EE 547 (PMP): Homework 3

Assigned: Thursday, January 22, 2015; Due: Wednesday, January 28, 2015

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Problem 1 (State Transition Matrix of an LTV Systems) Consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & t \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ t \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Show the fundamental matrix of system is

$$X = \begin{bmatrix} x_1(0) & x_2(0)[e^{2t}(\frac{t}{2} - \frac{1}{4}) + \frac{1}{4}] \\ 0 & x_2(0)e^{2t} \end{bmatrix} \tag{2}$$

Show that this can be further simplified as

$$X = \begin{bmatrix} 1 & e^{2t}(\frac{t}{2} - \frac{1}{4}) + \frac{1}{4} \\ 0 & e^{2t} \end{bmatrix} \tag{3}$$

b) Show the state transition matrix of system (1) as

$$\varphi(t, t_0) = \begin{bmatrix} 1 & -\frac{e^{-2t_0}}{4}[e^{-2t_0}(2t_0 - 1) + 1] + e^{-2t_0}\left[e^{2t}(\frac{t}{2} - \frac{1}{4}) + \frac{1}{4}\right] \\ 0 & e^{2(t-t_0)} \end{bmatrix} \tag{4}$$

(Hint: choose two linearly independent vectors of initial conditions, for examples, $x_{0,1}$ = [$x_1(0)$ 0] and $x_{0,2}$ = [0 $x_2(0)$])

c) Given an input u(t) = sin(t) and an arbitrary condition x_0 = [$x_1(0)$, $x_2(0)$], using the following expression

$$y(t) = C(t)\Phi(t, t_0)x_0 + \int_{t_0}^{t} C(t)\,\Phi(t, \tau)B(\tau)u(\tau)d\tau + D(t)u(t)$$

to show the output is

$$y = x_2(0)e^{2(t-t_0)} + \frac{1}{25}e^{2(t-t_0)}\{4\cos t_0 + 3\sin t_0 + 5t_0\cos t_0 + 10t_0\sin t_0\} - \frac{3}{25}\sin t - \frac{4}{25}\cos t - \frac{1}{5}t\cos t - \frac{2}{5}t\sin t \tag{5}$$

Problem 2 (Characteristic Polynomial) Derive by hand the characteristic polynomial of a matrix A as (6).

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 6 & 2 & 7 \\ 4 & 8 & 3 \end{bmatrix} \tag{6}$$

Problem 3 (Matrix Exponential of an LTI System) Consider a system as (7) with initial condition $x_1(0) = 1$ and $x_2(0) = 0.5$.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 2 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \tag{7}$$

a) Show the state transition matrix by Inverse Laplace Transform is

$$\varphi(t) = \begin{bmatrix} e^{-2t}(\cos 2t + \sin 2t) & e^{-2t}\sin 2t \\ -2e^{-2t}\sin 2t & e^{-2t}(\cos 2t + \sin 2t) \end{bmatrix} \tag{8}$$

b) Derive the system output to the step input $u(t) = 1$, $\forall t \geq 0$. You can verify with MATLAB or other software.

Problem 4 (Impulse Response of an LTI System) Consider a system as (9) with initial condition $x_1(0) = 1$ and $x_2(0) = 2$.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y &= \begin{bmatrix} -4 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + u(t) \end{aligned} \tag{9}$$

a) Derive the state transition matrix by MATLAB.
b) If an impulse input applies over the system, use the MATLAB **impulse** function and the method we learned to plot system's output.