EE 547 (PMP) Lab 4

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Demo Problem 1 (Eigenvalues, Jordan Form and Matrix Function) Given a square matrix A as (1).

Please write a MATLAB script to:

$$A = \begin{bmatrix} 6 & -12 & 8 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{1}$$

(a) Compute the characteristic polynomial and eigenvalues of matrix A.

(b) Evaluate the rank of matrix A.

(c) Evaluate the Jordan form of this matrix ($\bar{A}$) and corresponding transformation matrix Q.

(d) Evaluate A^{10} by Jordan form.

Individual Problem 1 Consider a system as (2) with initial condition x(0) = [1 0 1 0 1].

$$\dot{x} = \begin{bmatrix} 14 & -75 & 190 & -224 & 96 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \tag{2}$$

(a) Please determine matrix A first and then compute the characteristic polynomial and eigenvalues of matrix A.

(b) Evaluate the Jordan form of this matrix and corresponding transformation matrix Q.

(c) Solve zero-input response x(t) with Jordan form and given initial conditions.
(Hint: The solution of this system is of a unique form: $x = e^{At}x(0)$.)

Solution to Demo Problem 1:

(a) In lab 3, we learned **poly** and **eig** functions to evaluate eigenvalues of matrix A. Eigenvalues are actually the roots of characteristic polynomial. Therefore, another function **roots** can be applied to evaluate eigenvalues.

Eigenvalues: $\lambda_1 = \lambda_2 = \lambda_3 = 2$

Characteristic polynomials: $f(\lambda) = \lambda^3 - 6\lambda^2 + 12\lambda - 8$

(b) **rank(A)** returns the rank of matrix A.

rank(A) = 3

(c) To evaluate Jordan form and transformation matrix Q, we can use function **jordan**.
[Q,JA] = jordan(A)
where Q is transformation matrix and JA is Jordan form

$$Q = \begin{bmatrix} 4 & 4 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{3}$$

$$JA = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \tag{4}$$

(d) It is easier to evaluate the matrix exponential over Jordan form rather than original matrix because Jordan form is diagonal. Please note JA is used in (4) to denote $\bar{A}$.

$$A^{10} = (Q\bar{A}Q^{-1})^{10} = Q\bar{A}^{10}Q^{-1} \tag{5}$$

To evaluate (5), we define two functions as below:

$$f(\lambda) = \lambda^{10}$$
$$h(\lambda) = \beta_0 + \beta_1\lambda + \beta_2\lambda^2$$

$f(\lambda)$ is equal to $h(\lambda)$ on the spectrum of A, we can define the following system of three linear equations:

$$f(\lambda) = h(\lambda) \tag{6}$$
$f(\lambda)' = h(\lambda)'$ (first derivative)
$f(\lambda)" = h(\lambda)"$ (second derivative)

System (6) can be rewritten as:

$$\begin{aligned} \lambda^{10} &= \beta_0 + \beta_1\lambda + \beta_2\lambda^2 \\ 10\lambda^9 &= \beta_1 + 2\beta_2\lambda \\ (10)(9)\lambda^8 &= 2\beta_2 \end{aligned} \tag{7}$$

Since we have repeated eigenvalues $\lambda_1 = \lambda_2 = \lambda_3 = 2$, (7) can be rewritten as:

$$\begin{aligned} 2^{10} &= \beta_0 + 2\beta_1 + 4\beta_2 \\ (10)2^9 &= \beta_1 + 4\beta_2 \\ (10)(9)2^8 &= 2\beta_2 \end{aligned} \tag{8}$$

From (8), we can solve

$$\begin{aligned} \beta_0 &= 36864 \\ \beta_1 &= -40960 \\ \beta_2 &= 11520 \end{aligned} \tag{9}$$

$$A^{10} = h(A) = 36864I - 40960A + 11520A^2 = \begin{bmatrix} 67584 & -245760 & 225280 \\ 28160 & -101376 & 92160 \\ 11520 & -40960 & 36864 \end{bmatrix}$$

MATLAB Demo Code:

```
%% Lab 4: Eigenvalues, Jordan form, Matrix function
%% Developer: HRLin
close all;
clear all;

%% Define A matrix
A = [6 -12 8; 1 0 0; 0 1 0];

%% Eigenvalues, Jordan forms
ChaPoly = poly(A);       % return the characteristic polynomials of matrix A

rts = roots(ChaPoly);    % Another way to evaluate eignevalues

[VV,DD]=eig(A);          % return eigenvectors in V and eigenvalues in
diagonals of D

rankA=rank(A);           % to return rank value of matrix A

[Q,JA] = jordan(A)       % Q is transformation matrix and JA is Jordan form

%% betas
betac = [1 2 4 ; 0 1 4 ; 0 0 2];
lambdac = [2^10; 10*2^9; 10*9*2^8];
beta = inv(betac)*lambdac;

%% evaluate function
h = eye(size(A))*beta(1)+A*beta(2)+A^2*beta(3)
A^10
```