Chapter 3

STACKS AND QUEUES

All the programs in this file are selected from

Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed

Stacks

3.1; page 107 - 111

stack: a Last-In-First-Out (LIFO) list

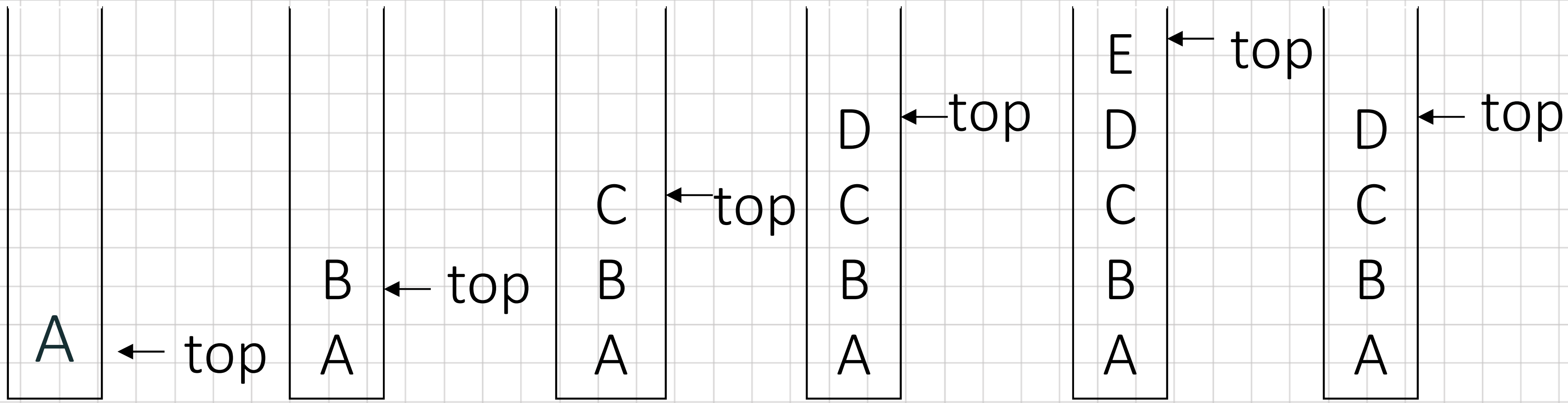
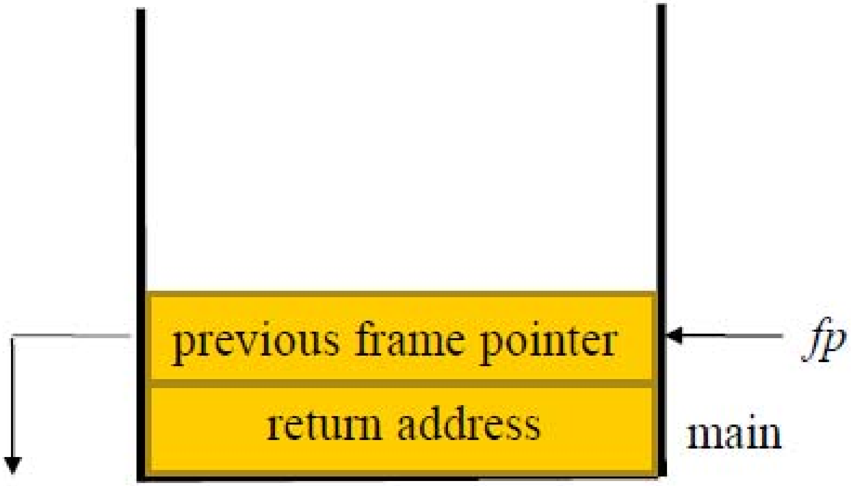


*Figure 3.1: Inserting and deleting elements in a stack (p.108)

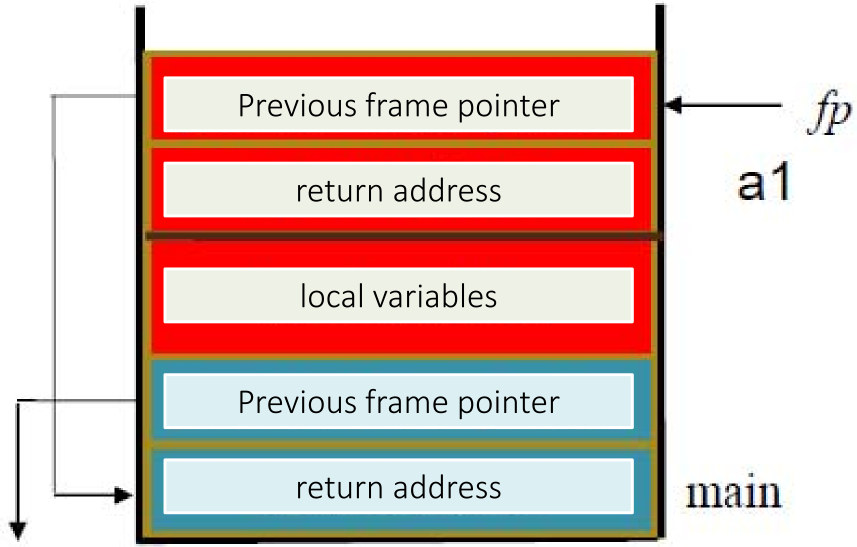
an application of stack: stack frame of function call (activation record)

fp: a pointer to current stack frame



system stack before a1 is invoked

(a)



system stack after a1 is invoked

(b)

*Figure 3.2: System stack after function call (p.109)

abstract data type for stack

ADT Stack is
  objects: a finite ordered list with zero or more elements.
  functions:
    for all stack ∈ Stack, item ∈ element, max_stack_size ∈ positive integer

```
Stack CreateS(max_stack_size) ::=
        create an empty stack whose maximum size is
        max_stack_size

Boolean IsFull(stack, max_stack_size) ::=
        if (number of elements in stack == max_stack_size)
        return TRUE
        else return FALSE

Stack Push(stack, item) ::=
        if (IsFull(stack)) stackFull
        else insert item into top of stack and return
```

```
Boolean IsEmpty(stack) ::=
        if(stack == CreateS(max_stack_size))
        return TRUE
        else return FALSE

Element Pop(stack) ::=
        if(IsEmpty(stack)) return
        else remove and return the item on the top
            of the stack.
```

*Structure 3.1: Abstract data type *Stack* (p.110)

Implementation: using array

```
Stack CreateS(max_stack_size) ::=
  #define MAX_STACK_SIZE 100 /* maximum stack size */
  typedef struct {
          int key;
          /* other fields */
          } element;

  element stack[MAX_STACK_SIZE];
  int top = -1;

  Boolean IsEmpty(Stack) ::= top< 0;

  Boolean IsFull(Stack) ::= top >= MAX_STACK_SIZE-1;
```

Add to a stack

```
void push(int *top, element item)
{
 /* add an item to the global stack */
    if (*top >= MAX_STACK_SIZE-1)  {
          stackFull( );
          return;
    }
    stack[++*top] = item;
}
```

*program 3.1: Add to a stack (p.109)

Delete from a stack

```
element pop(int *top)
{
 /*deleat and return the top element from the stack */

    if (*top == -1)
        return stackEmpty( );  /* returns and error key */
    return stack[(*top)--];
 }
```

*Program 3.2: Delete from a stack (p.111)

Queues

3.3; PAGE 114

Queue:
a First-In-First-Out (FIFO) list

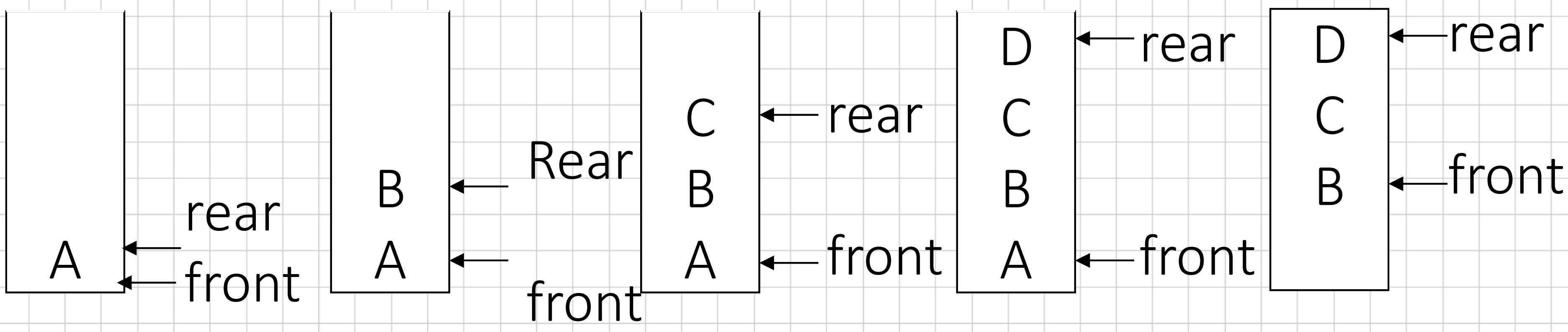


*Figure 3.4: Inserting and deleting elements in a queue (p.114)

Application: Job scheduling

front	rear	Q[0]	Q[1]	Q[2]	Q[3]	Comments
-1	-1					queue is empty
-1	0	J1				Job 1 is added
-1	1	J1	J2			Job 2 is added
-1	2	J1	J2	J3		Job 3 is added
0	2		J2	J3		Job 1 is deleted
1	2			J3		Job 2 is deleted

*Figure 3.5: Insertion and deletion from a sequential queue (p.117)

Abstract data type of queue

ADT Queue is
  objects: a finite ordered list with zero or more elements.
  functions:
    for all queue ∈ Queue, item ∈ element,
        max_ queue_ size ∈ positive integer

    Queue CreateQ(max_queue_size) ::=
        create an empty queue whose maximum size is
        max_queue_size

    Boolean IsFullQ(queue, max_queue_size) ::=
        if(number of elements in queue == max_queue_size)
        return TRUE
        else return FALSE

    Queue AddQ(queue, item) ::=
        if (IsFullQ(queue)) queue_full
        else insert item at rear of queue and return queue

```
Boolean IsEmptyQ(queue) ::=
        if (queue ==CreateQ(max_queue_size))
        return TRUE
        else return FALSE

  Element DeleteQ(queue) ::=
        if (IsEmptyQ(queue)) return
        else remove and return the item at front of queue.
```

*ADT 3.2: Abstract data type Queue (p.115)

Implementation 1: using array

Queue CreateQ(max_queue_size) ::=

```
# define MAX_QUEUE_SIZE 100/* Maximum queue size */
typedef struct {
            int key;
            /* other fields */
            } element;
element queue[MAX_QUEUE_SIZE];
int rear = -1;
int front = -1;
```

Boolean IsEmpty(queue) ::= front == rear
Boolean IsFullQ(queue) ::= rear == MAX_QUEUE_SIZE-1

Add to a queue

```
void addq(int *rear, element item)
{
/* add an item to the queue */
    if (*rear == MAX_QUEUE_SIZE - 1) {
        queue_full( );
        return;
    }
    queue [++*rear] = item;
}
```

*Program 3.5: Add to a queue (p.116)

Delete from a queue

```
element deleteq(int *front, int rear)
{
/* remove element at the front of the queue */
    if ( *front == rear)
        return queue_empty( );     /* return an error key */
    return queue [++ *front];
}
```

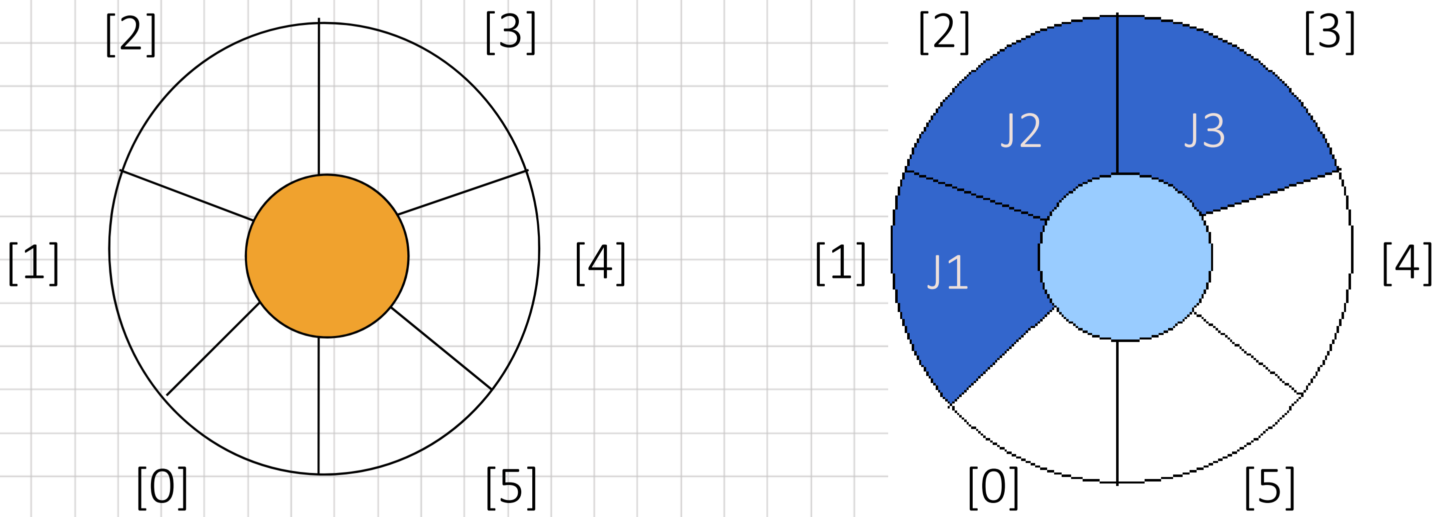
*Program 3.4: Delete from a queue(p.108)

problem: there may be available space when IsFullQ is true
I.E. movement is required.

Implementation 2:
regard an array as a circular queue

front: one position counterclockwise from the first element
rear: current end

EMPTY QUEUE



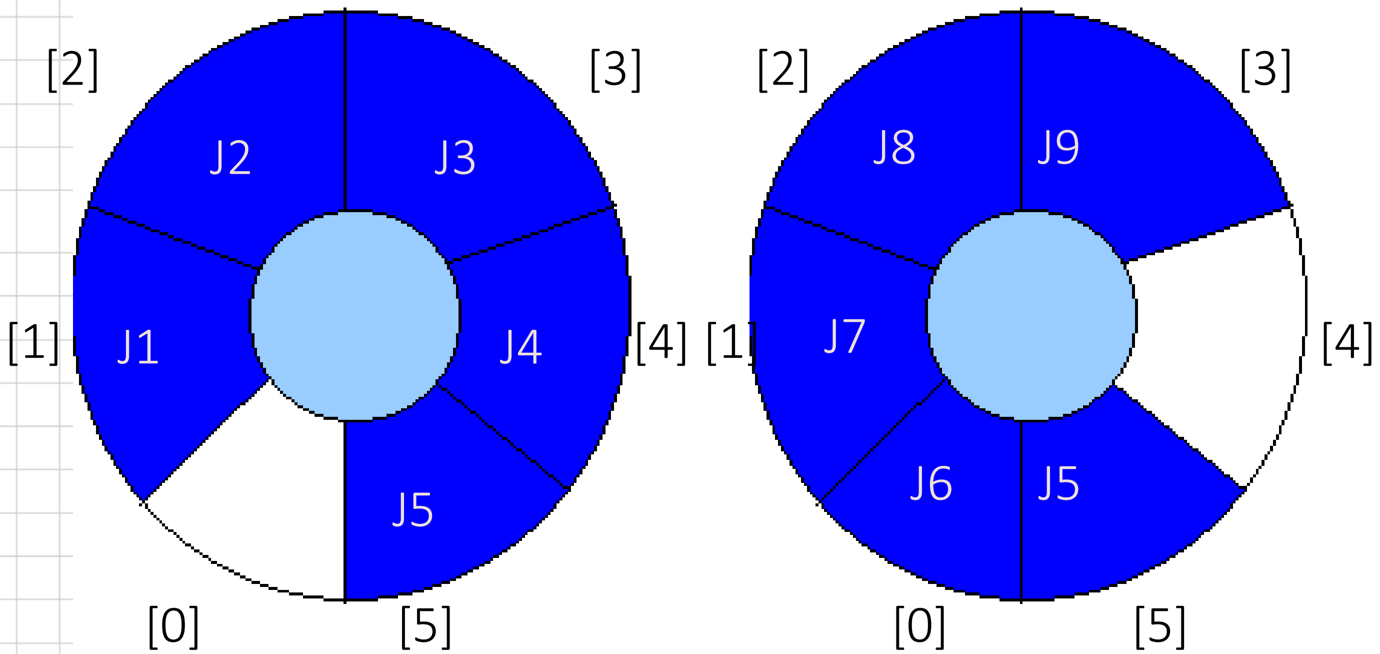
front = 0
rear = 0

front = 0
rear = 3

*Figure 3.6: Empty and nonempty circular queues (p.117)

Problem: one space is left when queue is full

FULL QUEUE FULL QUEUE



front =0
rear = 5

front =4
rear =3

*Figure 3.7: Full circular queues and then we remove the item (p.117)

Add to a circular queue

```
void addq(int front, int *rear, element item)
{
/* add an item to the queue */
    *rear = (*rear +1) % MAX_QUEUE_SIZE;
     if (front == *rear) /* reset rear and print error */
     return;
   }
     queue[*rear] = item;
}
```

*Program 3.7: Add to a circular queue (p.118)

Delete from a circular queue

```
element deleteq(int* front, int rear)
{
  element item;
  /* remove front element from the queue and put it in item */
     if (*front == rear)
        return queue_empty( );
                  /* queue_empty returns an error key */
     *front = (*front+1) % MAX_QUEUE_SIZE;
     return queue[*front];
}
```

*Program 3.8: Delete from a circular queue (p.119)

3.5 A Mazing Problem

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A Mazing Problem

entrance →

0	1	0	0	0	1	1	0	0	0	1	1	1	1	1
1	0	0	0	1	1	0	1	1	1	0	0	1	1	1
0	1	1	0	0	0	0	1	1	1	1	0	0	1	1
1	1	0	1	1	1	1	0	1	1	0	1	1	0	0
1	1	0	1	0	0	1	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	0	1	0	0	1	0	1
0	1	1	1	1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1	1	1	1	1	0	1
1	1	0	0	0	1	1	0	1	1	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	1	1	1	1	0
0	1	0	0	1	1	1	1	1	0	1	1	1	1	0

→ exit

1: blocked path 0: through path

*Figure 3.8: An example maze(p.123)

a possible representation

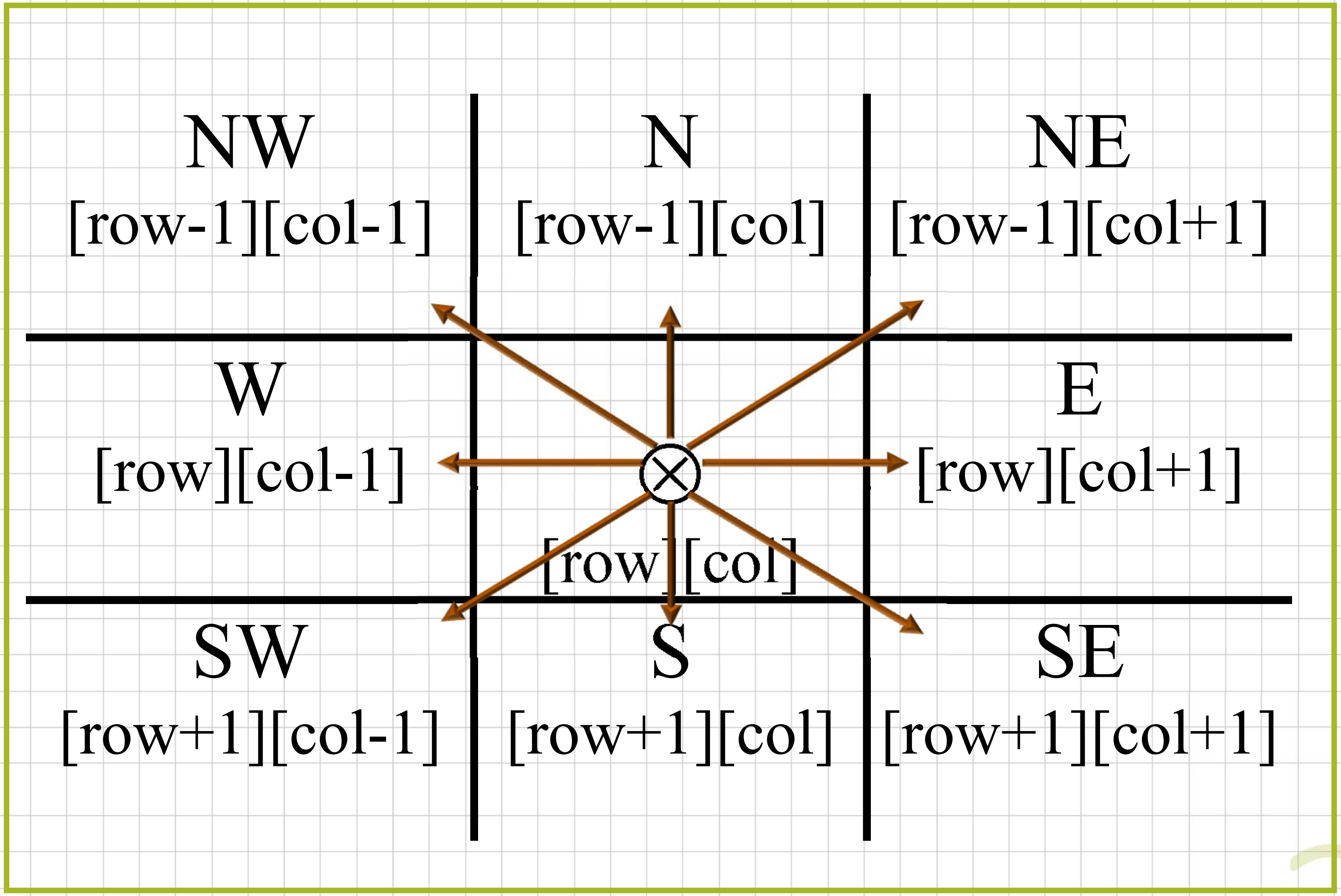


*Figure 3.9: Allowable moves (p.127)

a possible implementation

```
next_row = row + move[dir].vert;
next_col  = col  +  move[dir].horiz;
```

```
typedef struct {
        short int vert;
        short int horiz;
        } offsets;
offsets move[8];
/*array of moves for each direction*/
```

Name	Dir	move[dir].vert	move[dir].horiz
N	0	-1	0
NE	1	-1	1
E	2	0	1
SE	3	1	1
S	4	1	0
SW	5	1	-1
W	6	0	-1
NW	7	-1	-1

Use stack to keep pass history

```
#define MAX_STACK_SIZE 100
        /*maximum stack size*/

typedef struct {
        short int row;
        short int col;
        short int dir;
        } element;

element stack[MAX_STACK_SIZE];
```

Initialize a stack to the maze's entrance coordinates and direction to north;

```
while (stack is not empty){
  /* move to position at top of stack */
<row, col, dir> = delete from top of stack;

while (there are more moves from current position) {
    <next_row, next_col > = coordinates of next move;
    dir = direction of move;
    if ((next_row == EXIT_ROW)&& (next_col == EXIT_COL))
        success;
    if (maze[next_row][next_col] == 0 &&
         mark[next_row][next_col] == 0) {
```

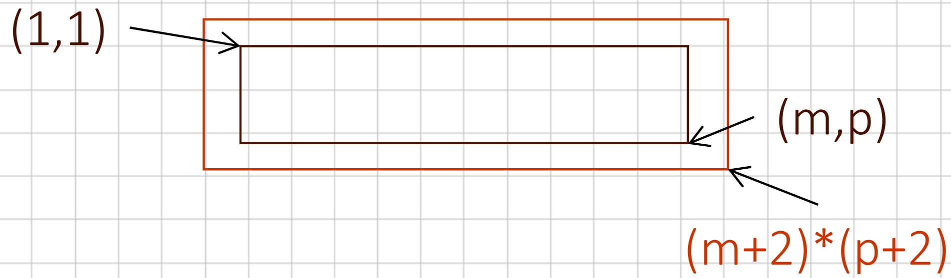
```
/* legal move and haven't been there */
        mark[next_row][next_col] = 1;
        /* save current position and direction */
        add <row, col, dir> to the top of the stack;
        row = next_row;
        col  = next_col;
        dir  = north;
    }
  }
}
printf("No path found\n");
```

*Program 3.11: Initial maze algorithm (p.126)

The size of a stack?

$$\begin{array}{l}
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}_{m*p} \\
\begin{matrix}
\;1 & \;0 & \;0 & \;0 & \;0 & \;0
\end{matrix}
\end{array}$$

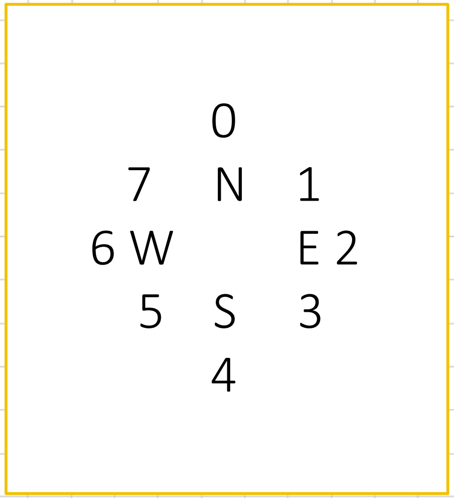
*Figure 3.11: Simple maze with a long path (p.127)




```
void path (void)
{
/* output a path through the  maze if such a path exists */
   int i, row, col, next_row, next_col, dir, found = FALSE;
   element position;
   mark[1][1] = 1; top =0;
   stack[0].row = 1; stack[0].col = 1; stack[0].dir = 1;

   while (top > -1 && !found) {
     position = delete(&top);
     row = position.row; col = position.col;
     dir  = position.dir;

     while (dir < 8 && !found) {
          /*move in direction dir  */
          next_row = row + move[dir].vert;
          next_col  = col + move[dir].horiz;
```



```
if (next_row==EXIT_ROW && next_col==EXIT_COL)
        found = TRUE;
    else if ( !maze[next_row][next_col] &&
            !mark[next_row][next_col] {
        mark[next_row][next_col] = 1;
        position.row = row; position.col = col;
        position.dir = ++dir;
        add(&top, position);
        row = next_row; col = next_col; dir = 0;
    }
    else ++dir;
  }
}
```

```
if (found) {
      printf("The path is :\n");
      printf("row col\n");
      for (i = 0; i <= top; i++)
           printf(" %2d%5d", stack[i].row, stack[i].col);
      printf("%2d%5d\n", row, col);
      printf("%2d%5d\n", EXIT_ROW, EXIT_COL);
   }
   else printf("The maze does not have a path\n");
}
```

*Program 3.12:Maze search function (p.128)

Evaluation of Expressions

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Evaluation of Expressions

X = a / b - c + d * e - a * c

a = 4, b = c = 2, d = e = 3

Interpretation 1:
((4/2)-2)+(3*3)-(4*2)=0 + 8+9=1

Interpretation 2:
(4/(2-2+3))*(3-4)*2=(4/3)*(-1)*2=-2.66666...

How to generate the machine instructions corresponding to a given expression?

precedence rule + associative rule

Token	Operator	Precedence[1]	Associativity
( ) [ ] -> .	function call array element struct or union member	17	left-to-right
-- ++	increment, decrement[2]	16	left-to-right
-- ++ ! - - + & * sizeof	decrement, increment[3] logical not one's complement unary minus or plus address or indirection size (in bytes)	15	right-to-left
(type)	type cast	14	right-to-left
* / %	mutiplicative	13	Left-to-right

+ -	binary add or subtract	12	left-to-right
<< >>	shift	11	left-to-right
> >= < <=	relational	10	left-to-right
== !=	equality	9	left-to-right
&	bitwise and	8	left-to-right
^	bitwise exclusive or	7	left-to-right
|	bitwise or	6	left-to-right
&&	logical and	5	left-to-right
[illegible]	logical or	4	left-to-right

?:	conditional	3	right-to-left
= += -= /= *= %= <<= >>= &= ^= ⊠	assignment	2	right-to-left
,	comma	1	left-to-right

1.The precedence column is taken from Harbison and Steele.
2.Postfix form
3.prefix form

*Figure 3.12: Precedence hierarchy for C (p.130)

user compiler

Infix	Postfix
2+3*4	234*+
a*b+5	ab*5+
(1+2)*7	12+7*
a*b/c	ab*c/
(a/(b-c+d))*(e-a)*c	abc-d+/ea-*c*
a/b-c+d*e-a*c	ab/c-de*ac*-

*Figure 3.13: Infix and postfix notation (p.131)

Postfix: no parentheses, no precedence

Token	Stack			Top
	[0]	[1]	[2]	
6	6			0
2	6	2		1
/	6/2			0
3	6/2	3		1
-	6/2-3			0
4	6/2-3	4		1
2	6/2-3	4	2	2
*	6/2-3	4*2		1
+	6/2-3+4*2			0

*Figure 3.14: Postfix evaluation (p.131)

Postfix Evaluation

Assumptions:

operators: +, -, *, /, %

operands: single digit integer

```
#define MAX_STACK_SIZE 100 /* maximum stack size */
#define MAX_EXPR_SIZE 100 /* max size of expression */

typedef enum{1paran, rparen, plus, minus, times, divide,
                    mod, eos, operand} precedence;

int stack[MAX_STACK_SIZE]; /* global stack */
char expr[MAX_EXPR_SIZE]; /* input string */
```

```
int eval(void)
{
/* evaluate a postfix expression, expr, maintained as a
   global variable, '\0' is the the end of the expression.
   The stack and top of the stack are global variables.
   get_token is used to return the token type and
   the character symbol. Operands are assumed to be single
character digits */

  precedence token;
  char symbol;
  int op1, op2;
  int n = 0;  /* counter for the expression string */
  int top = -1;

  token = get_token(&symbol, &n);
  while (token != eos)  {
     if (token == operand)
          push(&top, symbol-'0');   /* stack insert */
```

exp: character array

```
else {
        /* remove two operands, perform operation, and
           return result to the stack */
    op2 = pop(&top);  /* stack delete */
    op1 = pop(&top);
    switch(token) {
        case plus: push(&top, op1+op2); break;
        case minus: push(&top, op1-op2); break;
        case times: push(&top, op1*op2); break;
        case divide: push(&top, op1/op2); break;
        case mod: push(&top, op1%op2);
    }
  }
  token = getToken (&symbol, &n);
}
return pop(&top); /* return result */
}
```

*Program 3.13: Function to evaluate a postfix expression (p.133)

```
precedence getToken(char *symbol, int *n)
{
/* get the next token, symbol is the character
   representation, which is returned, the token is
   represented by its enumerated value, which
   is returned in the function name */

  *symbol =expr[(*n)++];
  switch (*symbol)  {
    case '(' : return lparen;
    case ')' : return rparen;
    case '+': return plus;
    case '-' : return minus;
```

```
    case        '/' :     return divide;
    case        '*' :     return times;
    case        '%' :     return mod;
    case        '\0' :    return eos;
    default  :            return operand;
            /* no error checking, default is operand */
    }
}
```

*Program 3.14: Function to get a token from the input string (p.134)

Infix to Postfix Conversion (Intuitive Algorithm)

(1) Fully parenthesize expression

a / b - c + d * e - a * c -->

((((a / b) - c) + (d * e)) - a * c))

(2) All operators replace their corresponding right parentheses.

((((a / b) - c) + (d * e)) - a * c))

/ - * + * -

(3) Delete all parentheses.

ab/c-de*+ac*-

two passes

The orders of operands in infix and postfix are the same.

a + b * c, * > +

Token	Stack [0]	Stack [1]	Stack [2]	Top	Output
a				-1	a
+	+			0	a
b	+			0	ab
*	+	*		1	ab
c	+	*		1	abc
eos				-1	abc*+

*Figure 3.15: Translation of a+b*c to postfix (p.135)

a $*_1$ (b +c) $*_2$ d

Token	Stack [0]	Stack [1]	Stack [2]	Top	Output
a				-1	a
$*_1$	$*_1$			0	a
(	$*_1$	(		1	a
b	$*_1$	(		1	ab
+	$*_1$	(	+	2	ab
c	$*_1$	(	+	2	abc
)	$*_1$	match)		0	abc+
$*_2$	$*_2$	$*_1 = *_2$		0	abc+$*_1$
d	$*_2$			0	abc+$*_1$d
eos	$*_2$			0	abc+$*_1$d$*_2$

* Figure 3.16: Translation of a*(b+c)*d to postfix (p.135)

Rules

(1) **Operators are taken out of the stack** as long as their in-stack precedence (ISP) is higher than or equal to the incoming precedence (ICP) of the new operator.

(2) (has low in-stack precedence, and high incoming precedence.

	(	)	+	-	*	/	%	eos
isp	0	19	12	12	13	13	13	0
icp	20	19	12	12	13	13	13	0

```
precedence stack[MAX_STACK_SIZE];

/* isp and icp arrays -- index is value of precedence
lparen, rparen, plus, minus, times, divide, mod, eos */

static int isp [ ] = {0, 19, 12, 12, 13, 13, 13, 0};
static int icp [ ] = {20, 19, 12, 12, 13, 13, 13, 0};
```

isp: in-stack precedence

icp: incoming precedence

```
void postfix(void)
{
/* output the postfix of the expression. The expression
    string, the stack, and top are global */
   char symbol;
   precedence token;
   int n = 0;
   int top = 0; /* place eos on stack */
   stack[0] = eos;

   for (token = getToken(&symbol, &n); token != eos;
                     token = geToken(&symbol, &n)) {
     if (token == operand)
        printf (“%c”, symbol);
     else if (token == rparen ){
```

```
/*unstack tokens until left parenthesis */
    while (stack[top] != lparen)
      printToken(pop(&top));
     pop(&top); /*discard the left parenthesis */
  }
  else{
   /* remove and print symbols whose isp is greater
      than or equal to the current token's icp */
   while(isp[stack[top]] >= icp[token] )
      printToken(pop(&top));
   push(&top, token);
  }
 } //for
 while ((token = pop(&top)) != eos)
     printToken(token);
 print("\n");
}
```

$\theta(n)$

$f(n)=\theta(g(n))$ iff there exist positive constants c_1, c_2, and n_0 such that $c_1 g(n) \le f(n) \le c_2 g(n)$ for all n, $n \ge n_0$.

$f(n)=\theta(g(n))$ iff $g(n)$ is both an upper and lower bound on $f(n)$.

*Program 3.15: Function to convert from infix to postfix (p.137)

Infix	Prefix
a*b/c a/b-c+d*e-a*c a*(b+c)/d-g	/*abc -+-/abc*de*ac -/*a+bcdg

(1) evaluation
(2) transformation

*Figure 3.17: Infix and postfix expressions (p.138)

Multiple stacks and queues

Two stacks

m[0], m[1], ..., m[n-2],m[n-1]

bottommost stack 1 → ← bottommost stack 2

More than two stacks (n)

memory is divided into n equal segments

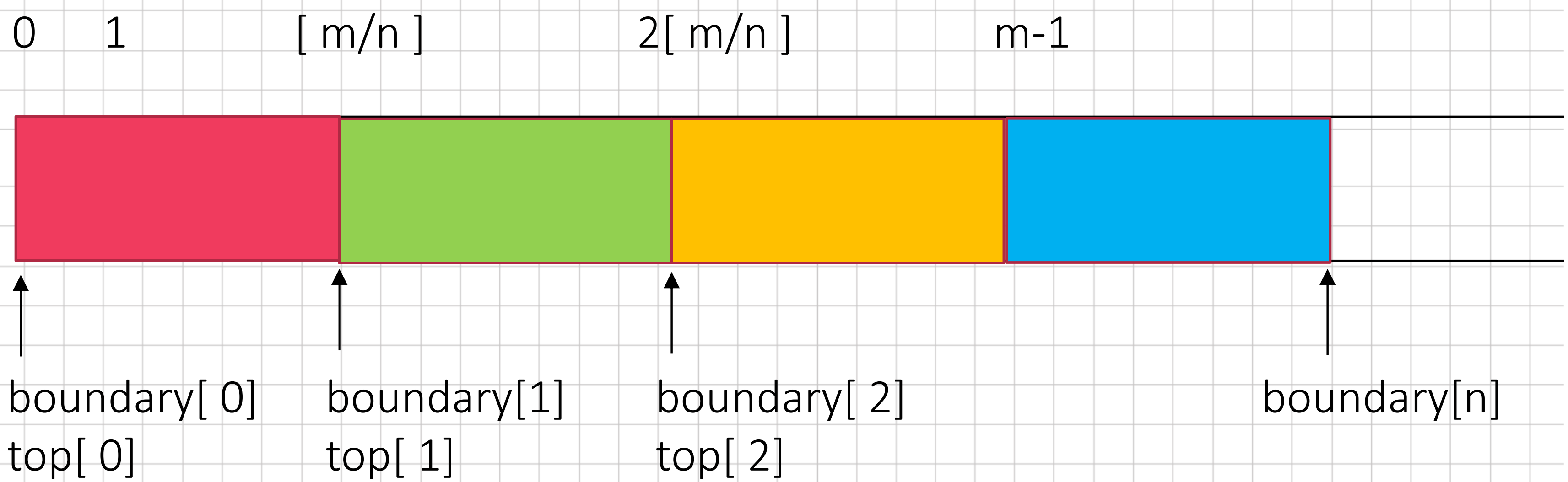
boundary[stack_no]

$0 \leq$ stack_no < MAX_STACKS

top[stack_no]

$0 \leq$ stack_no < MAX_STACKS

Initially, boundary[i]=top[i].



All stacks are empty and divided into roughly equal segments.

*Figure 3.18: Initial configuration for *n* stacks in memory [m]. (p.129)

```
#define MEMORY_SIZE 100    /* size of memory */
#define MAX_STACK_SIZE 100
            /* max number of stacks plus 1 */
/* global memory declaration */
element memory[MEMORY_SIZE];
int top[MAX_STACKS];
int boundary[MAX_STACKS];
int n; /* number of stacks entered by the user */
```

*(p.139)

```
top[0] = boundary[0] = -1;
for (i = 1; i < n; i++)
  top[i] =boundary[i] =(MEMORY_SIZE/n)*i;
boundary[n] = MEMORY_SIZE-1;
```

*(p.139)

```
void push(int i, element item)
{
   /* add an item to the ith stack */
   if (top[i] == boundary [i+1])
      stackFull(i);                    may have unused storage
      memory[++top[i]] = item;
}
```

*Program 3.16:Add an item to the stack stack-no (p.140)

```
element pop(int i)
{
   /* remove top element from the ith stack */
   if (top[i] == boundary[i])
     return stackEmpty(i);
   return memory[top[i]--];
}
```

*Program 3.17:Delete an item from the stack stack-no (p.141)