Newton School of Technology

Set Theory (Part – 3)

By- Piyush Jain

CS01: Mathematics-I

Join the class

Quick Revision:

In the last class you read:

- Principle of Inclusion Exclusion
- Demorgan’s Law
- Cartesian Product
- Relation on Sets
- Empty Relation
- Reflexive Relation
- Symmetric Relation
- Transitive Relation

Homework

What is the formula for the union of 4 sets, i.e., n(A U B U C U D)?

Symmetric Difference Explanation

Homework

$N \times Z = \{(a,b) | a \in N,\ b \in Z \wedge a=b\}$ So, does order matter?

It does, as the first element is a natural number and the second number is a integer.

Equivalence Relations

- A relation **R** on **A** is an equivalence relation if:

– Reflexive: for all element a in **A**, (a , a) belongs to **R**

– Symmetric: If a and b in **A** with (a , b) belongs to **R** ⇒ (b , a) belongs to **R**

– Transitive: If a, b and c in **A** with (a, b) and (b,c) belongs to **R** ⇒ (a,c) belongs to **R**

Alternatively; using a notation R or ~

– Reflexive: for all element a in A, a R a

– Symmetric: If a and b in A with a R b ⇒ b R a

– Transitive: If a, b and c in A with a R b and b R c ⇒ a R c

Equivalence Relation: Gender Relation

Let A be the set of all people.

We define the relation ~ by:

a ~ b if and only if a and b have the same gender.

Is it a equivalence relation?

Verification of Equivalence Relation:

Reflexive: Each person has the same gender as themselves. So, $a \sim a$ for any $a \in A$.

Symmetric: If $a \sim b$,meaning a and b have the same gender, then $b \sim a$ also holds because gender is mutual.

Transitive: If $a \sim b$ and $b \sim c$, meaning a and b have the same gender.

Equivalence Classes and Partitioning

An **equivalence class** of an element $a \in A$ is defined as:

$$[a] = \{x \in A : x \sim a\}$$

This means that the equivalence class $[a]$ contains all elements x in the set A that are related to a by the relation $\sim$.

Equivalence Classes: Gender

Equivalence Classes:

[Adam] = {Set of all males}

[Eve] = Set of all females.

Explanation:

- The equivalence class [Adam] := { set of all people Whose gender is male}
- The equivalence class [Eve] := { groups all people who are female}

The set A is partitioned into **disjoint equivalence classes**. This Means:

- Every element of A belongs to exactly one equivalence class.
- The equivalence classes are disjoint: if [a1]= [a2], then [a1] ∩ [a2] = ∅.
- The union of all equivalence classes covers the entire set A.

Quotient Set: A/ ~

The **quotient set** A/ ~ is the set of all equivalence classes under the relation ~. It is defined as:

$$A/\sim = \{[a1], [a2], [a3], \ldots\}$$

This means the quotient set consists of all distinct equivalence classes formed by the relation ~.

Properties of the Quotient Set:

1. The quotient set is a **set of sets**: it contains equivalence classes as its elements.
2. The quotient set provides a way to group elements of A based on the relation~.
3. The quotient set partitions the set A into disjoint subsets, with each subset corresponding to an equivalence class

Choosing a Class Representative

In each equivalence class, we need to pick

one representative element.

How do we choose a representative?

- From each equivalence class [a], we pick

one element $r \in [a]$ to represent that entire class.

- The representative is a single element that

symbolizes all the elements of the equivalence class.

- The set of all representatives forms a subset of A, where each representative represents a unique equivalence class.

Who is the Representative of the Class?

Let’s now apply this idea to our classroom.

Who is the representative of our classroom?

- Imagine that the equivalence classes are groups of students in the class with similar characteristics (e.g., students who have the same hobby, major, or study habits).
- Each group has one representative who represents all the students in that group.
- The representative is selected in a way that they reflect the interests of the entire group.

Thus, the representative is the element that **symbolizes** the entire equivalence class in our classroom setting.

Thank You!