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# **Computation of Clustered Argumentation Frameworks via Boolean Satisfiability**

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# Abstract

English abstract of your thesis



# Kurzfassung

Deutsche Kurzfassung der Abschlussarbeit



# Acknowledgements

Thanks to everyone who made this thesis possible





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# List of Acronyms and Symbols

<b>AF</b>	Argumentation Framework
<b>AI</b>	Artificial Intelligence
<b>ASP</b>	Answer Set Programming
<b>cf</b>	Conflict-Free
<b>adm</b>	Admissible
<b>stb</b>	Stable
<b>BFS</b>	Breadth First Search
<b>DFS</b>	Depth First Search





# 1 Introduction

We all encounter arguments in our lives frequently. When talking to friends, listening to political discussions, or even making decisions in our head. These arguments can get heated and complex since humans have different beliefs and motivations. Finding a common ground or a "correct" conclusion is complicated and sometimes impossible. However, these imperfections are what make us humans. Artificial Intelligence (AI), conversely, needs to act precisely and logically [7]. That is why much research is being done on knowledge representation and reasoning [6, 15].

When observing arguments objectively, we can distinguish between facts and conclusions. A fact represents a specific state in the real world. A conclusion on the other hand is a fact claimed by the logical relation of the promises. The relations are opposing facts (f.e. *the square  $x$  is red* and *the square  $x$  is blue*), which are contradicting each other. While accepting the correctness of facts is very important, refuting facts is even more critical in an argument.

If a fact or, i.e., an argument  $a$  is a counterargument of another argument  $b$ , we can say that  $a$  attacks  $b$ . With this generalization, we can abstract our model with directed graphs. The arguments are represented as nodes, and the attacks as directed edges [9]. With this abstraction, we can define AFs and use them to evaluate conclusions [11]. Most of the time, we do not operate on real-world cases, but on abstract examples. This means, that we do not care about the argument which is represented by a specific node. But drawing a conclusion from an AF can be challenging and tears down to the definition of semantics.

A semantic defines a subset of argument sets that satisfy the semantic-specific rules. Dung already defined different semantics [8] like Conflict-Free (cf), Admissible (adm) and Stable (stb). According to Dungs definitions, a set  $S$  is cf if there are no attacks between the arguments in  $S$ . The conflict-free set is mainly a building block for the other semantics, which means that the conflict-free set is always a superset of admissible and stable. A stable set, is a conflict-free set, if for every argument, which is not in  $S$ , has an attacker which is in  $S$ . Finally, an admissible set is a conflict-free set, where each argument in  $S$  has a defender in  $S$ . The specific rules can be defined via a boolean formula. They can be used to encode the AFs to be solvable with different boolean solvers like Answer Set Programming (ASP) [3] or, as in our case, with a Boolean Satisfiability Solver (SAT-Solver) [1]. Unfortunately, drawing a conclusion from an AF can be challenging, e.g., it can be NP-complete and sometimes even be beyond NP to decide whether an argument is acceptable under a specific argumentation semantics [10]. In fact, the complexity of proofing faithfulness or spuriousness of an AF is  $\Pi_2^P$  [16]. This means, that to obtain a result, multiple instances or calls of a SAT-Solver need to be invoked.

One of the first papers describing the concept was written by Dung [8] in 1995. Since then, there has been more and more interest in AFs due to the artificial intelligence community [5]. The argumentation systems and semantics have been modified and improved over the years, and another abstraction layer has been added. This specific abstraction layer is called *clustering* and generalizes multiple arguments into one bundled cluster [16]. Clustering is a technique to reduce the number of arguments without changing the conclusion, which in this instance would be the sets produced by a specific semantic. When producing a clustered (*abstract*) AF, which produces the same semantic sets as the non-clustered (*concrete*) AF, the abstract AF is defined to be *faithful*. While each concrete semantic set has a directly mapped abstract semantic set, not every abstract semantic set has to have a directly mapped concrete semantic set. If we create an abstract AF that produces a semantic set that cannot be mapped to a concrete semantic set, we call it *spurious*.

When reducing the amount of arguments with clustering we have to pay attention to not abstract crucial facts, and thus, falsify accepted sets from the concrete AF or accept refuted ones. This would lead to a spurious abstraction and the abstract AF would not represent the concrete AF anymore. To preserve the representation of the concrete AF, we need to show faithfulness.

For instance, let us consider a real-world example like the weather. We can define arguments:

- *a*: The sky is blue
- *b*: The atmosphere scatters the sunlights and makes the sky appear blue.
- *c*: There exist photographs of a blue sky.
- *d*: Photographs can be fake.
- *e*: At sunrise the sky appears to be orange.
- *f*: Observations can alter, depending on the time.

With this knowledge basis, we can create a concrete AF  $G = (A, R)$ . Where we abstract the arguments into nodes and transform the opposing statement into attacks as shown in 1.1. If we apply another layer of abstraction, we obtain, f.e. the abstract AF  $G' = (A', R')$  defined in 1.2. Here we created a single Cluster consisting of the singletons  $\{a, b, c, e, f\}$ .

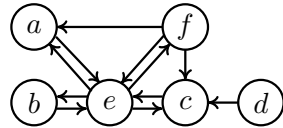


Figure 1.1: AF G concrete

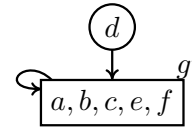


Figure 1.2: AF G' abstract

Now we can compute the sets of the according semantic (cf, adm, stb). To reduce cluttering, we keep this example to the stable semantic. The stable sets of the AF  $G$  defined in 1.1 are  $\text{stb} = [\{d, e\}, \{b, d, f\}]$ .

By computing the stable semantic sets of the abstract AF  $G'$   $\text{stb} = [\{d\}, \{d, g'\}]$ , we can observe that it is spurious due to the extension  $\{d\}$ , since it cannot be mapped to one of the concrete stable extensions.

When concretizing the argument  $\{c\}$ , we create a new AF  $G'' = (A'', R'')$  1.3, which has the following stable extensions:  $\text{stb} = [\{g', d\}]$ . This Extension can be mapped to both stable extensions of the concrete AF  $G$ , by expanding the cluster  $g'$  with  $\{e\}$  or  $\{b, f\}$ . Thus, we created a faithful abstract AF.

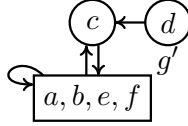


Figure 1.3: AF  $G''$

When producing an AF with multiple layer of abstractions, the concrete problem can be hard to map. To still have an understanding of the structure to some extend, extracting single arguments of the cluster by concretizing them can be helpful. This also allows the user to have a direct impact to the outcome and produce customized faithful AFs.

Creating abstract, faithful AFs can be challenging and is the main focus of this paper. We created one of the first tools [4] to produce an abstract AFs based on a concrete AFs. We cover different setups and usages, including different semantics and base functionalities:

- Generate semantic sets of a concrete- or abstract AF. The sets calculated iteratively or all at once. The covered semantics are cf, adm, and stb, which can be selected throughout the project independently by a parameter in the command line.
- Determine faithfulness or spuriousness by providing two AFs. We provide two approaches, Breadth First Search (BFS) and Depth First Search (DFS), which alter the procedure. While BFS calculates all the semantic sets of the two AFs first and then compares them, DFS calculates iteratively a semantic set of the abstract AF and then verifies it with the concrete AF. The algorithm selection is done via a command line parameter.
- Concretize a set of arguments (i.e., pull out arguments from the cluster) given the concrete AF and an abstract AF (faithful or spurious), and provide faithfulness (by concretizing other arguments not specified in the concretize list as well). The user provides the concretized arguments via a command line parameter.
- ...

*TODO: Further contributions*

*TODO: give pointers to why are non-trivial to obtain*

*TODO: Choice of methods to obtain results*

*TODO: How big AFs are still feasible to solve*

## 2 Background

### 2.1 Argumentation Frameworks

Argumentation frameworks were first formally described by Dung in 1995 [9]. They represent an information state, where various conclusions can be drawn from. An AF  $G = (A, R)$  consists of two parameters: a set of arguments  $A$ , and a collection of relations  $R$ , called attacks which describe the conflicts between the arguments.

They are mostly used in the fields of AI, f.e. in automated reasoning and logic programming [12, 17]. But do also find their applications in other fields like Natural Language Processing [2], Trust and Reputation Systems [13], and even in Game Theory and Strategic Reasoning [14].

AFs are represented by directed graph, where the nodes are an abstraction of the arguments  $A$ , and the arrows represent the attacks  $R$ . Let us define an AF  $G = (A, R)$  with the arguments  $A = \{a, b, c, d, e\}$  and the attacks  $R = [(a, b), (b, b), (a, c), (c, a), (c, d), (d, e), (e, d)]$ . With the arguments and attacks, we can create a directed graph 2.1

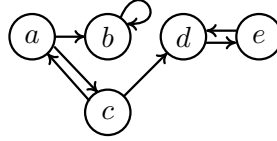


Figure 2.1: AF G

To be able to conclude something, out of an abstract AF, we need to define semantics. A semantic defines a subset of argument sets that satisfy the semantic-specific rules. Dung already defined different semantics [8] like *cf*, *adm* and *stb*. According to Dungs definitions, a set  $S$  is *cf* if there are no attacks between the arguments in  $S$ . The conflict-free set is mainly a building block for the other semantics, which means that the conflict-free set is always a superset of admissible and stable. In the example 2.1 the *conflict-free* sets are:  $\{a\}$ ,  $\{c\}$ ,  $\{d\}$ ,  $\{e\}$ ,  $\{a, d\}$ ,  $\{a, e\}$ ,  $\{c, e\}$ ,

An admissible set is a conflict-free set, where each argument in  $S$  has a defender in  $S$ . In the example 2.1 the *admissible* sets are:  $\{a\}$ ,  $\{c\}$ ,  $\{e\}$ ,  $\{a, d\}$ ,  $\{a, e\}$ ,  $\{c, e\}$ ,

Finally, a stable set is a conflict-free set, if for every argument, which is not in  $S$ , has an attacker which is in  $S$ . In the example 2.1 the *stable* sets are:  $\{a, d\}$ ,  $\{a, e\}$ ,

The specific rules can be defined via a boolean formula. They can be used to encode the AFs to be solvable with different boolean solvers like ASP [3] or, as in our case, with a SAT-Solver [1]. Unfortunately, drawing a conclusion from an AF can be challenging,

e.g., it can be NP-complete and sometimes even be beyond NP to decide whether an argument is acceptable under a specific argumentation semantics [10].

## **2.2 Clustering of Argumentation Frameworks**

*TODO: What are clusters*

*TODO: Why do we need clusters*

*TODO: Definition of Semantics*

*TODO: Complexity*

## **2.3 SAT-Solver**

*TODO: What are SAT-Solvers*

*TODO: Complexity of SAT-Problems*

*TODO: Where and how do we use SAT-Solvers in the research*

# 3 Algorithm

## 3.1 Concretizing Singletons

Concretizing a list of arguments is done iteratively by deep copying the abstract AF  $F'$  to create a new AF  $F''$  and mutating it. The mutation is guided by five steps considering the unchanged abstract AF  $F'$  and the concrete AF  $F$ . To improve the understanding of each step, we accompany the explanation with the example depicted in 3.3, where we concretize the arguments  $a$  and  $b$ .

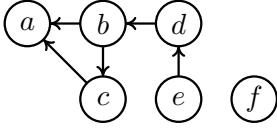


Figure 3.1: Concrete AF  $F$

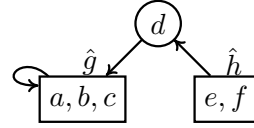


Figure 3.2: Abstract AF  $F'$

Figure 3.3: Example: Concretization of arguments

**Step 1:** Each argument needing concretization is first removed from the parent cluster and added as a singleton in  $F''$ . If an argument is not part of a cluster, we remove it and continue with the filtered list. We do not consider attacks in this step since they depend on the concrete- and abstract AFs.

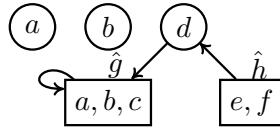


Figure 3.4: Concretized AF  $F''$  after Step 1

**Step 2:** We add the new attacks from all concretized arguments to the remaining clusters. We must do this after removing the arguments from the clusters because if an argument  $a$  attacks argument  $b$  in the concrete AF, and  $b$  is part of the cluster  $F'$  in the abstract AF, by concretizing  $b$ , the attack  $(a, F')$  would not be persistent anymore.

**Step 3:** After adding the new attacks, we need to check which attacks from  $F'$  are still persistent in  $F''$ . If an attack does not persist through the concretization, we remove it

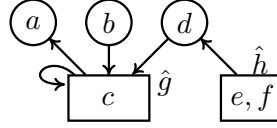


Figure 3.5: Concretized AF  $F''$  after Step 2

in  $F''$ . An attack is not persistent anymore; if we remove one of the arguments being attacked or attacked by argument  $a$  from the cluster  $f$  and no other attack exists, s.t.  $a$  is attacked from/attacking an argument within  $f$ . Selfattacks of clusters could also change by the concretization of arguments. Therefore, we need to check the clusters from which the arguments are concretized.

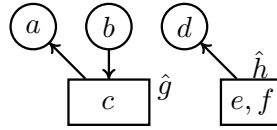


Figure 3.6: Concretized AF  $F''$  after Step 3

**Step 4:** In this step we add the new attacks between the singletons. Due to the fact, that we copied all the attacks from  $F'$ , we only have to take into consideration the attacks from or to the concretized singletons. So instead of iterating over all singletons of the AF, we can limit the attack creation to the concretized singletons.

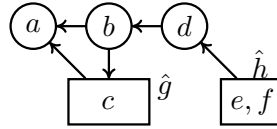


Figure 3.7: Concretized AF  $F''$  after Step 4

**Step 5:** The last step is to clean up the argumentation framework  $F''$  by removing all empty clusters and mutating the clusters with exactly one singleton to the mentioned singleton.

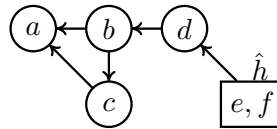


Figure 3.8: Concretized AF  $F''$  after Step 5



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**Algorithm 1** Concretizing Singletons Pseudocode

---

**Require:**  $A : AF(a_1, r_1)$  ▷ Abstract Clustered AF  
**Require:**  $C : AF(a_2, r_2)$  ▷ Abstract Concrete AF  
**Require:**  $e : list(Arguments)$  ▷ concretizer list

- 1: **for**  $e_i$  in  $e$  **do**
- 2:     **if**  $e_i$  not in  $C \vee e_i$  not in  $A_C$  **then** ▷  $A_C$  = Cluster in  $A$
- 3:          $e.remove(e_i)$
- 4:     **end if**
- 5: **end for**
- 6:  $N \leftarrow A$  ▷  $N$  = Concretized Cluster
- 7:  $N.addSingletons(e)$  ▷ Step 1
- 8:  $N_C.removeArguments(e)$
- 9: **for**  $e_i$  in  $e$  **do** ▷ Step 2
- 10:     **for**  $e_i$  attacks  $A_c$  **do**
- 11:          $N[e_i].attacks.append(A_c)$
- 12:     **end for**
- 13: **end for**
- 14: **for**  $r_i$  in  $A_r$  **do** ▷ Step 3
- 15:     **if**  $r_i$  not persists in  $C$  **then**
- 16:          $A_r.remove(r_i)$
- 17:     **end if**
- 18: **end for**
- 19: **for**  $e_i$  in  $e$  **do** ▷ Step 4
- 20:     **for**  $e_i$  attacks  $C_a$  **do**
- 21:          $N[e_i].attacks.append(C_a)$
- 22:     **end for**
- 23: **end for**
- 24: **for**  $c_i$  in  $N_c$  **do** ▷ Step 5
- 25:     **if**  $c_i.argAmount == 1$  **then**
- 26:          $c_i \leftarrow Singleton$
- 27:     **else if**  $c_i.argAmount == 0$  **then**
- 28:          $N_c.remove(c_i)$
- 29:     **end if**
- 30: **end for**

---

### 3.2 Computation of Concretizer List

*TODO: Explain how concretizer list is computed*

### 3.3 Algorithmic Approach to Compute Faithful Clusterings

*TODO: Concretize singletons of clustered AF algorithm*

### **3.4 Heuristics and Refinements**

*TODO: Define every Heuristic and refinement we used for each semantic*

## 4 Implementation

### 4.1 Creating AFs

*TODO: Explain AF creation algorithms (Random + Grid-Based)*

### 4.2 BFS and DFS Approach

*TODO: BFS and DFS approach in current research + when BFS is better than DFS*

### 4.3 Generating Semantic Sets

*TODO: Semantic sets generation algorithm*

### 4.4 Faithful/Spurious Determination

*TODO: Determine faithful/spurious algorithm*



## 5 Related Works



## 6 Conclusion





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