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Set Theory

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CS01: Mathematics-I

Rapid fire

Is collection of beautiful cities a set?

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Rapid fire

Is collection of beautiful cities a set?

→ **No**

Subjective and varies from person to person

Rapid fire

Is collection of best movies a set?

Rapid fire

Is collection of best movies a set?

→ **No**

Subjective and varies from person to person

Rapid fire

Is collection of large numbers a set?

Rapid fire

Is collection of large numbers a set?

→ **No**

Subjective and varies from person to person

Set

- Set is a collection of well defined objects.
- Sets are usually denoted by capital letters A, B, C, X, Y, Z…
- The elements of a set are represented by small letters a, b, c, x, y, z…
- **Cardinality of a set :** Number of elements in a set.

Example:

- Collection of vowels a, e, i, o, u : A Set
- Collection of good movies : Not a Set

Example

Identify if these are sets or not.

a. Prime factors of 210, namely, 2,3,5 and 7

b. The collection of keywords of Python Language.

c. Most dangerous animals of the world.

d. The collection of good cricket players of India.

Rapid fire

"If we have the set of all prime numbers less than 30, what are the elements of this set?"

Rapid fire

"If we have the set of all prime numbers less than 30, what are the elements of this set?"

A = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 }

Representation of a Set

Roster Form

- Set V of all vowels in the English alphabet
  V = {a, e, i, o, u}

- Set O of odd positive integers less than 10
  O = {1, 3, 5, 7, 9}

- Set of positive integers less than 100
  {1, 2, 3, … , 99}

Set Builder Form

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- The general form of this notation is **$\{x \mid x$ has property $P\}$** and is read **"the set of all x such that x has property P."**
- $O = \{x \in Z \mid x$ is an odd positive integer less than $10\}$
- $O = \{x \in Z+ \mid x$ is odd and $x < 10\}$.
- $O = \{x \in Z \mid x = 2k + 1, k \in Z$ and $x < 10\}$

Notations

- $N = \{1, 2, 3, \dots\} \rightarrow$ Set of all natural numbers
- $Z = \{\dots, -2, -1, 0, 1, 2, \dots\} \rightarrow$ Set of all integers
- $Z+ = \{1, 2, 3, \dots\} \rightarrow$ Set of all positive integers
- $Q = \{p/q \mid p \in Z, q \in Z, \text{ and } q \neq 0\} \rightarrow$ Set of all rational numbers
- $R \rightarrow$ Set of all real numbers
- $R+ \rightarrow$ Set of all positive real numbers
- $C \rightarrow$ Set of all complex numbers.

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Match the following :

1. $\{1,-2\}$
2. $\{2,3,5,7\}$
3. $\{3,-3\}$
4. $\{-3,-2,-1,0,1\}$
5. $\{a,b,c,d\}$

A. {x: x is a prime number less than 10}

B. {x: x is an integer, greater than -4 and less than 2}

C. {x: x is an element of a,b,c,d}

D. {x: solution set of the equation $x^2 + x – 2 = 0$}

E. {x: x is an integer and $x^2 – 9 = 0$}

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E. {x: x is an integer and $x^2 - 9 = 0$}

Equal Sets

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- Two sets are equal if and only if they have the same elements.
- The sets **{1, 3, 5}** and **{3, 5, 1}** are equal, because they have the same elements.
- Note that the order in which the elements of a set are listed does not matter.
- It does not matter if an element of a set is listed more than once, so **{1, 3, 3, 3, 5, 5, 5, 5}** is the same as the set **{1, 3, 5}** because they have the same elements.

Empty Set

- A set that has no elements is called the **empty set**, or **null set**, and is denoted by ∅ **or { }**.
- The set of all positive integers that are greater than their squares is the null set.
- {0} is not a null set, since it contains 0 as an element.
- {∅} is not a null set, since it contains an empty set as an element.

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Example:

Which of the following are null sets:

I. A = {x | x is a positive integer less than 1}

II. B = $\{x \in N : x = 3n,\ n \in N\}$

III. C = $\left\{x \in Z : x^2 + 5x + 6 = 0\right\}$

IV. D = {x | x is an even prime number greater than 2}

Example:

Which of the following are null sets:

I. A = {x | x is a positive integer less than 1}

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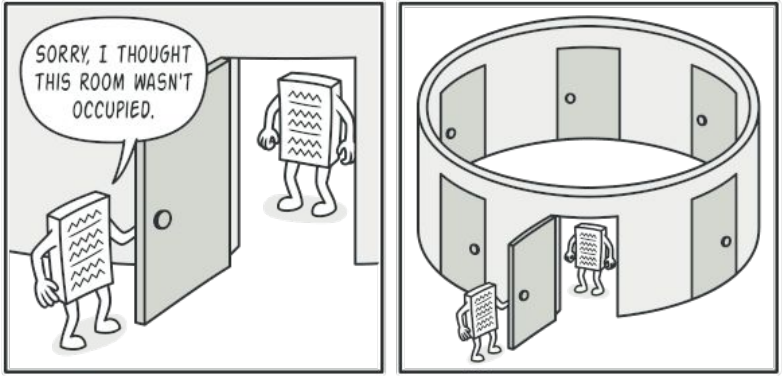
IV. D = {x | x is an even prime number greater than 2}

Singleton Set

- A set with one element is called a singleton set.
- **{∅}** is a singleton set, not an empty set. The single element of the set {∅} is the empty set itself!
- The empty set '∅' can be thought of as an empty folder in a computer file system and the set consisting of just the empty set '**{∅}**' can be thought of as a folder with exactly one folder inside, namely, the empty folder.



Finite Set

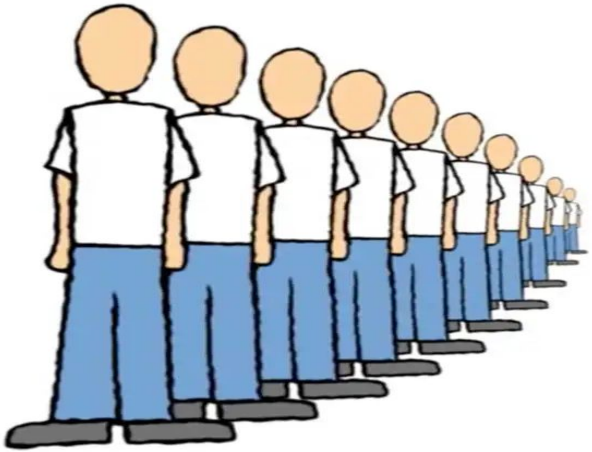
- A set whose elements can be listed or counted.

- **Example:** Days in a week.

- Set Representation:
  D = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

Infinite Set

A set which consist of infinite number of elements.



Example

Which of the following is/are infinite sets?

a) Set of natural numbers
b) Set of points on a line
c) Set of real numbers between 1 and 2
d) All of the above

Example

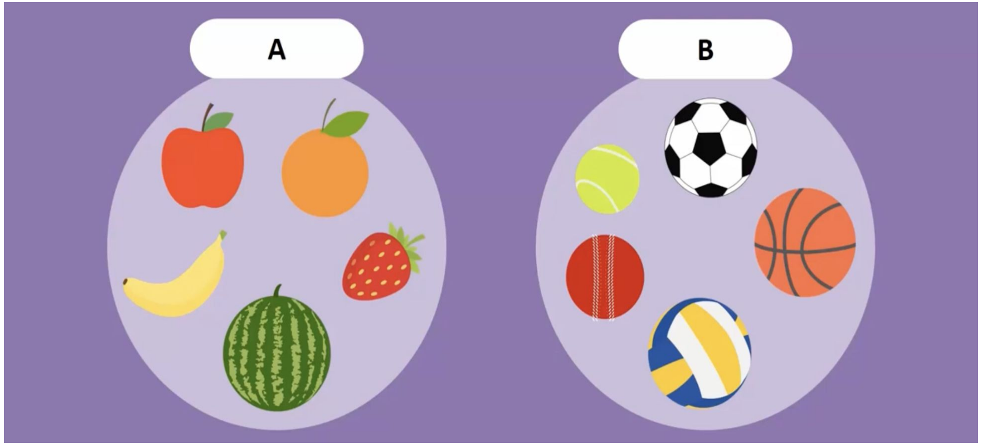
Which of the following is/are infinite sets?

a) Set of natural numbers
b) Set of points on a line
c) Set of real numbers between 1 and 2
d) All of the above

Equivalent Sets

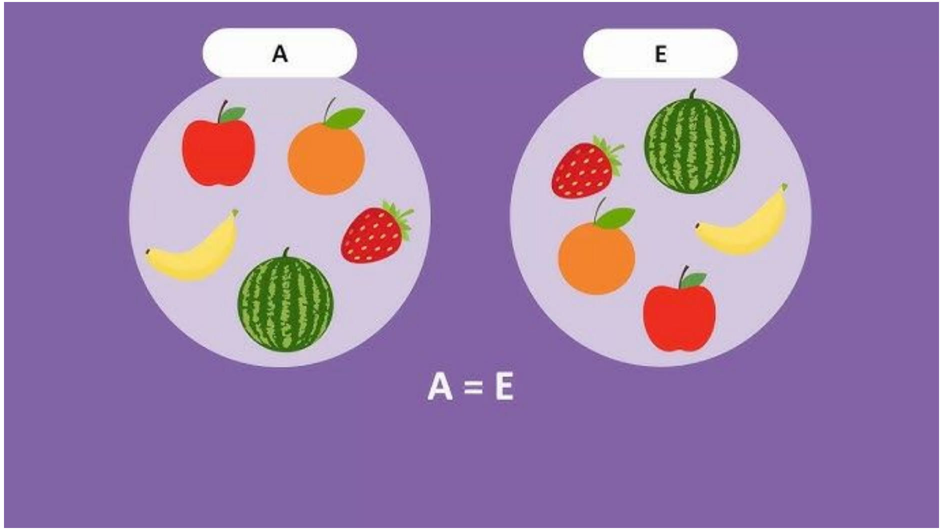
Two finite sets A and B are equivalent if their cardinality is same. i.e. n(A) = n(B).

Example: {5, 10, 15, 20, 25}, & {a, e, i, o, u} are equivalent.



Equal Sets

Two sets A and B are said to be equal if every element of A is a member of B and every element of B is a member of A.



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“If two sets are equal, they must also be equivalent.”

True or False?

Rapid fire

“If two sets are equal, they must also be equivalent.”

→ **True**

Rapid fire

“If two sets are equivalent, they must also be equal.”

True or False?

Rapid fire

“If two sets are equivalent, they must also be equal.”

→ **False**

Subsets and Power Set

Subsets

- The set A is a subset of B, if and only if every element of A is also an element of B.
- We use the notation $A \subseteq B$ to indicate that A is a subset of the set B.
- **Showing that A is a Subset of B →**
  - To show that $A \subseteq B$, show that if x belongs to A, then x also belongs to B.
- **Showing that A is Not a Subset of B →**
  - To show that $A \nsubseteq B$, find a single $x \in A$ such that $x \notin B$.

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For every set S,

(i) $\varnothing \subseteq S \rightarrow$ True or False

(ii) $S \subseteq S \rightarrow$ True or False

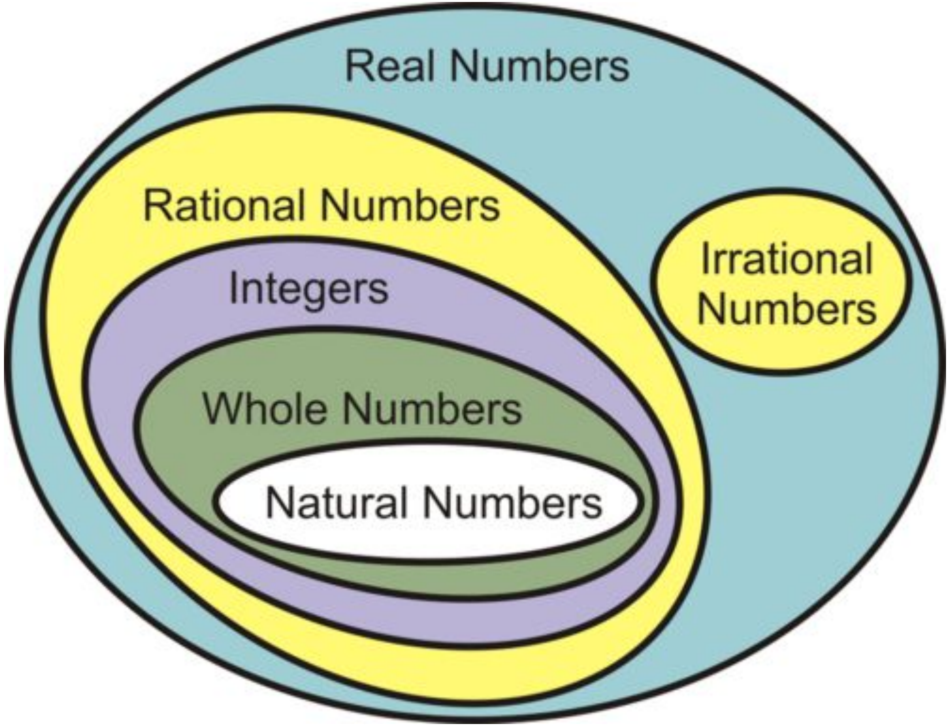
Rapid fire

For every set S,

(i) $\varnothing \subseteq S \rightarrow$ **True**

(ii) $S \subseteq S \rightarrow$ **True**

Subsets



N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

Z+ : the set of positive integers

Q+ : the set of positive rational numbers

R+ : the set of positive real number

Example

Q. Find number of possible subsets of a set A= {1,2,3}

Example

Q. Find number of possible subsets of a set A= {1,2,3}

Ans. 8

1. $\emptyset$ (empty set)
2. $\{1\}$
3. $\{2\}$
4. $\{3\}$
5. $\{1, 2\}$
6. $\{1, 3\}$
7. $\{2, 3\}$
8. $\{1, 2, 3\}$

Proper Subset

- A set A is a **proper subset** of set B if:
  - Every element of A is in B
  - A is not equal to B
- **Notation:** $A \subset B$ → A is proper subset of B → A is strictly contained in B.

Power Set

- The **power set** of a given set S is the set of all possible subsets of S, including the empty set and set S itself.
- **Notation**: The power set of set S is denoted as **P(S)**.
- **Number of elements in a power set = 2^n**

  where, **n** is the number of elements in a set.
- **Example 1:** Let S = {1, 2} → |S| = 2

  Then, P(S) = {{}, {1}, {2}, {1,2}} → |P(S)| = 2^2 = 4
- **Example 2:** Let S = {1, 2, 3} → |S| = 3

  Then, P(S) = {{}, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}} → |P(S)| = 2^3 = 8

Example

What is the power set of the set $A = \{1, \{2\}\}$?

Example

What is the power set of the set A = {1, {2}}?

P(A)= {{}, {1}, {{2}}, {1,{2}}}

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Is null set a subset of every set?

Rapid fire

Is null set a subset of every set?

→ **Yes**

Rapid fire

Can number of elements in a set and its power set be equal?

Rapid fire

Can number of elements in a set and its power set be equal?

→ **No**

Set Operations

Example

Imagine you are working in a small **e-commerce** company called Myntra**,** that sells various products online. You have two sets of customers :

- Customers who **bought electronics item** :
  - Set A= {Rohan, Manu, Aakash, Riya, Aditya}
- Customers who have **bought clothing items :**
  - Set B= {Aakash, Riya, Pankaj, Rahul, Ishaan}

Question : Find the users who have bought both items from your website.

Set Intersection

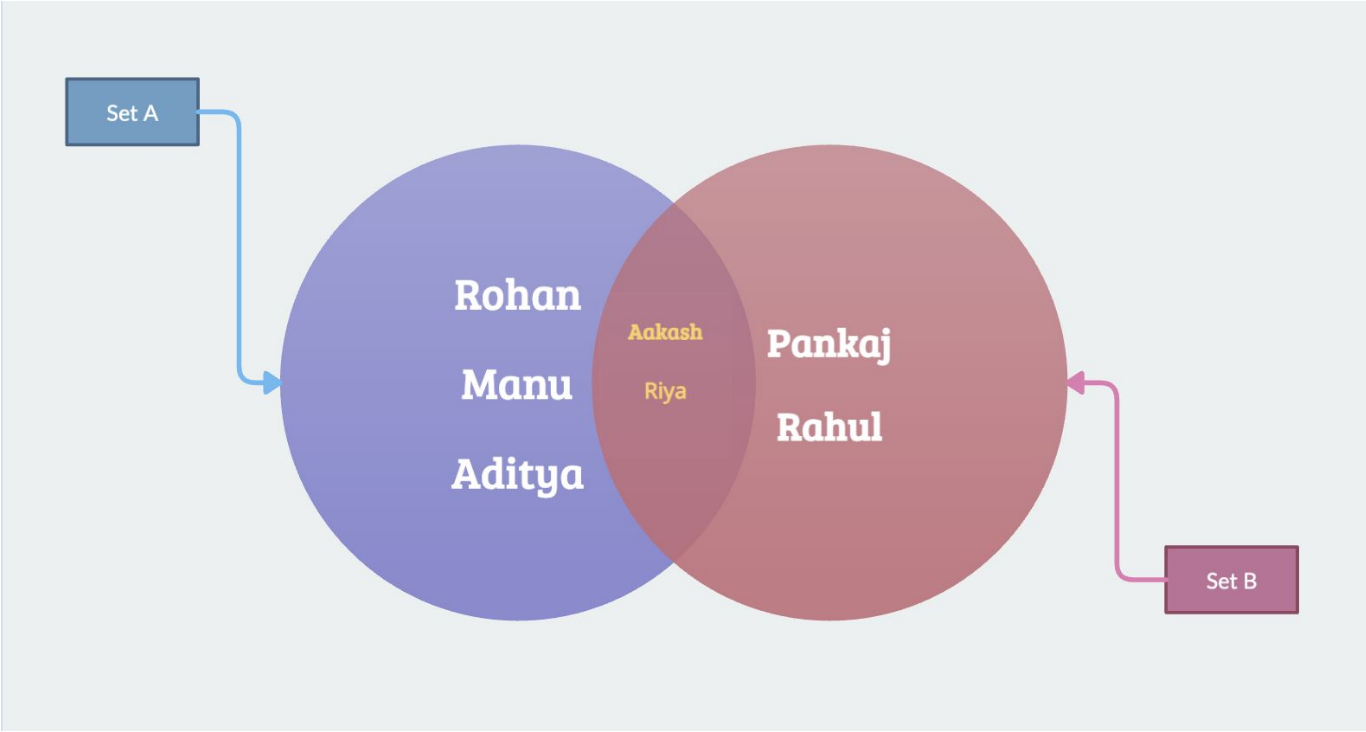
Set A= {Rohan, Manu, **Aakash**, **Riya**, Aditya}

Set B= {**Aakash**, **Riya**, Pankaj, Rahul}

$A \cap B = \{x : x \in A \wedge x \in B\}$

A∩B= { Aakash, Riya}

Venn Diagram Representation



Example:

Imagine you are working in a small **e-commerce** company called Myntra**,** that sells various products online. You have three sets of customers :

- Customers who **bought electronics item** :
  - Set A={Rohan, Manu, Aakash, Riya, Aditya}
- Customers who have **bought clothing items :**
  - Set B={Aakash, Riya, Pankaj, Rahul, Ishaan}
- Customers who have bought **snacks items:**
  - Set C= {Monika, Riya, Aditya, Aryan }

Questions :

A. Find the set of customers who are **most loyal** to you (have bought all different items from you)

B. Set of customers who are purchasing **at least two different items** from you.

Example:

Ans:

A. {Riya}
B. {Aaditya, Riya, Aakash}

Example:

Imagine you are working in a small **e-commerce** company called Myntra, that sells various products online. You have three sets of customers :

- Customers who **bought electronics item** :
    - Set A={Rohan, Manu, Aakash, Riya, Aditya}
- Customers who have **bought clothing items :**
    - Set B={Aakash, Riya, Pankaj, Rahul, Ishaan}
- Customers who have bought **snacks items:**
    - Set C= {Monika, Riya, Aditya, Aryan }

Questions :

A. Find the set of all the customers who have bought any product on your website?

Example:

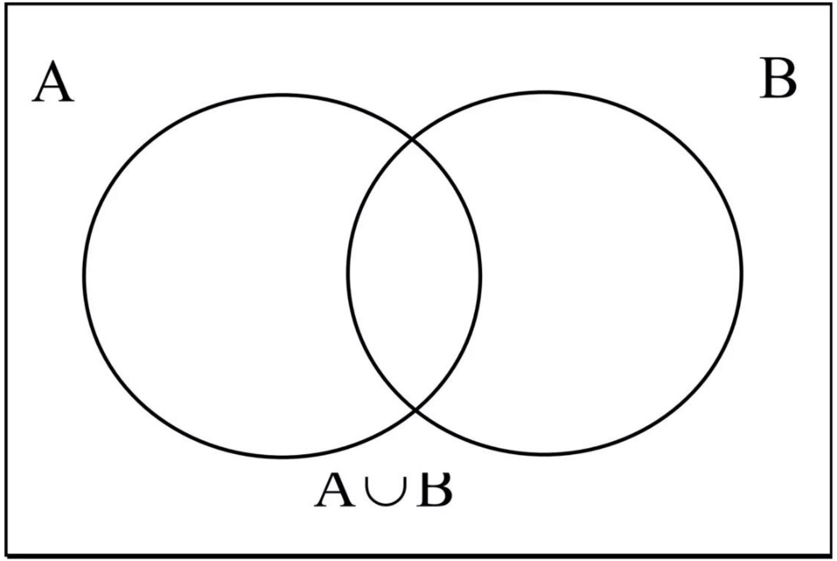
Ans:

A. {Rohan, Manu, Aakash, Riya, Aditya, Pankaj, Rahul, Ishaan, Monika, Aryan}

Set Union

$$A \cup B$$

$A \cup B = \{x : x \in A \lor x \in B\}$



Example:

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  - Set B={Aakash, Riya, Pankaj, Rahul, Ishaan}
- Customers who have bought **snacks items:**
  - Set C= {Monika, Riya, Aditya, Aryan }

Questions :

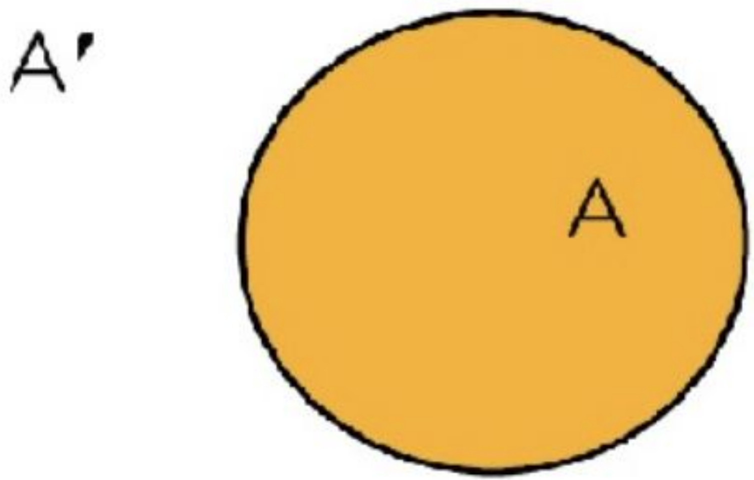
A. Find the set of all customers who have not bought electronics item from you?

Example:

Ans:

A. A’= {Pankaj, Rahul, Ishaan, Monika, Aryan}

Set Complement



Example:

Imagine you are working in a small **e-commerce** company called Myntra, that sells various products online. You have three sets of customers :

- Customers who **bought electronics item** :
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- Customers who have **bought clothing items :**
  - Set B={Aakash, Riya, Pankaj, Rahul, Ishaan}

Questions :

A. Find the set of all customers who have bought electronics item but not the clothing items from your website ?

B. Find all the customers who have bought clothing items but not the electronics items from your website?

Example:

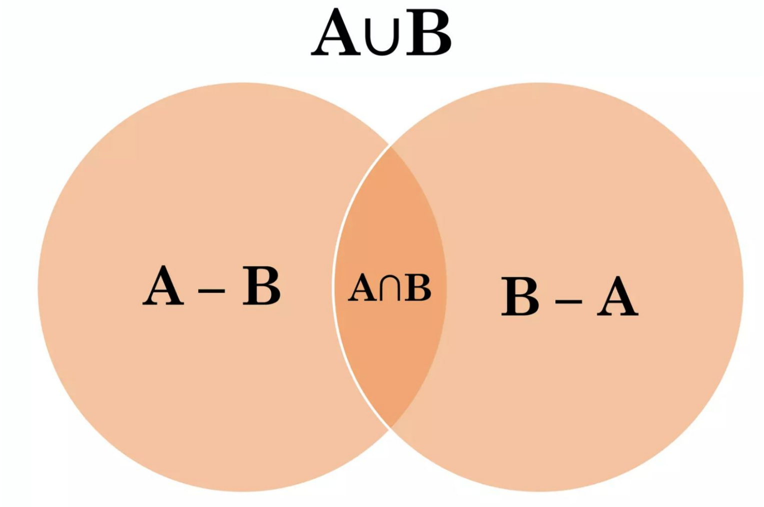
Ans:

A. A-B= {Rohan, Manu, Aaditya}
B. B-A= {Pankaj , Rahul, Ishaan}

Set Difference

- **Note:** The backslash notation '\' in set theory means set difference.

$$A - B = A \setminus B = \{x \in A \mid x \notin B\}$$



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Is A-B equal to B-A for two sets A and B?

Rapid fire

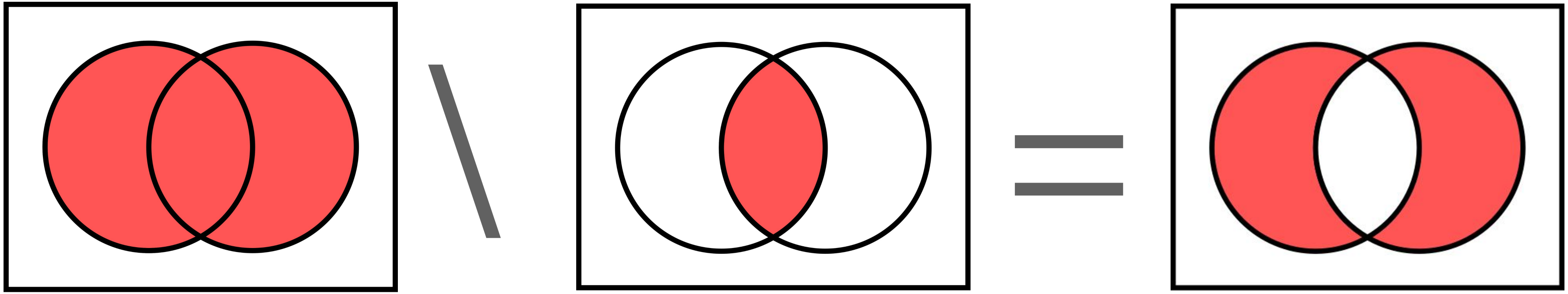
Is A-B equal to B-A for two sets A and B?

No

Symmetric Difference

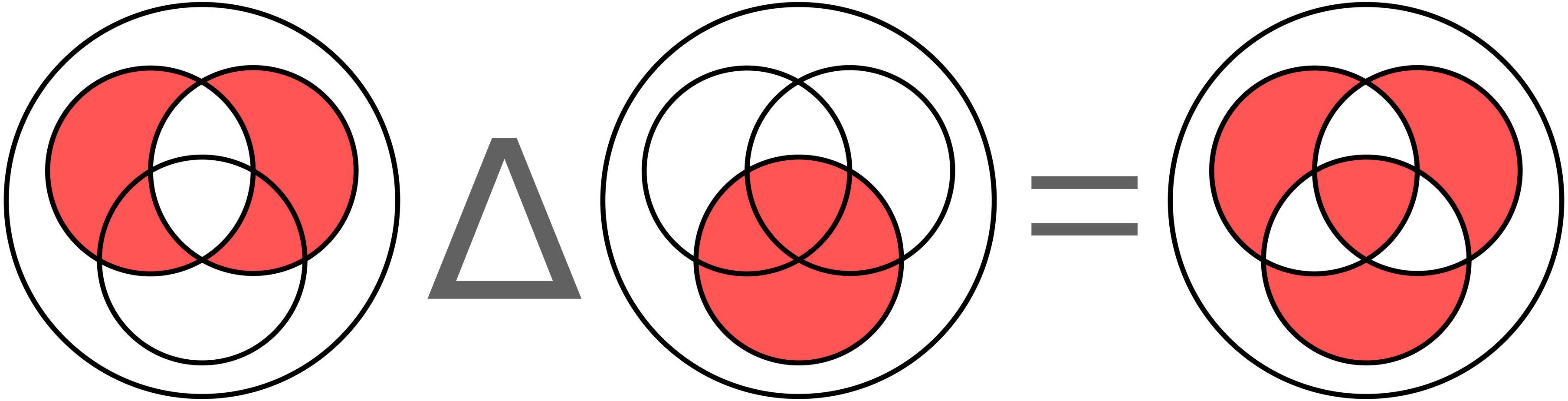
- The symmetric difference is the set of elements that are in either set, but not in the intersection.
- **Symmetric Difference of 2 sets:** (A Δ B)
- Symmetric difference is commutative. → A Δ B = B Δ A

$$A \Delta B = (A \setminus B) \cup (B \setminus A) \qquad A \Delta B = (A \cup B) \setminus (A \cap B)$$



Symmetric Difference

- **Symmetric Difference of 3 sets:** ((A Δ B) Δ C)
- Symmetric difference is associative. → (A Δ B) Δ C = A Δ (B Δ C)



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What is $A \Delta \varnothing$?

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What is $A \Delta \emptyset$?

Answer:

$$A \Delta \emptyset = A$$

Rapid fire

What is $A \Delta A$?

Rapid fire

What is A Δ A ?

Answer:

$$A \Delta A = \varnothing$$

Disjoint Sets (Mutually Exclusive Sets)

Disjoint sets, also known as mutually exclusive sets, are sets that have no elements in common. In other words, if two sets A and B are disjoint, their intersection is empty.

$$A \cap B = \emptyset$$

Disjoint Sets (Mutually Exclusive Sets)

Examples:

1. **Example 1**: Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. Here, A and B are disjoint since they have no elements in common.
2. **Example 2**: Let $C = \{x, y\}$ and $D = \{y, z\}$. Here, C and D are not disjoint because they share the element y.