Logic and Propositions

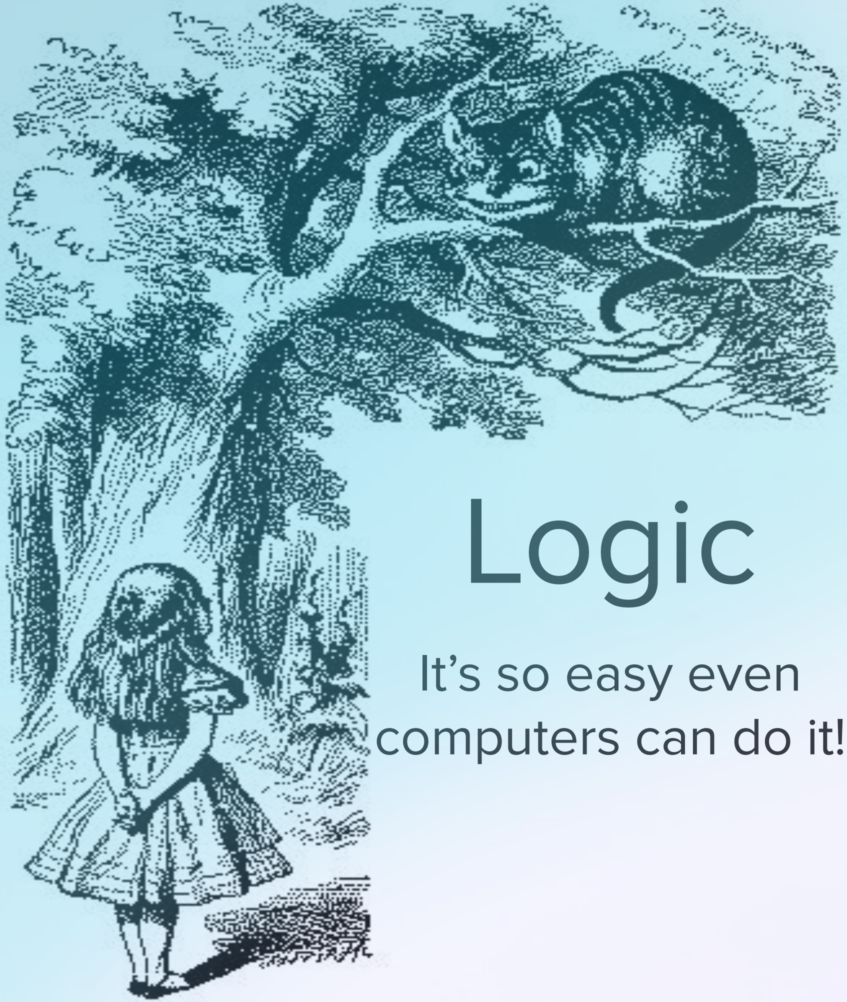
By- Piyush Jain

CSA102: Mathematics-I

Quick Revision

In the last class, we studied:

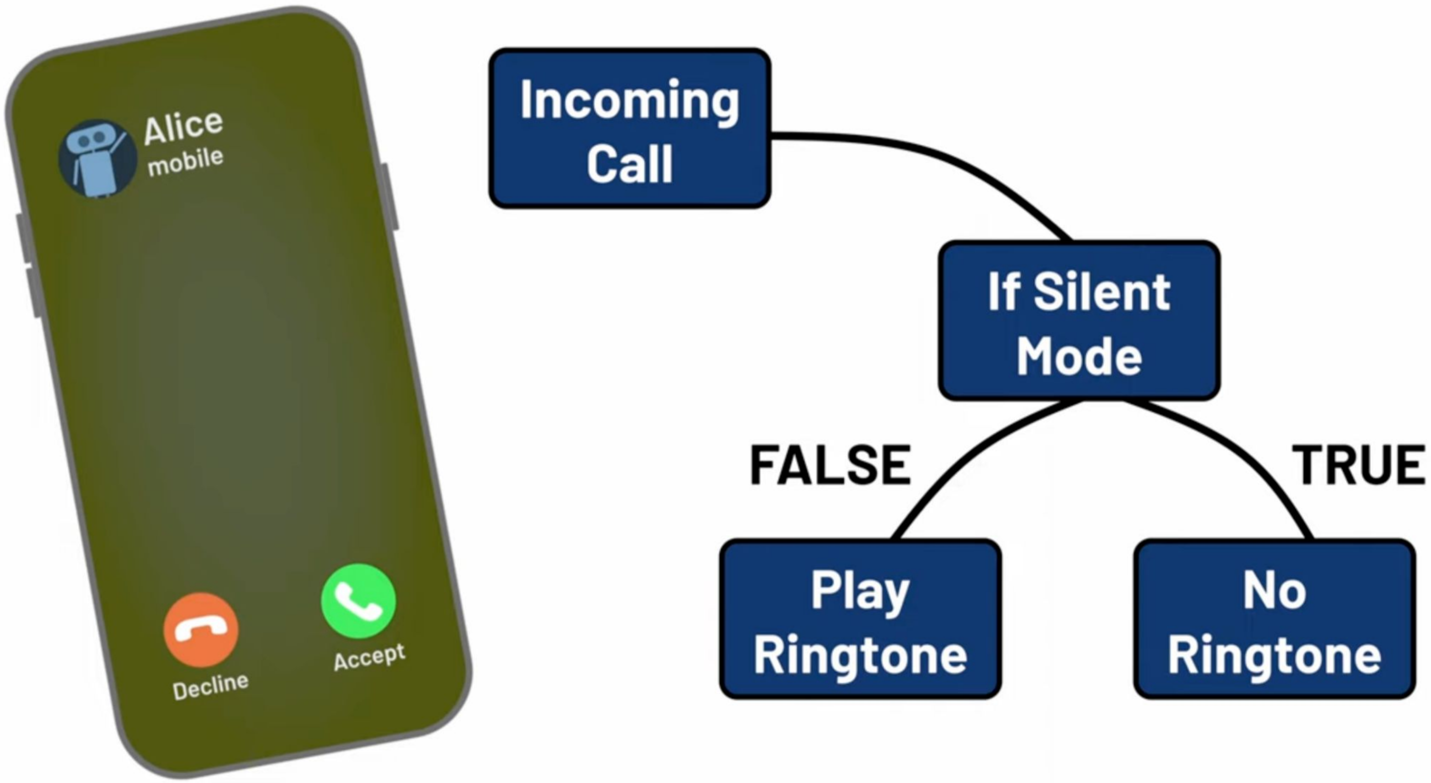
- Modulo Arithmetic
- Divisibility rules



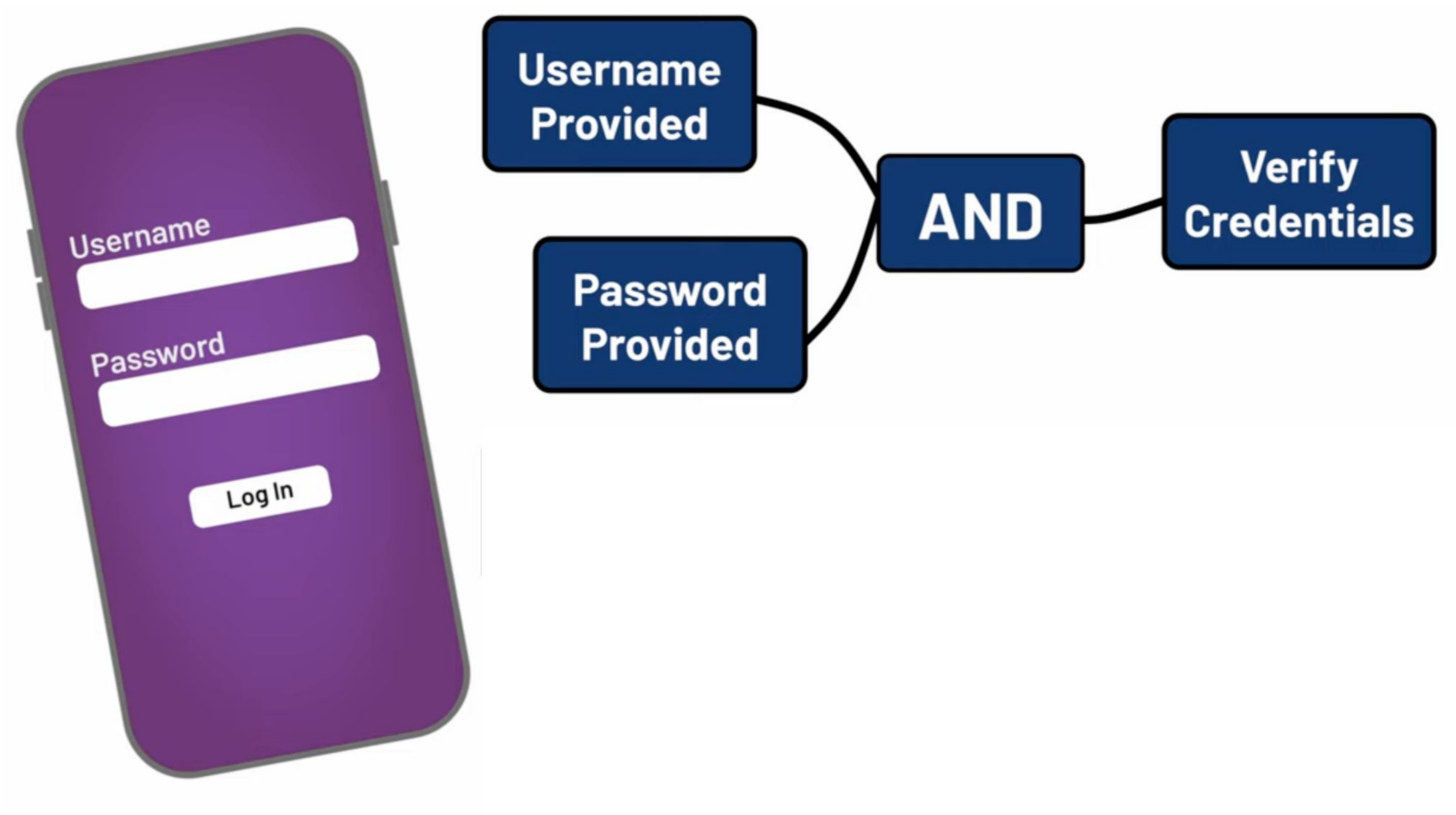
Logic

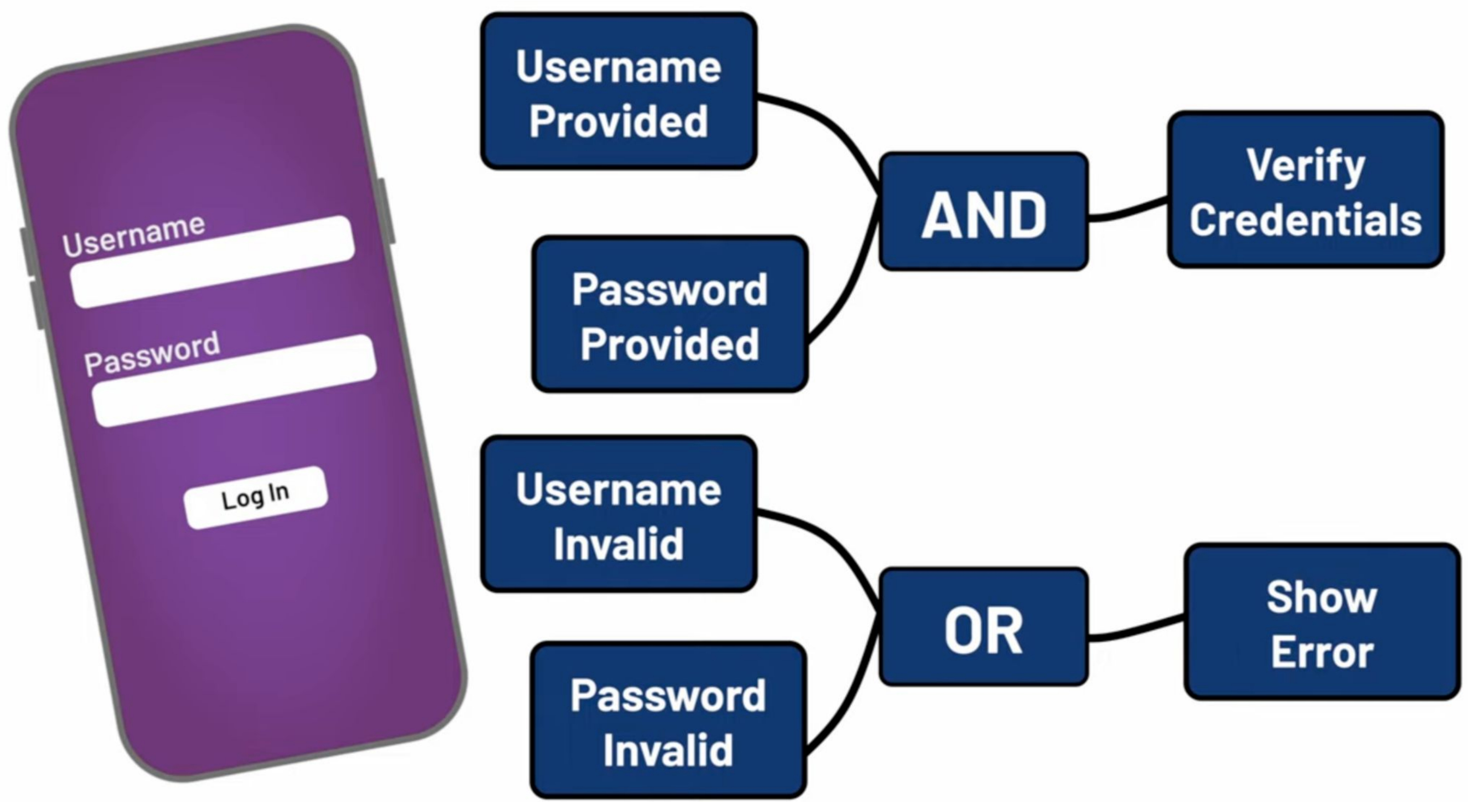
It’s so easy even computers can do it!


Alice
mobile
Decline
Accept


Alice
mobile
Decline
Accept
Incoming Call
If Silent Mode
FALSE
TRUE
Play Ringtone
No Ringtone


Username
Password
Log In

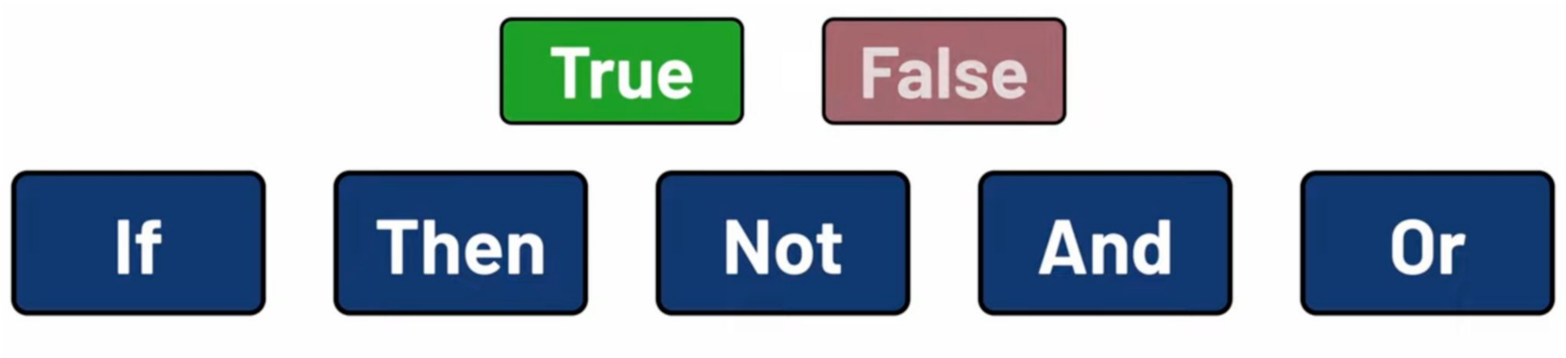

Username
Password
Log In
Username Provided
Password Provided
AND
Verify Credentials


Username
Password
Log In
Username Provided
Password Provided
AND
Verify Credentials
Username Invalid
Password Invalid
OR
Show Error

Logic

- If
- Then
- Not
- And
- Or

Logic



Logic

- Logic is the **formal science of reasoning**, helping us distinguish between **valid** and **invalid** arguments.
- It forms the foundation of **mathematical thinking**, **computer programming**, **problem-solving**, and **everyday decision-making**.

We will explore:

  - Propositions — What makes a statement logically valid?
  - Logical Connectives — How can we combine simple statements?
  - Negation of Statements — What does it mean to negate a statement?

Propositions

Statement (Proposition)

A proposition is a declarative sentence that is either true or false, but not both.

✓ **True:** "2 + 2 = 4"

✗ **Variable-dependent:** "x + y > 0"

? **Paradox:** "I am lying"

Statement (Proposition)

A *Proposition* is a sentence that is either **True** or **False**

Examples:

$2 + 2 = 4$ True

$3 \times 3 = 8$ False

787009911 is a prime

Non-examples:

$x+y>0$

$x^2+y^2=z^2$

They are true for some values of x and y but are false for some other values of x and y.

Example

Decide which of the following is a proposition or not.

• The number 4 is even and less than 12.	Proposition (T)
• New Delhi is the capital of India.	Proposition (T)
• Covid19 is a Bacteria.	Proposition (F)
• 5 is an even number.	Proposition (F)
• I am lying.	Not a Proposition. It’s a Paradox.

Example

Proposition: For every integer $n>1$, the number $n^2 + n + 41$ is prime.

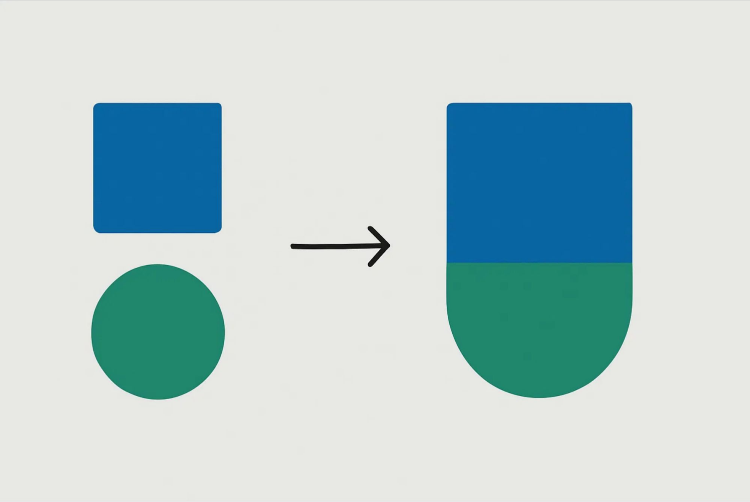
- Is it true? If not, find a counterexample.

Simple and Compound Propositions

Simple Propositions

Atomic statements that cannot be broken down further.

“It is raining.”



Compound Propositions

Combinations of simple propositions using logical connectives.

“It is hot and sunny.”

Simple and Compound Propositions

1) **Simple** - a single, complete, statement -
    a) New Delhi is the capital of India.
    b) Covid19 is a Bacteria.
    c) 5 is an even number.

2) **Compound** - two or more conjoined statements -
    a) The number 4 is even ***and*** less than 12.
    b) ***Either*** it is a Maths class, ***or*** it is a programming class.
    c) ***If*** Husky is a dog, ***then*** Husky is a mammal.

Logical connectives / operators

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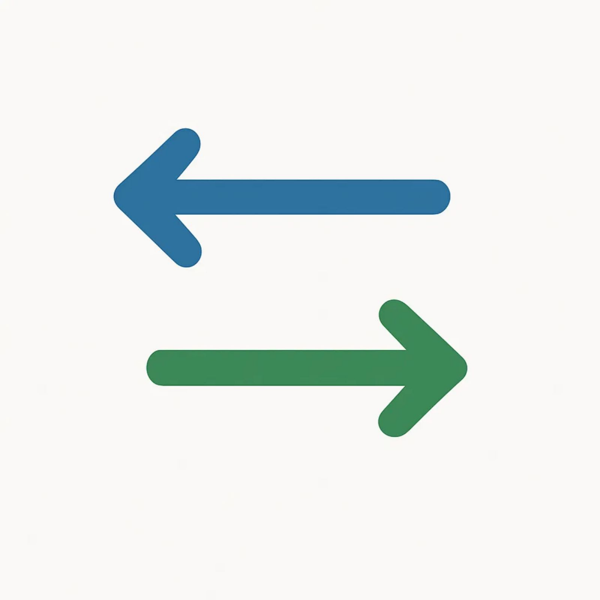
Inverting Truth: The Negation (¬)

⇄ Symbol: ¬p

Meaning: Inverts the truth value of a proposition.

Truth Table

P	¬P
T	F
F	T



Inverting Truth: The Negation (¬)

A statement that is false whenever the given statement is true, and true whenever the given statement is false.

Example :

- p : The Anti-Terrorism law is constitutional.
- $\neg p$: The Anti-Terrorism law is unconstitutional.

Example

Proposition P: Pakistan is independent.

What is ¬P?

- Pakistan is not independent.

At Least One: The Disjunction (∨)

Symbol: p ∨ q

Meaning: True if at least one of p or q is true.

Truth Table

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Example

P(n) : n is a multiple of 4 or n is a multiple of 6

Determine whether the disjunction P(n) is true or false for the following values of n:

1. $n = 6$
2. $n = 9$
3. $n = 12$
4. $n = 5$

Both Must Be True: The Conjunction (∧)

Symbol: $p \land q$

Meaning: True only if both p and q are true.

Truth Table

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

Example

Is the following proposition true: "All rectangles are squares, and there is no life outside Earth."?

Solution:

Lazy Evaluation

```python
if 2+2==5 and 5%0==0:
    print("True")
else:
    print("False")
```

Example

Consider the following statements, and determine whether it is true or false-

- Ice floats in water and 2 + 2 = 4 (T)
- New Delhi is in Nepal and 5 is odd (F)
- 5 - 3 = 1 or 2 * 2 = 4 (T)

Example

Let p, q, and r be the propositions

p : You get an A on the final exam.

q : You do every assignments of maths.

r : You get an A in maths.

Write these propositions using p, q, and r and logical connectives.

1. You get an A in maths, but you do not do every assignment of maths.
2. You get an A on the final, you do every assignment of maths, and you get an A in maths.
3. You get an A on the final, but you don’t do every assignment of maths; nevertheless, you get an A in maths.

Example

Give the truth table for the formal proposition p ^ ¬q.

Example

Give the truth table for the formal proposition (p V q) ^ (¬p ^ q).

Mapping Truth: Constructing Truth Tables

Purpose of Truth Tables

Systematically evaluate the truth values of compound statements for all possible combinations of input values.

1 List Variables

Identify all simple propositions (p, q, r, etc.)

2 Create Rows

Generate all possible T/F combinations (2^n rows for n variables)

3 Build Columns

Add columns for sub-expressions and final result

4 Evaluate

Apply logical operators step by step

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How many distinct logical operators possible operating on two propositional variables?

Light Switches

Imagine a room with two light switches controlling the same light. What is the condition for the light to be ON?

The light is on if exactly one of the switches is on.

Switch 1: ON (1)

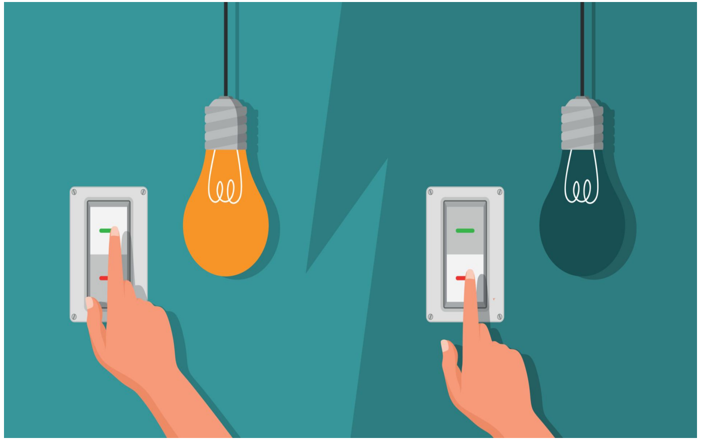
Switch 2: OFF (0)

Light: ON

Switch 1: OFF (0)

Switch 2: ON (1)

Light: ON



Exclusive OR

Exclusive OR (⊕)

Symbol: $p \oplus q$

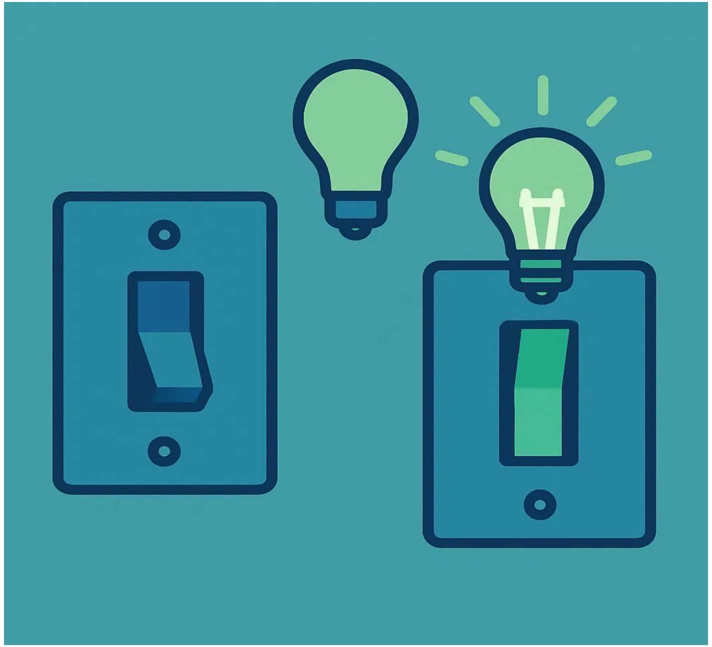
Meaning: "Either p or q, but not both."

> Light is ON if exactly one switch is ON

Truth Table

P	Q	P ⊕ Q
T	T	F
T	F	T
F	T	T
F	F	F

$$p \oplus q \equiv (p \land \neg q) \lor (\neg p \land q)$$



Passing the Exam

Statement- “If I teach Maths, then all students will pass.”

Let:

- T: “I teach Maths.”
- P: “All students will pass.”

Under what condition(s) will this proposition be considered false?

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If-Then/ Implication

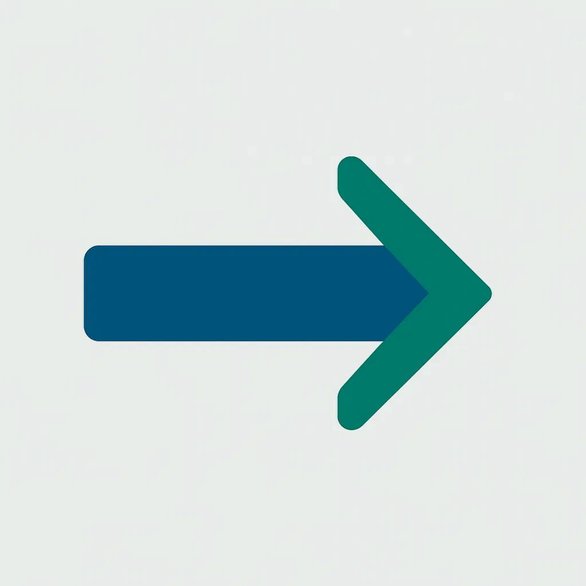
Cause and Effect: Implication (⇒)

→ Symbol: $p \Rightarrow q$

Meaning: False only when p is true and q is false.

Rewording:

- "If p, then q"
- "p is sufficient for q"
- "q is necessary for p"



Truth Table

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

Cause and Effect: Implication (⇒)

- It can also be expressed as:

"If P, then Q"	"P implies Q"	"If P, Q"
"P only if Q"	"Q if P"	"Q whenever P"
"P is sufficient for Q"	"a sufficient condition for Q is P"	
"Q is necessary for P"	"a necessary condition for P is Q"	
"Q follows from P"	"Q unless ~P"	"Q provided that P"

Example

Is $\mathbf{p \implies Q}$ true or false?

- P : "n is divisible by 6 and n is divisible by 8"
- Q : "n is divisible by 12"

Example

Let p, q, and r be the propositions

p : You get an A on the final exam.
q : You do every assignments of maths.
r : You get an A in maths.

Write these propositions using p, q, and r and logical connectives.

1. To get an A in maths, it is necessary for you to get an A on the final exam.
2. Getting an A on the final exam and doing every assignment of maths is sufficient for getting an A in maths.
3. You will get an A in maths if you either do every assignment of maths or you get an A on the final exam.

Key Takeaways

Today we learnt:

- Propositions
- Logical Connectives:
    - Negation
    - Conjunction
    - Disjunction
    - Exclusive OR
    - Implication

Thank You!