Logic and Propositions

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CSA102: Mathematics-I

Quick Revision

Simplifying Logic

What is the negation of the statement -

“Ramesh speaks Hindi and Ramesh speaks Punjabi”?

Example

Statement: Ramesh is in the football team and the basketball team.

Negation: Ramesh is not in the football team or Ramesh is not in the basketball team.

Statement: Ankur Sir is available in office hours or Badal Sir is available in office hours.

Negation: Ankur Sir is not available in office hours and Badal Sir is not available in office hours.

Example

What is an equivalent statement to “not (A and not B)"?

1. not A and B
2. A or not B
3. A and not B
4. not A or B

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What is equivalent to “not (A and not (B or not C))"?

1. not A or not B and not C
2. not A or not B or not C
3. not A or B or not C
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Beyond Individuals

Aryan says that “All white lions weigh more than 100kg”.

What statement could Ritik say to contradict him?

Beyond Individuals

Consider the following statements about a number x:

1. x is a multiple of 2.
2. x is a multiple of 3.
3. x is a multiple of 6.

Is it possible that for some integer x -

A. None of them are true?
B. Exactly one of them is true?
C. Exactly two of them are true?
D. All three statements are true?

Quantifiers

Quantifiers in Logic

- Quantifiers: which quantifies over a range of values
    - Have True and False values distributed across range
- Two types of quantifiers:
    - **Universal quantifier (All/every)**
        - Ex: All the apples in this basket are red.
    - **Existential quantifier (At least one)**
        - This is used when you want to say that there's at least one thing in a group that meets a certain condition.
        - Ex: There is at least one orange in this box.

Quantifiers in Logic

Universal Quantifier (∀)

"For all," "For every," "Each"

> **Example:**
>
> "All students in this class are good at math."
>
> $\forall x \ (S(x) \Rightarrow M(x))$

Existential Quantifier (∃)

"There exists," "For some," "At least one"

> **Example:**
>
> "Some birds can fly."
>
> $\exists x \ (B(x) \land F(x))$

Quantifiers in Logic

⇄ Negation Rules

$\neg(\forall x\ P(x)) \equiv \exists x\ \neg P(x)$

$\neg(\exists x\ P(x)) \equiv \forall x\ \neg P(x)$

Original:

"All elephants are tall and heavy."

Negation:

"There exists an elephant that is short or light."

Original:

"Some students understand logic."

Negation:

"All students don't understand logic."

Example

Suresh says that in every region there is a town where all inhabitants are happy. Ramesh wants to say that Suresh is wrong. Which of the following sentences should Ramesh say?

1. There is a region where there is a town where all inhabitants are unhappy.
2. In every region in all towns all inhabitants are unhappy.
3. In every region there is a town where at least one inhabitant is unhappy.
4. There is a region where in all towns at least one inhabitant is unhappy.

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Example

Mohit claims that every student in a group knows Hindi, English, or both. Which of the following sentences asserts that Mohit's statement is wrong (no more, no less)?

1. All students know at most one of these two languages.
2. There is a student who does not know Hindi and does not know English.
3. There is a student who does not know Hindi or does not know English (or both).
4. Every student who knows Hindi does not know English.

Example

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Dual of a Compound Proposition

- The dual of a compound proposition is a transformation of the original expression where:
  - AND ($\wedge$) is replaced by OR ($\vee$)
  - OR ($\vee$) is replaced by AND ($\wedge$)
  - True (T) is replaced by False (F)
  - False (F) is replaced by True (T)
- However, negations ($\neg$) and variables (like p, q) remain unchanged.
- If you are given a compound proposition **P**, its **dual** is denoted as $\mathbf{P^D}$.

- **Why is Duality Important?**
  - Validate logical equivalences.
  - Understand symmetry in logic rules.

Dual of a Compound Proposition

- Dual of $(p \land q) \lor r$ is $(p \lor q) \land r$.
- Dual of $(p \lor T) \land (\neg q \lor r)$ is $(p \land F) \lor (\neg q \land r)$.
- Dual of $(p \lor F) \land T$ is $(p \land T) \lor F$.
- The dual of the dual gives the original proposition back.

  $\mathbf{(P^D)^D = P}$

- Duality is not the same as logical negation.
  - Dual of $p \lor q$ is $p \land q$.
  - Negation of $p \lor q$ is $\neg p \land \neg q$ (De Morgan's law).

Example: Knights, Knaves, and Logic

On a certain island, there are only two types of islanders:

- **knights**, who always tell the truth, and
- **knaves**, who always lie.

You meet two islanders named Aditya and Bhumika.

Aditya: Bhumika is a Knight and I am a knight.

Bhumika: Aditya is a Knave or I am a Knave.

Example: Knights, Knaves, and Logic

Question: What types of islanders are Aditya and Bhumika?

- Aditya is a knight, Bhumika is a knight.
- Aditya is a knave, Bhumika is a knave.
- Aditya is a knight, Bhumika is a knave.
- Aditya is a knave, Bhumika is a knight.

Example: Knights, Knaves, and Logic

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- Aditya is a knave, Bhumika is a knave.
- Aditya is a knight, Bhumika is a knave.
- Aditya is a knave, Bhumika is a knight.

Self Test

The Island of Truth and Lies

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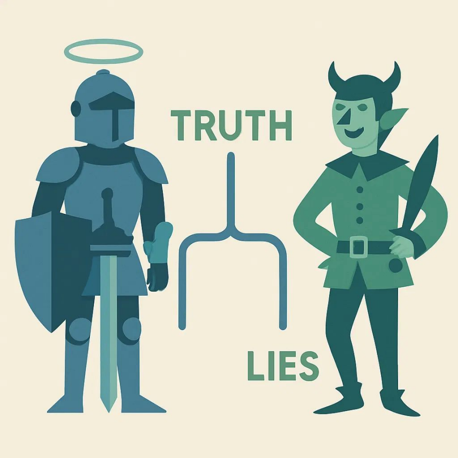
The Rules

Knights: Always tell the truth

Knaves: Always lie

Goal: Determine who is what!



Your Turn!

Ramesh says:

"If I am a Knight, then Suresh is a Knave."

Suresh says:

"Ramesh and I are the same type."

Can you solve this puzzle?

Prove:

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Prove:

TABLE 7 Logical Equivalences Involving Conditional Statements.

$$p \to q \equiv \neg p \vee q$$
$$p \to q \equiv \neg q \to \neg p$$
$$p \vee q \equiv \neg p \to q$$
$$p \wedge q \equiv \neg(p \to \neg q)$$
$$\neg(p \to q) \equiv p \wedge \neg q$$
$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$
$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$
$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$
$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

Key Takeaways

Today we learnt:

- Negation of operators
- Logical Equivalences
- Quantifiers

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Thank You!