EE 547 (PMP): Homework 5

Assigned: Thursday, February 12, 2015. Due: Wednesday, February 18, 2015.

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Problem 1 (Controllability and Observability) Consider a continuous-time linear time-invariant system as (1). Please write a MATLAB script to:

$$\dot{x} = \begin{bmatrix} -9 & -31 & -51 & -40 & -12 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \qquad (1)$$

$$y = [95 \ 92 \ -3 \ 60 \ -72]x + [0]u$$

(a) Check if the system is asymptotically stable.

(b) Evaluate the controllability and observability matrices of the system, C and O.

(c) Check if the system is controllable and observable by the ranks of controllability and observability matrices, C and O.

(d) Find the controllability and observability gramians, W_c and W_o, of this system.

(e) Please derive input that drives initial state $x_0 = [-50,40,-300,-100,200]^T$ into $x_1 = [0,0,0,0,0]^T$ within 8 seconds.

$$u(t) = -B^T e^{A^T(t_1-t)} W_C^{-1}(t_1)[e^{At_1}x_0 - x_1] \qquad (2)$$

where $W_C(t)$ is the controllability gramian.

(f) Use **lsim** function to simulate this controllable system with time span defined as **tspan = 0:0.1:8**. The input is the derived input in previous step. Please plot state variable on one chart and derived input on another chart.

Problem 2 (Lyapunov Controllability Test) Consider a continuous-time linear time-invariant system as (3). Please write a MATLAB script to:

$$\dot{x} = \begin{bmatrix} -9 & 9 & -5 \\ 7 & -9 & 7 \\ 2 & 2 & -6 \end{bmatrix} x + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u$$

(3)

(a) Check if the system is asymptotically stable.

(b) Check the controllability of this system by Lyapunov test.

(Hint: you may want to refer lab 5.)

(c) Derive the controllability and observability matrices of the system, C and O.

(d) Check if the system is controllable and observable by the ranks of controllability and observability matrices, C and O.

(e) Find the controllability and observability gramians, W_c and W_o, of this system.

(f) Please derive inputs that drive initial state $x_0 = [10,-20,30]^T$ into $x_1 = [0,0,0]^T$ within 8 seconds. (Note: We have multiple inputs in this system.)

(g) Use **lsim** function to simulate this controllable system with time span defined as **tspan = 0:0.1:8**. The input is the derived input in previous step. Please plot state variable on one chart and derived input on another chart.