Logic and Propositions

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CSA102: Mathematics-I

Newton School of Technology

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Back to Basics: A Quick Logic Recap

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What We've Learned

Propositions are declarative sentences that are either true or false. We combine them using logical connectives to create complex statements.

Essential Logical Connectives

Symbol	Name	Meaning
$\neg$	NOT	Negation
$\wedge$	AND	Conjunction
$\vee$	OR	Disjunction
$\oplus$	XOR	Exclusive OR
$\Rightarrow$	IF-THEN	Implication

Passing the Exam

Statement- “If I teach Maths, then all students will pass.”

Let:

- T: “I teach Maths.”
- P: “All students will pass.”

Under what condition(s) will this proposition be considered false?

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If-Then/ Implication

Cause and Effect: Implication (⇒)

→ Symbol: $p \Rightarrow q$

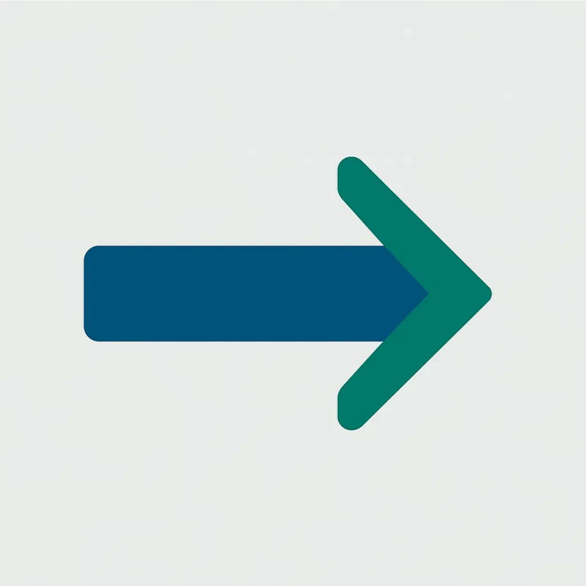
Meaning: False only when p is true and q is false.

Rewording:

- "If p, then q"
- "p is sufficient for q"
- "q is necessary for p"

Truth Table

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T



Cause and Effect: Implication (⇒)

- It can also be expressed as:

“If P, then Q”	“P implies Q”	“If P, Q”
“P only if Q”	“Q if P”	“Q whenever P”
“P is sufficient for Q”	“a sufficient condition for Q is P”	
“Q is necessary for P”	“a necessary condition for P is Q”	
“Q follows from P”	“Q unless ~P”	“Q provided that P”

Example

Statement- “You can receive an A in Maths only if your score on the final is at least 90%.”

- P: “You can receive an A in Maths.”
- Q: “Your score on the final is at least 90%.”

If you receive an A in Maths, then you know that your score on the final is at least 90%. If you do not receive an A, you may or may not have scored at least 90% on the final.

This is written as - **P⟹Q**

Example

Is $\mathbf{P \Longrightarrow Q}$ true or false?

- P : "n is divisible by 6 and n is divisible by 8"
- Q : "n is divisible by 12"

Example

Let p, q, and r be the propositions

p : You get an A on the final exam.

q : You do every assignments of maths.

r : You get an A in maths.

Write these propositions using p, q, and r and logical connectives.

1. To get an A in maths, it is necessary for you to get an A on the final.
2. Getting an A on the final and doing every assignment of maths is sufficient for getting an A in maths.
3. You will get an A in maths if you either do every assignment of maths or you get an A on the final.

Example: Knights, Knaves, and Logic

On a certain island, there are only two types of islanders:

- **knights**, who always tell the truth, and
- **knaves**, who always lie.

You meet two islanders named Aditya and Bhumika.

Aditya: Bhumika is a Knight and I am a knight.

Bhumika: Aditya is a Knave or I am a Knave.

Example: Knights, Knaves, and Logic

Question: What types of islanders are Aditya and Bhumika?

- Aditya is a knight, Bhumika is a knight.
- Aditya is a knave, Bhumika is a knave.
- Aditya is a knight, Bhumika is a knave.
- Aditya is a knave, Bhumika is a knight.

Example: Knights, Knaves, and Logic

Question: What types of islanders are Aditya and Bhumika?

- Aditya is a knight, Bhumika is a knight.
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- Aditya is a knave, Bhumika is a knight.

Knights, Knaves, and Logic

On the same island, Ramesh and Suresh also lives. Given below is a statement. Classify them as Knights and Knaves.

- **Ramesh:** If I am a knight then Suresh is a knave.
- **Suresh:**

Is Suresh a knight or a knave ?

(Suresh is quiet,busy enjoying his chocolate.)

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Transforming Implications

Original: $P \Rightarrow Q$

"If P, then Q"

Converse: $Q \Rightarrow P$

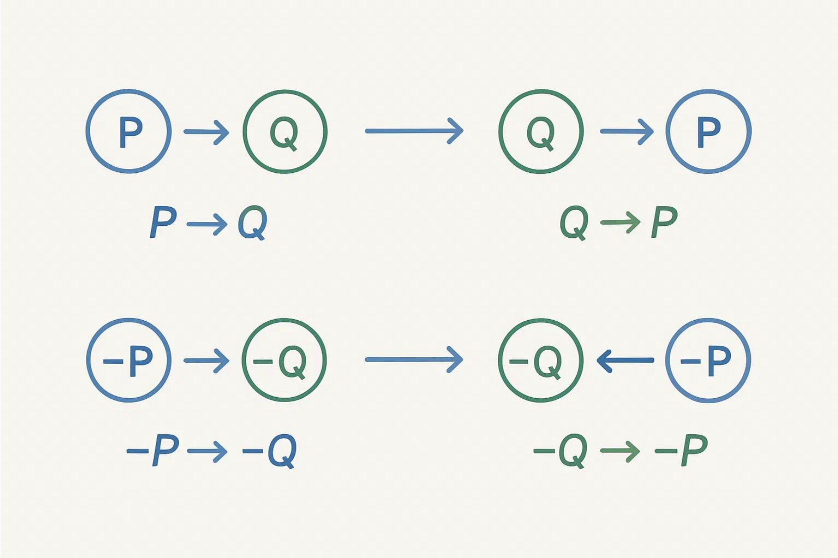
"If Q, then P"

Inverse: $\neg P \Rightarrow \neg Q$

"If not P, then not Q"

Contrapositive: $\neg Q \Rightarrow \neg P$

"If not Q, then not P"



💡 **Key Insight:** Original ≡ Contrapositive!

The Spectrum of Truth

Tautology

A statement that is **always true**, regardless of truth values.

`Example: P ∨ ¬P (P or not P)`

Contradiction

A statement that is **always false**, regardless of truth values.

`Example: P ∧ ¬P (P and not P)`

Contingency

A statement that can be **either true or false**, depending on circumstances.

`Example: P ⊕ Q (P exclusive or Q)`

Simplifying Logic

What is the negation of the statement -

“Ramesh speaks Hindi and Ramesh speaks Punjabi”?

DeMorgan's Laws in Action

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DeMorgan's First Law

$\neg(P \land Q) \equiv \neg P \lor \neg Q$

"Not (P and Q)" equals "Not P or Not Q"

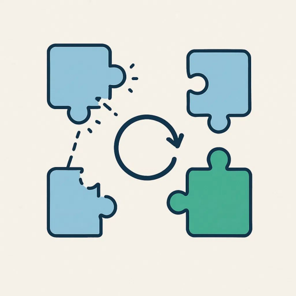
DeMorgan's Second Law

$\neg(P \lor Q) \equiv \neg P \land \neg Q$

"Not (P or Q)" equals "Not P and Not Q"

The Magic of Negation

DeMorgan's Laws help us "push" negation through parentheses by flipping the connective!



Original Statement:

"It is not true that it's raining AND cold."

DeMorgan's Equivalent:

"It's not raining OR it's not cold."

Original Statement:

"It's not true that I'll go to beach OR mountains."

DeMorgan's Equivalent:

"I won't go to beach AND I won't go to mountains."

Example

Statement: Ramesh is in the football team and the basketball team.

Negation: Ramesh is not in the football team or Ramesh is not in the basketball team.

Statement: Ankur Sir is available in office hours or Badal Sir is available in office hours.

Negation: Ankur Sir is not available in office hours and Badal Sir is not available in office hours.

Example

What is an equivalent statement to “not (A and not B)"?

1. not A and B
2. A or not B
3. A and not B
4. not A or B

Example

What is an equivalent statement to “not (A and not B)"?

1. not A and B
2. A or not B
3. A and not B
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Example

What is equivalent to “not (A and not (B or not C))"?

1. not A or not B and not C
2. not A or not B or not C
3. not A or B or not C
4. not A or B and not C

Example

What is equivalent to “not (A and not (B or not C))"?

1. not A or not B and not C
2. not A or not B or not C
3. not A or B or not C
4. not A or B and not C

Example

Draw the truth table for the expression:

$$(p \land \neg q) \lor (\neg p \land q)$$

Logical Equivalence

The Power of Sameness: Understanding Logical Equivalence

Definition

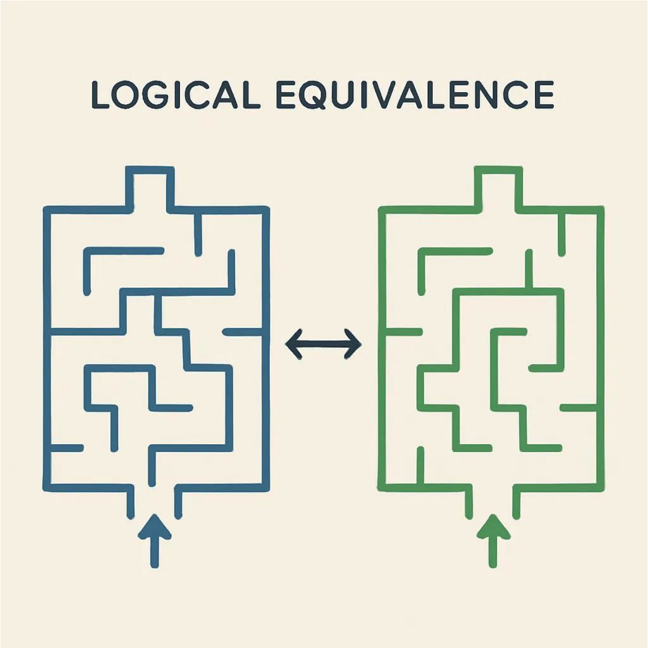
Two statements are **logically equivalent** if they have identical truth values in all cases.

`Symbol: ≡`

Key Example:

$p \Rightarrow q \equiv \neg p \vee q$

"If p then q" equals "Not p or q"



The Power of Sameness: Understanding Logical Equivalence

Commutative

$p \wedge q \equiv q \wedge p$

Associative

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Identity

$p \wedge T \equiv p$

Distributive

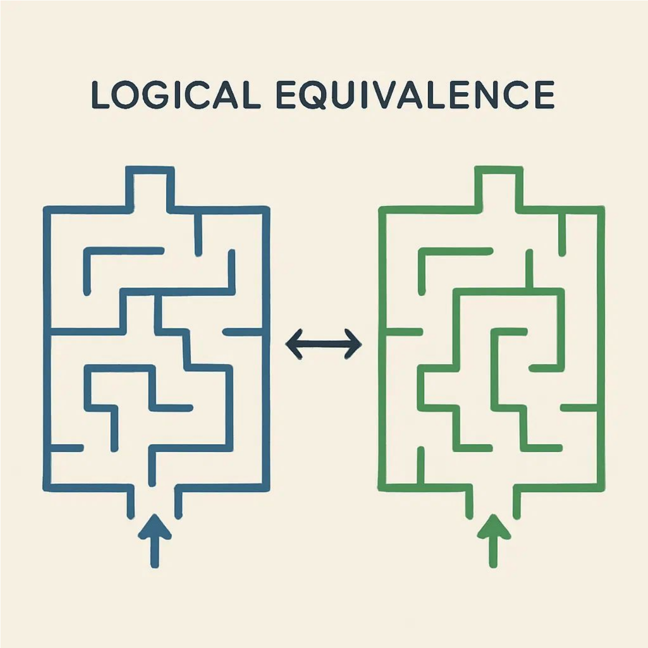
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Domination

$p \vee T \equiv T$

Absorption

$p \wedge (p \vee q) \equiv p$



Are They Equivalent? A Logical Challenge

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Test Your Understanding

Determine if these pairs of statements are logically equivalent. Use truth tables or equivalence laws!

Challenge 1

Statement A: $\neg(p \wedge q)$

Statement B: $\neg p \vee \neg q$

Challenge 2

Statement A: $p \Rightarrow q$

Statement B: $\neg q \Rightarrow \neg p$

Challenge 3

Statement A: $p \wedge (p \vee q)$

Statement B: p

Challenge 4

Statement A: $\neg(p \vee q)$

Statement B: $\neg p \wedge \neg q$

Are They Equivalent? A Logical Challenge

Test Your Understanding

Determine if these pairs of statements are logically equivalent. Use truth tables or equivalence laws!

Challenge 1

Statement A: $\neg(p \land q)$

Statement B: $\neg p \lor \neg q$

Hint: Think DeMorgan's Laws!

Challenge 2

Statement A: $p \Rightarrow q$

Statement B: $\neg q \Rightarrow \neg p$

Hint: What's the contrapositive?

Challenge 3

Statement A: $p \land (p \lor q)$

Statement B: p

Hint: Absorption law in action!

Challenge 4

Statement A: $\neg(p \lor q)$

Statement B: $\neg p \land \neg q$

Hint: Another DeMorgan's Law!

Thank You!