EE 547 (PMP) Lab 3

Wednesday, January 21, 2015

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Demo Problem 1 (Matrix Exponential of an LTI Systems) Given a square matrix A as (1). Please write a MATLAB script to :

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \tag{1}$$

(a) Compute the determinant of matrix A, define matrix (sI-A) and find $(sI-A)^{-1}$.

(b) Compute matrix exponential e^{At}. (Hint: expm)

(c) Repeat step (b) by using inverse Laplace transform of $(sI-A)^{-1}$. (Hint: ilaplace)

(d) Compute the characteristic polynomial and eigenvalues of matrix A.

(e) Evaluate and plot diagonal terms of e^{At}.

Individual Problem 1 Consider a system as (2) with initial condition $x_1(0) = 1$ and $x_2(0) = 0.5$.

$$\dot{x} = \begin{bmatrix} 0 & -2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2}$$

(a) The solution of this system is of a unique form: $x = e^{At}x(0)$. As you learned in the lecture, matrix exponential is the state transition matrix. Please compute e^{At} by appropriate functions.

(b) Please derive the eigenvalues and characteristic polynomial of this matrix A by MATLAB script.

(c) Please evaluate and plot $x_1(t)$ and $x_2(t)$. The tspan is set as [0:0.01:5]. (Hint: use functions **eval** and **subs** to evaluate a function with symbol.)

Now consider system (3) with an impulse input.

$$\dot{x} = \begin{bmatrix} 0 & -2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \delta(t)$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3}$$

(d) Please use **impulse** function and the method we learned in step (c) to plot system's output. (Note: **impulse** function in MATLAB assumes zero initial state values)

Solution **to Demo Problem 1:**

(a) Initial set up:

   i. Define parameters and symbols

      syms s

      syms t

   ii. Define A matrix

(b) Compute the determinant of matrix A and find $(sI\text{-}A)^{-1}$.

   detA = **det**(A)

   ChaMatrix = **inv**(s*eye(size(A))-A);

(c) Compute matrix exponential e^{At}.

   expAt = **expm**(A*t)

(d) Inverse Laplace transform of $(sI\text{-}A)^{-1}$.

   DexpAt = **ilaplace**(ChaMatrix,s,t)

(e) Derive characteristic polynomial and eigenvalues of matrix A

   ChaPoly = **poly**(A);

   This returns the coefficients of characteristic polynomial in descending power.

   [V,D]=**eig**(A);

   This returns eigenvectors as column vector of matrix V. D is a diagonal matrix with diagonal terms as eigenvalues.

(f) Evaluate and plot diagonal terms of e^{At}.

   eval(subs(expAt(1,1),t,tspan)

   Since e^{At} is a matrix, the element to be plotted has to be defined clearly. **subs** is to replace symbol t with the values defined in tspan. Please note after **subs**, the class of output is *sym*, not *double*. **eval** is to return the function output in terms of *double*.

(g) The specific outputs are plotted in Figure **1**.

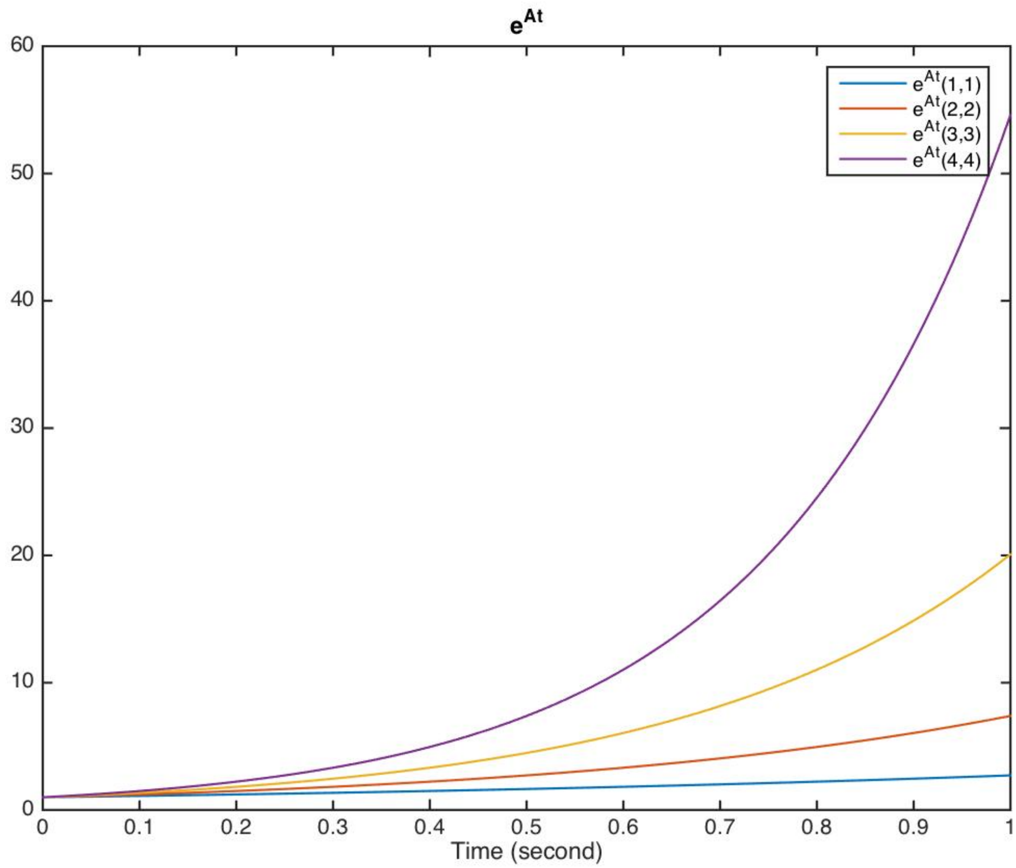

Figure 1 Diagonal terms of e^{At}. Please note e^{At} is a 4 by 4 matrix in this case.

MATLAB Code:

```
%% Lab 3: Matrix exponential and inverse Laplace transformation
%% Developer: HRLin

close all;
clear all;
clc;
%% Define parameters for individual problem
tspan = 0:0.01:1;
syms s; % to create symbole s for Laplace transform
syms t; % to create symbole t for time

%% Define A matrix
A = [1 0 0 0; 2 2 0 0; 0 0 3 3; 0 0 0 4]

%% MATLAB function practice
detA = det(A);  % determinant of matrix A

ChaMatrix = inv(s*eye(size(A))-A);

expAt = expm(A*t);  %calculate exponential of matrix At

DexpAt = ilaplace(ChaMatrix,s,t);    %derive exp(At) by inverse Laplace

ChaPoly = poly(A);

[V,D] = eig(A);
%% Define input
u = cos(tspan);

%% Output Figures

figure(1)
plot(tspan,eval(subs(expAt(1,1),t,tspan)),tspan,eval(subs(expAt(2,2),t,tspan)
)...

,tspan,eval(subs(expAt(3,3),t,tspan)),tspan,eval(subs(expAt(4,4),t,tspan)))
title('e^A^t');
xlabel ('Time (second)');
legend('e^A^t(1,1)','e^A^t(2,2)','e^A^t(3,3)','e^A^t(4,4)')
print -djpeg 'e^(At).jpeg'
```