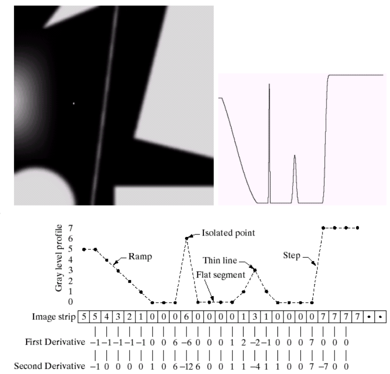
- First derivative - measure local changes to the signal
- Second derivatives - measure changes to the changes (i.e., curvature)



## Derivative Operators

a b
c

**FIGURE 3.38**
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point.
(c) Simplified profile (the points are joined by dashed lines to simplify interpretation).






## Derivative Operators

- Consider a 1-d signal $f(t)$. How is it changing?
- Recall from calculus that

$$\frac{d}{dt}f(t) = \lim_{h\to 0}\frac{f(t+h)-f(t)}{h}$$

- But for sampled signals, we can't measure values closer together than one sample. So,

$$\frac{d}{dt}f(t) \approx \frac{f(t+1)-f(t)}{1}$$

- This is called a *forward difference*



## Derivative Operators

- Can also measure a derivative from the other side:

$$\frac{d}{dt}f(t) = \lim_{h\to 0}\frac{f(t)-f(t-h)}{h}$$

- Again setting $h$ to one sample spacing,

$$\frac{d}{dt}f(t) \approx \frac{f(t)-f(t-1)}{1}$$

- This is called a *backwards difference*



## Derivative Operators

- Can also compromise between forward and backwards derivatives by measuring from both sides:

$$\frac{d}{dt}f(t) = \lim_{h\to 0}\frac{f(t+h)-f(t-h)}{2h}$$

- Again setting $h$ to one sample spacing,

$$\frac{d}{dt}f(t) \approx \frac{f(t+1)-f(t-1)}{2}$$

- This is called a *central difference*



## Kernels for Derivatives

| | | |
|---|---|---|
| Forward difference: | $\frac{d}{dt}f(t) \approx \frac{f(t+1)-f(t)}{1}$ | $\begin{array}{\|c\|c\|}\hline \boxed{-1} & +1 \\ \hline\end{array}$ |
| Backward difference: | $\frac{d}{dt}f(t) \approx \frac{f(t)-f(t-1)}{1}$ | $\begin{array}{\|c\|c\|}\hline -1 & \boxed{+1} \\ \hline\end{array}$ |
| Central difference: | $\frac{d}{dt}f(t) \approx \frac{f(t+1)-f(t-1)}{2}$ | $\frac{1}{2}\begin{array}{\|c\|c\|c\|}\hline -1 & \boxed{0} & +1 \\ \hline\end{array}$ |

## Second Derivatives

- Forward difference:

$$\frac{d}{dt}f(t) \approx f(t+1) - f(t)$$

- Backwards difference:

$$\frac{d}{dt}f(t) \approx f(t) - f(t-1)$$

- Difference *of the differences*:

$$\begin{aligned}\frac{d^2}{dt^2}f(t) &\approx [f(t+1) - f(t)] - [f(t) - f(t-1)] \\ &= f(t+1) - 2f(t) + f(t-1)\end{aligned}$$

| | | |
|---|---|---|
| +1 | -2 | +1 |



## Two Dimensions

Differentiate in one dimension, ignore the other

$\frac{\partial}{\partial x}$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| -1 | 0 | +1 |
| 0 | 0 | 0 |

$\frac{\partial}{\partial y}$

| | | |
|---|---|---|
| 0 | -1 | 0 |
| 0 | 0 | 0 |
| 0 | +1 | 0 |

$\frac{\partial^2}{\partial x^2}$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| +1 | -2 | +1 |
| 0 | 0 | 0 |

$\frac{\partial^2}{\partial y^2}$

| | | |
|---|---|---|
| 0 | +1 | 0 |
| 0 | -2 | 0 |
| 0 | +1 | 0 |



## Better Approach for Multiple Dimensions

Differentiate in one dimension, *smooth* in the other (handles noise better)

$\frac{\partial}{\partial x}$

| | | |
|---|---|---|
| -1 | 0 | +1 |
| -1 | 0 | +1 |
| -1 | 0 | +1 |

$\frac{\partial}{\partial y}$

| | | |
|---|---|---|
| -1 | -1 | -1 |
| 0 | 0 | 0 |
| +1 | +1 | +1 |

$\frac{\partial^2}{\partial x^2}$

| | | |
|---|---|---|
| +1 | -2 | +1 |
| +1 | -2 | +1 |
| +1 | -2 | +1 |

$\frac{\partial^2}{\partial y^2}$

| | | |
|---|---|---|
| +1 | +1 | +1 |
| -2 | -2 | -2 |
| +1 | +1 | +1 |



## Edge Detectors

Prewitt Kernels (Uniform averaging in opposite direction)

$\frac{\partial}{\partial x}$

| | | |
|---|---|---|
| -1 | 0 | +1 |
| -1 | 0 | +1 |
| -1 | 0 | +1 |

$\frac{\partial}{\partial y}$

| | | |
|---|---|---|
| -1 | -1 | -1 |
| 0 | 0 | 0 |
| +1 | +1 | +1 |

Sobel Kernels (Center-weighted averaging in opposite direction)

$\frac{\partial}{\partial x}$

| | | |
|---|---|---|
| -1 | 0 | 1 |
| -2 | 0 | +2 |
| -1 | 0 | 1 |

$\frac{\partial}{\partial y}$

| | | |
|---|---|---|
| -1 | -2 | -1 |
| 0 | 0 | 0 |
| +1 | +2 | +1 |



## The Gradient

The *gradient* of a multivariate function is a vector defined by

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \vdots \end{bmatrix}$$

- Orientation = direction of greatest change
- Magnitude (length) = steepness in that direction

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

By far the most common way to do edge detection.



## The Laplacian

- The Laplacian is the sum of the second derivatives in $x$ and $y$.
- Sort of an "overall curvature" for multiple dimensions

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ +1 & -2 & +1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & +1 & 0 \\ 0 & -2 & 0 \\ 0 & +1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & +1 & 0 \\ +1 & -4 & +1 \\ 0 & +1 & 0 \end{bmatrix}$$

- Same as the pure-sharpening part of the unsharp mask kernel
- Places where the Laplacian changes from positive to negative are also good potential edges.