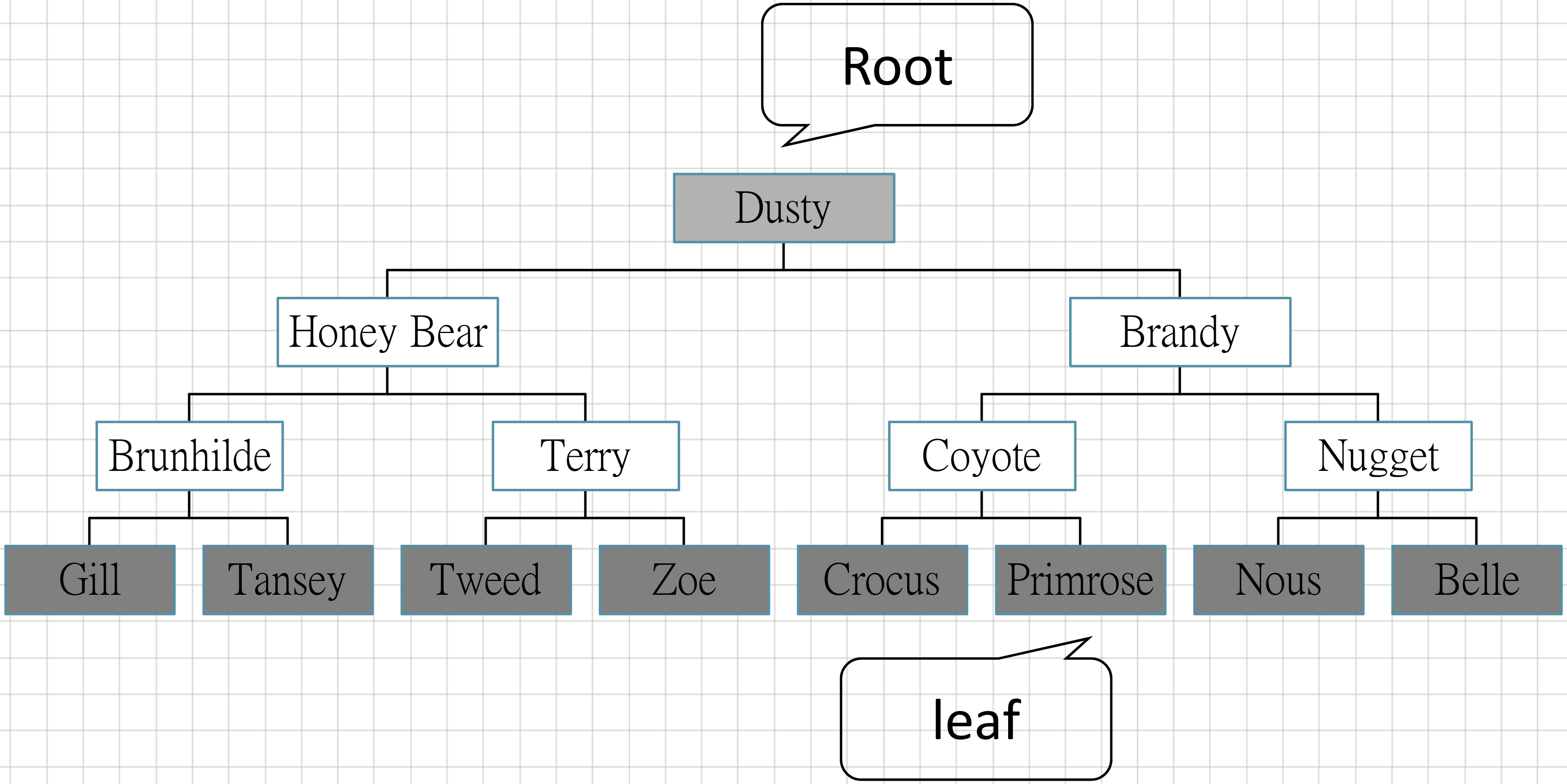
Trees

CHAPTER 5

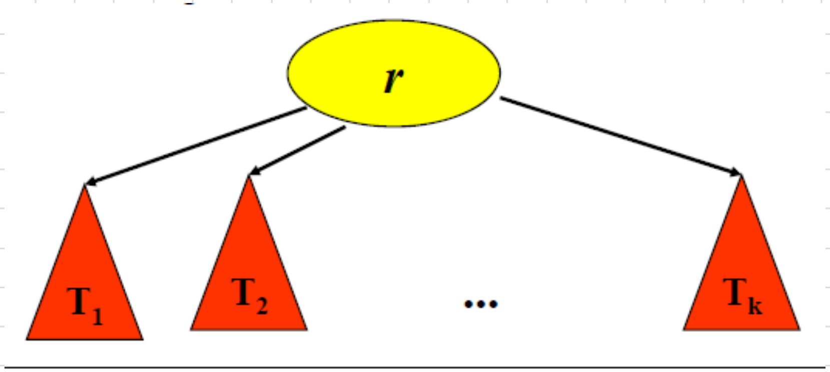
All the programs in this file are selected from
Ellis Horowitz, Sartaj Sahni, and Susan Anderson-Freed
"Fundamentals of Data Structures in C",

Trees


Root
Dusty
Honey Bear
Brandy
Brunhilde
Terry
Coyote
Nugget
Gill
Tansey
Tweed
Zoe
Crocus
Primrose
Nous
Belle
leaf

Definition of Tree

- A tree is a finite set of one or more nodes such that:
- There is a specially designated node called the root.
- The remaining nodes are partitioned into k>=0 disjoint sets T1, ..., Tk, where each of these sets is a tree.
- We call T1, ..., Tk the subtrees of the root.
- K-way tree



Level and Depth

node (13)

degree of a node

leaf (terminal)

Non-terminal

parent

children

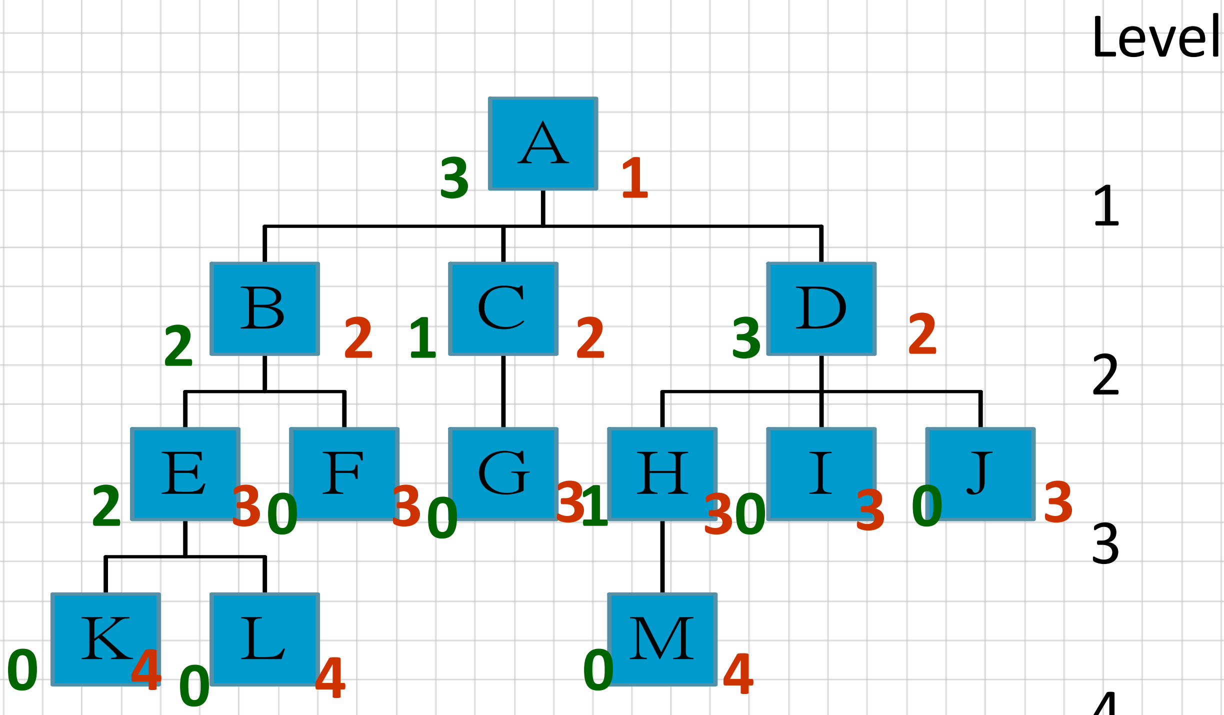
sibling

degree **of a tree** (3)

ancestor

level of a node

height of a tree (4)



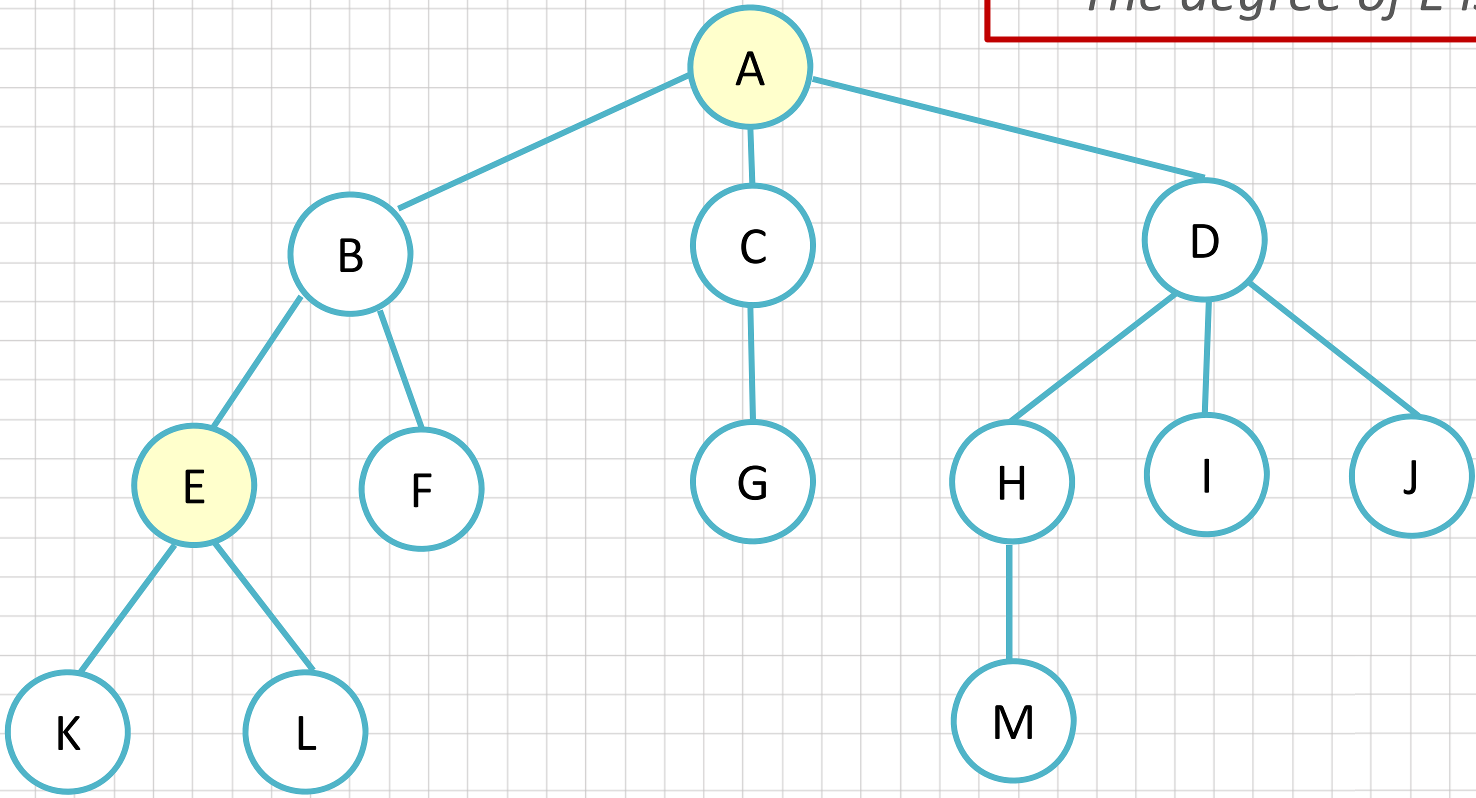
Terminology (1)

Degree :

the number of subtrees of the node

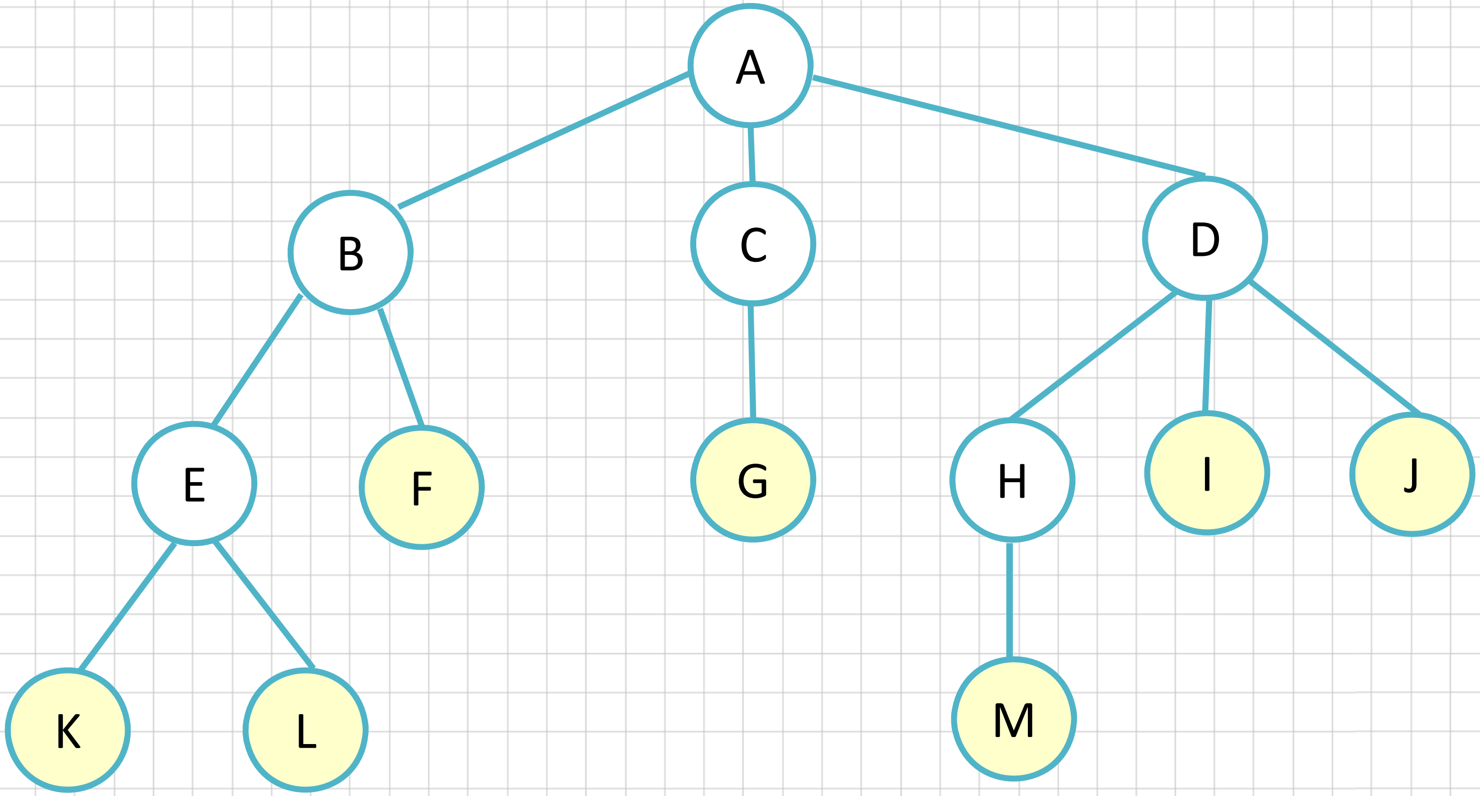
The degree of A is 3

The degree of E is 2.



Terminology (2)

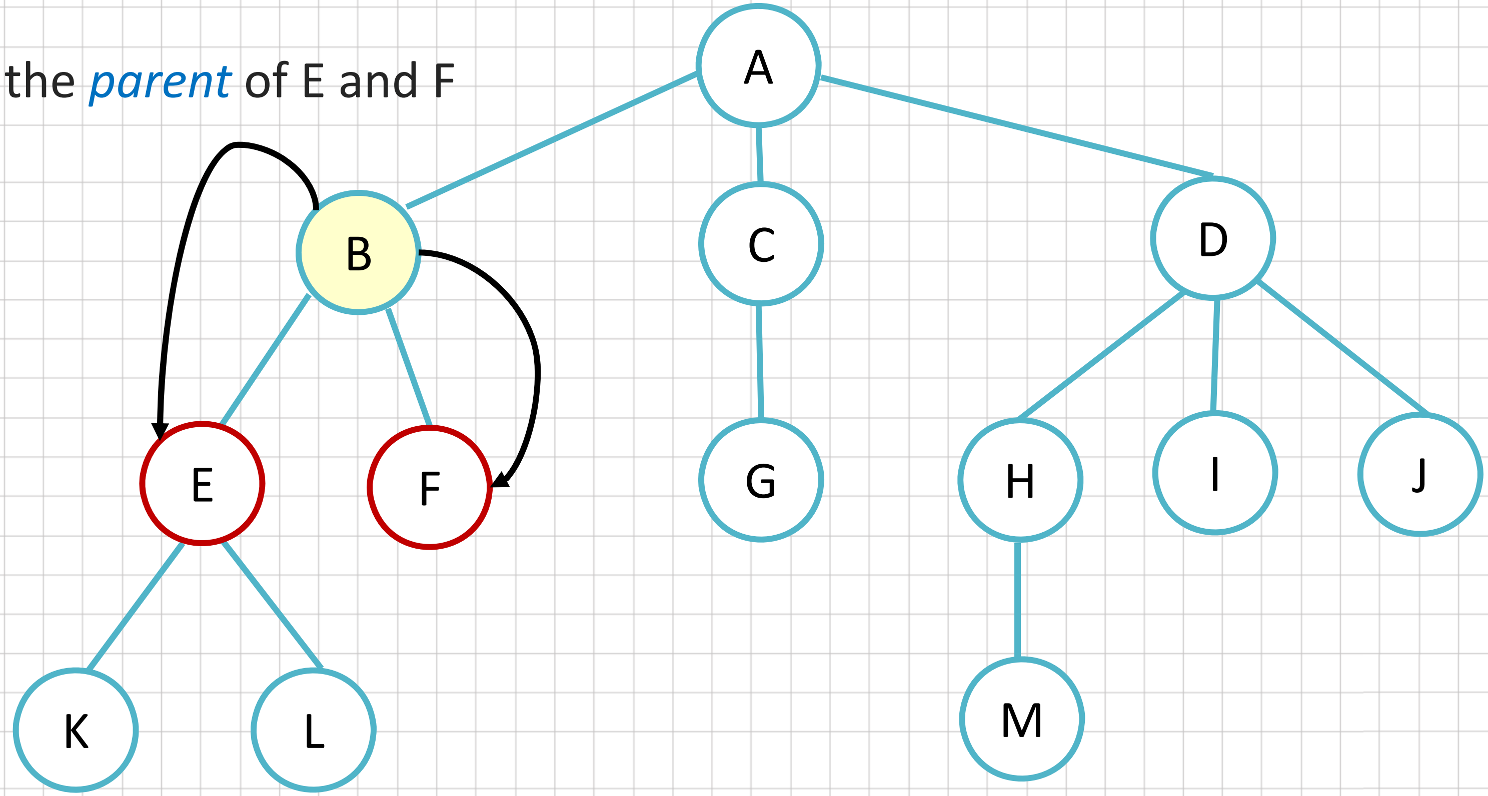
The node with degree 0 is a leaf or terminal node.



Terminology (3)

A node that has subtrees is the *parent* of the roots of the subtrees.

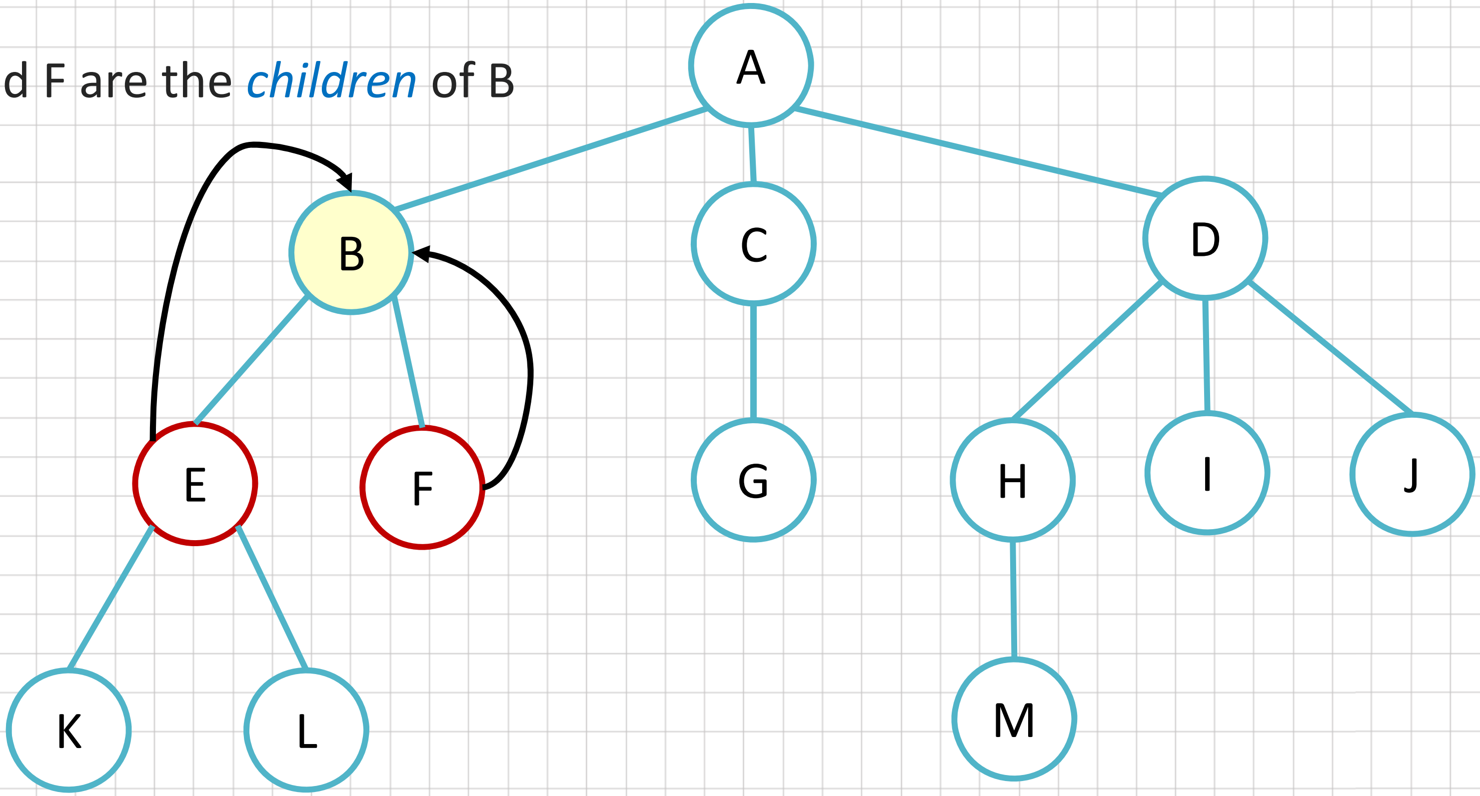
B is the *parent* of E and F



Terminology (4)

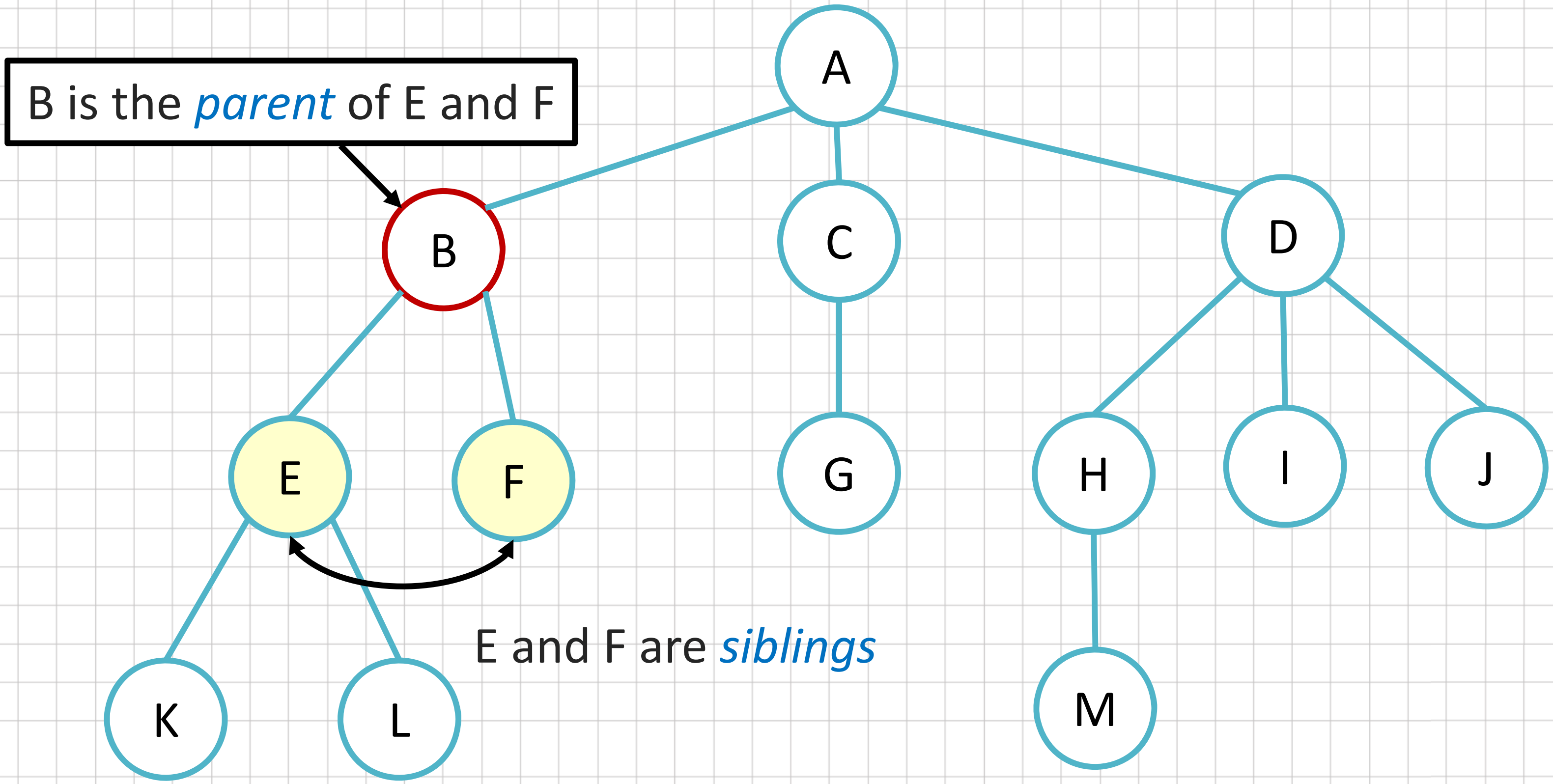
The roots of these subtrees are the *children* of the node.

E and F are the *children* of B



Terminology (5)

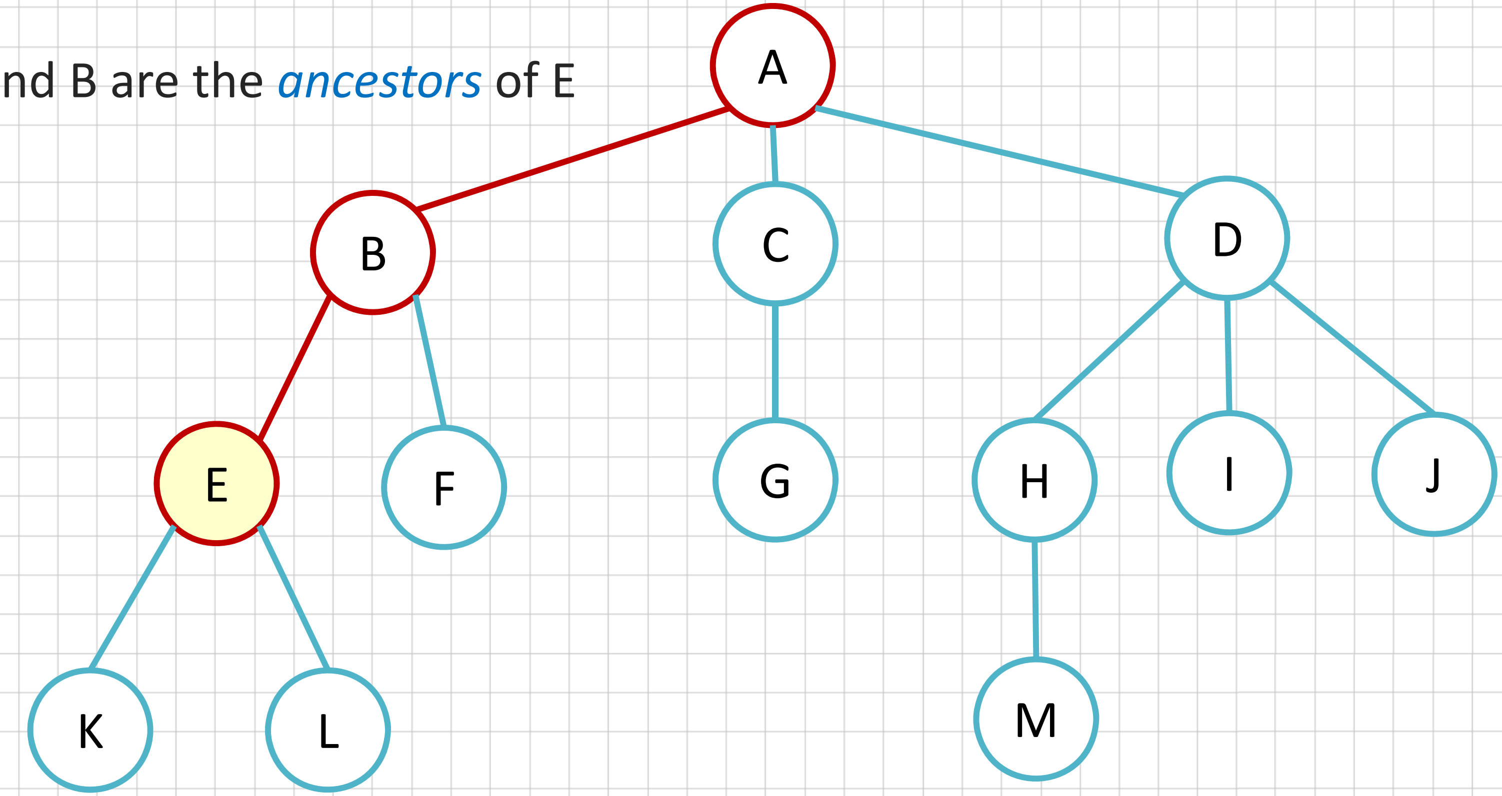
Children of the same parent are *siblings*.



Terminology (6)

The ancestors of a node are all the nodes along the path from the root to the node.

A and B are the *ancestors* of E

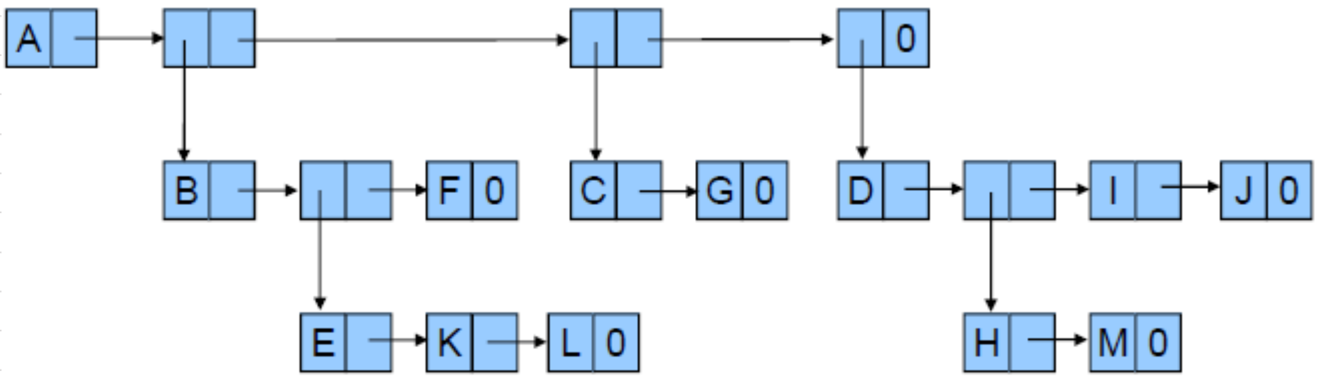


Representation of Trees

List Representation

(A (B (E (K, L), F), C (G), D (H (M), I, J)))

The root comes first, followed by a list of sub-trees



Representation of Trees

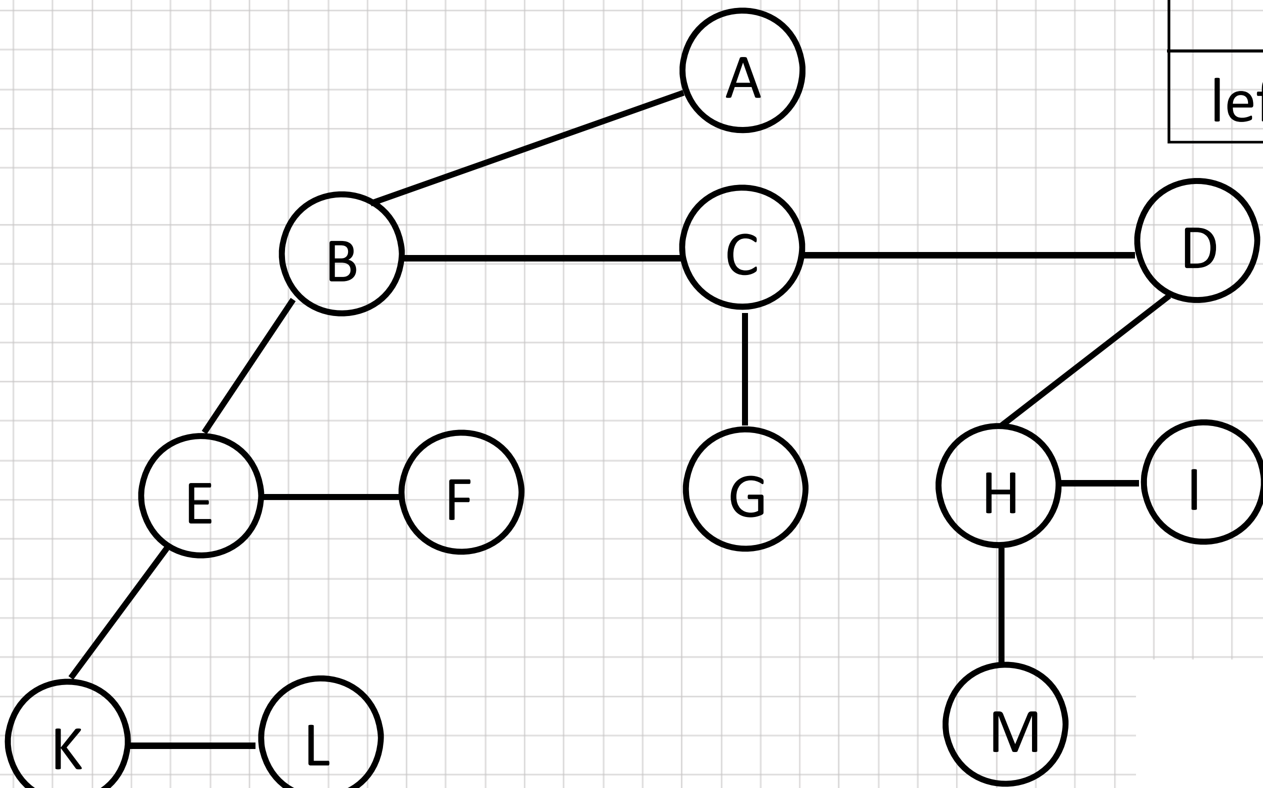
Another Representation

data	link 1	link 2	...	link n

How many link fields are needed in such a representation?

Left Child - Right Sibling

data	
left child	right sibling



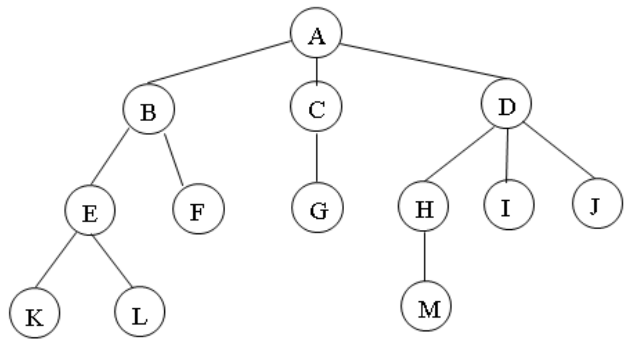
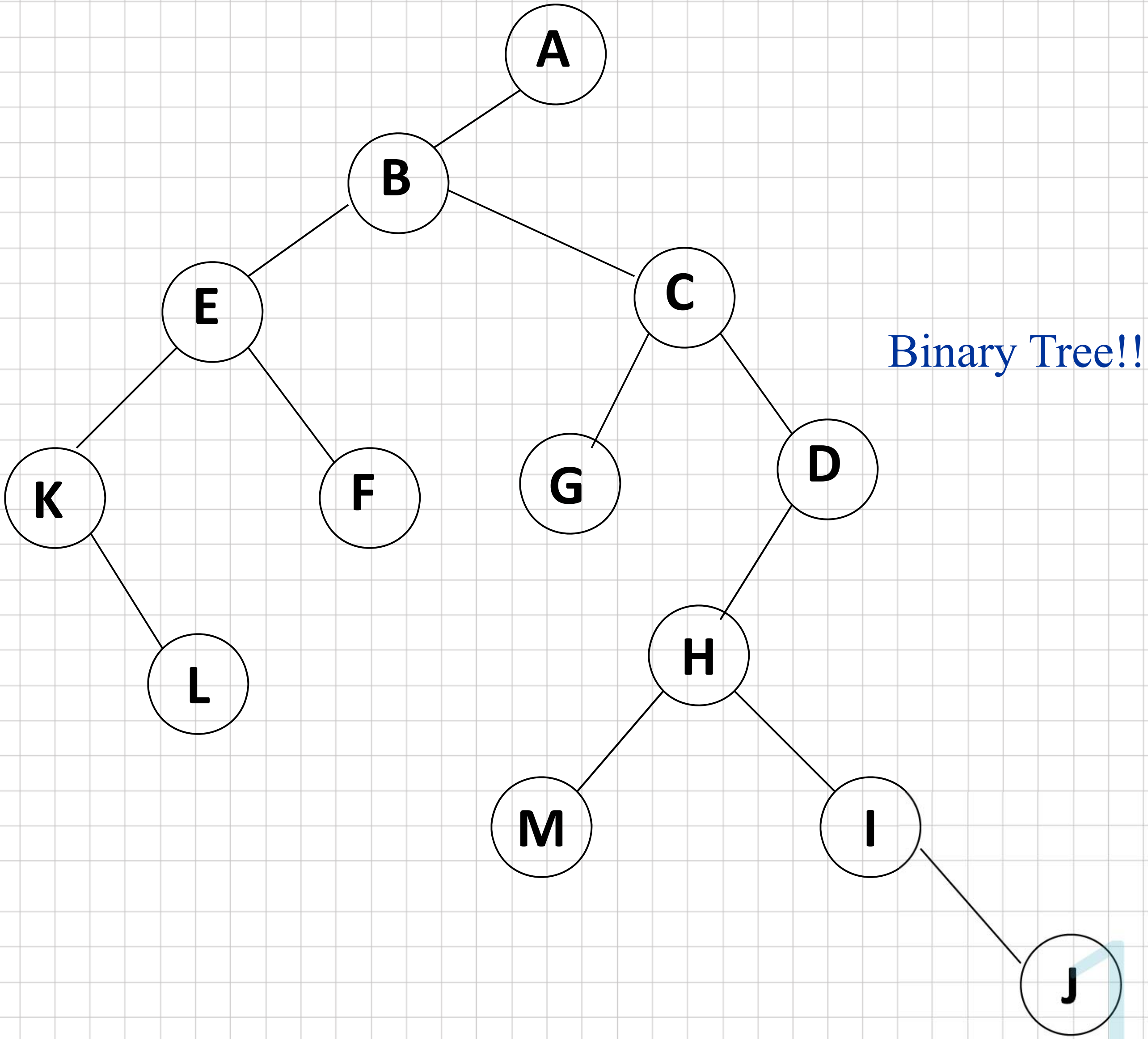


Figure 5.7: Left child-right sibling tree representation of a tree (p.197)



Binary Trees

A binary tree is **a finite set** of nodes that is either empty or consists of a **root** and **two** disjoint binary trees called the left subtree and the right subtree.

1. Any tree **can** be transformed into a binary tree.

   by left child-right sibling representation

2. The left subtree and the right subtree are **distinguished**.

ADT Binary Tree

objects:

An empty set of node

A finite set of nodes consisting of :

1. *a root node*
2. *left Binary_Tree*
3. *right Binary_Tree.*

functions:

Bintree Create()

Boolean IsEmpty(bt)

BinTree MakeBT(bt1, item, bt2)

Bintree Lchild(bt)

element Data(bt)

Bintree Rchild(bt)

Abstract Data Type Binary Tree

ADT *Binary_Tree*(abbreviated BinTree) is

objects: a finite set of nodes either empty or consisting of a root node, left Binary_Tree, and right Binary_Tree.

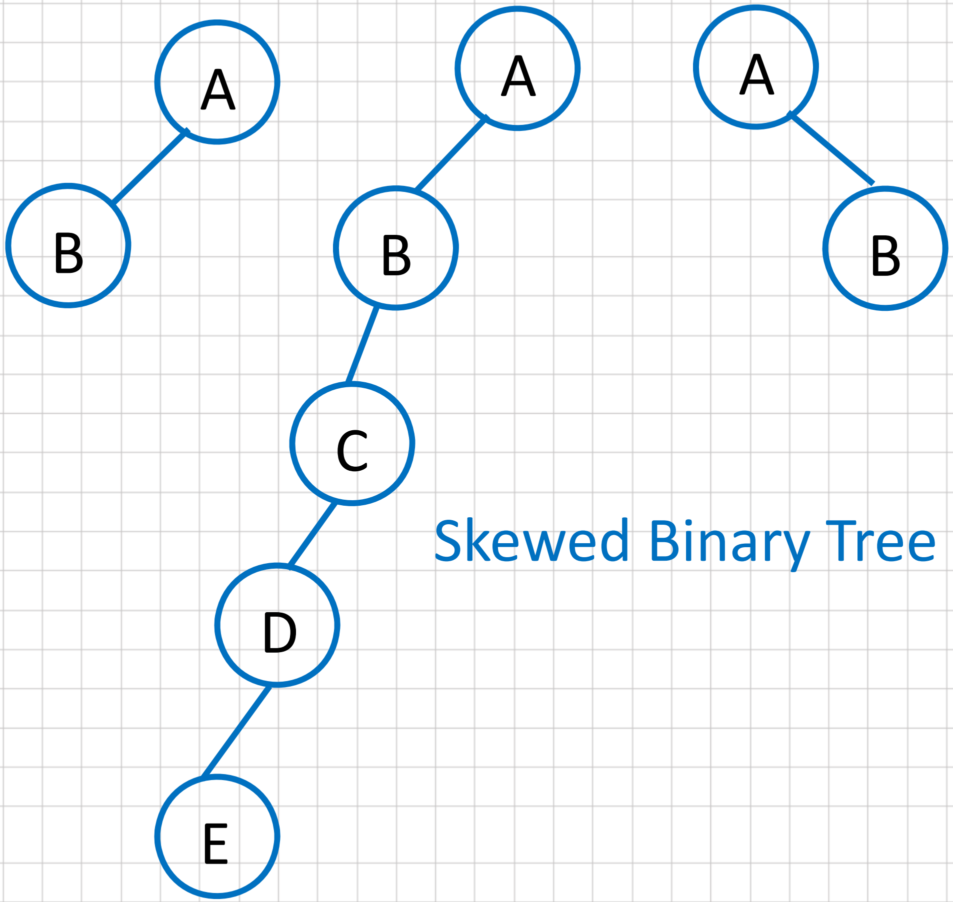
functions:

for all bt, bt1, bt2 $\in$ BinTree, item $\in$ element

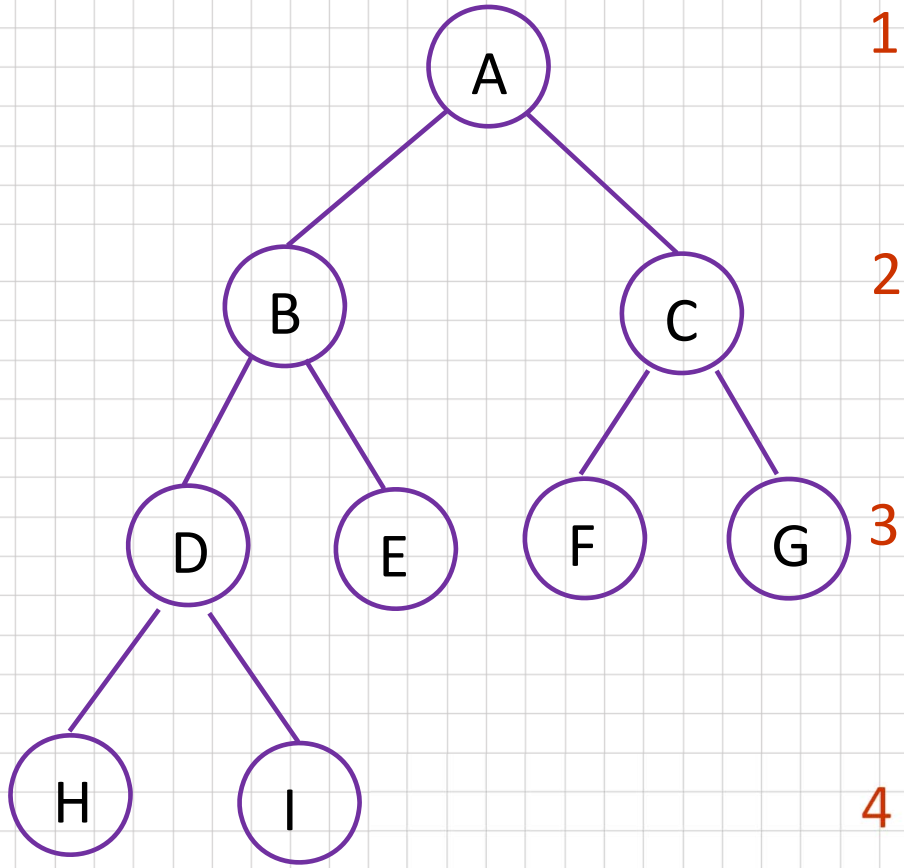
Bintree Create()	::= creates an empty binary tree
Boolean IsEmpty(bt)	::= **if** (bt==empty binary tree) **return** TRUE else return FALSE

BinTree MakeBT(bt1, item, bt2)	::= **return** a binary tree whose left subtree is bt1, whose right subtree is bt2, and whose root node contains the data item
Bintree Lchild(bt)	::= **if** (IsEmpty(bt)) **return** error **else return** the left subtree of bt
element Data(bt)	::= **if** (IsEmpty(bt)) **return** error **else return** the data in the root node of bt
Bintree Rchild(bt)	::= **if** (IsEmpty(bt)) **return** error **else return** the right subtree of bt

Samples of Trees



Skewed Binary Tree

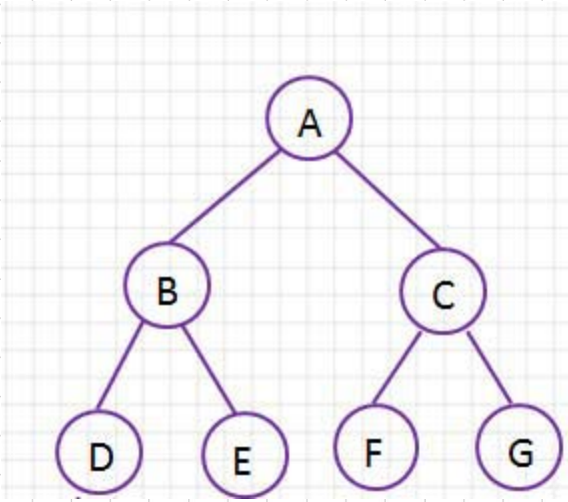


Complete Binary Tree

Maximum Number of Nodes in BT

The **maximum number** of nodes on level **i** of a binary tree is 2^{i-1}, i>=1.

The maximum nubmer of nodes in a binary tree of depth k is $2^k - 1$, k>=1.



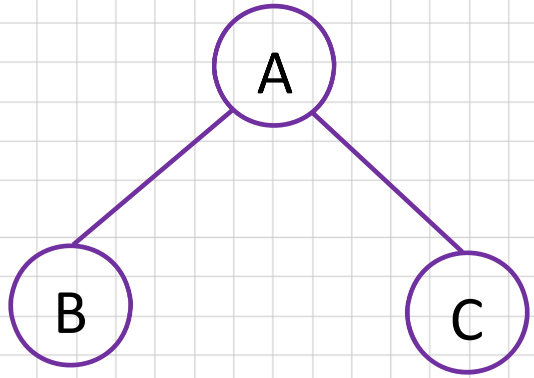
1
2
3

Prove by induction.

$$\sum_{i=1}^{k} 2^{i-1} = 2^k - 1$$

Relations between Number of Leaf Nodes and Nodes of Degree 2

For any nonempty binary tree, T, if n0 is the number of leaf nodes and n2 the number of nodes of degree 2, then n0=n2+1



proof:

Let n and B denote the total number of nodes & branches in T.

Let $n0$, $n1$, $n2$ represent the nodes with no children, single child, and two children respectively.

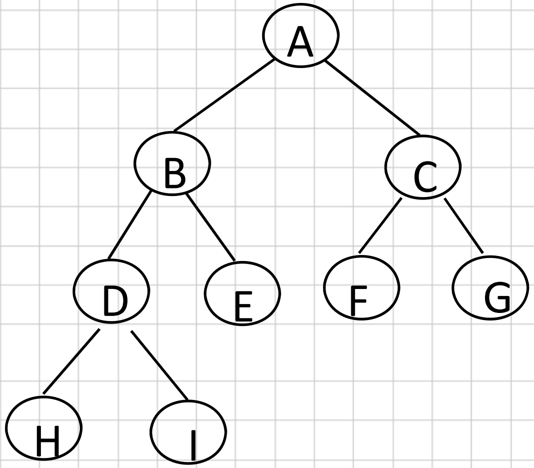
$n= n0+n1+n2$, $B+1=n$, $B=n1+2n2$ ==> $n1+2n2+1= n$,

$n1+2n2+1= n0+n1+n2$ ==> $n0=n2+1$

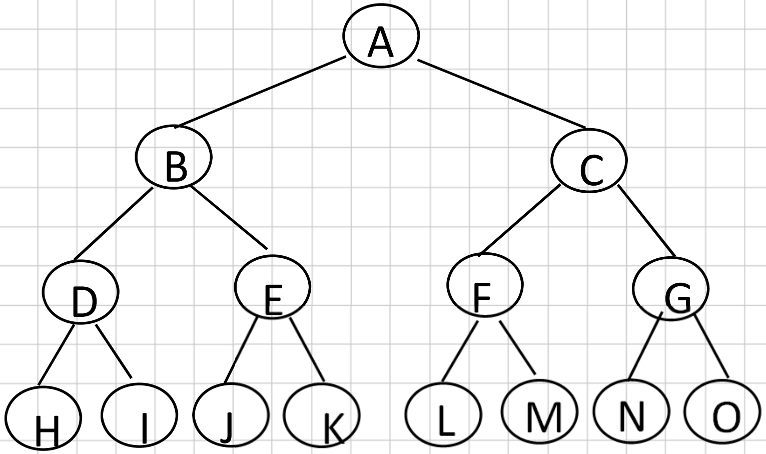
Full BT VS Complete BT

A **full binary tree** of depth k is a binary tree of depth k **having 2^k -1 nodes**, k>=0.

A binary tree with n nodes and depth k is complete *iff* its nodes correspond to the nodes numbered from 1 to n in the full binary tree of depth k.



Complete binary tree



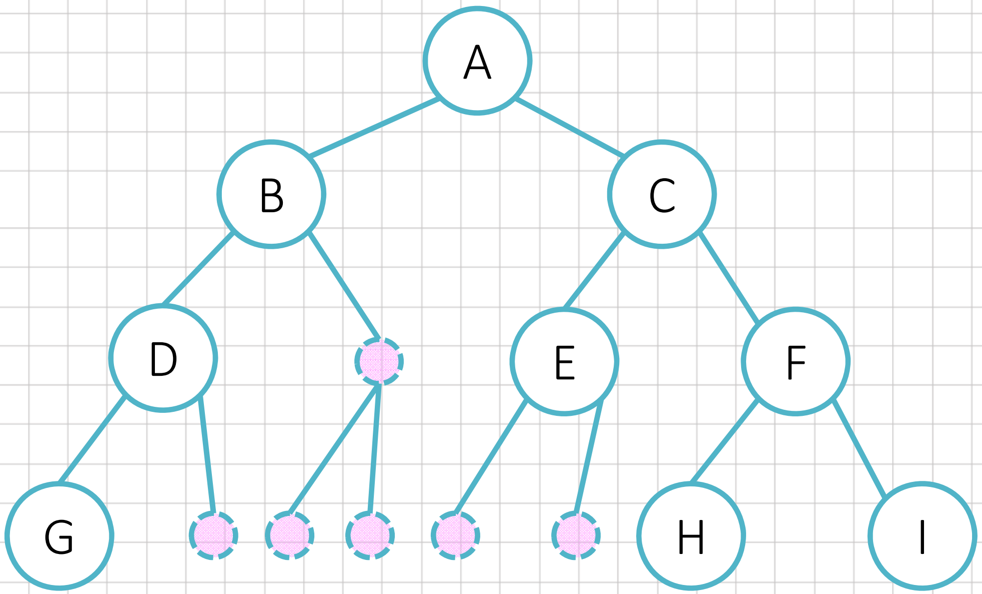
Full binary tree of depth 4

Binary Tree Representations

If **a complete binary tree** with n nodes (depth = **log n + 1**) is represented sequentially, then for any node with index i, 1<=i<=n, we have:

- parent(i) is at i/2 if i!=1. If i=1, i is at the root and has no parent.
- left_child(i) is at 2i if 2i<=n. If 2i>n, then i has no left child.
- right_child(i) is at 2i+1 if 2i +1 <=n. If 2i +1 >n, then i has no right child.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	B	C	D		E	F	G						H	I



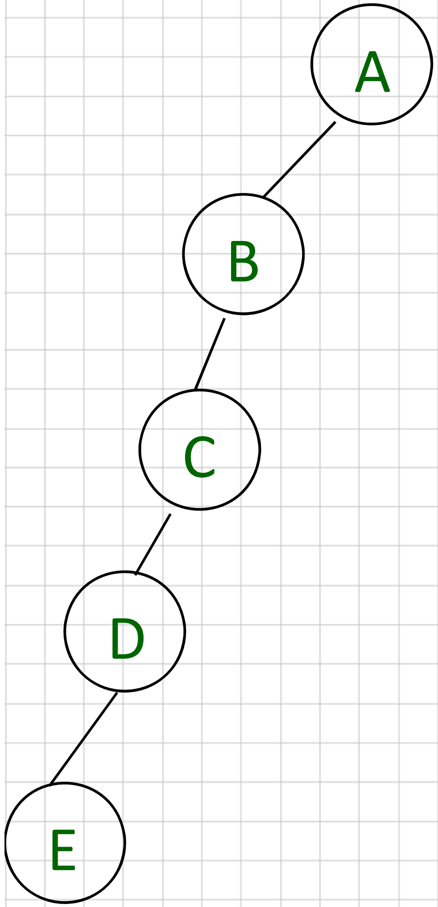
由上至下，
由左至右編號

Sequential Representation (array)

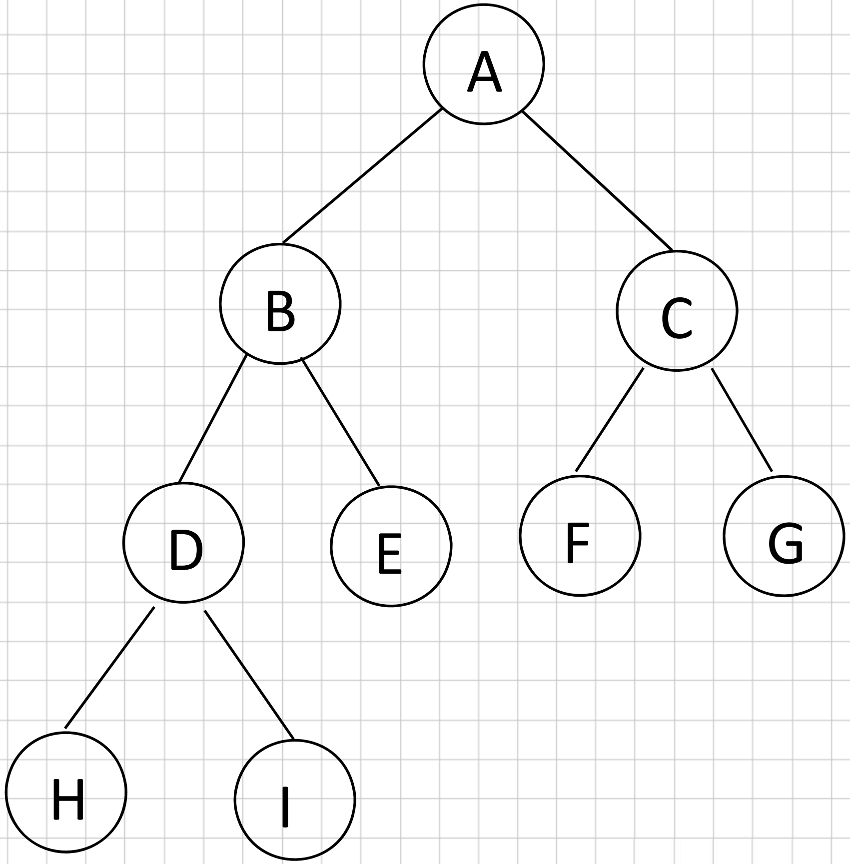
(+) Easy to find the parent, left child, and right child of a node.

(-) Waste space if a tree is not complete.
(-) insertion/deletion problem

- parent(i) is at i/2
- left_child(i) is at 2i
- right_child(i) is at 2i + 1



[1]	A
[2]	B
[3]	--
[4]	C
[5]	--
[6]	--
[7]	--
[8]	D
[9]	--
.	.
[16]	E

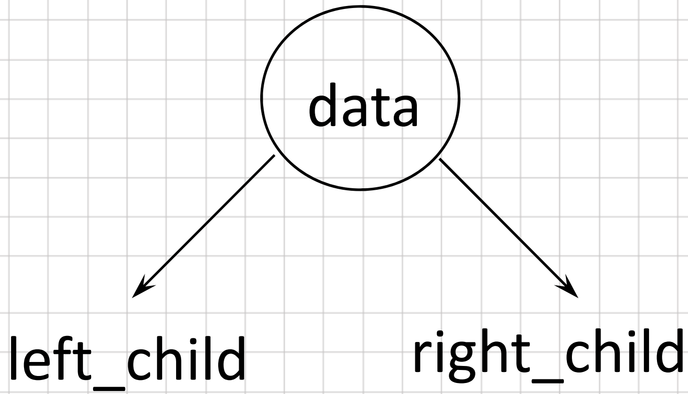


[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

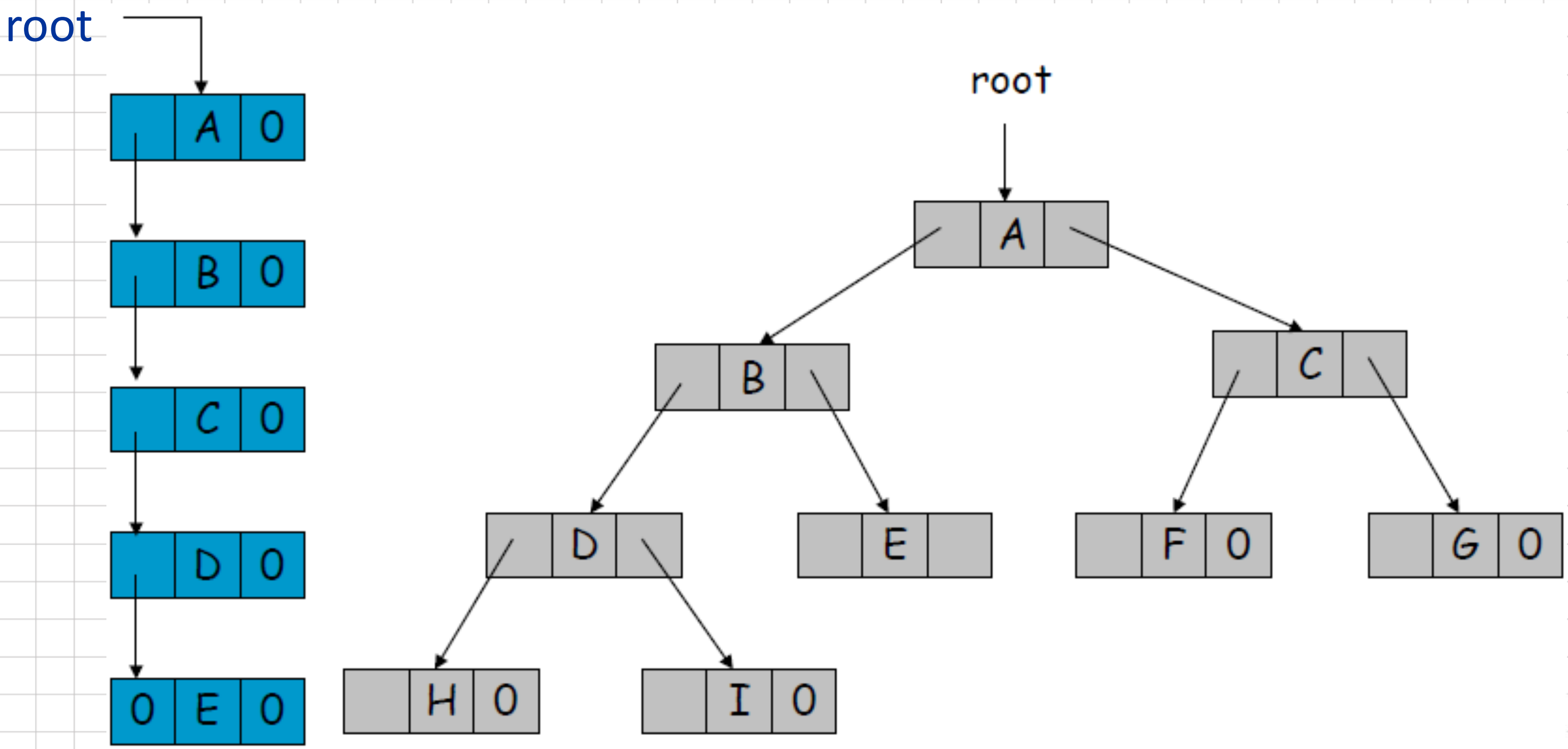
Linked Representation

```
typedef struct node *tree_pointer;
typedef struct node {
    int data;
    tree_pointer left_child, right_child;
};
```

left_child	data	right_child



Linked List Representation For The Binary Trees



Binary Tree Representation Summary

	Array representation	Linked representation
Determine the locations of the parent, left child and right child	Easy	Difficult
Space overhead	Much	Little
Insertion and deletion	Difficult	easy

Binary Tree Traversals

Traversal: Visiting each node **exactly** once

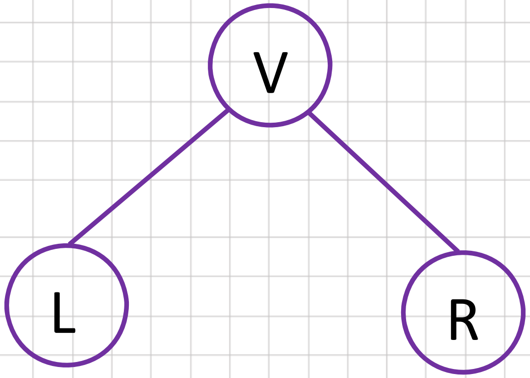
Let L, V, and R stand for moving left, visiting the node, and moving right.

There are six possible combinations of traversal

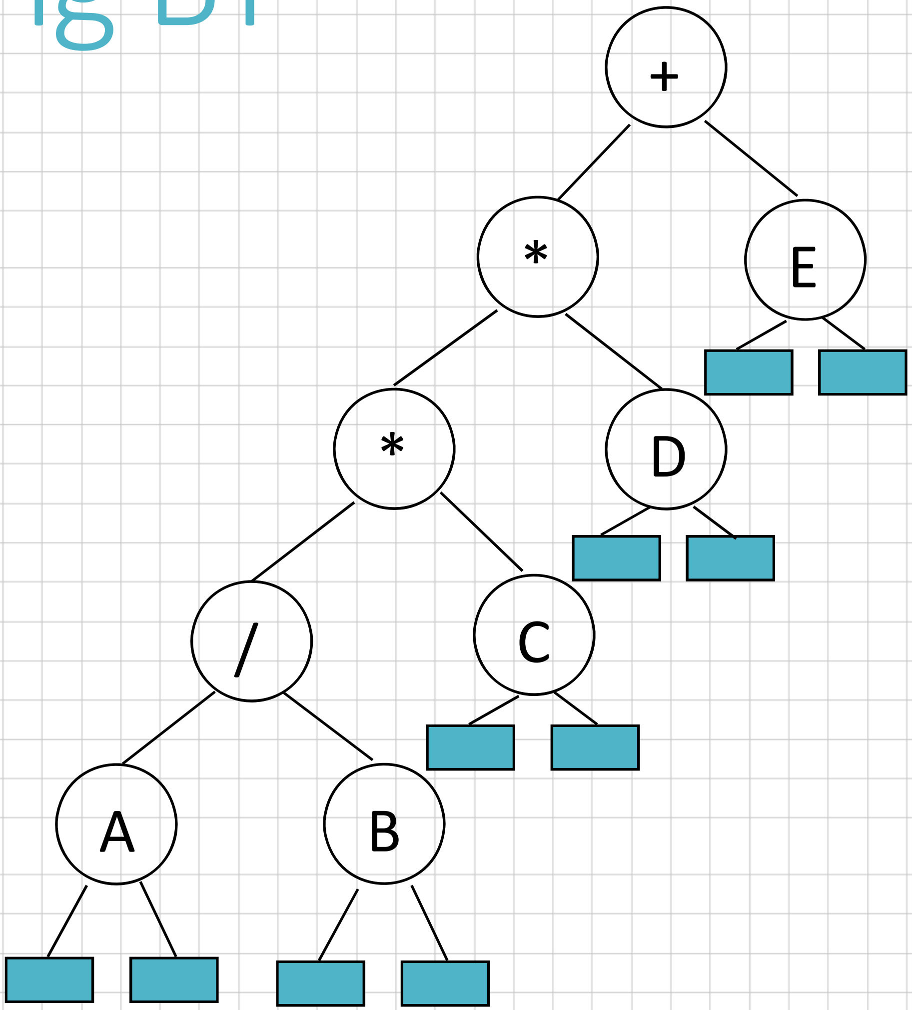
- LVR, LRV, VLR, VRL, RVL, RLV

Adopt convention that we traverse left before right, only 3 traversals remain

- LVR, LRV, VLR
- inorder, postorder, preorder



Arithmetic Expression Using BT



inorder traversal

A / B * C * D + E

infix expression

preorder traversal

+ * * / A B C D E

prefix expression

postorder traversal

A B / C * D * E +

postfix expression

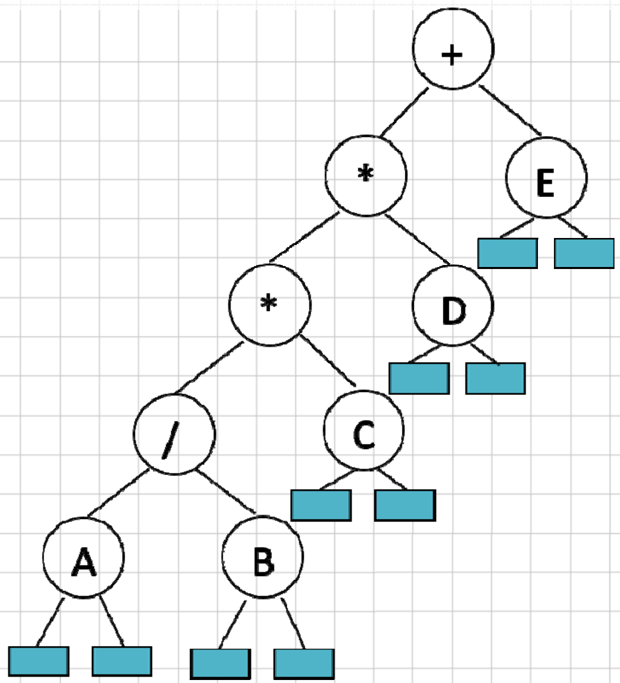
level order traversal

+ * E * D / C A B

Inorder Traversal (recursive version)

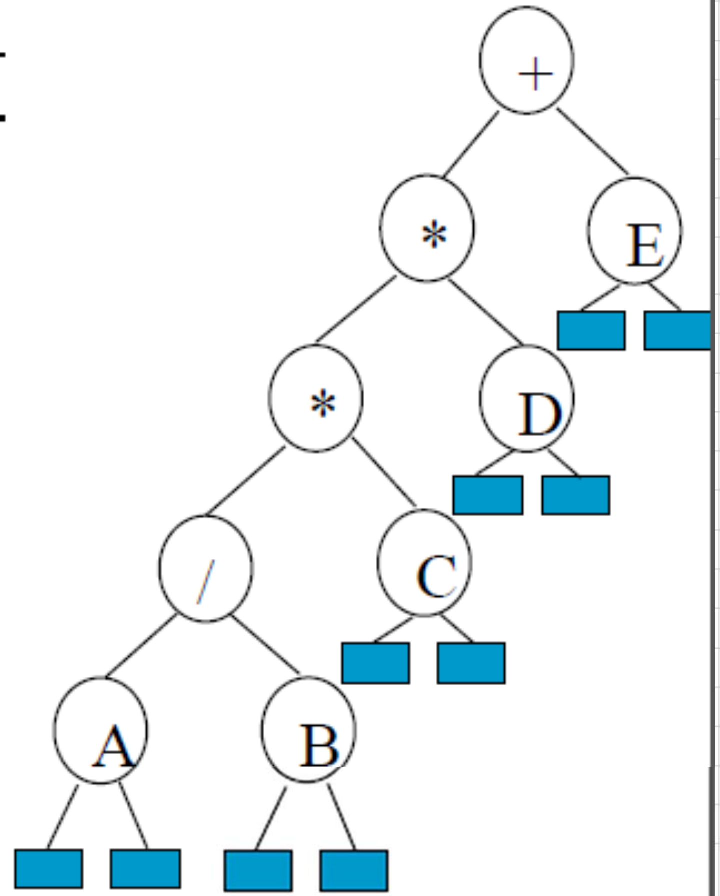
```
void inorder(tree_pointer ptr)
{
    if (ptr) {
        inorder(ptr->left_child);
        printf("%d", ptr->data);
        indorder(ptr->right_child);
    }
}
```

A / B * C * D + E



Trace Operations of Inorder Traversal

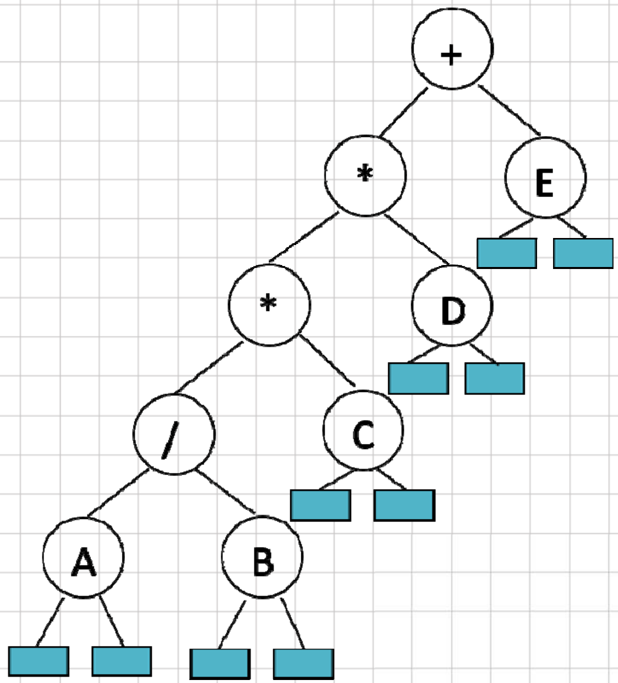
Call of inorder	Value in root	Action	Call of inorder	Value in root	Action
1	+		11	C	
2	*		12	NULL	
3	*		11	C	cout
4	/		13	NULL	
5	A		2	*	cout
6	NULL		14	D	
5	A	cout	15	NULL	
7	NULL		14	D	cout
4	/	cout	16	NULL	
8	B		1	+	cout
9	NULL		17	E	
8	B	cout	18	NULL	
10	NULL		17	E	cout
3	*	cout	19	NULL	



Preorder Traversal (recursive version)

```
void preorder(tree_pointer ptr)
{
    if (ptr) {
        printf("%d", ptr->data);
        preorder(ptr->left_child);
        predorder(ptr->right_child);
    }
}
```

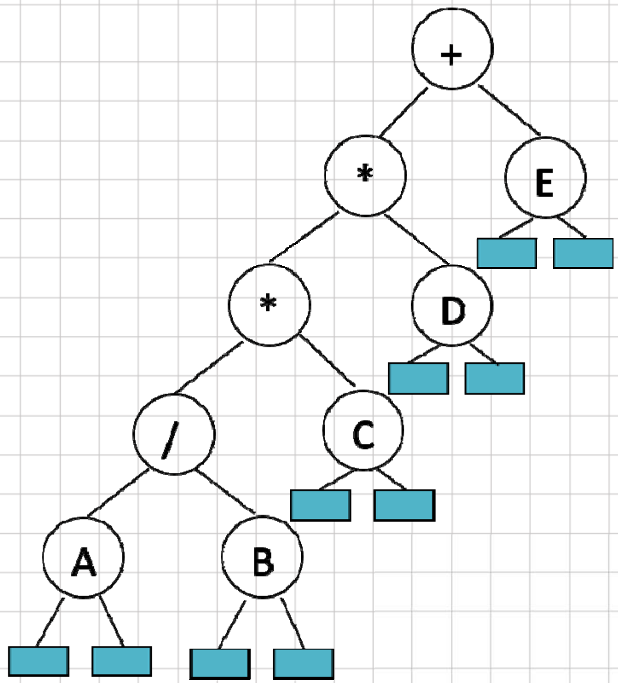
+ * * / A B C D E



Postorder Traversal (recursive version)

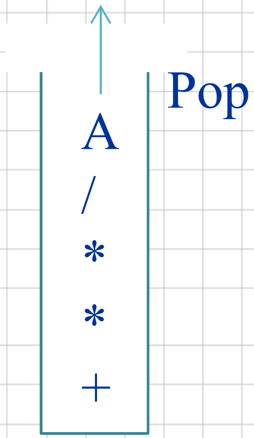
```
void postorder(tree_pointer ptr)
{
    if (ptr) {
        postorder(ptr->left_child);
        postdorder(ptr->right_child);
        printf("%d", ptr->data);
    }
}
```

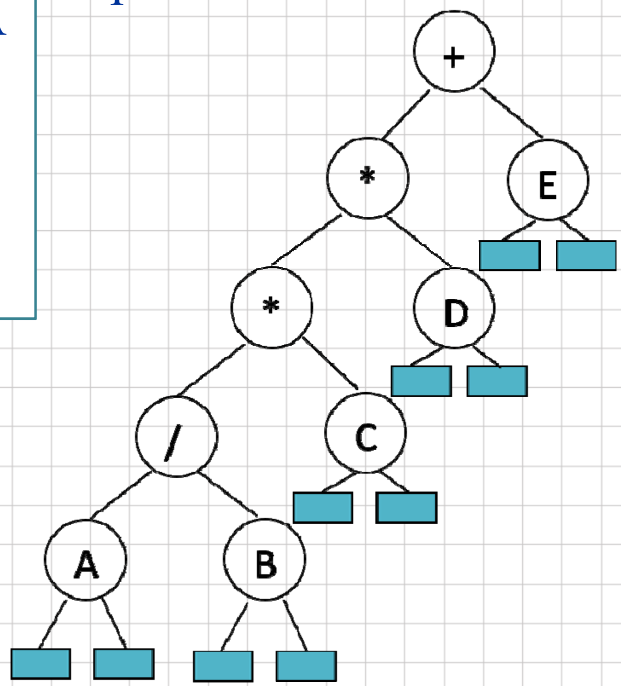
A B / C * D * E +



Iterative Inorder Traversal (using stack)

```
void iter_inorder(tree_pointer node){
  int top= -1; /* initialize stack */
  tree_pointer stack[MAX_STACK_SIZE];
  for (;;) {
   for (; node; node=node->left_child)
     add(&top, node);/* add to stack */
   node= delete(&top); /* delete from stack */
   if (!node) break; /* empty stack */
   printf("%D", node->data);
   node = node->right_child;
  }
}
```



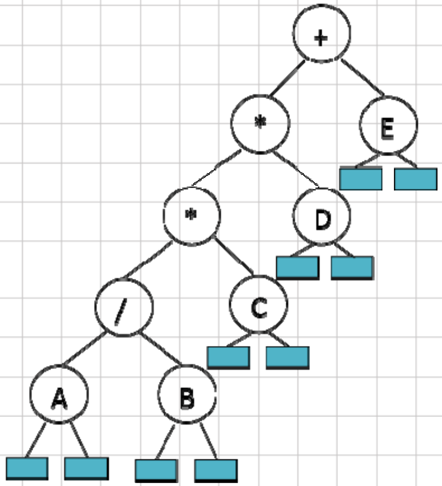


O(n)

A / B * C * D + E

Level Order Traversal(using queue)

```
void level_order(tree_pointer ptr)
{
  int front = rear = 0;
  tree_pointer queue[MAX_QUEUE_SIZE];
  if (!ptr) return; /* empty queue */
  addq(front, &rear, ptr);
  for (;;) {
    ptr = deleteq(&front, rear);
    if (ptr) {
      printf("%d", ptr->data);
      if (ptr->left_child)
        addq(front, &rear, ptr->left_child);
      if (ptr->right_child)
        addq(front, &rear, ptr->right_child);
    }
    else break;
  }
}
```

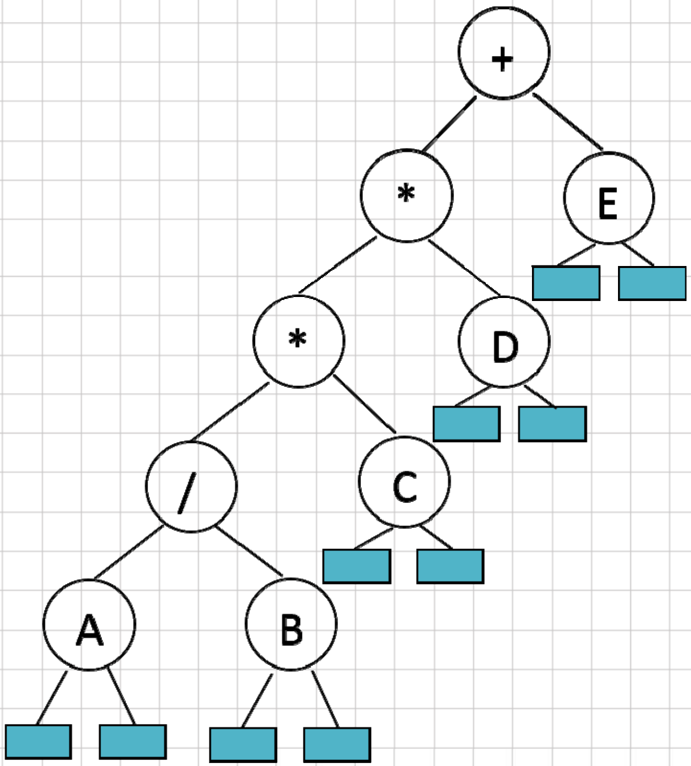


+ * E * D / C A B

Copying Binary Trees

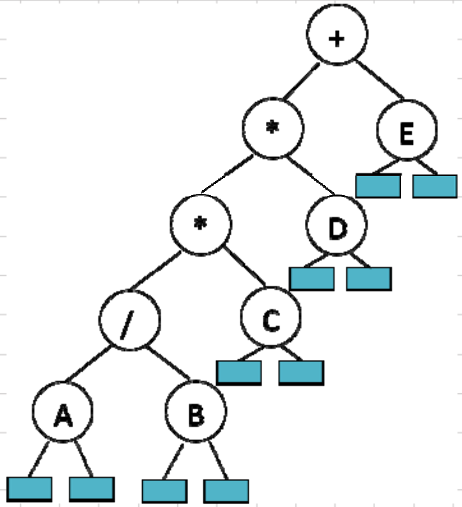
```
tree_poointer copy(tree_pointer original){
tree_pointer temp;
if (original) {
    temp=(tree_pointer) malloc(sizeof(node));
    if (IS_FULL(temp)) {
      fprintf(stderr, “the memory is full\n”);
      exit(1);
    }
    temp->left_child=copy(original->left_child);
    temp->right_child=copy(original->right_child);
    temp->data=original->data;
    return temp;
}
return NULL;
}
```

postorder



the same topology and data

Equality of Binary Trees




```
int equal(tree_pointer first, tree_pointer second)
{
/* function returns FALSE if the binary trees first and
   second are not equal, otherwise it returns TRUE */
  return ((!first && !second)
            || (first && second && (first->data == second->data)
            &&  equal(first->left_child, second->left_child)
            &&  equal(first->right_child, second->right_child)))
}
```

Propositional Calculus Expression

A variable is an expression.

If x and y are expressions, then ¬x, x∧y, x∨y are expressions.

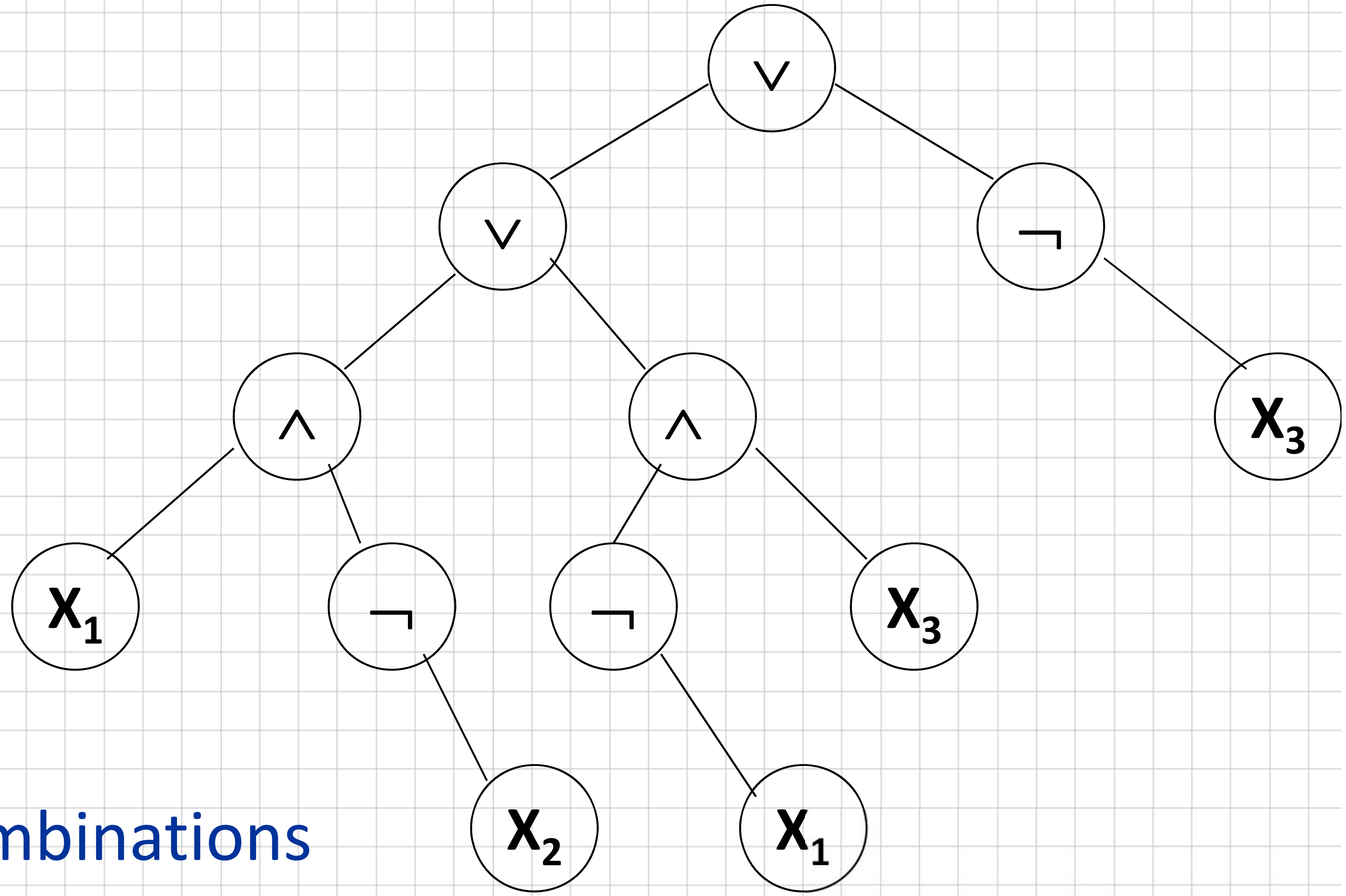
Parentheses can be used to alter the normal order of evaluation (¬ > ∧ > ∨).

Example: $x1 \vee (x2 \wedge \neg x3)$

Satisfiability problem: Is there an assignment to make an expression true?

$$(x_1 \wedge \neg x_2) \vee (\neg x_1 \wedge x_3) \vee \neg x_3$$

(t,t,t)
(t,t,f)
(t,f,t)
(t,f,f)
(f,t,t)
(f,t,f)
(f,f,t)
(f,f,f)



2^n possible combinations
for n variables

postorder traversal (postfix evaluation)

node structure

left_child	*data*	*value*	*right_child*

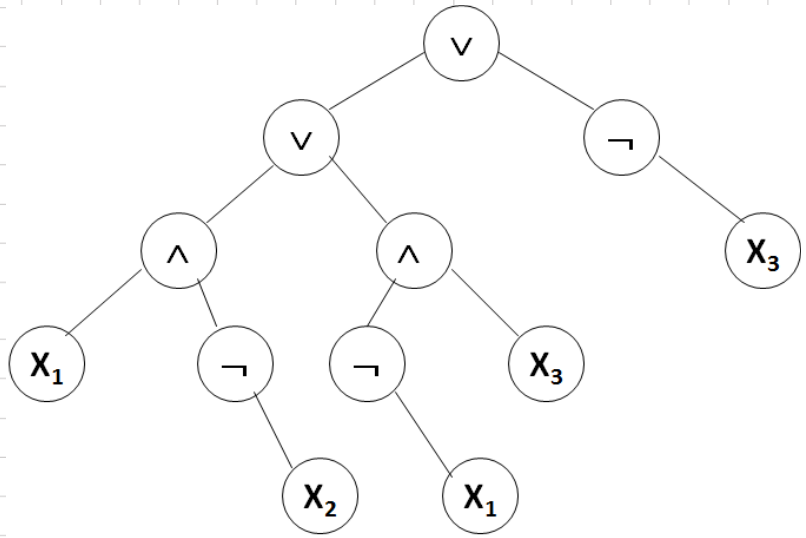
```
typedef   emun {not, and, or, true, false } logical;
typedef   struct node *tree_pointer;
typedef   struct node {
          tree_pointer  left_child;
          logical        data;
          short int      value;
          tree_pointer  right_child;
          } ;
```

First version of satisfiability algorithm

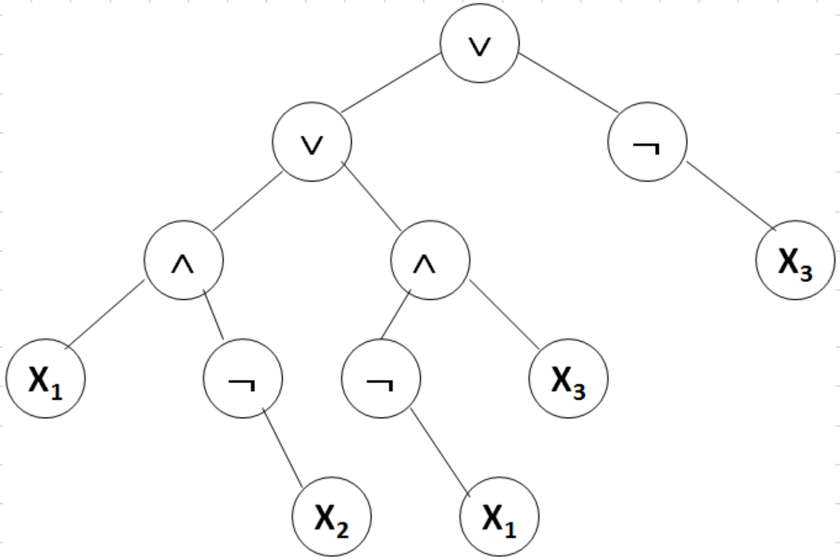
```
for (all 2^n possible combinations) {
      generate the next combination;
      replace the variables by their values;
      evaluate root by traversing it in postorder;
      if (root->value) {
            printf(<combination>);
            return;
  }
}
printf("No satisfiable combination \n");
```

Post-order-eval function

```
void post_order_eval(tree_pointer node)
{
/* modified post order traversal to evaluate a propositional
calculus tree */
   if (node) {
      post_order_eval(node->left_child);
      post_order_eval(node->right_child);
      switch(node->data) {
        case not:  node->value =
              !node->right_child->value;
              break;
```



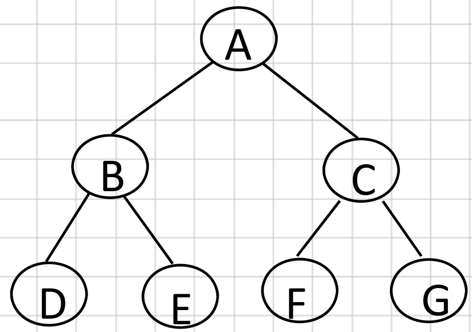
```
case and:    node->value =
          node->right_child->value &&
          node->left_child->value;
          break;
    case or:      node->value =
          node->right_child->value | |
          node->left_child->value;
          break;
     case true:   node->value = TRUE;
          break;
     case false:  node->value = FALSE;
    }
  }
}
```



Threaded Binary Trees

Too many null pointers in current representation of binary trees

- n: number of nodes
- number of non-null links: n - 1
- total links: 2n
- null links: 2n - (n - 1) = n + 1



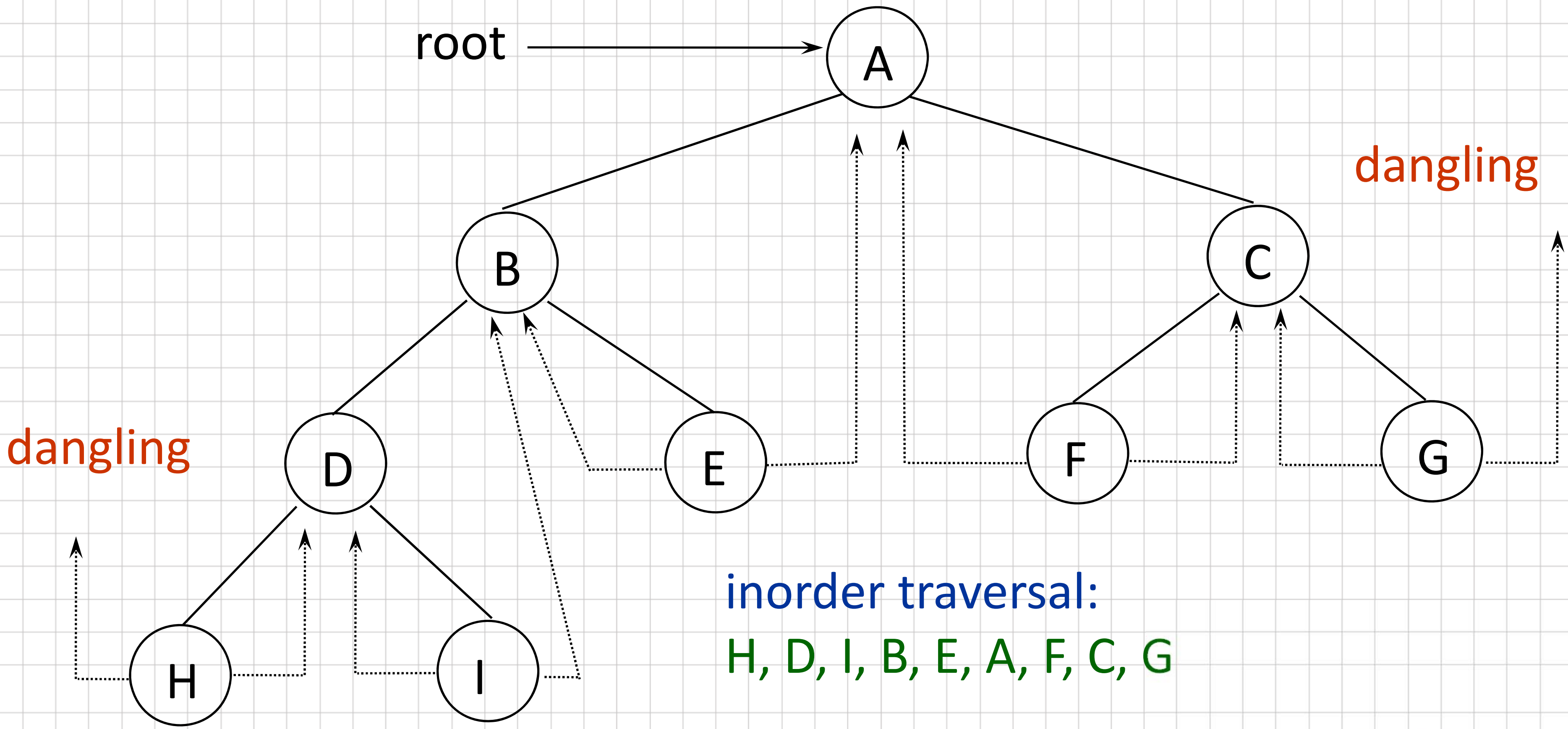
Replace these null pointers with some useful “threads”.

Threaded Binary Trees *(Continued)*

If `ptr->left_child` is null,
replace it with a pointer to the node that would be visited *before* `ptr` in an ***inorder traversal***

If `ptr->right_child` is null,
replace it with a pointer to the node that would be visited *after* `ptr` in an ***inorder traversal***

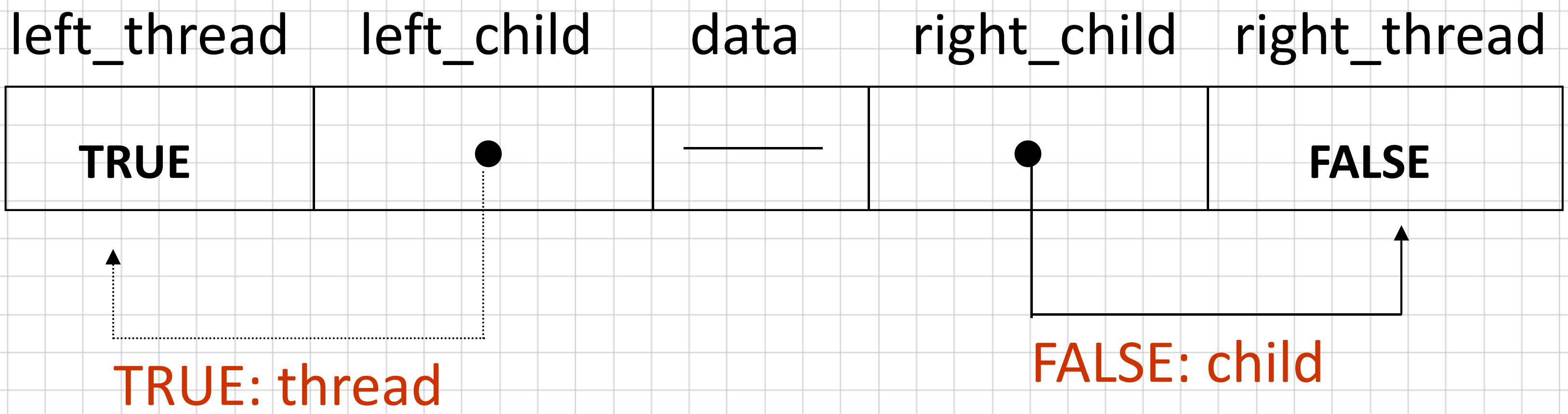
A Threaded Binary Tree



inorder traversal:

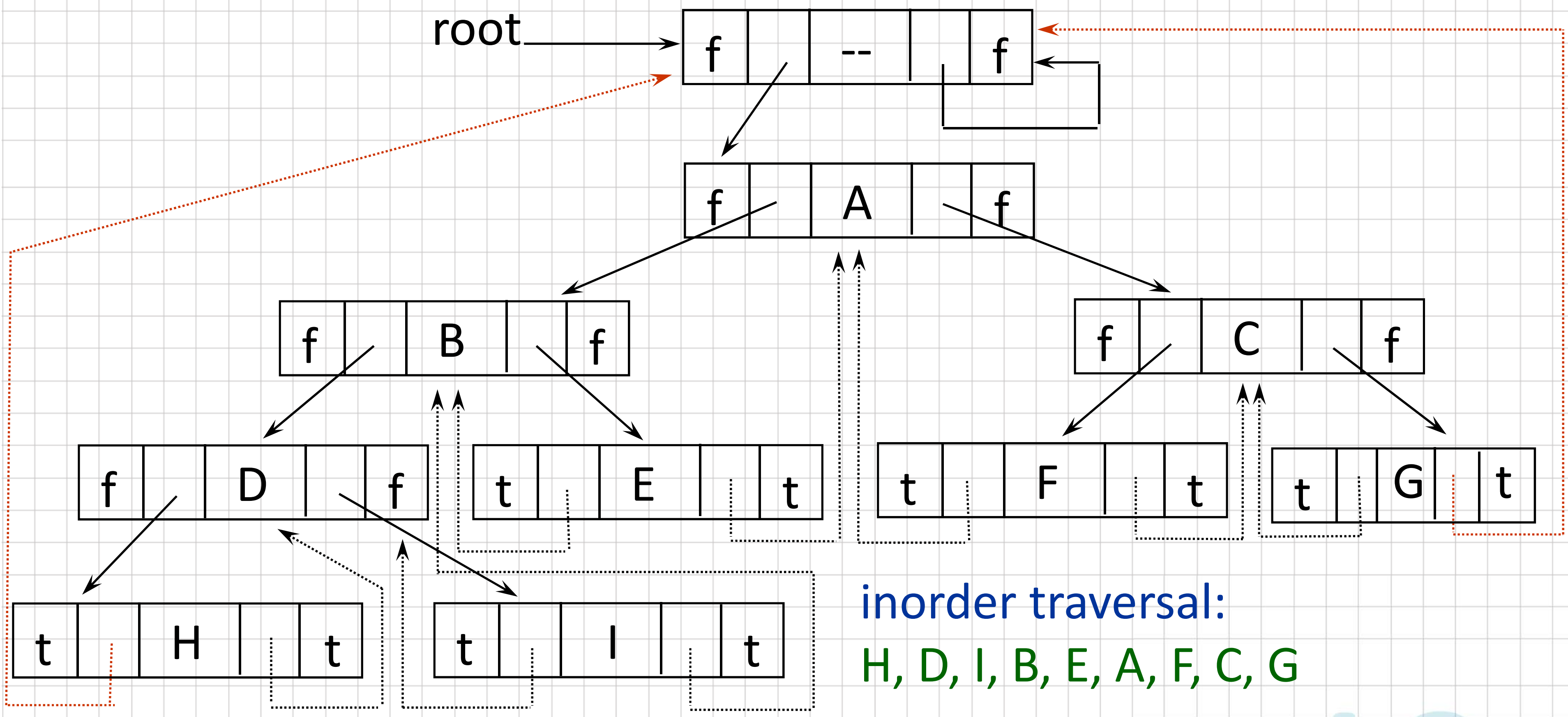
H, D, I, B, E, A, F, C, G

Data Structures for Threaded BT




```
typedef struct threaded_tree
  *threaded_pointer;
typedef struct threaded_tree {
    short int left_thread;
    threaded_pointer left_child;
    char data;
    threaded_pointer right_child;
    short int right_thread;   };
```

Memory Representation of A Threaded BT



inorder traversal:

H, D, I, B, E, A, F, C, G

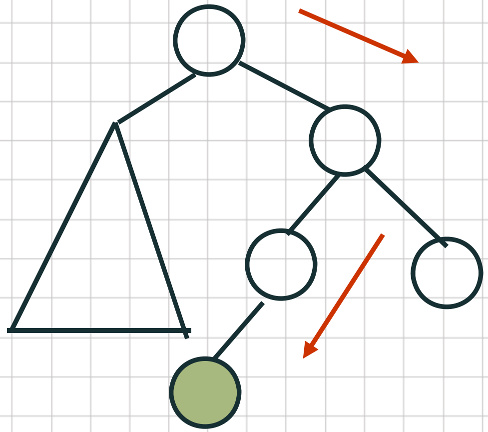
Inorder Traversal of Threaded BT

```
void tinorder(threaded_pointer tree)
{
   /* traverse the threaded binary tree inorder */
   threaded_pointer temp = tree;
   for (;;) {
      temp = insucc(temp);
      if (temp==tree) break;
      printf("%3c", temp->data);
   }
}
```

O(n)

Next Node in Threaded BT

```
threaded_pointer insucc(threaded_pointer tree)
{
  threaded_pointer temp;
  temp = tree->right_child;
  if (!tree->right_thread)
    while (!temp->left_thread)
      temp = temp->left_child;
  return temp;
}
```



Inserting Nodes into Threaded BTs

Insert **`child`** as the right child of node **`parent`**

1. change `parent->right_thread` to FALSE
2. set `child->left_thread` and `child->right_thread` to TRUE
3. set `child->left_child` to point to `parent`
4. set `child->right_child` to `parent->right_child`
5. change `parent->right_child` to point to `child`

```
3. child->left_child= parent
4. child->right_child=parent->right_child
5. parent->right_child = child
```

Examples

empty Case (a)

Insert a node D as a right child of B.

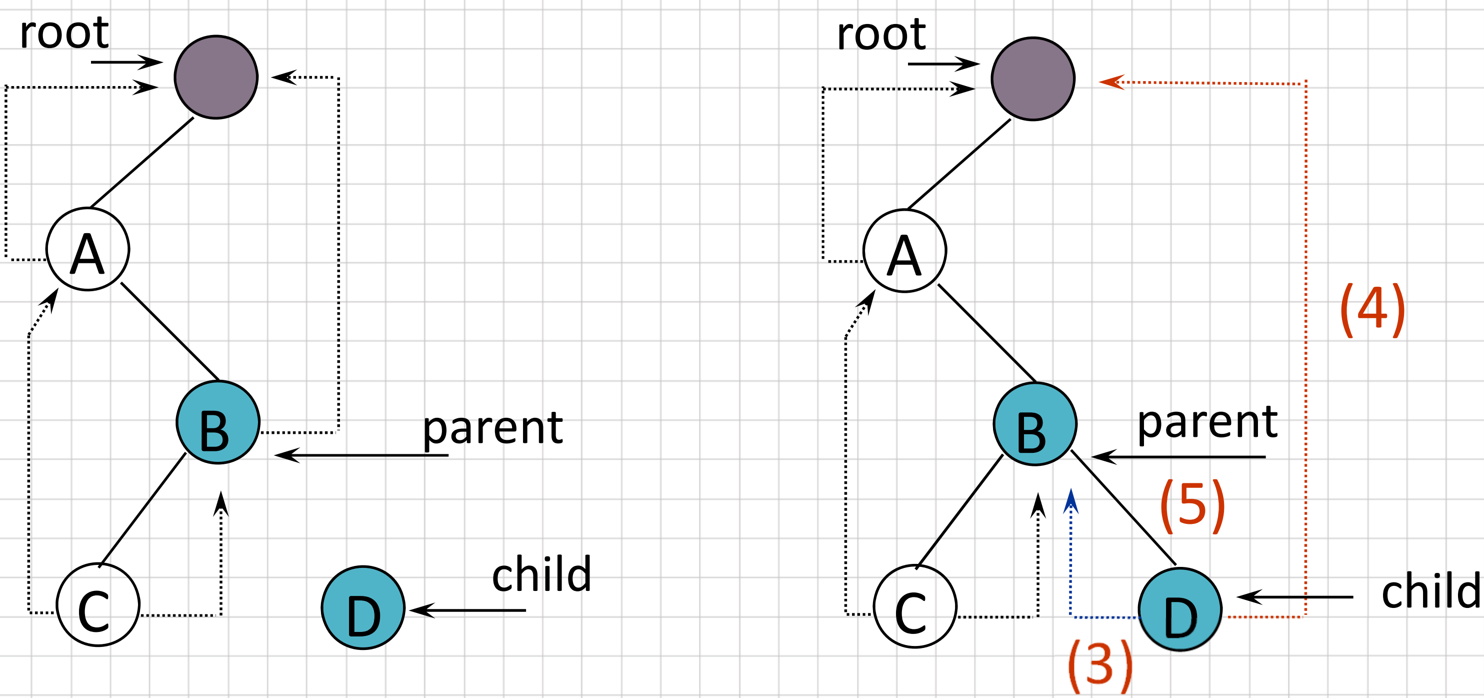
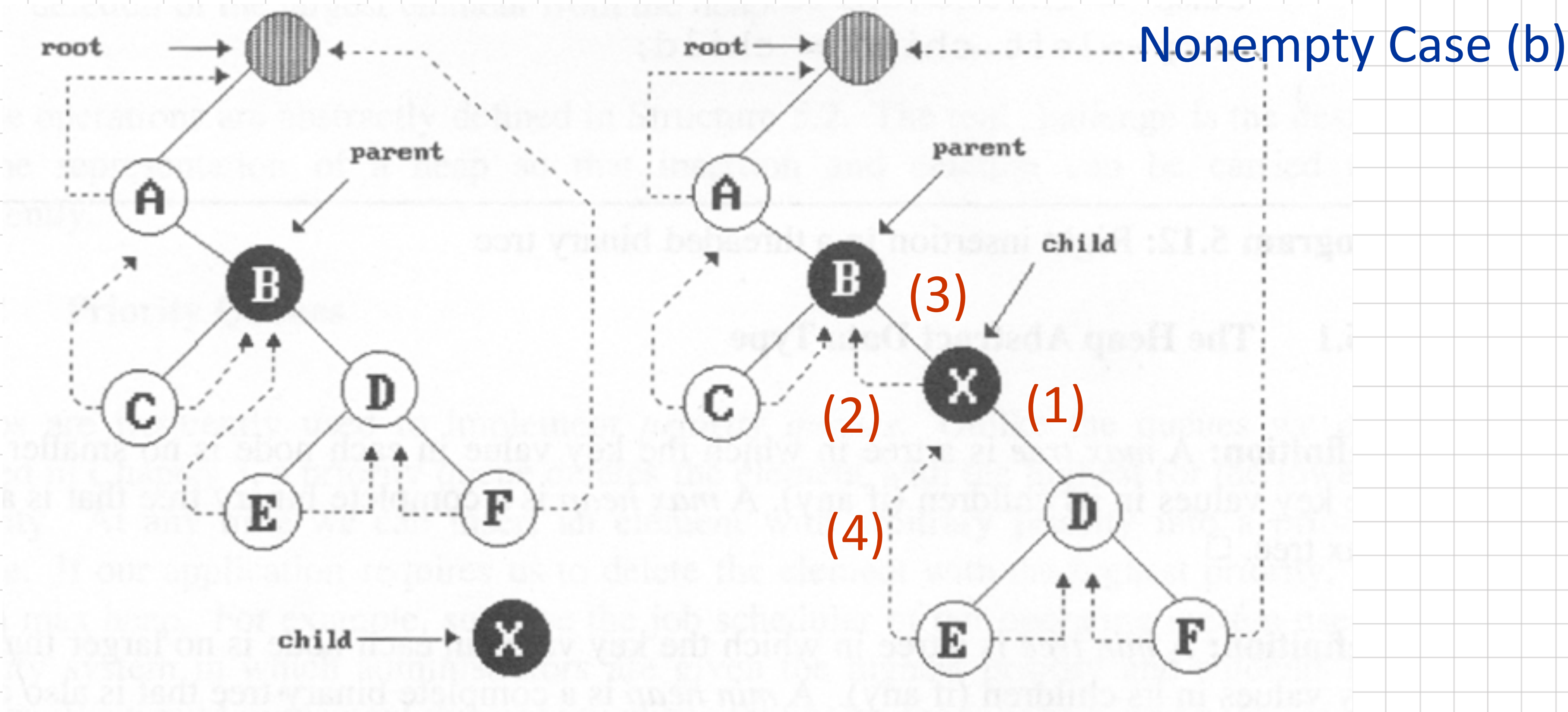


Figure 5.24: Insertion of child as a right child of parent in a threaded binary tree (p.222)



1. child->right_child= parent->right_child
2. Child->left_child = parent, child->left_thread = true
3. parent->right_child = child
4. temp = insucc(child), temp->left_child = child
   temp->left_thread = true

Right Insertion in Threaded BTs

```
void insert_right(threaded_pointer parent, threaded_pointer child)
{
  threaded_pointer temp;
  child->right_child = parent->right_child;
  child->right_thread = parent->right_thread;
  child->left_child = parent;              case (a)
  child->left_thread = TRUE;
  parent->right_child = child;
  parent->right_thread = FALSE;
  if (!child->right_thread) {              case (b)
  temp = insucc(child);
    temp->left_child = child;
  }
}
```

max tree :

a tree in which the key value in each node is no smaller than the key values in its children.

max heap :

a complete binary tree that is also a max tree.

min tree :

a tree in which the key value in each node is no larger than the key values in its children.

min heap :

a complete binary tree that is also a min tree.

Operations:

creation(empty heap)
insertion(new element,heap)
deletion(largest element,heap)

HEAP

Chapter 5.6

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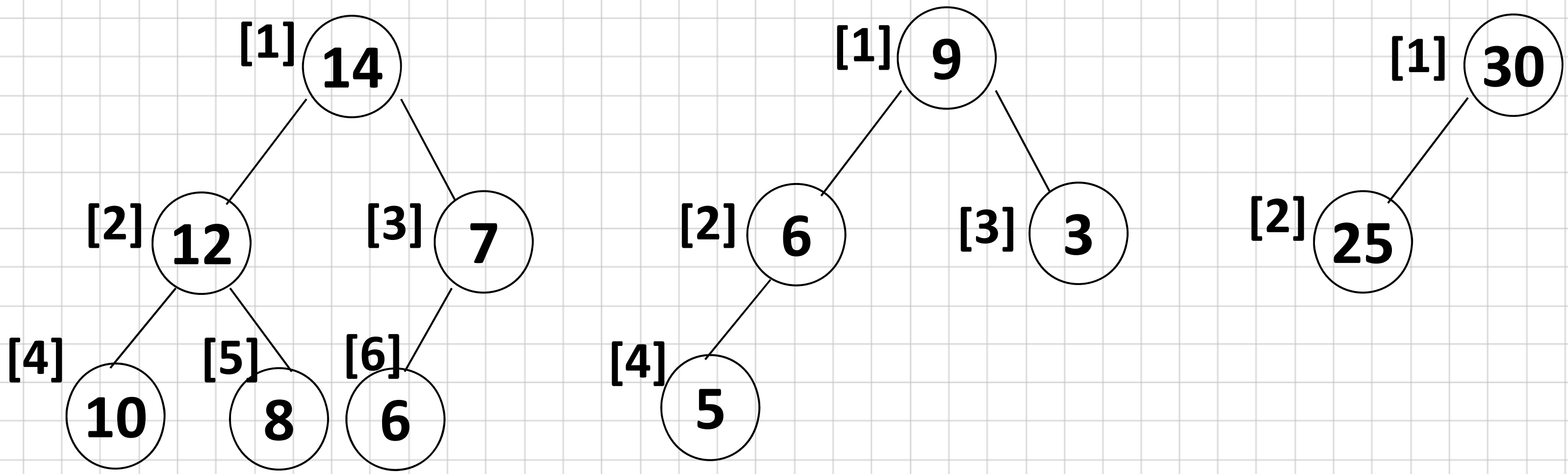
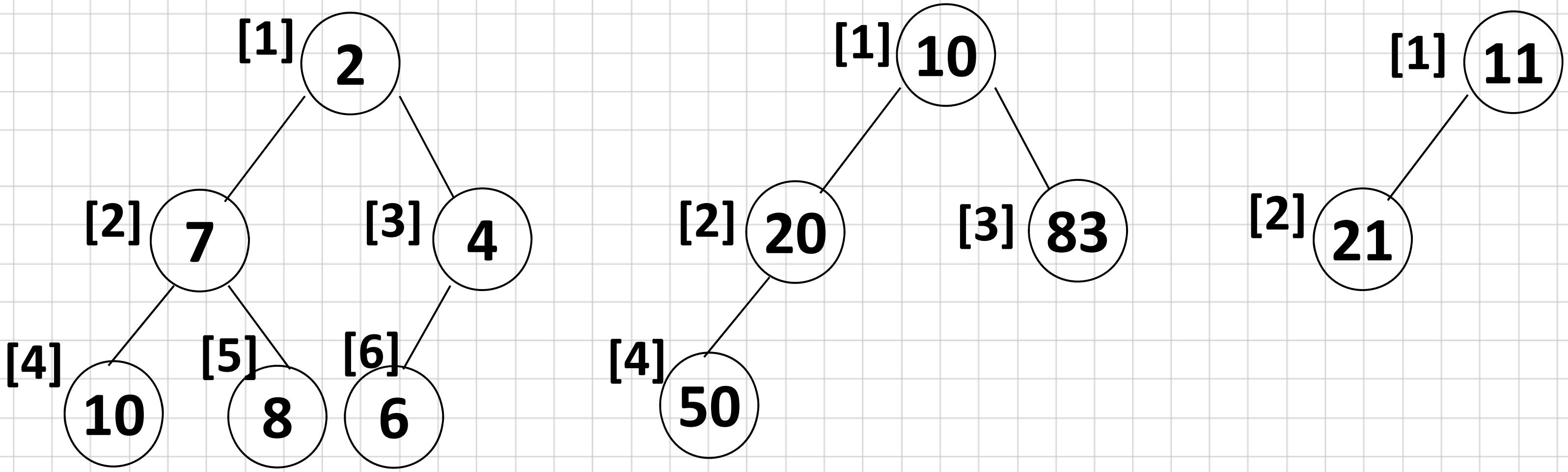
Figure 5.25: Max heaps (p.225)



Property:

The root of max heap (min heap) contains the largest (smallest).

Figure 5.26: Min heaps (p.225)



ADT for Max Heap

ADT MaxHeap

objects:

complete binary

the value in each node is at least as large as those in its children

functions:

MaxHeap Create(max_size)

Boolean HeapFull(heap, n)

MaxHeap Insert(heap, item, n)

Boolean HeapEmpty(heap, n)

Element Delete(heap,n)

ADT for Max Heap

ADT MaxHeap

objects:

a complete binary tree of n > 0 elements organized so that the value in each node is at least as large as those in its children

functions:

for all heap belong to MaxHeap, item belong to Element, n, max_size belong to integer

MaxHeap Create(max_size)

::= create an empty heap that can hold a maximum of max_size elements

Boolean HeapFull(heap, n)

::= if (n==max_size) **return** TRUE
else return FALSE

MaxHeap Insert(heap, item, n)
::= if (!HeapFull(heap,n)) insert item into heap and
return the resulting heap,
else return error

Boolean HeapEmpty(heap, n)
::= if (n>0) **return** FALSE
else return TRUE

Element Delete(heap,n)
::= if (!HeapEmpty(heap,n))
return one instance of the largest element in the heap and
remove it from the heap,
else return error

Application: priority queue

- machine service
  - amount of time (min heap)
  - amount of payment (max heap)
- factory
  - time tag

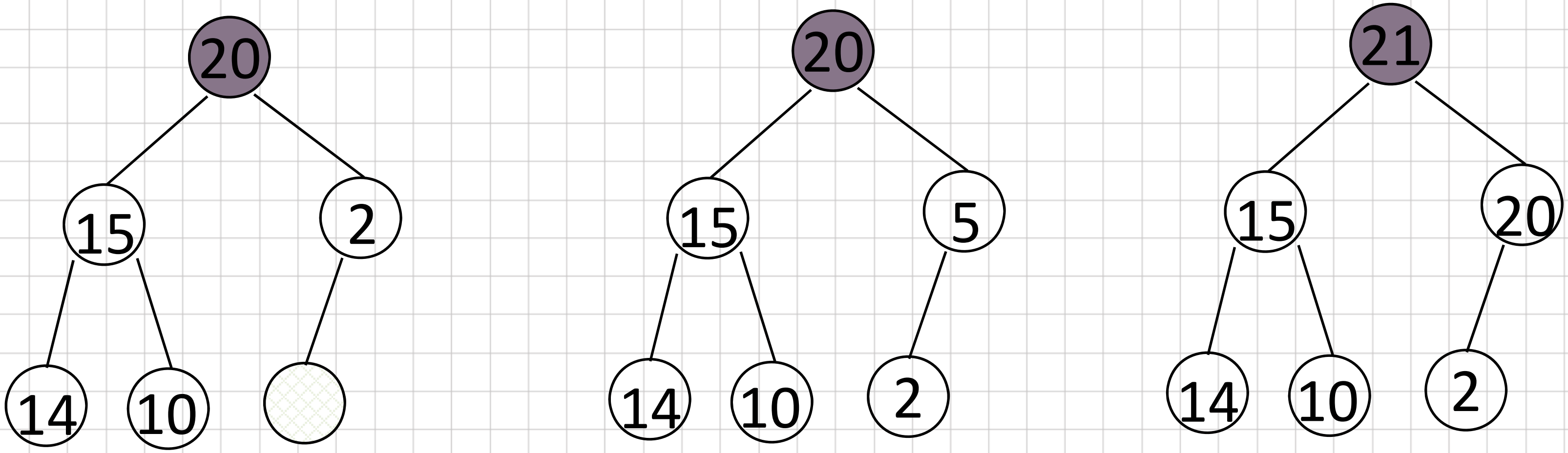
Data Structures for finding the min and max entry

1. unordered linked list
2. unordered array
3. sorted linked list
4. sorted array
5. heap

*Figure 5.27: Priority queue representations (p.221)

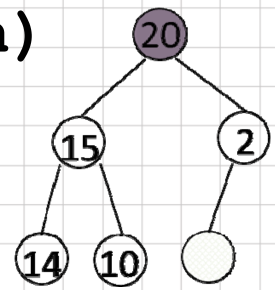
Representation	Insertion	Deletion
Unordered array	$\Theta(1)$	$\Theta(n)$
Unordered linked list	$\Theta(1)$	$\Theta(n)$
Sorted array	$O(n)$	$\Theta(1)$
Sorted linked list	$O(n)$	$\Theta(1)$
Max heap	$O(\log_2 n)$	$O(\log_2 n)$

Example of Insertion to Max Heap



initial location of new node   insert 5 into heap   insert 21 into heap

Insertion into a Max Heap



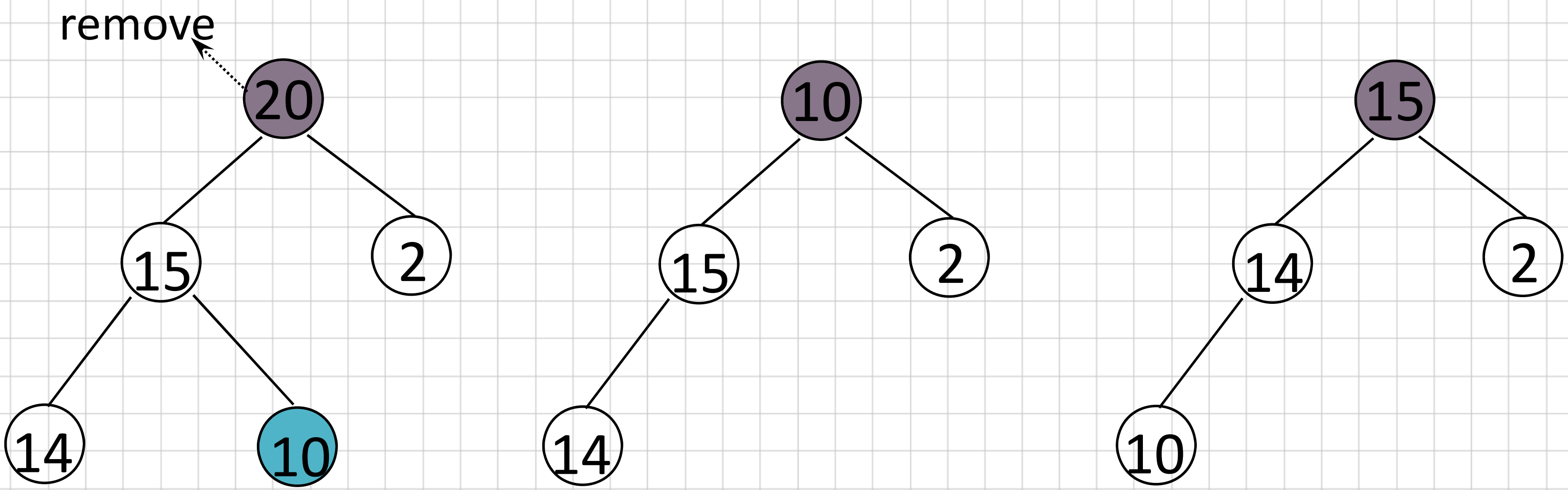
Insert 21

```
void insert_max_heap(element item, int *n)
{
  int i;
  if (HEAP_FULL(*n)) {
    fprintf(stderr, "the heap is full.\n");
    exit(1);
  }
  i = ++(*n);
  while ((i!=1)&&(item.key>heap[i/2].key)) {
    heap[i] = heap[i/2];
    i /= 2;
  }
  heap[i]= item;
}
```

$2^k-1=n \Longrightarrow k=\lceil \log_2(n+1) \rceil$

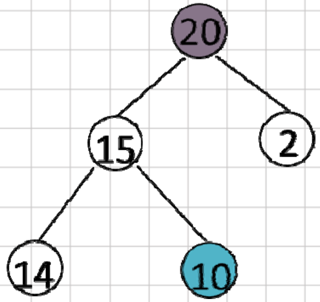
$O(\log_2 n)$

Example of Deletion from Max Heap



Deletion from a Max Heap

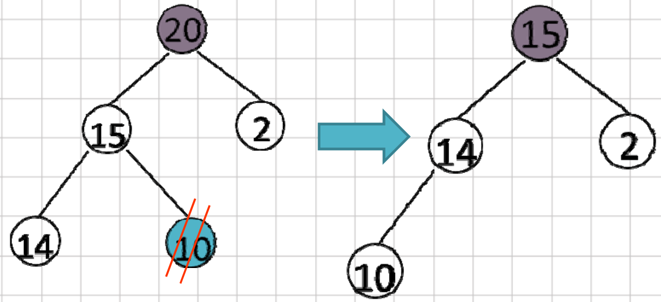
```
element delete_max_heap(int *n)
{
  int parent, child;
  element item, temp;
  if (HEAP_EMPTY(*n)) {
    fprintf(stderr, “The heap is empty\n”);
    exit(1);
  }
  /* save value of the element with the
     highest key */
  item = heap[1];
  /* use last element in heap to adjust heap */
  temp = heap[(*n)--];
  parent = 1;
  child = 2;
```



1	2	3	4	5
20	15	2	14	10

1	2	3	4	~~5~~
20	15	2	14	~~10~~

temp = 10




```
while (child <= *n) {
    /* find the larger child of the current
       parent */
    if ((child < *n)&&
        (heap[child].key<heap[child+1].key))
      child++;
    if (temp.key >= heap[child].key) break;
    /* move to the next lower level */
    heap[parent] = heap[child];
    parent = child;
    child *= 2;
  }
  heap[parent] = temp;
  return item;
}
```

1	2	3	4
15	15	2	14

1	2	3	4
15	14	2	14

1	2	3	4
15	14	2	10

Heap

1. a min (max) element is deleted. $O(\log_2 n)$
2. deletion of an arbitrary element $O(n)$
3. search for an arbitrary element $O(n)$
4. Heap sort for ascending/descending order $O(n \log_2 n)$
5. Construct a heap $O(n \log_2 n)$
6. Return a max(min) key $O(1)$

1. Every element has a unique key.
2. The keys in a nonempty left subtree (right subtree) are smaller (larger) than the key in the root of subtree.
3. The left and right subtrees are also binary search trees.

Binary Search Tree

Chapter 5.7

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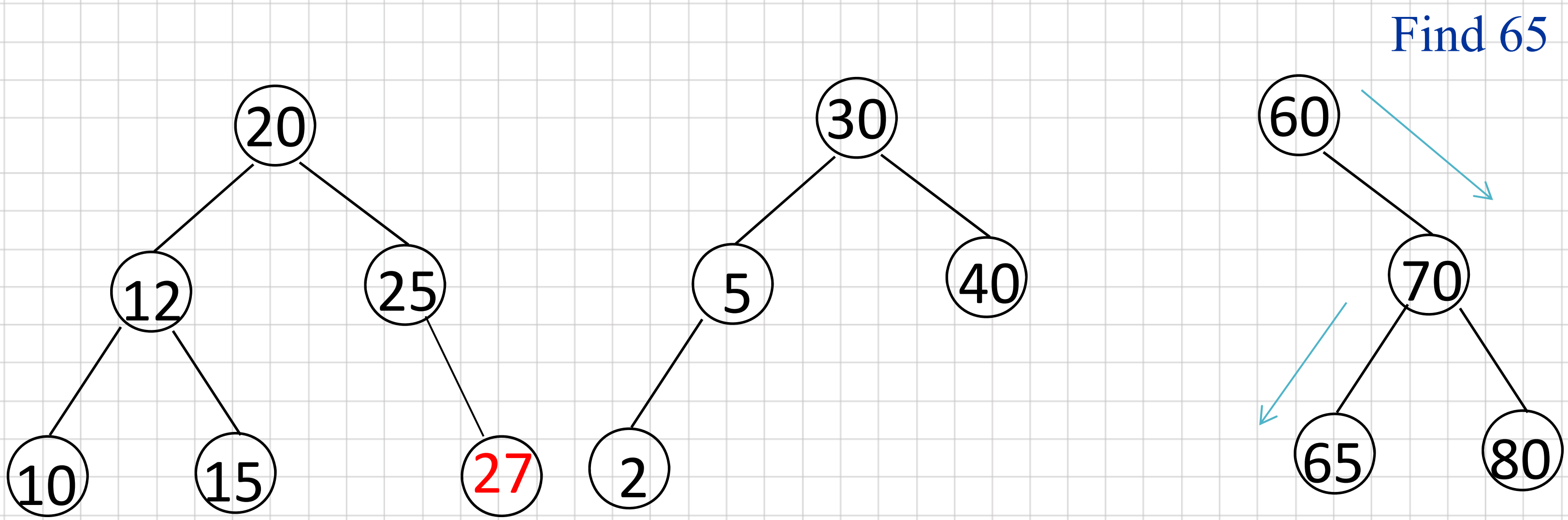
Examples of Binary Search Trees



Searching a Binary Search Tree

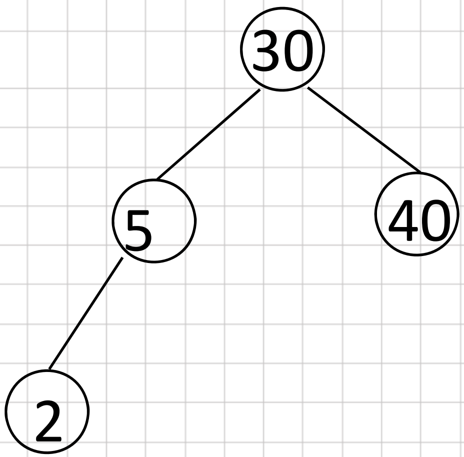
```
tree_pointer search(tree_pointer root,
                           int key)
{
/* return a pointer to the node that
  contains key. If there is no such
  node, return NULL */

   if (!root) return NULL;
   if (key == root->data) return root;
   if (key < root->data)
        return search(root->left_child,
                         key);
   return search(root->right_child,key);
}
```

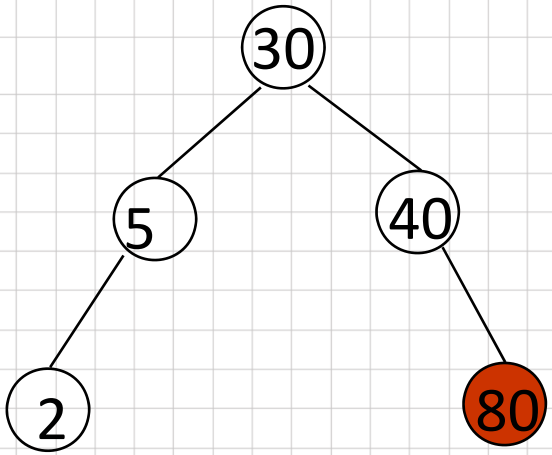
Another Searching Algorithm

```
tree_pointer search2(tree_pointer tree,
  int key)
{
   while (tree) {
      if (key == tree->data) return tree;
      if (key < tree->data)
            tree = tree->left_child;
      else tree = tree->right_child;
   }
   return NULL;
}
```

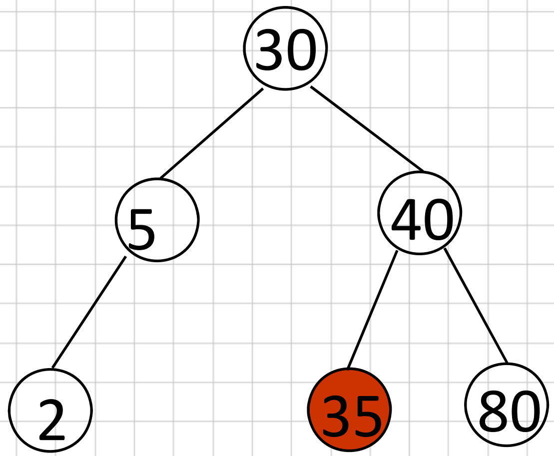
O(h) or O($\log_2 n$)

Insert Node in Binary Search Tree





Insert 80

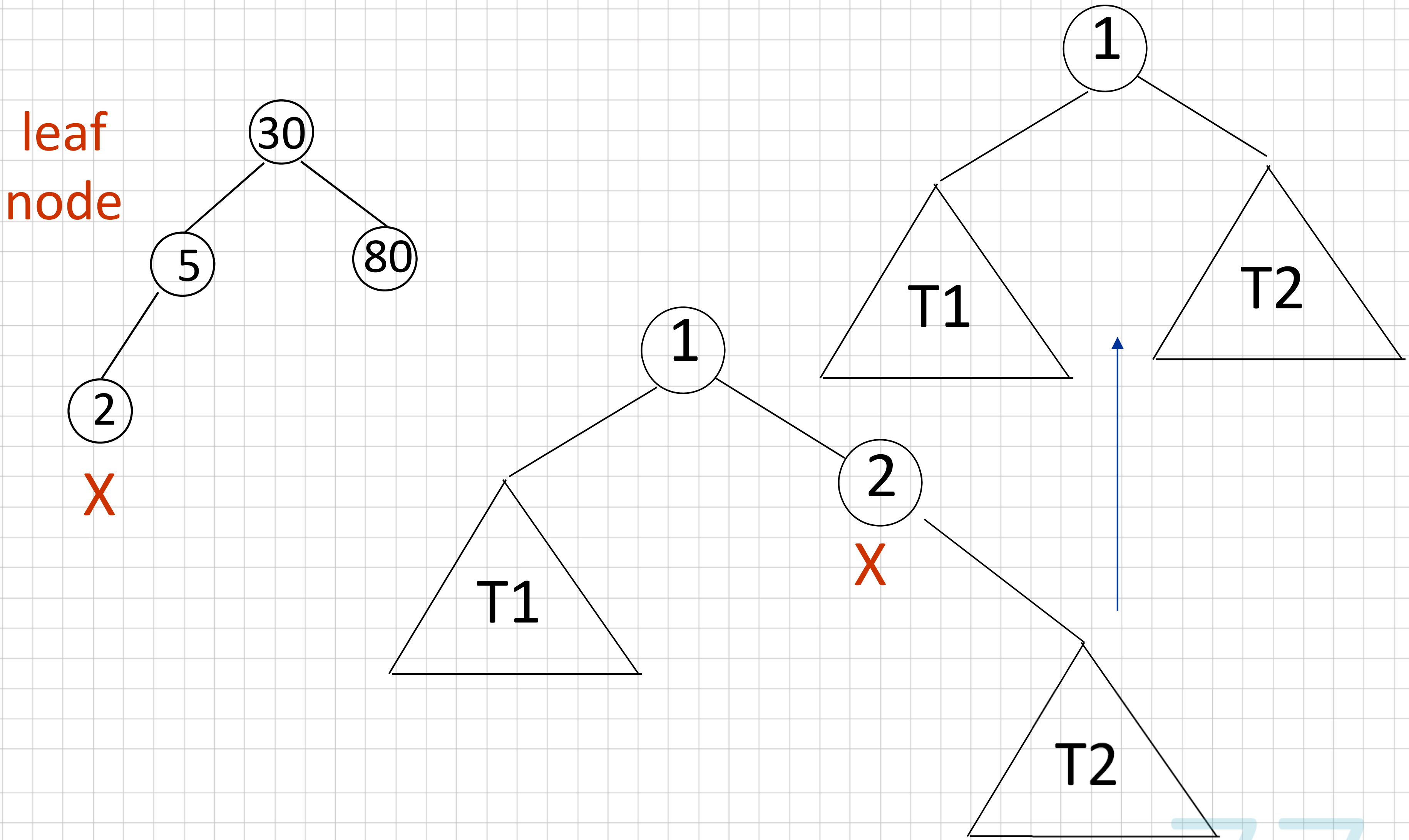


Insert 35

Insertion into A Binary Search Tree

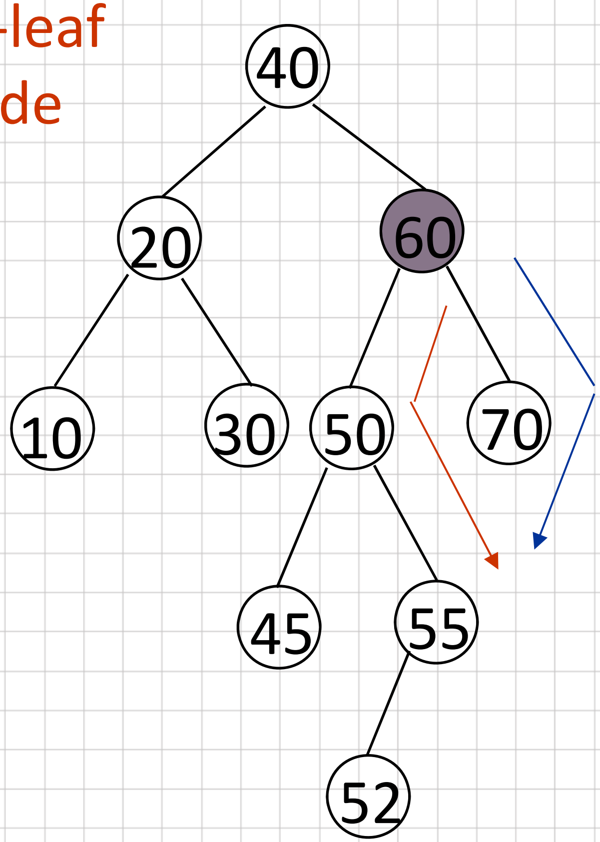
```
void insert_node(tree_pointer *node, int num)
{ tree_pointer ptr,
  temp = modified_search(*node, num);
  if (temp || !(*node)) {
   ptr = (tree_pointer) malloc(sizeof(node));
   if (IS_FULL(ptr)) {
     fprintf(stderr, "The memory is full\n");
     exit(1);
   }
   ptr->data = num;
   ptr->left_child = ptr->right_child = NULL;
   if (*node)
     if (num<temp->data) temp->left_child=ptr;
        else temp->right_child = ptr;
   else *node = ptr;
  }
}
```

Deletion for A Binary Search Tree

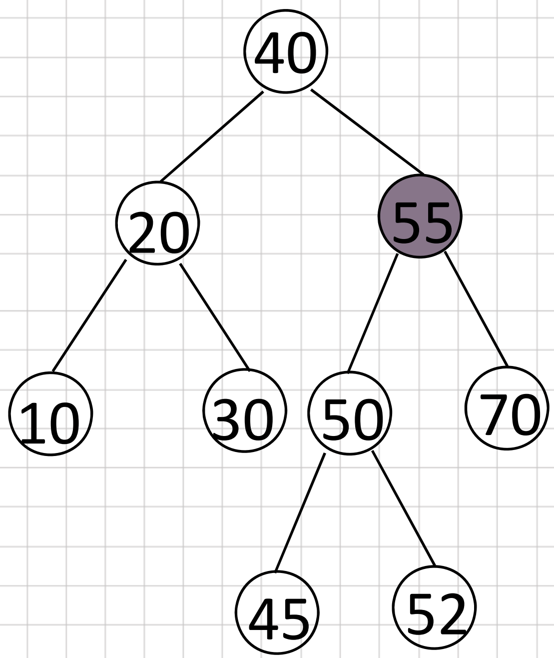


Deletion for A Binary Search Tree

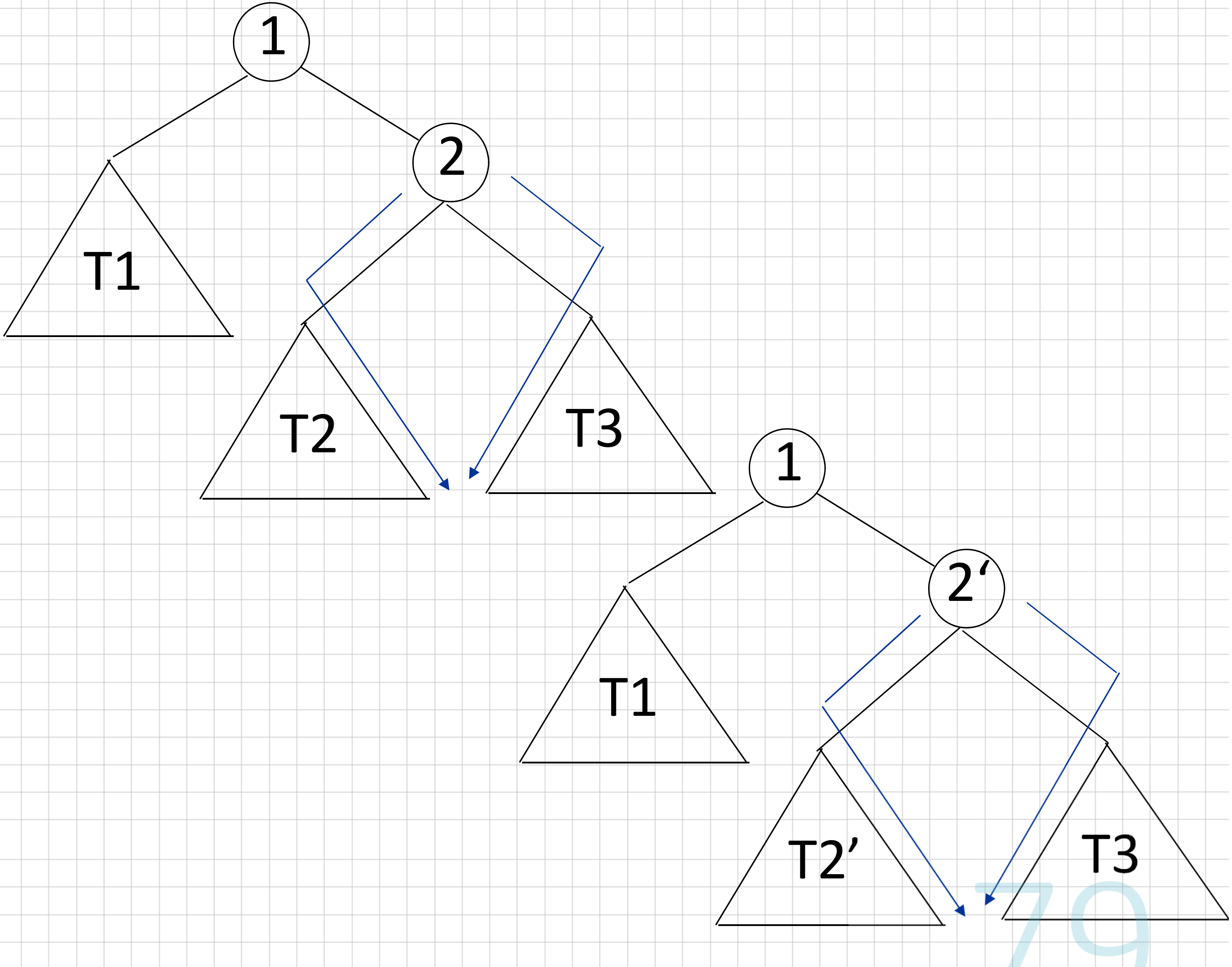
non-leaf node



Before deleting 60



After deleting 60


1
2
T1
T2
T3
1
2‘
T1
T2’
T3

Selection Trees

Chapter 5.8

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(1) winner tree
(2) loser tree

winner tree

sequential allocation scheme (complete binary tree)

Each node represents the smaller of its two children.

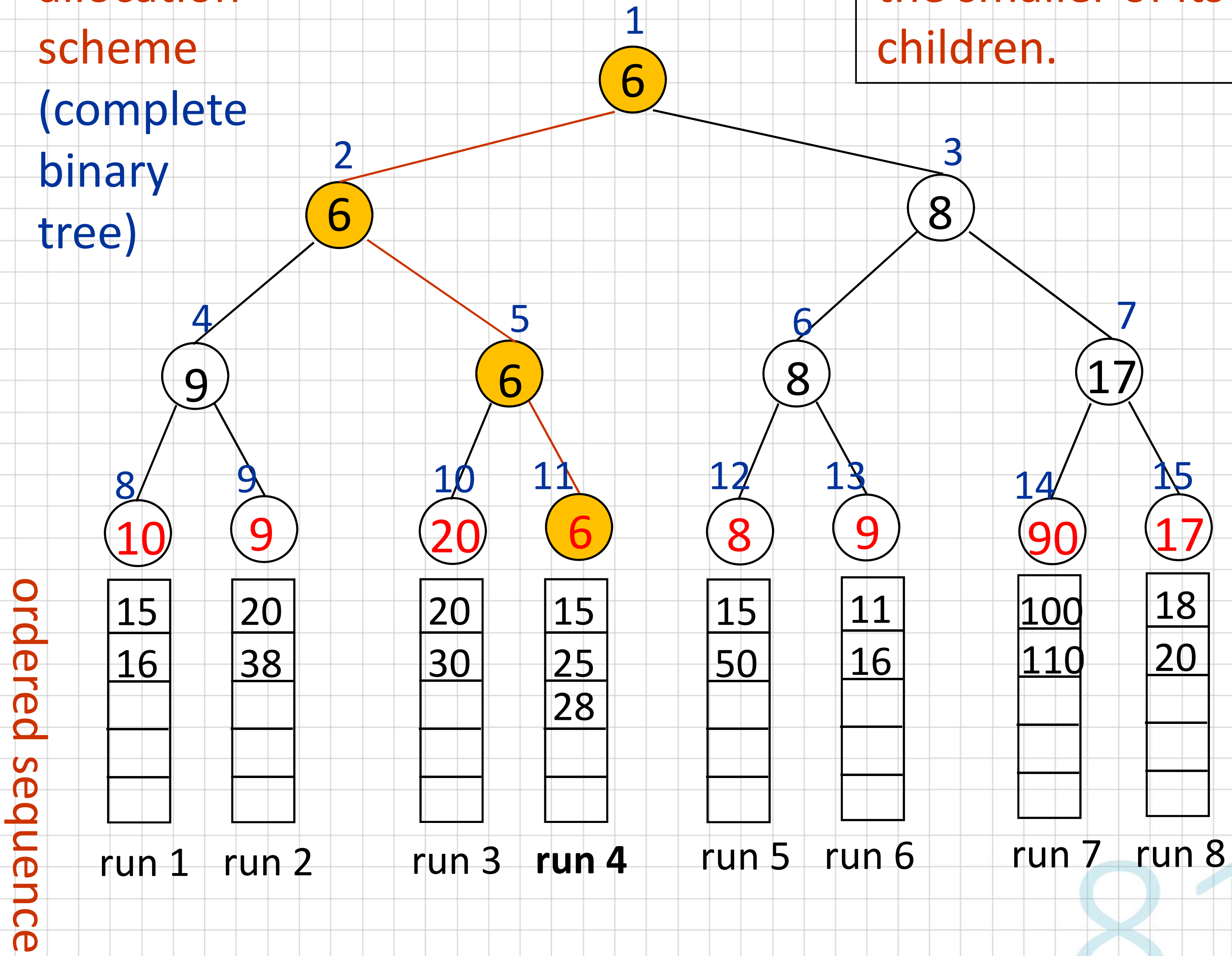
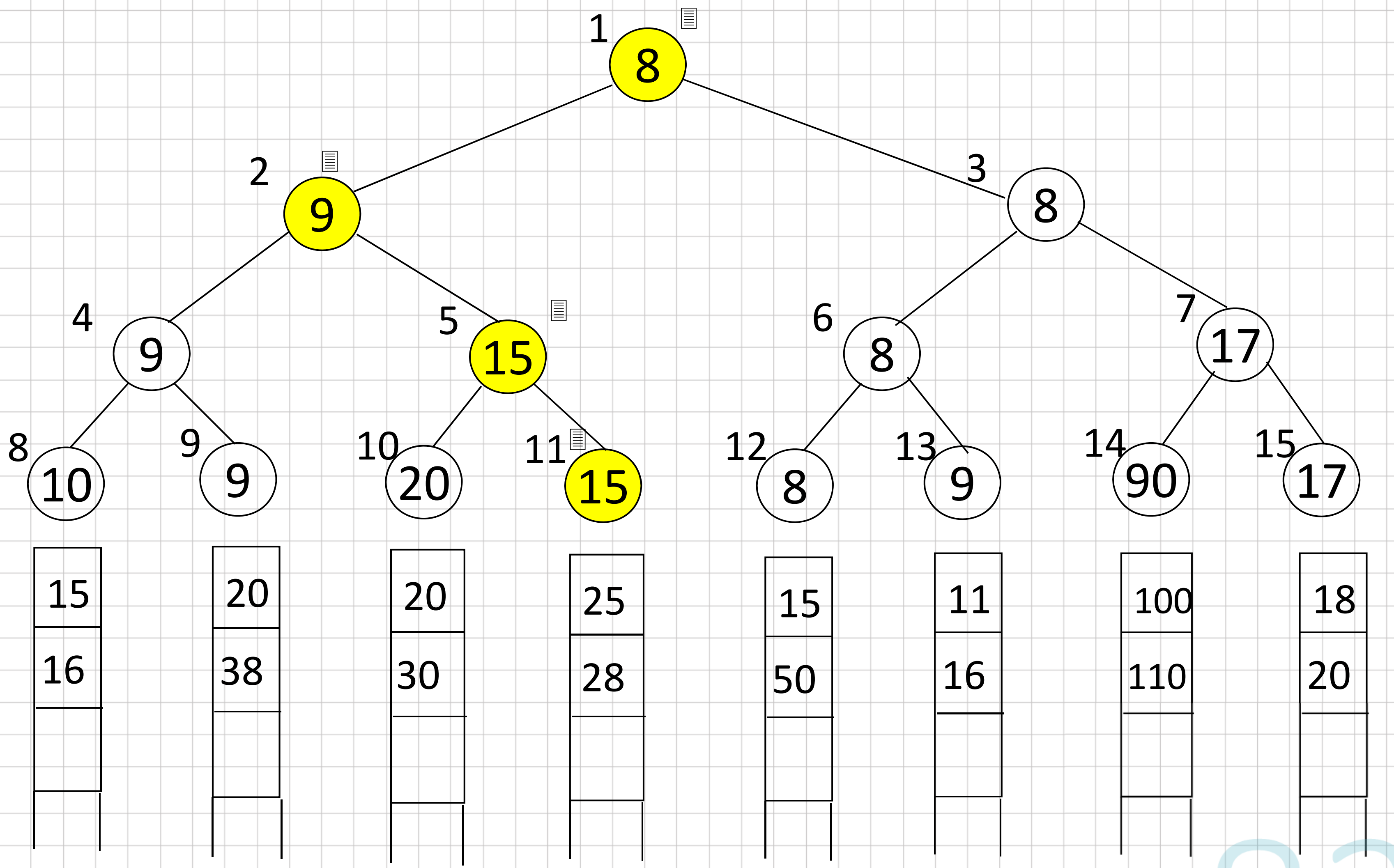


***Figure 5.35:** Selection tree of Figure 5.33 after one record has been output and the tree restructured(nodes that were changed are ticked)



Analysis

K: # of runs

n: # of records

setup time: O(K) (K-1)

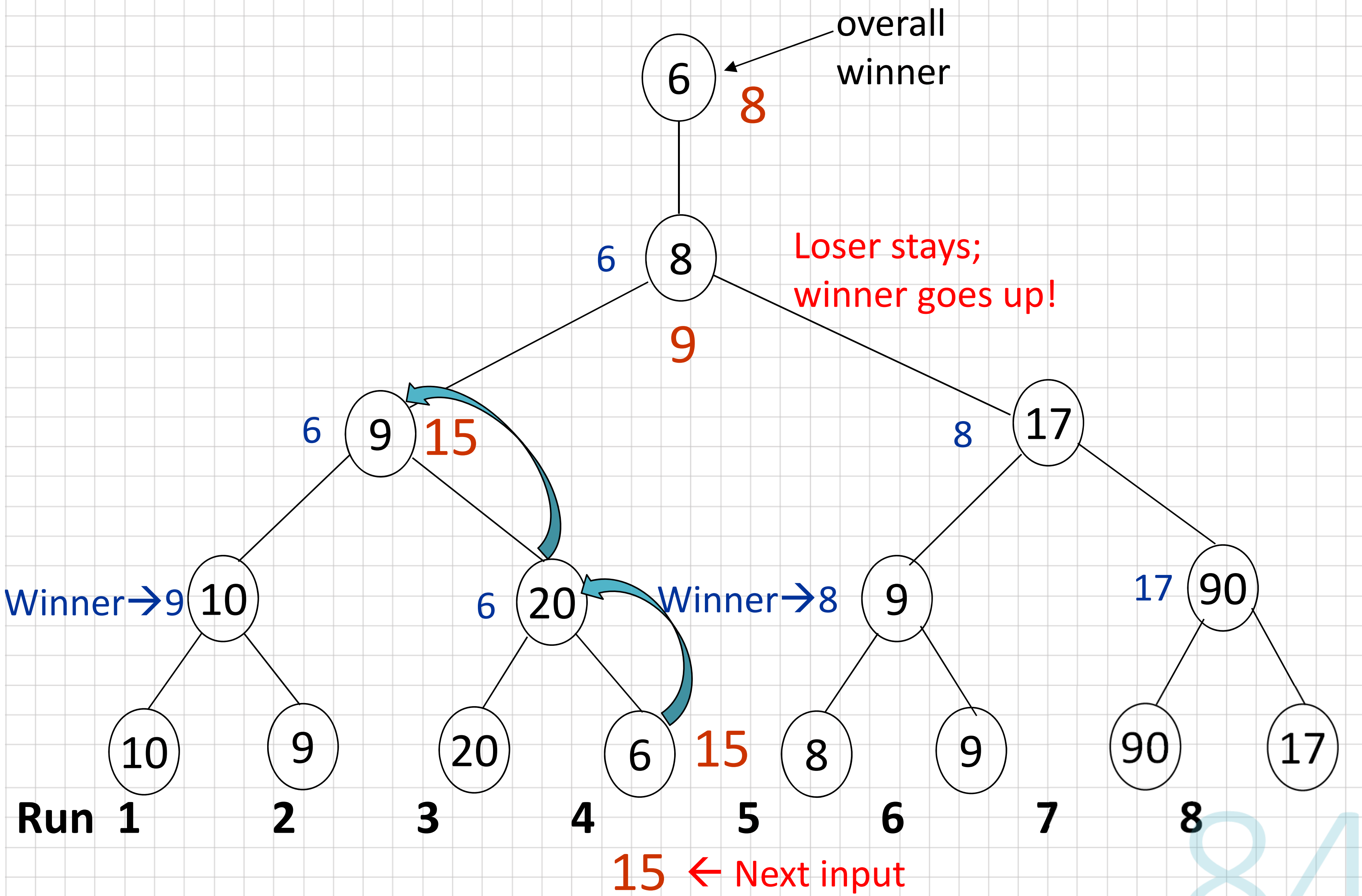
restructure time: $O(\log_2 K)$ $\lceil \log_2(K+1) \rceil$

merge time: $O(n\log_2 K)$

slight modification: tree of loser

consider the parent node only (vs. sibling nodes)

***Figure 5.36: Tree of losers** corresponding to Figure 5.34 (p.244)



Forest

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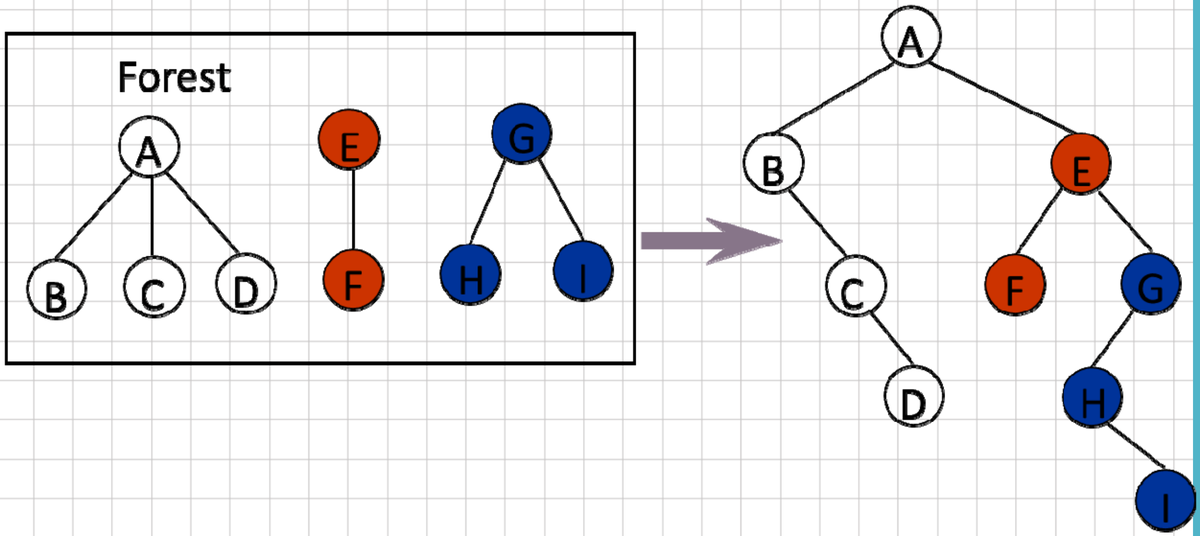
- A forest is a set of n >= 0 disjoint trees



Transform a forest into a binary tree

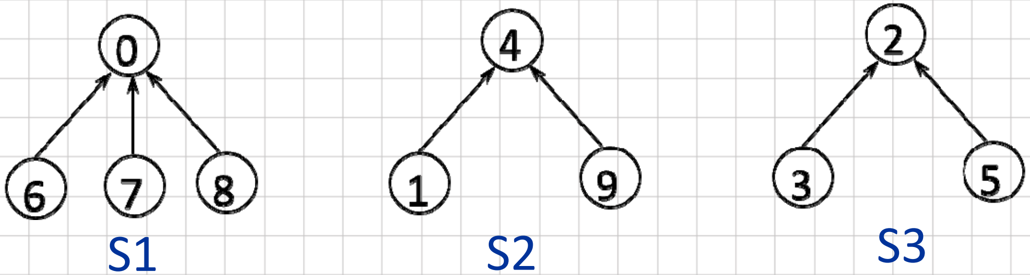
T1, T2, ..., T*n*: a forest of trees
B(T1, T2, ..., Tn): a binary tree corresponding to this forest

algorithm
(1) empty, if n = 0
(2) has root equal to root(T1)
has left subtree equal to B(T11,T12,...,T1*m*)
has right subtree equal to B(T2,T3,...,Tn)

Forest Traversals

- Preorder
  - If F is empty, then return
  - Visit the root of the first tree of F
  - Taverse the subtrees of the first tree in tree preorder
  - Traverse the remaining trees of F in preorder
- Inorder
  - If F is empty, then return
  - Traverse the subtrees of the first tree in tree inorder
  - Visit the root of the first tree
  - Traverse the remaining trees of F is indorer

- $S_1=\{0, 6, 7, 8\}$, $S_2=\{1, 4, 9\}$, $S_3=\{2, 3, 5\}$



Disjoint set: $S_i \cap S_j = \phi$

- Two operations considered here
  - *Disjoint set union*

    $S_1 \cup S_2=\{0,6,7,8,1,4,9\}$
  - *Find*(*i*): Find the set containing the element *i*.

    $3 \in S_3$, $8 \in S_1$

Set Representation

Chapter 5.10

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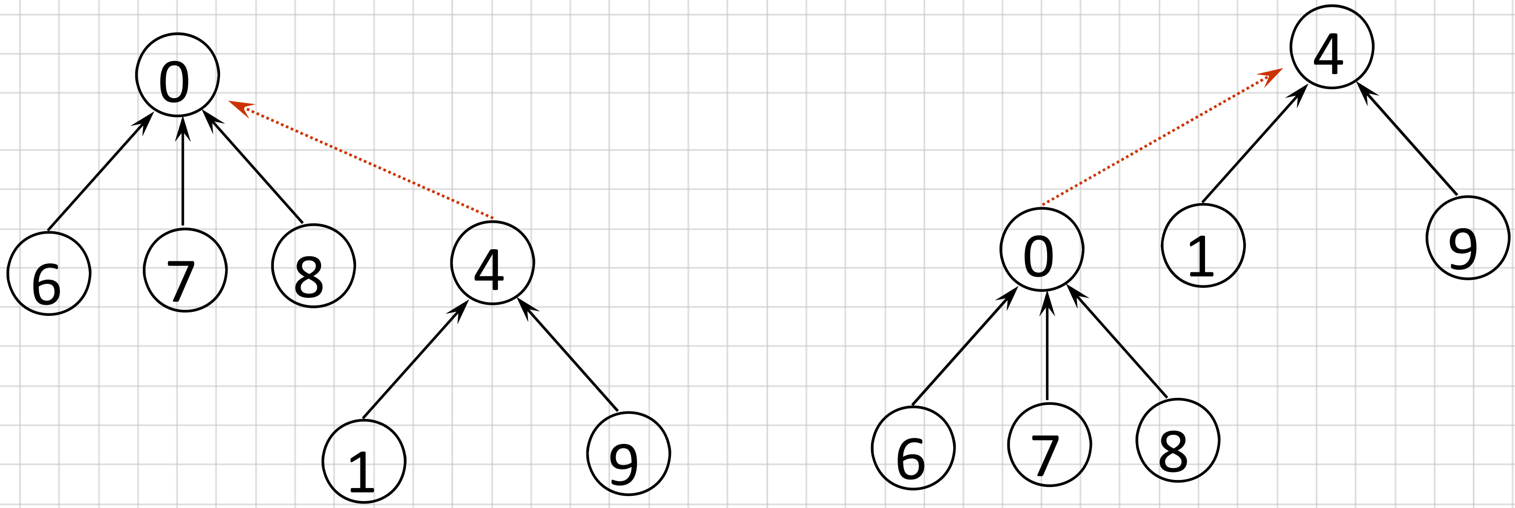
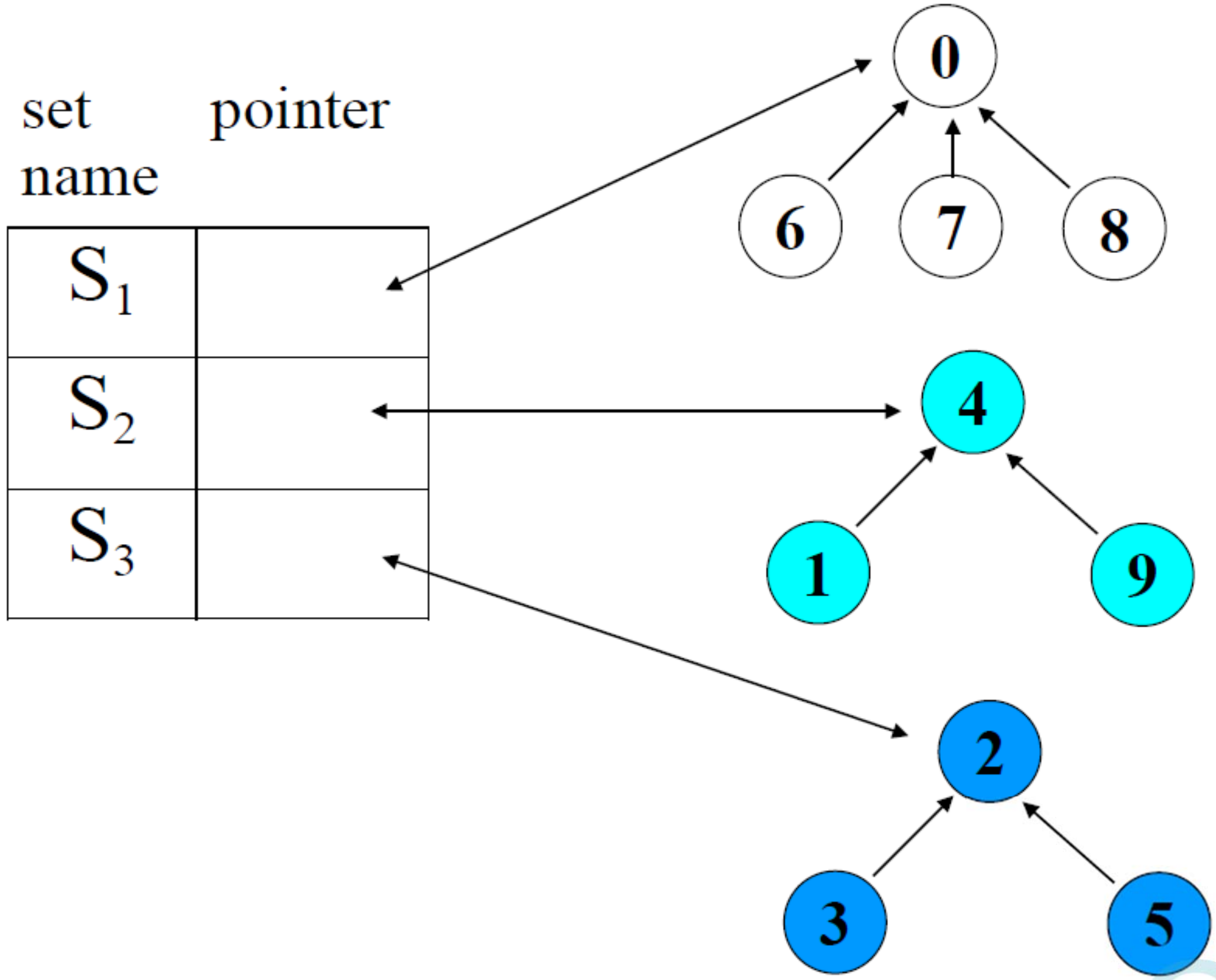
Disjoint Set Union

Make one of trees a subtree of the other



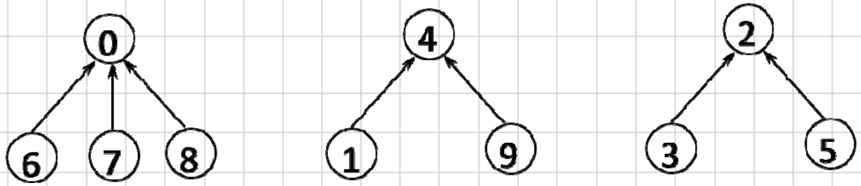
Possible representation for S_1 *union* S_2

Data Representation of S1, S2and S3



Array Representation for Set

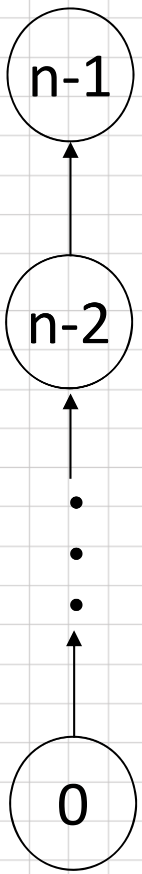
i	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
parent	-1	4	-1	2	-1	2	0	0	0	4




```
int find(int i)
{
    for (; parent[i]>=0; i=parent[i]);
    return i;
}

void union(int i, int j)
{
    parent[i]= j;
}
```

*Figure 5.41:Degenerate tree (p.252)



degenerate tree

union(0,1)
union(1,2).
.
.
union(n-2,n-1)

union operation
O(n) -> n-1 unions

find(0)
find(1)
.
.
.
find(n-1)

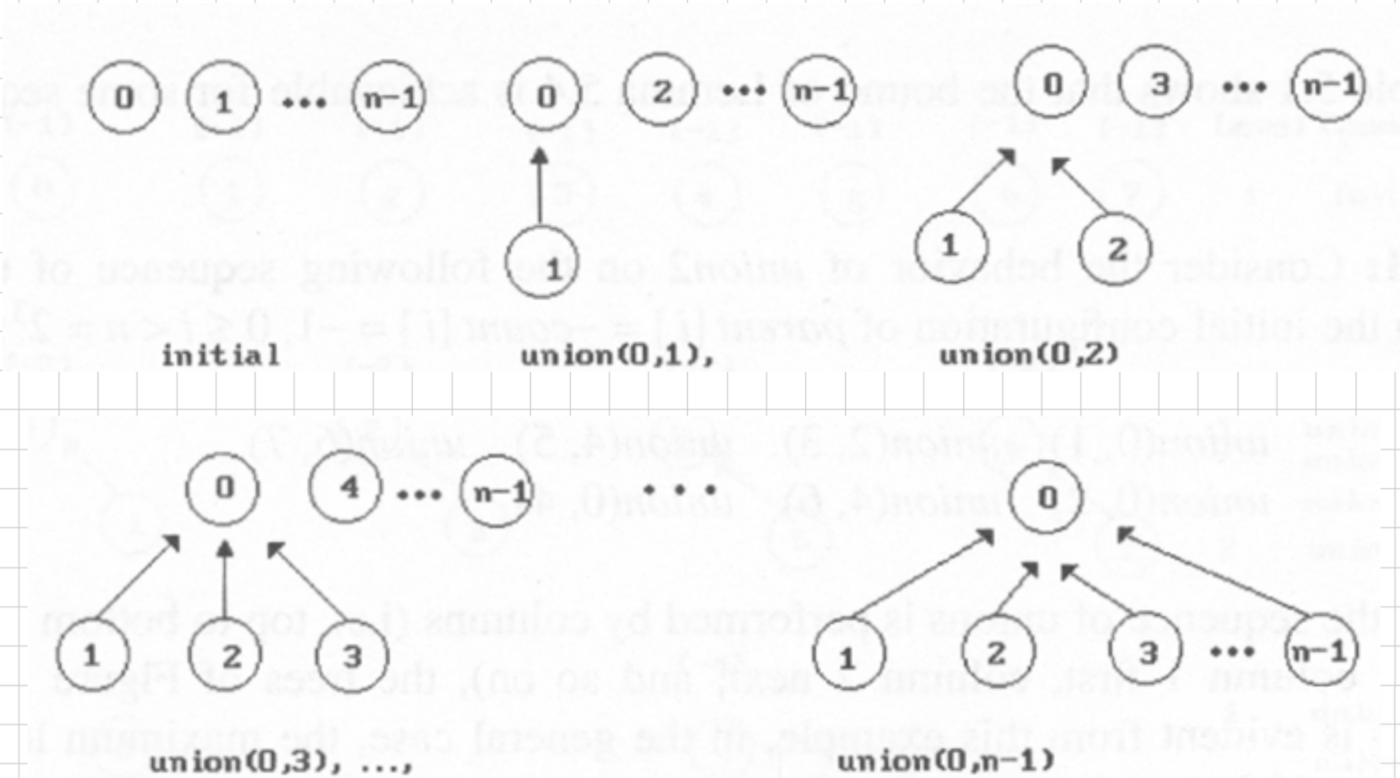
find operation
$O(n^2)$

Weighting Rule

Definition [Weighting rule for union(i, j)]:
If the number of nodes in the tree with root i is less than the number in the tree with root j, then make j the parent of i; otherwise make i the parent of j.

***Figure 5.42:**Trees obtained using the weighting rule(p.252)

weighting rule for union(i,j): if # of nodes in i < # in j then j the parent of i

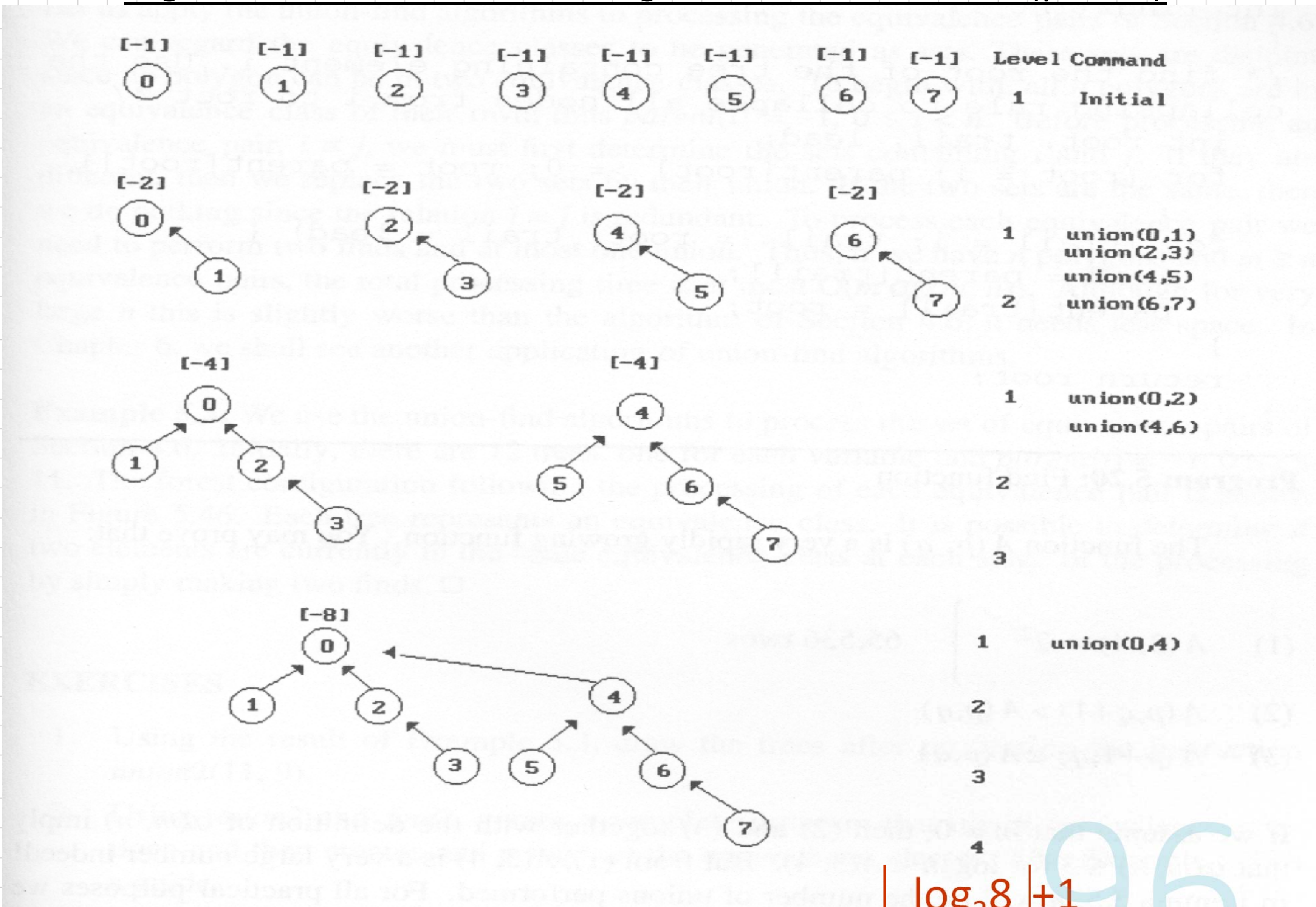


Modified Union Operation

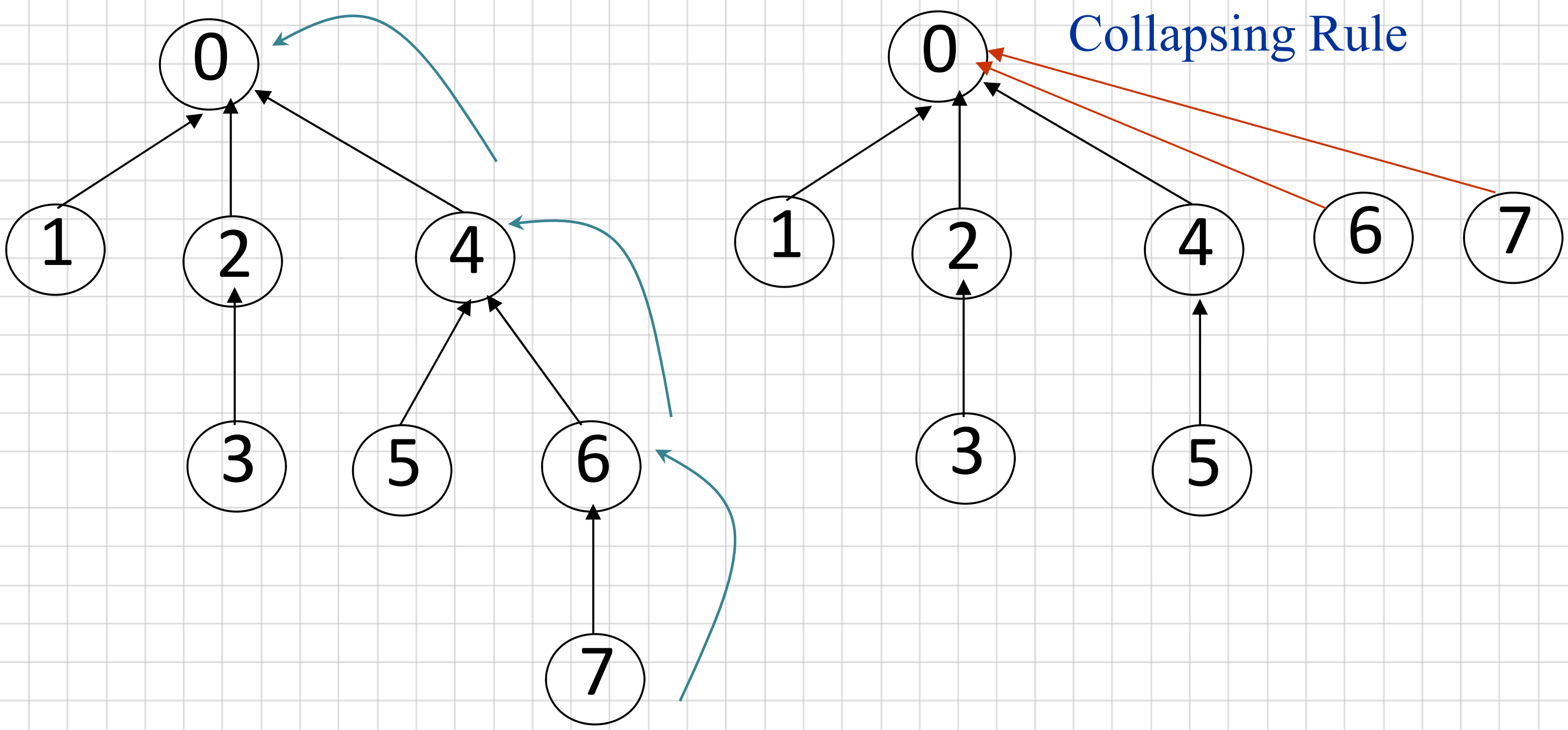
```
void union_modified(int i, int j)
{      Keep a count in the root of tree
    int temp = parent[i]+parent[j];
    if (parent[i]>parent[j]) {
        parent[i]=j;      i has fewer nodes.
        parent[j]=temp;
    }
    else {  j has fewer nodes
        parent[j]=i;
        parent[i]=temp;
    }
}
```

If the number of nodes in tree i is less than the number in tree j, then make j the parent of i; otherwise make i the parent of j.

Figure 5.43: Trees achieving worst case bound (p.254)



$\lfloor \log_2 8 \rfloor + 1$



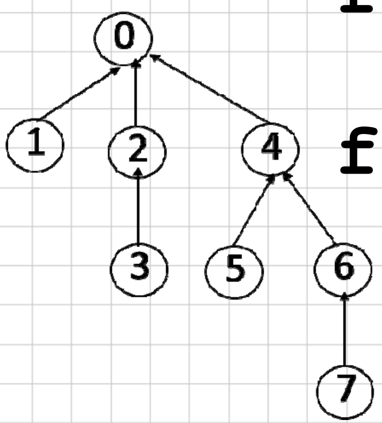
	find(7)	find(7)	find(7)	find(7)	find(7)	find(7)	find(7)	find(7)
go up	3	1	1	1	1	1	1	1
reset	2 (or 3)							

12 (or 13) moves (vs. 8 finds * 3 = 24 moves)

Modified *Find*(*i*) Operation

```
int find_modified(int i)
{
    int root, trail, lead;
    for (root=i; parent[root]>=0;
                            root=parent[root]);
    for (trail=i; trail!=root;
                            trail=lead) {
        lead = parent[trail];
        parent[trail]= root;
    }
    return root:
}
```

i	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
parent	-7	0	0	2	0	4	4	6



If j is a node on the path from i to its root then make j a child of the root

Applications

Find equivalence class $i \equiv j$

Find S_i and S_j such that $i \in S_i$ and $j \in S_j$
(two finds)

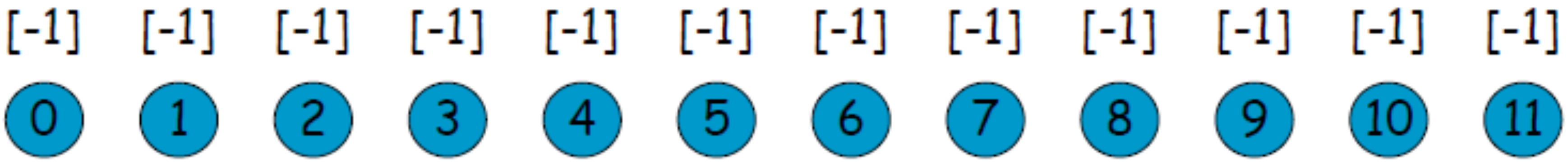
$S_i = S_j$ do nothing

$S_i \neq S_j$ union(S_i , S_j)

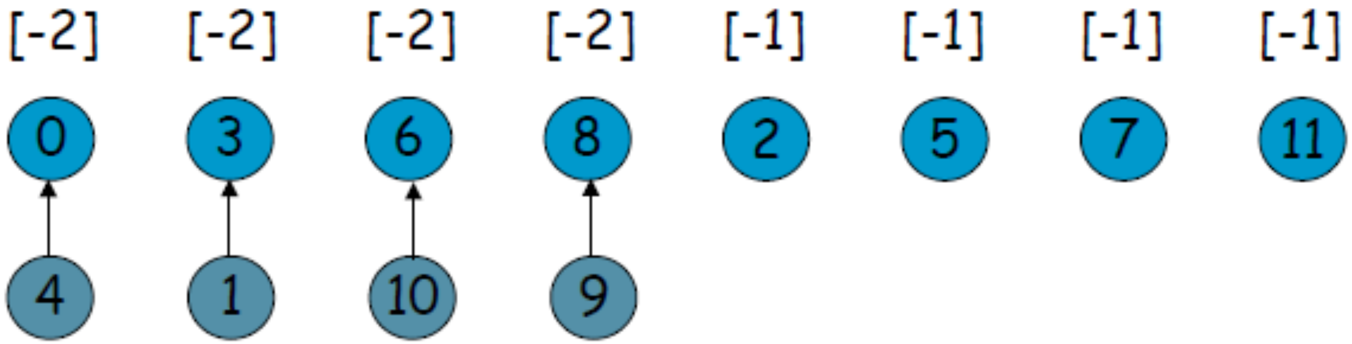
example

$0 \equiv 4$, $3 \equiv 1$, $6 \equiv 10$, $8 \equiv 9$, $7 \equiv 4$, $6 \equiv 8$,
$3 \equiv 5$, $2 \equiv 11$, $11 \equiv 0$

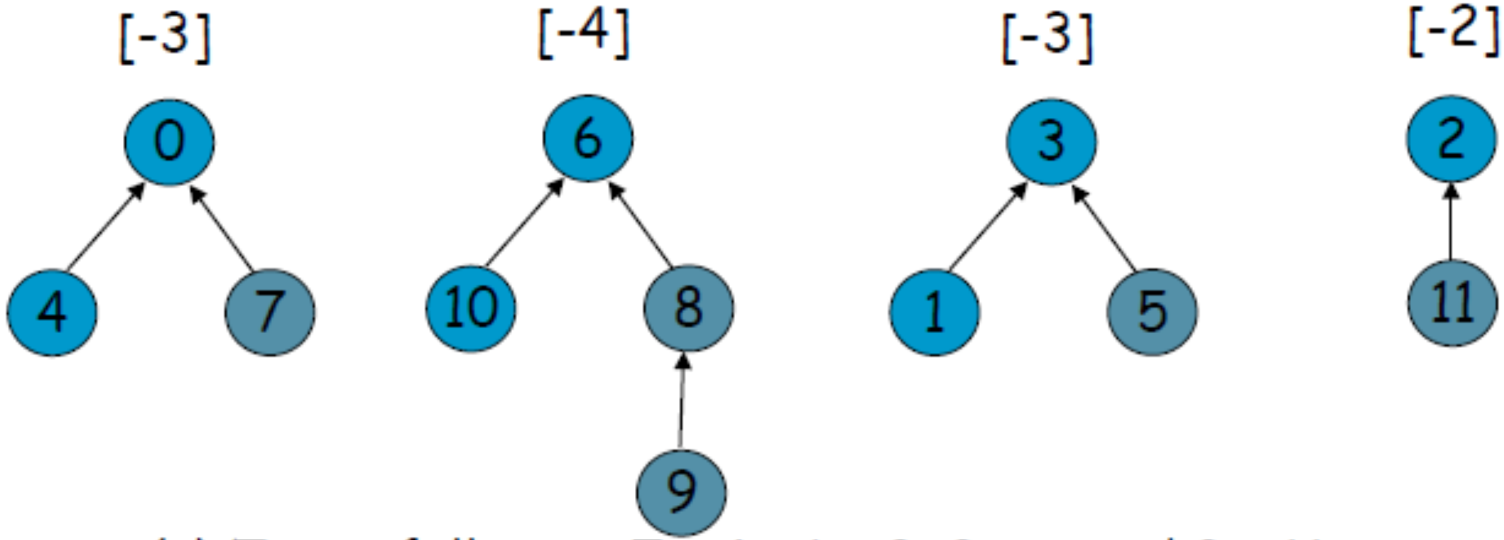
{0, 2, 4, 7, 11}, {1, 3, 5}, {6, 8, 9, 10}



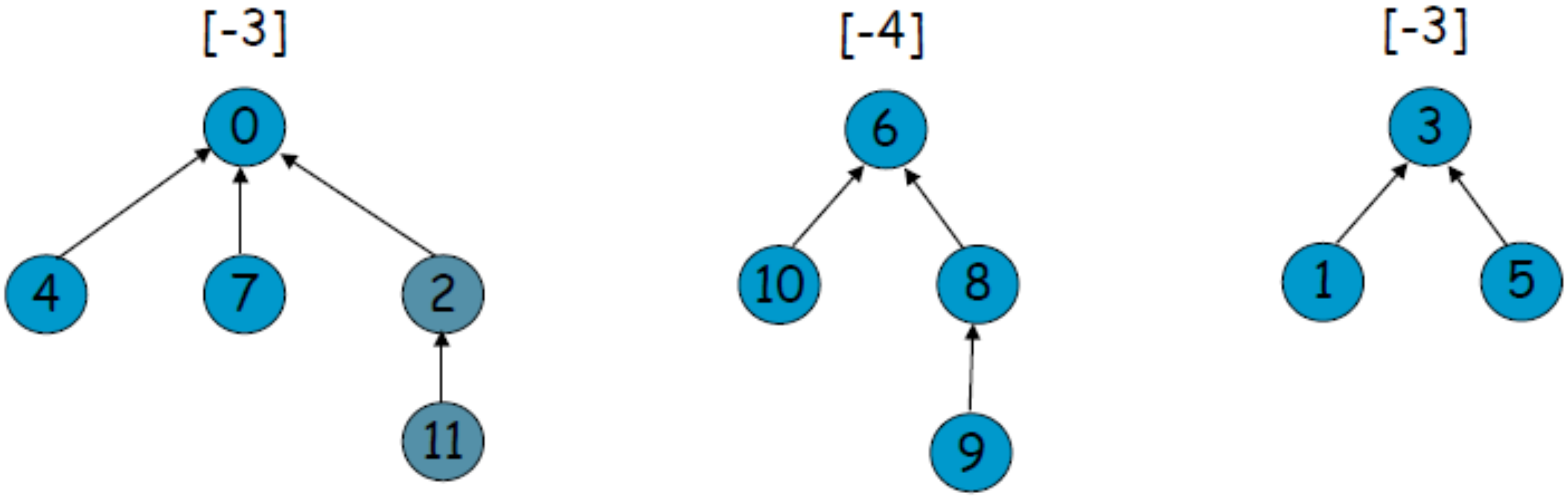
(a) Initial trees



(b) Height-2 trees following $0 \equiv 4$, $3 \equiv 1$, $6 \equiv 10$, and $8 \equiv 9$



(c) Trees following 7≡4, 6≡8, 3≡ 5, and 2≡11

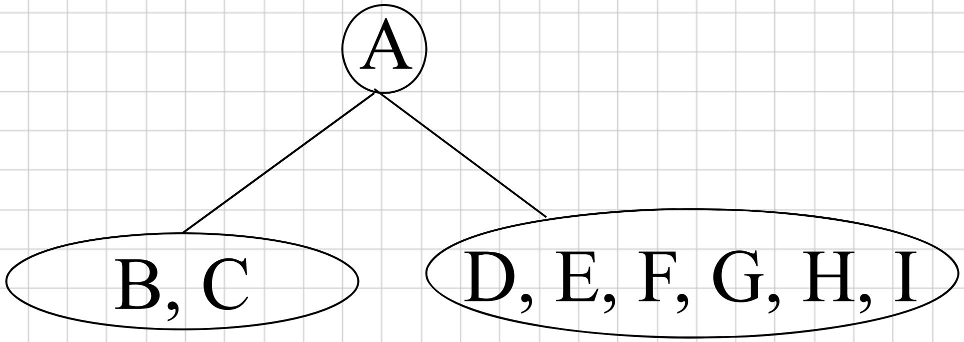


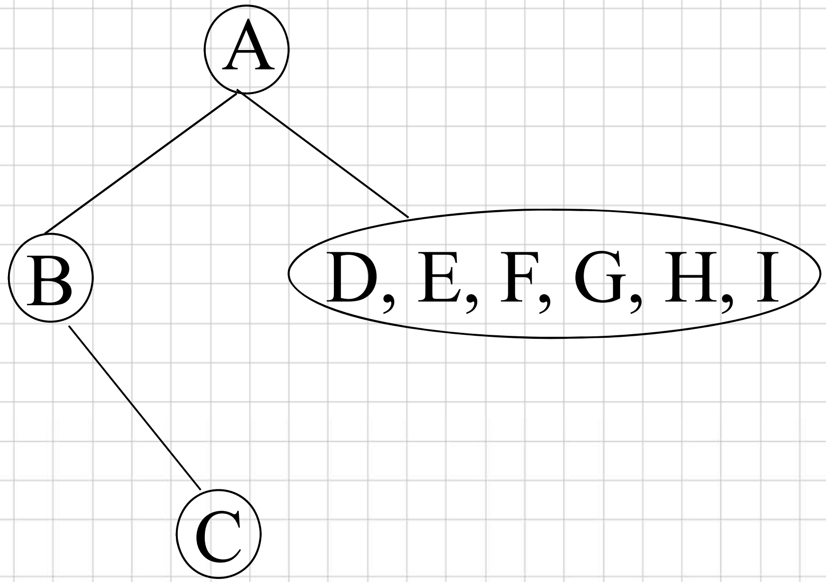
(d) Trees following 11≡0

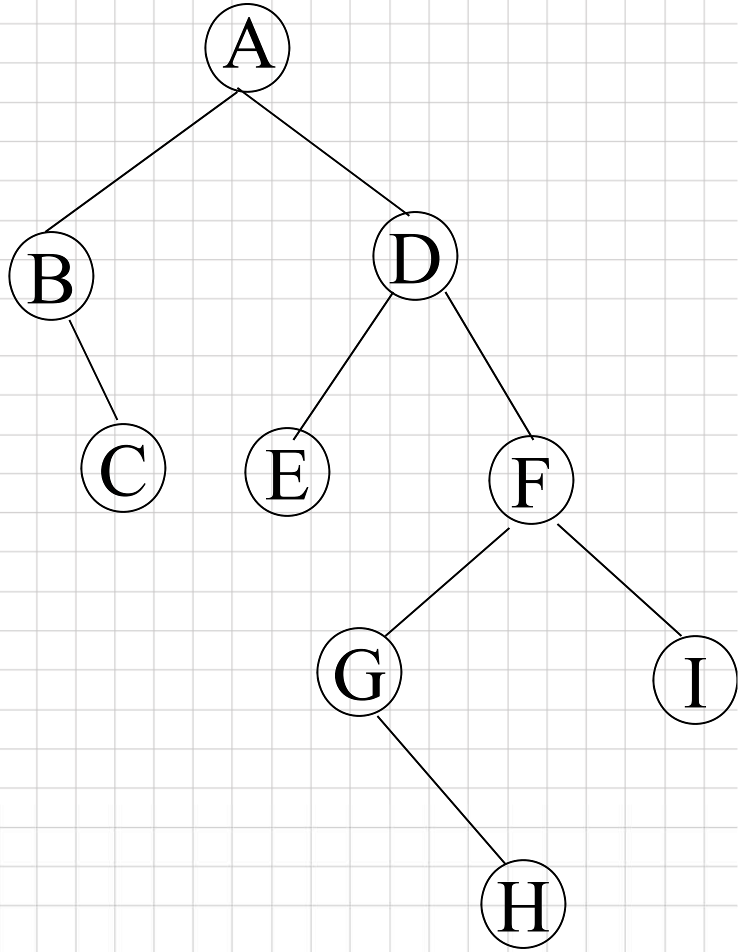
Uniqueness of a Binary Tree

We introduced preorder, inorder, and postorder traversal of a binary tree. Now suppose we are given a sequence (e.g., inorder sequence BCAEDGHFI), does the sequence uniquely define a binary tree?

preorder: A B C D E F G H I
inorder: B C A E D G H F I







Distinct Binary Trees