Ratio & Proportion

Concept Of Ratio

- The comparative relation between two amounts/ quantities of same type is called ratio.
- Let an amount be x and another is y, then the ratio between them is $x : y$ or $x \div y$.
- If a : b : : c : d, then a and d are called extremes and b and c are called means.

$\therefore$ Product of extremes = Product of means.

i.e., ad = bc

Note:

It does not change the ratio, when we multiply or divide antecedent and consequent of the ratio by a same non-zero number as-

e.g. $a : b = \frac{a}{b} = \frac{a \times c}{b \times c} = ac : bc = a : b$

Definition of different types of ratio

1. **Mixed ratio** – Let $x : y$ and $P : Q$ be two ratios, then $Px : Qy$ is called mixed ratio.

2. **Duplicate ratio** – The mixed ratio of two equal ratios is called the duplicate ratio as

duplicate ratio of $a : b$ is $a^2 : b^2$

3. **Subduplicate ratio** – The square root of a certain ratio is called its subduplicate.

The subduplicate ratio of $a : b = \sqrt{a} : \sqrt{b}$

4. **Triplicate ratio** – The cube of a certain ratio is called triplicate ratio.

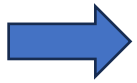
The triplicate ratio of $a : b = a^3 : b^3$

5. **Subtriplicate ratio** – The cube root of a certain ratio is called subtriplicate ratio as -

The Subtriplicate ratio of $a : b = \sqrt[3]{a} : \sqrt[3]{b}$

6. **Inverse ratio** – The reciprocal of quantities of ratio is called its inverse. Reciprocal or inverse ratio of $a : b$  $= \frac{1}{a} : \frac{1}{b}$

7. **Invertendo** – The proportion in which antecedent and consequent quantities change their places, is called invertendo, as -



$\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

8. **Alternendo** – If a : b :: c : d is a proportion then its alternendo is a : c :: b : d. i.e alternendo of $\frac{a}{b} = \frac{c}{d}$ is $\frac{a}{c} = \frac{b}{d}$

Componendo and dividendo – If there is a proportion a : b :: c : d then its componendo and dividendo is

$$(a + b) : (a - b) :: (c + d) : (c - d) \text{ or, } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

1. Three numbers are in the ratio 3:4:5. The sum of the largest and smallest equals the sum of the second and 52. The smallest number is

(a) 20 (b) 27 (c) 39 (d) 52

2. A and B have money in ratio 2:1. If A gives Rs.2 to B , the money will be in the ratio 1:1. What were the initial amount they had

(a) Rs.12 & Rs.6 (b) Rs.8 & Rs.4 (c) Rs.6 & Rs.3 (d) None

3. (a) 4A=5B=8C the A:B:C will be

(a) 8:5:10 (b) 15:8:10 (c) 10:8:5 (d) 12:10:11

(b) If A:B= ½:1/3 , B:C= 1/5:1/3 then (A+B):(B+C) is equal to

(a) 5:8 (b) 9:10 (c) 15:16 (d) 6:15

(c) If A:B=3:4 , B:C= 5:7 , & C:D= 8:9 then A:D= ?

(a) 10:21 (b) 21:10 (c) 7:3 (d) 3:7

4. Rs.33630 is divided among A,B & C in such a manner that the ratio of the amount of A to that of B is 3:7 and the ratio of amount of B to that of C is 6:5. The amount of money received by B is

(a) Rs.14868 (b) Rs.16257 (c) Rs.13290 (d) Rs.12390

5. The ratio of number of boys to number of girls in a school of 1650 students is 6:5. If 124 new boys are admitted and few new girls are admitted, the ratio changes to 4:3. What is the ratio of newly admitted girls to newly admitted boys.

(a) 9:62 (b) 32:33 (c) 5:32 (d) 18:37

6. The ratio of monthly income of X and Y is 5:4 and that of their monthly expenditure is 9:7. If the income of Y is equal to the expenditure of X, then what is the ratio of the savings of X and Y.

(a) 9:8 (b) 7:6 (c) 11:10 (d) 1:1

7.Atul purchased Bread costing Rs 20 and gave a 100 rupee note to the shopkeeper gave the balance gave the balance money in coins of denomination Rs 2, Rs 5 and Rs 10. If these coins are in the ratio 5:4:1, then how many Rs 5 coins did the shopkeeper give?

(a) 5 (b) 6 (c) 8 (d) 4

8.P pays Q a sum of Rs.150 using coins of Rs.2, Rs.5 & Rs.10. He uses a total of 50 coins. If the ratio of Rs.2 & Rs.5 coins used is 5:2, then how many coins of Rs.10 used in the payment

(a) 5 (b) 1 (c) 2 (d) 11

9. The ratio of the monthly incomes of A and B is 11 : 13 and the ratio of their expenditures is 9:11. If both of them manage to save Rs. 4000 per month, then find the difference in their income (in Rs.)

(a) Rs.2000 (b) Rs.3200 (c) Rs.4000 (d) Rs.2400

Proportion

Proportion: When two ratios are equal to each other, then they are called proportional as

a : b = c : d, then, a, b, c and d are in proportion.

Mean Proportion – Let x be the mean proportion between a and b, then a : x :: x : b (Real condition)

$$\therefore \frac{a}{x} = \frac{x}{b} \Rightarrow x^2 = ab$$

$$\therefore x = \sqrt{ab}$$

So, mean proportion of a and b $= \sqrt{ab}$

Third proportional – Let 'x' be the third proportional of a and b then,

a : b :: b : x (Real condition)

i.e. $\frac{a}{b} = \frac{b}{x} \Rightarrow ax = b^2$

$$\therefore x = \frac{b^2}{a}$$

$\therefore$ Third proportional of a and b $= \frac{b^2}{a}$

Fourth Proportional – Let x be the fourth proportional of a, b and c, then a : b :: c : x (Real condition)

$$\Rightarrow \frac{a}{b} = \frac{c}{x} \Rightarrow ax = bc$$

$$\therefore x = \frac{bc}{a}$$

$$\therefore \text{Fourth proportional of a, b and c} = \frac{bc}{a}$$

First Proportional – Let x be the first proportional of a, b and c. then, x : a :: b : c (Real condition)

$$\therefore \frac{x}{a} = \frac{b}{c} \Rightarrow cx = ab$$

$$\therefore x = \frac{ab}{c}$$

10.In a proportion the 1^{st}, 2^{nd}, and 4^{th} terms are 51,68 and 108 respectively. What is the third term.

(a) 65 (b) 78 (c) 93 (d) 81

11. What will be the third proportion of P and 12 when 8, P, 16 and 18 are in proportion.

(a) 16 (b) 18 (c) 22 (d) 25

12. Find the mean proportion between (3+ root of 5) & (15 – root of 125)

13. What should be added to 3, 11, 4 & 14 to make them in proportion

(a) 4 (b) 6 (c) 1 (d) 2

14. What is the least number should be subtracted from 14, 36, 20 & 54 so that these numbers becomes proportional.

(a) 3 (b) 4 (c) 2 (d) 5

15. When x is added to each of the numbers 7, 11, 18 & 23 then the number so obtained are in proportion. What is the mean proportion between |x-1| and |2x-10|

(a) 68 (b) 78 (c) 98 (d) 48

16. The train fare, bus fare and air fare between two places are in the ratio 5:8:12, the number of passenger travelled by them are in the ratio 3:4:5 . If the total fare collected on a particular day from these modes of transportation is Rs.107000. Find the fare collected from air passengers.

(a) Rs.45000 (b) Rs.60000 (c) Rs.72000 (d) Rs.38000

17. Rs. 13000 is divided among X, Y and Z such that 2 times of X's share is equal to 3 times of Ys share which is equal to 4 times of Z's share. What is the share of Y?

(a) Rs.4000 (b) Rs.5000 (c) Rs.5280 (d) Rs.4740