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Basic Arithmetic Foundations

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CS01: Mathematics-I

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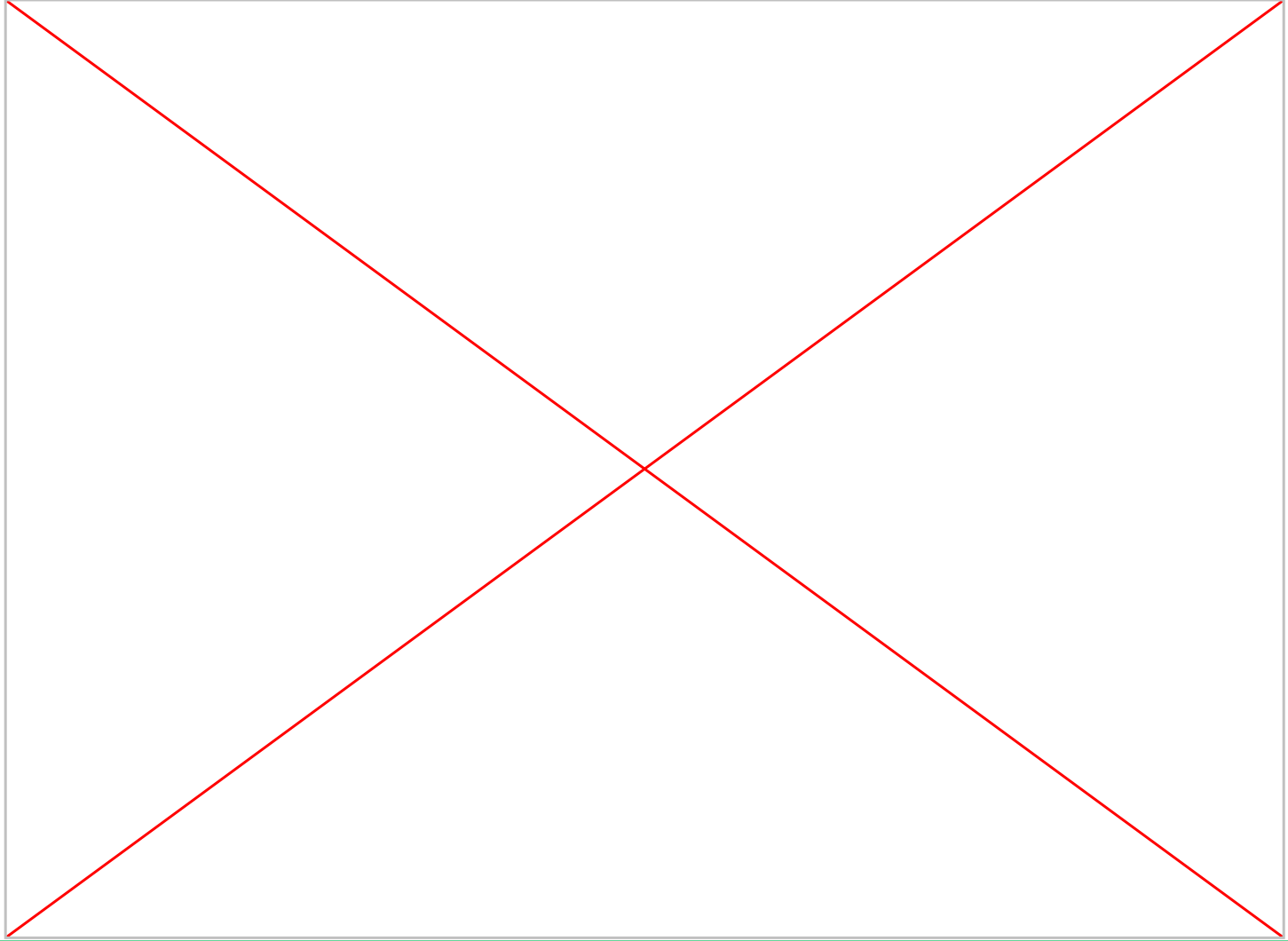
Join the class

Example

1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9

What is the sum of all of the numbers in this square?



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Strategic Arithmetics

Arithmetics:

- Deals with the basic operations of numbers, such as addition, subtraction, multiplication, and division.

Strategic Arithmetics:

- Doing arithmetic operations more fast and with little calculation on
  - Numbers using rearrangement and regrouping
  - Fractions understanding their real life analogy

Example

Which is larger?

$$27 + 28 + 29 + 30 + 31$$

or

$$29 \times 5$$

Example

Which is larger?

$$27 + 28 + 29 + 30 + 31$$

or

$$29 \times 5$$

Ans. Both are equal.

The Hack

- Whenever you have-
  - **Sum** of values like: $27 + 28 + 29 + 30 + 31$
  - **Product** of values like: $199 * 39$

- Always be ready to rearrange and regroup them to find some pattern to solve it fast and easy

$$27 + 28 + 29 + 30 + 31 = (29 - 2) + (29 - 1) + 29 + (29 + 1) + (29 + 2)$$

$$199 * 39 = (200 - 1) * (40 - 1) = 8000 - 200 - 40 + 1 = 7761$$

Example

- What's the value of this expression?

$$\frac{10 + 11 + 12 + 13 + 14 + 15}{25}$$

Example

4	5	6	1	2
2	3	5	6	1
1	2	4	5	6
6	1	2	3	5
5	6	1	2	4

What is the sum of all of the numbers in this square?

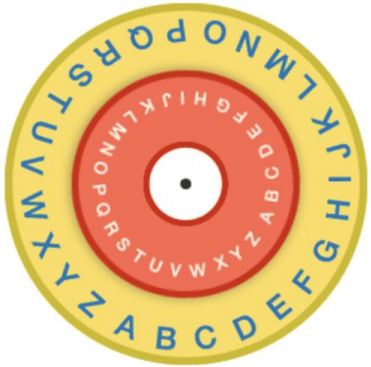
Example

4	5	6	1	2
2	3	5	6	1
1	2	4	5	6
6	1	2	3	5
5	6	1	2	4

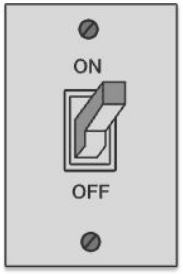
What is the sum of all of the numbers in this square?

Ans: 88

Number Theory



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



A cycle of 2 values

A cycle of 3 values

A cycle of 7 values

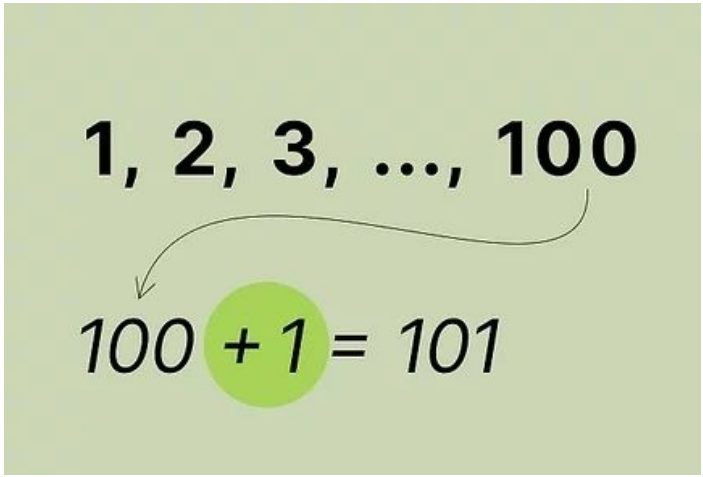
A cycle of 12 values

Natural Numbers

- Natural numbers are the numbers that we generally use for counting, or enumerating items.
- **Natural Numbers:** 1, 2, 3, 4, 5, 6, 7, …
- The set of Natural numbers is denoted by $\mathbb{N}$.

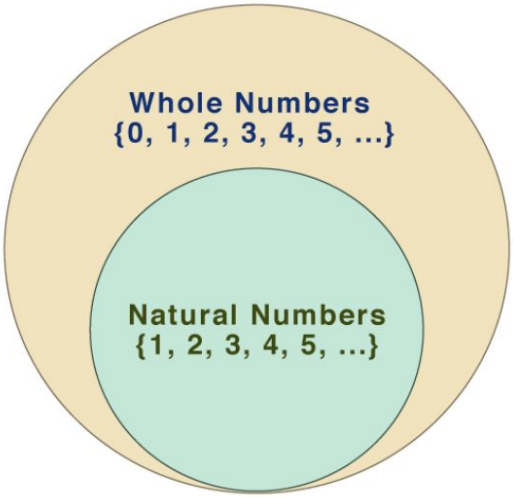
Sum of n natural numbers

$$S = \frac{n(n+1)}{2}$$



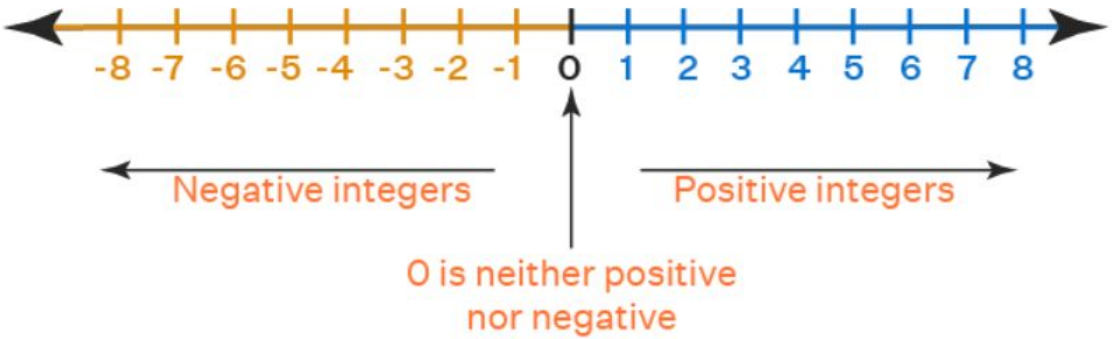
Whole Numbers

- Whole numbers are all the non-negative integers. This means the set includes 0 and all positive counting numbers.
- **Whole Numbers:** 0, 1, 2, 3, 4, 5, 6, 7, …
- The set of Whole numbers is denoted by **W**.



Integers

- The set of integers adds the opposites of the natural numbers to the set of whole numbers: {...,−3,−2,−1, 0, 1, 2, 3,...}.
- 0 is neither positive nor negative.
- The set of Integers is denoted by $\mathbb{Z}$ (from German "Zahlen").



Rapid Fire

- Even + Even = ?
- Even x Even = ?
- Even / Even = ?

Rapid Fire

- Even + Even = Even
- Even x Even = Even
- Even / Even can be Even or Odd or Not an Integer

Rapid Fire

- Odd + Odd = ?
- Odd x Odd = ?
- Odd / Odd = ?

Rapid Fire

- Odd + Odd = Even
- Odd x Odd = Odd
- Odd / Odd can be Odd or a non Integer

Rapid Fire

- Even + Odd = ?
- Even x Odd = ?
- Even / Odd = ?
- Odd / Even = ?

Rapid Fire

- Even + Odd = Odd
- Even x Odd = Even
- Even / Odd can be Even or Odd or Not an Integer
- Odd / Even is usually not an Integer

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Strategic Arithmetics on fractions

Rational Numbers (Fractions)

- A rational number (fraction) is any number that can be written in the form

$$\frac{p}{q}$$

where **p, q** $\in \mathbb{Z}$

q ≠ 0

- 1 slice out of 8 slices of Pizza → ⅛ fraction

Example

Which number is bigger?

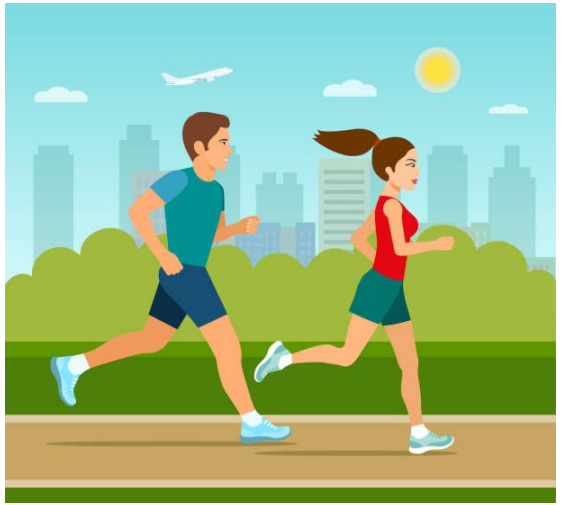
$$\frac{50}{100} \quad \text{or} \quad \frac{51}{101}$$

Example: The Big Race

Kartik, Abhi, Kritika and Manasa are getting ready for the university race.

Example: The Big Race

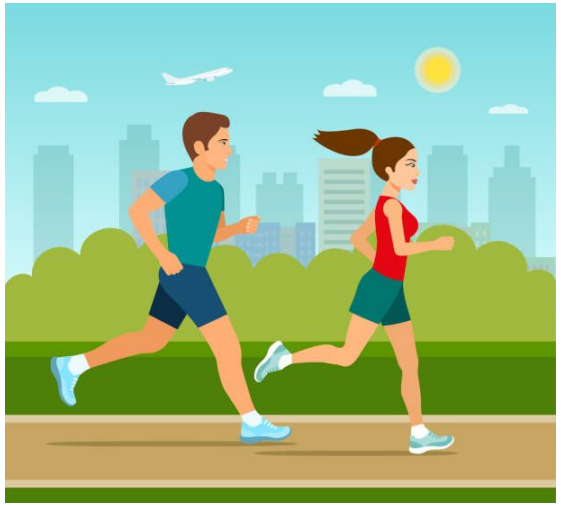
While **Abhi** and **Manasa** have been training on their own, **Kartik** and **Kritika** have been practicing for months against each other before the **big race**. So far they have had **100** practice races and **Kartik** and **Kritika** have won half the races each. What fraction of races has **Kritika** won so far?



Example: The Big Race

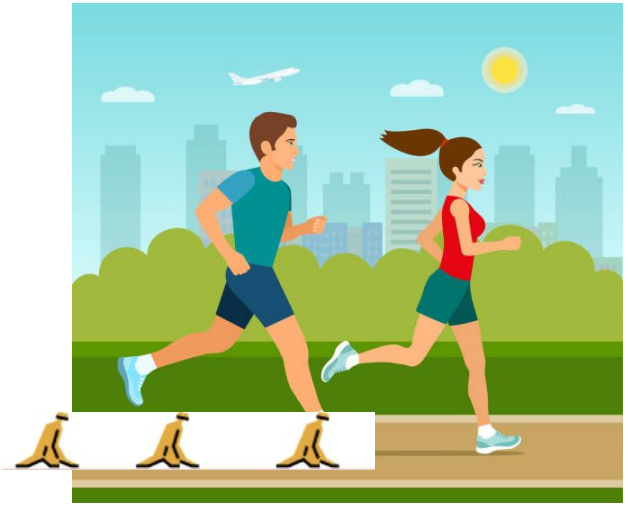
While **Abhi** and **Manasa** have been training on their own, **Kartik** and **Kritika** have been practicing for months against each other before the **big race**. So far they have had **100** practice races and **Kartik** and **Kritika** have won half the races each. What fraction of races has **Kritika** won so far?

$$\frac{50}{100}$$



Example: The Big Race

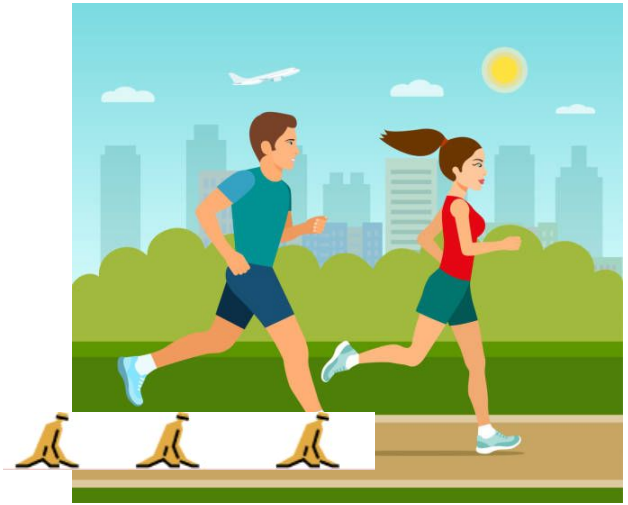
Kritika is getting frustrated with being neck to neck with **Kartik**. She decides to use a **secret weapon** and then easily win their next practice match. What fraction of races has she now won?



Example: The Big Race

Kritika is getting frustrated with being neck to neck with **Kartik**. She decides to use a **secret weapon** and then easily win their next practice match. What fraction of races has she now won?

$$\frac{51}{101}$$



Example: The Big Race

Who is now in the lead?

Kartik or Kritika

Example: The Big Race

Who is now in the lead?

Ans. Kritika

Attention

Earlier matches won by Kritika:

$$\frac{50}{100}$$

Result: **Not leading**

Matches won by Kritika now:

$$\frac{51}{101}$$

Result: **Leading**

Which number should be bigger?

$$\frac{50}{100} \quad \text{or} \quad \frac{51}{101}$$

Example: The Big Race

The problem we gave you earlier was which of these two fractions is bigger:

$$\frac{50}{100} \quad \text{or} \quad \frac{51}{101}$$

Can you see a connection between that problem and Kartik and Kritika's practice matches?

Example

Is this statement true or false?

$$\frac{50}{100} < \frac{51}{101} < \frac{52}{102} < \frac{53}{103} < \frac{54}{104}$$

Comparing Fractions using Cross Multiplication

$$\frac{a}{b} \quad \text{and} \quad \frac{c}{d}$$

🔄 Step 1: Cross Multiply

- Multiply **diagonally**:
  - Left cross: $a \times d$
  - Right cross: $b \times c$

🔍 Step 2: Compare the Products

- If $a \times d > b \times c$, then: $\frac{a}{b} > \frac{c}{d}$
- If $a \times d < b \times c$, then: $\frac{a}{b} < \frac{c}{d}$
- If $a \times d = b \times c$, then: $\frac{a}{b} = \frac{c}{d}$

Comparing Fractions using Cross Multiplication

Compare $\frac{3}{7}$ and $\frac{4}{9}$

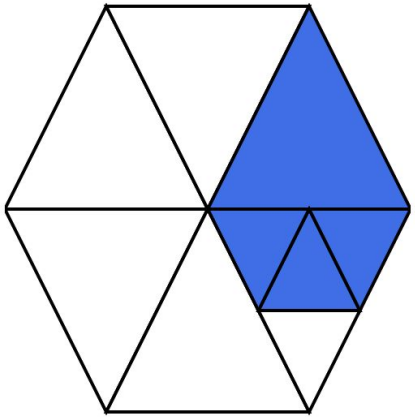
- $3 \times 9 = 27$
- $4 \times 7 = 28$
- $27 < 28 \rightarrow$ So, $\frac{3}{7} < \frac{4}{9}$

- This method avoids converting to decimals or finding LCMs.
- It works because you're effectively comparing both fractions after multiplying by the same denominator. 💡

Does the cross multiplication method for comparing fractions work for negative fractions too?🤔

Example

What fraction of the shape is shaded?



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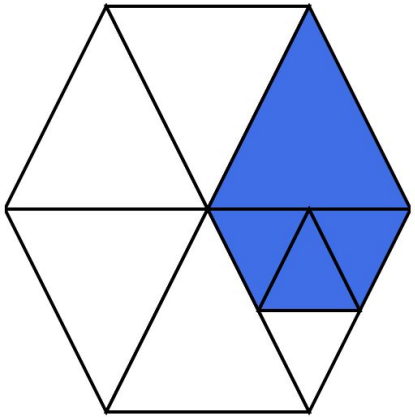
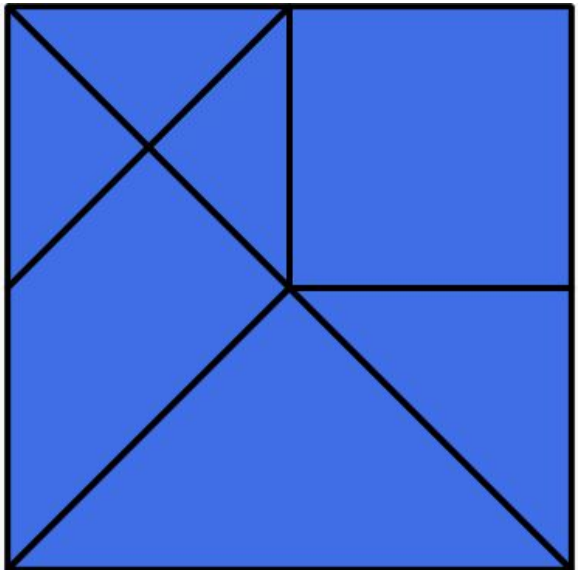


Ans. 7/24

Example

Remove color from parts of the square so that $3/4$ of it remains shaded.



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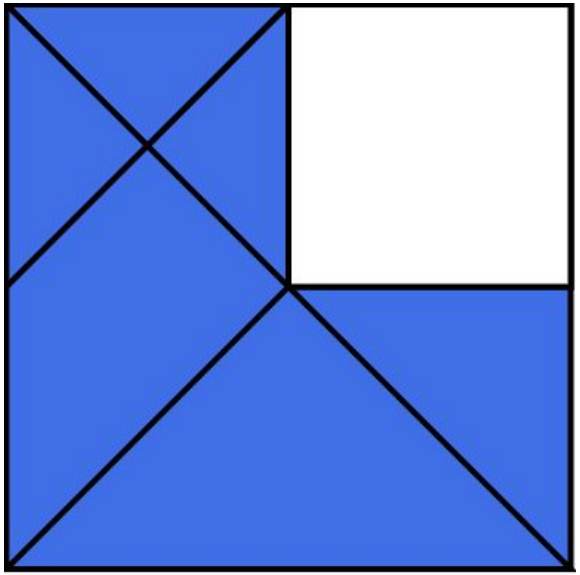
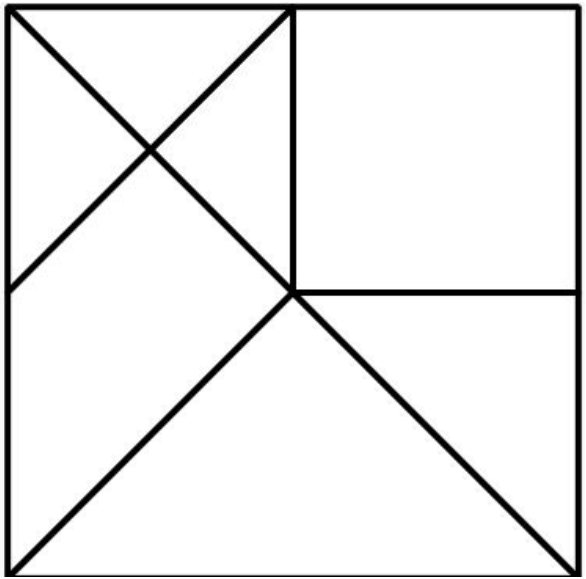


Example

Shade ⅜ of the square.



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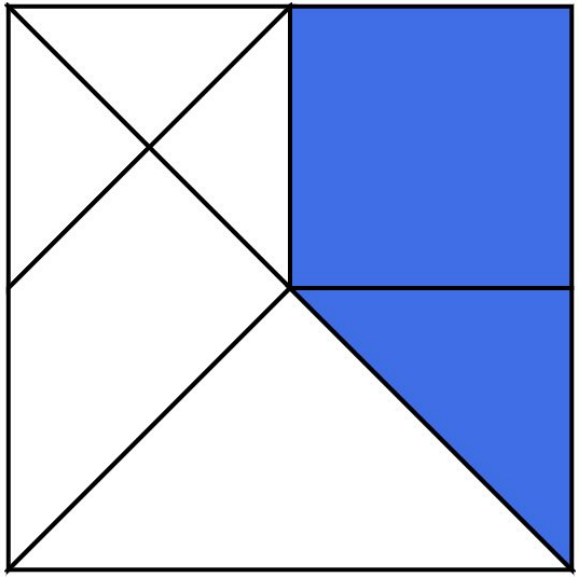

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Properties of Number System

Addition on Natural Numbers satisfies

- **Commutative Property**
  $a+b=b+a$

- **Associative Property**
  $(a+b)+c=a+(b+c)$

Addition on Integers satisfies

- **Commutative Property**
  $a+b=b+a$

- **Associative Property**
  $(a+b)+c=a+(b+c)$

- **Additive Identity**
  $a+0=a$

- **Additive Inverse**
  $a+(-a)=0$

Addition on Rational Numbers satisfies

- **Commutative Property**
  $a+b=b+a$

- **Associative Property**
  $(a+b)+c=a+(b+c)$

- **Additive Identity**
  $a+0=a$

- **Additive Inverse**
  $a+(-a)=0$

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Multiplication on Natural Numbers satisfies

- **Commutative Property**
  a x b = b x a

- **Associative Property**
  (a x b) x c = a x (b x c)

- **Multiplicative Identity**
  a x 1 = a

Multiplication on Integers satisfies

- **Commutative Property**
  a x b = b x a

- **Associative Property**
  (a x b) x c = a x (b x c)

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Multiplication on Rational Numbers satisfies

- **Commutative Property**
  a x b = b x a

- **Associative Property**
  (a x b) x c = a x (b x c)

- **Multiplicative Identity**
  a x 1 = a

- **Multiplicative Inverse (Division)**
  a x b = 1 where b = 1/a which is a rational number

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Quiz Quiz Quiz

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See You Guys in Next Session :)