EE 547 (PMP) Midterm

Assigned: Thursday, February 4, 2015. Due: Wednesday, February 11, 2015.

Professor Linda Bushnell, UW EE

Problem 1 Consider a non-linear system as (1).

$$\dot{x} = \begin{bmatrix} f_1(x_1,x_2,x_3,u) \\ f_2(x_1,x_2,x_3,u) \\ f_3(x_1,x_2,x_3,u) \end{bmatrix} = \begin{bmatrix} -9x_1 - 4x_2 - (1+x_3)x_3 + \sin x_3 + \sin(u) \\ (x_2x_3 - 4)x_1 - 10\sin x_2 + 3cosx_3 + x_3^2 \sin(u) \\ 9x_1 + (x_1^2 - 4)x_3 - 10x_2 + u \end{bmatrix}$$

$$y = \begin{bmatrix} g_1(x_1,x_2,x_3,u) \\ g_2(x_1,x_2,x_3,u) \\ g_3(x_1,x_2,x_3,u) \end{bmatrix} = \begin{bmatrix} x_1 + x_2x_3 + \sin(u) \\ x_2 + x_1x_3 + u^2 \\ x_3 + x_2x_3 + \cos(u) \end{bmatrix} \tag{1}$$

(a) Linearize the system around equilibrium point $x_{eq} = (-0.1,0.1,-0.2)^T$ and $u_{eq} = 0$. Find the state space representation of the linearized system (i.e., find A, B, C, D matrices.)

(b) Find the transfer functions of this system. Are these transfer functions proper rational functions?

(c) Is the system BIBO stable?

(d) Evaluate the Jordan form of the matrix A and corresponding transformation matrix Q.

(e) Find the state transition matrix from the Jordan form.

(f) Given the initial state $x_{ini} = (-0.1,0.1,-0.2)^T$, evaluate and plot zero-input responses of the system in time span: tspan = 0:0.01:3.

(g) Following previous step, evaluate the complete impulse response of this system. Check if the system is asymptotically stable by solving the Lyapunov equation, where Q is a symmetric positive-definite matrix:

$$A^T P + PA = -Q$$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 2 Consider two systems:

$$\dot{x} = \begin{bmatrix} -5 & -9 & 4 \\ 2 & -9 & 2 \\ 9 & -10 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} u \qquad \dot{x} = \begin{bmatrix} -82 & -8 & 54 \\ -174 & -33 & 130.5 \\ -138 & -16 & 93 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ -7 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad y = \begin{bmatrix} -4 & -1 & 3.5 \\ 1 & 0 & -0.5 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(a) Find the transfer functions of these two systems in explicit form.

(b) Are these two systems zero-state equivalent?

Problem 3 Consider a matrix A:

$$A = \begin{bmatrix} -4 & 3 & -1 & 10 & 3 \\ -6 & -9 & 5 & 7 & -6 \\ -8 & -5 & 2 & -9 & 2 \\ -4 & -5 & 6 & -1 & 2 \\ -6 & 8 & -8 & -4 & 2 \end{bmatrix}$$

a) Evaluate eigenvalues and characteristic polynomial of matrix A.

b) Evaluate Jordon form and transformation matrix Q of matrix A.

c) Evaluate the function $f_2(A) = A^5+10A^4+251A^3+1658A^2+12462A+23160\,\mathbf{I}_{5x5}$.