April 13th 2017 PHYS 375: Stars Prabhakar Uthayakumar
Final Project – Individual Report 20523430

1. Description of the Code

Creating a star consists of solving five coupled differential equations. The Runge-Kutta method with an adaptive step size was used to integrate these stellar equations with respect to radius. The step size (dr) is initially 10000 m, until one hundredth of the current star radius is larger, in which case the step size is changed to 0.01Rstar. Before the integration begins, a central temperature and a central density are chosen. The integration is started at 0.0001 m to avoid divergences during the calculations. The integration is stopped when the change in optical depth is 2/3, described as:

$$\delta\tau = \tau(\infty) - \tau(R_{star}) = 2/3$$

At this point, the luminosity obtained from integration is compared to the luminosity calculated using the obtained star radius and surface temperature.

$$4\pi\sigma {R_{star}}^2 {T_{star}}^4$$

A trial function can be defined as the normalized difference between the two luminosities.

$$f(\rho_c) = \frac{L_{star} - 4\pi\sigma {R_{star}}^2 {T_{star}}^4}{\sqrt{4\pi\sigma {R_{star}}^2 {T_{star}}^4 L_{star}}}$$

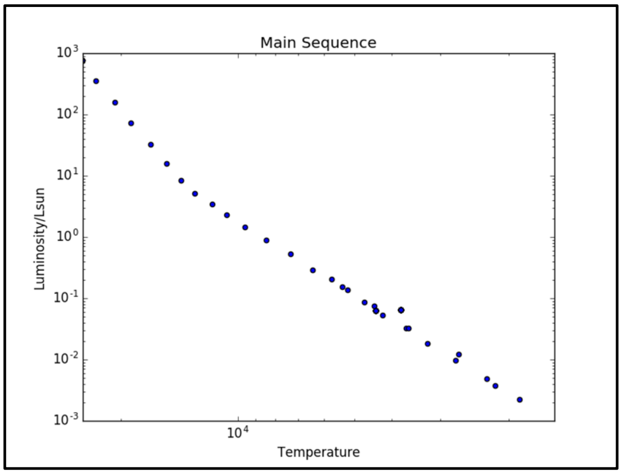
The central density corresponding to the zero crossing of this function represents a more accurate value to use for the star integration. This is achieved by using a bisection method, which essentially approaches the crossing density from the left (low density) and from the right (high density). The bisection is stopped when the difference between the densities is within 0.5 percent. At this point, the last central density is chosen and the integration is repeated using the new central density. It should be noted that the surface temperature was calculated by setting the trial function to zero, with the surface luminosity and star radius.

$$T_{star} = \sqrt[4]{\frac{L_{star}}{4\pi\sigma {R_{star}}^2}}$$

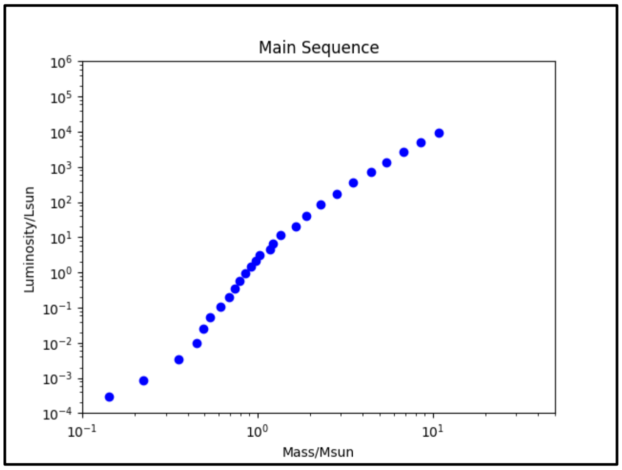
To obtain a main sequence, this integration was done for a range of central temperatures (10 million K – 40 million K).

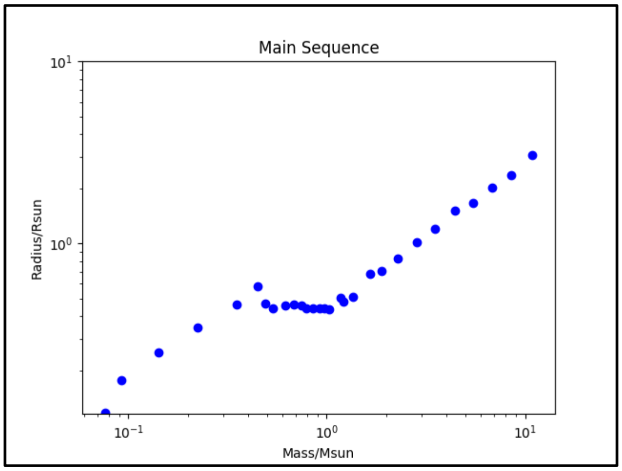
2. Main sequences

The HR diagram of the generated main sequence stars is displayed below



Below are the plots showing the Mass- Luminosity relation (left) and the Mass – Radius relation (right) created with the generated stars for the HR diagram above. The mass-luminosity plot agrees with the expression in the text, the luminosity increases with respect to increasing mass. Looking at the mass- radius plot, there is an unexpected plateau region between 0.5 to 1 solar masses. The radii below 0.5 of the solar mass and above 1 solar mass follow the expected increasing trend.

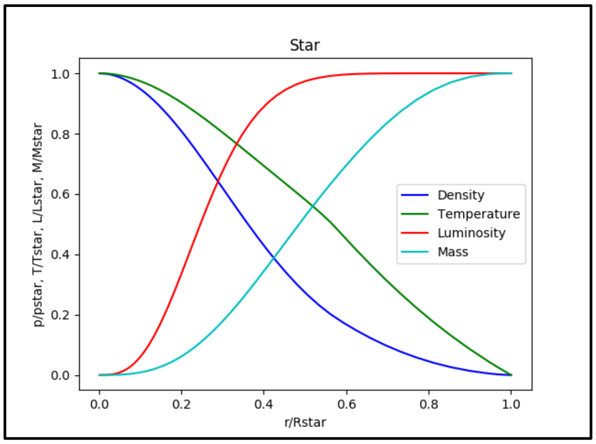


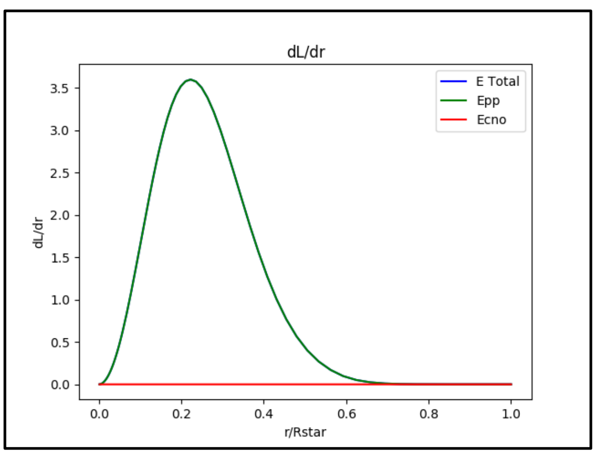


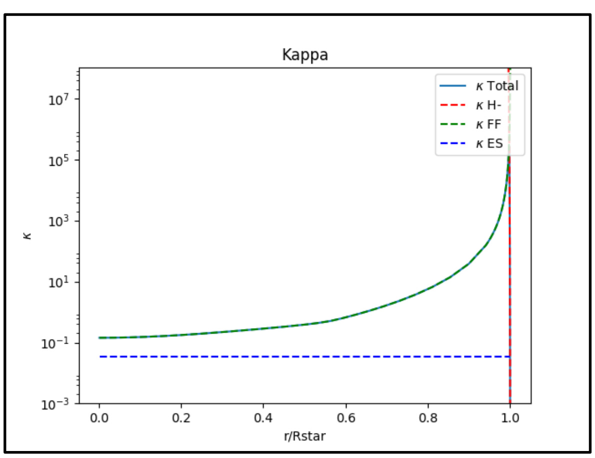
3. Individual Stars

Below are the stellar equation plots for two stars of different masses.

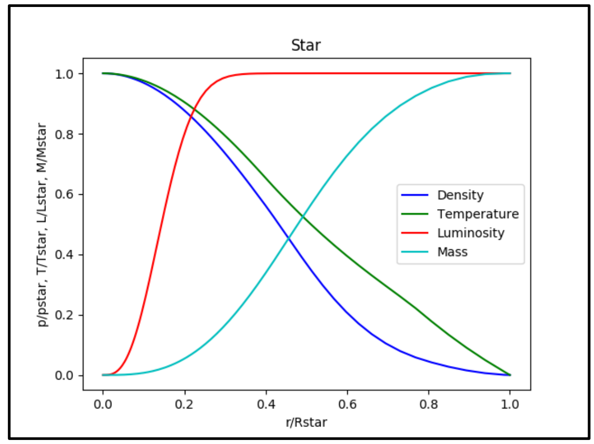
0.7 M_{sun}, 0.5 R_{sun}, 1.0 T_{sun}

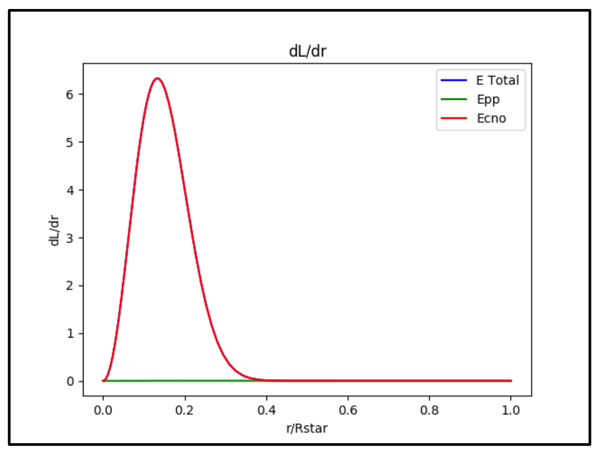


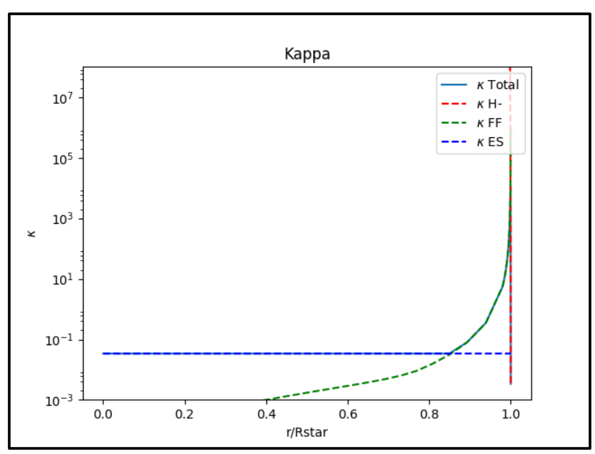




14 M_{sun}, 3.0 R_{sun}, 6.3 T_{sun}







There are a few noticeable differences between the two stars looking at the first plots. The larger star's luminosity plateaus much quicker with respect to radius, in comparison to the smaller star. There is a distinct bump in the temperature curves, occurring closer to the centre for the smaller star than the other. This may be a result of the star switch from a radiation zone to a convection zone. The smaller star should have a convection zone first from the center, then switching into radiation. The case is opposite for the larger star, given its mass is larger than 1.5 solar masses

Comparing the dL/dr plots, the energy production in the lesser massive star is completely dominated by proton-proton chain fusion. On the other hand, the CNO cycle dominates energy production for the larger star. Looking at the opacity plot for the less massive star, the free-free opacity is the most dominant throughout the interior of the star. For the more massive star, the electron scattering opacity is dominant from the center to about the 0.85 of the star radius, after which the free-free opacity quickly dominates. In both cases, the opacity due to H^- arises at the surface of the stars.