6.1 ~~X(ω)~~ $X(e^{j\omega})$ is periodic with period 2π.

6.2 a) $y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, e^{j\omega_0(n-k)}$$

$$y(n) = e^{j\omega_0 n} \sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega_0 k}$$

$$y(n) = e^{j\omega_0 n} H(j\omega_0)$$

~~b) b) $y[n] = x[n] * h[n]$~~

~~$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[k-n]$~~

b) $y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \right) e^{-j\omega n}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]\, e^{-j\omega n}$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \cancel{h[k]}\, x[k]\, h[n-k]\, e^{-j\omega n}$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} x[k] \left[\sum_{n=-\infty}^{\infty} h[n-k]\, e^{-j\omega(n-k)} \right] e^{-j}$$

$$\Rightarrow \left[\sum_{k=-\infty}^{\infty} x[k]\, e^{-j\omega k} \right] H(j\omega)$$

$$\Rightarrow \quad X(j\omega)\, H(j\omega).$$

$$\therefore \quad Y(j\omega) = X(j\omega)\, H(j\omega)$$

6.3

(b) <u>Time Invariant</u>

$$y[n] = \sum_m b_m x[n-m] - \sum_l a_l y[n-l]$$

$\Rightarrow$ For input $x[n] \rightarrow$ output is $y[n]$

(i) let $x[n-n_0]$ be the input and its output be $g[n]$

$$\Rightarrow g[n] = \sum_m b_m x[n-n_0-m] - \sum_l a_l g[n-l] \quad -①$$

(ii) Let the shifted output be $y[n-n_0]$.

$$y[n-n_0] = \sum_m b_m \, x[n-n_0-m] - \sum_l a_l \, y[n-n_0-l] \quad -②$$

So; for the $x[n-n_0]$ as input, output ~~we~~ is $g[n]$ and when the output is shifted by 'n_0'; the system reacts in the same manner as if input is shifted by 'n_0'.

$$\therefore g[n] = y[n-n_0]$$

$\Rightarrow$ The System is Time-Invariant

b) $$y_1[n] + \sum_l a_l y_1[n-l] = \sum_m b_m x_1[n-m] \quad -①$$

$$y_2[n] + \sum_l a_l y_2[n-l] = \sum_m b_m x_2[n-m] \quad -②$$

Let $a x_1[n] + b x_2[n]$ be the input.

$\Rightarrow F(a x_1[n] + b x_2[n]) = ?$

Scanned by CamScanner

$$a\left(y_1[n] + b y_2[n]\right) + \sum_{l} a_l \left(a y_1(n-l) + b y_2(n-l)\right)$$

$$= \sum_{m} b_m \left(a x_1(n-m) + b x_2(n-m)\right)$$

$$= F\left(a x_1[n]\right) + F\left(b x_2[n]\right)$$

$$\Rightarrow F\left(a x_1[n] + b x_2[n]\right)$$

$$\therefore F\left(a x_1[n] + b x_2[n]\right) = a\, F\left(x_1[n]\right) + b\, F\left(x_2[n]\right)$$

$\Rightarrow$ Linear System.

e) 6.3(c) is Low pass filter and repeating with period '2π'.

$\rightarrow$ Low pass because it allows frequency around Zero.

6.3(d) is high-pass and repeating with period 2π.

$\rightarrow$ It allows all frequency other than around Zero.

f) $$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1+0.9e^{-j\omega}}$$, High pass filter

$$H(e^{j\omega}) = \frac{1}{1+0.9e^{-j\omega}}$$

$$\cancel{h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{jn\omega}\,d\omega}$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{1+0.9e^{-j\omega}}\, e^{jn\omega}\,d\omega$$

(ii) $y[n] = x[n] + 0.9\,y[n-1]$

$h[n] = 0 \; ; \; n < 0$

Let $x[n] = \delta[n]$

$\Rightarrow y[0] = x[0] \; ; \; n = 0$

$\Rightarrow h[0] = 1$; Since $\left\{ \begin{array}{l} \delta(0) = 1 ; \\ h[0] = y[0] \end{array} \right.$

$\Rightarrow y[1] = x[1] + 0.9\,y[0]$

$\Rightarrow y[1] = 0.9$

$\Rightarrow h[1] = 0.9$

$\Rightarrow h[n] = 0.9\,h[n-1] \; ; \; n \geq 1$ $\quad$ ($\because \delta[n] = 1$; for $n=0$ only)

$h[0] = 1$

$\boxed{h[n] = (0.9)^n} \; ; \; n \geq 0$

$$\Rightarrow h[n] = \begin{cases} (0.9)^n & ; \; n \geq 0 \\ 0 & ; \; n < 0 \end{cases}$$

(ii) $y[n] = x[n] - 0.9\, y[n-1]$

let $x[n] = \delta[n]$

$\Rightarrow y[n] = h[n]$

$\Rightarrow h[n] = \delta[n] - 0.9\, h[n-1]$

$\Rightarrow h[0] = 1 - 0.9\, h[-1]$

$\therefore h[n] = 0;\ \forall\ n < 0$

$\therefore h[0] = \cancel{0}\ 1$

$h[n] = -0.9\, h[n-1]$ for $n > 1$

$\because \delta[n] = 0\ \forall\ n \neq 0$

$\Rightarrow h[n] = (-0.9)^n$

$$\therefore h[n] = \begin{cases} (-0.9)^n ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$