EE 547 (PMP) Midterm

Problem 1 Consider a non-linear system as (1).

$$\dot{x} = \begin{bmatrix} f_1(x_1, x_2, x_3, u) \\ f_2(x_1, x_2, x_3, u) \\ f_3(x_1, x_2, x_3, u) \end{bmatrix} = \begin{bmatrix} -9x_1 + \sin x_1 - 4x_2 - (1 + x_1)x_3 + \cos\ u \\ -10 \sin x_1 + (x_1 x_3 - 4)x_2 + 3sinx_3 + x_3^2 cosu \\ 9x_1 + (x_1^2 - 4)x_2 - 10x_3 + u \end{bmatrix}$$

$$y = \begin{bmatrix} g_1(x_1, x_2, x_3, u) \\ g_2(x_1, x_2, x_3, u) \\ g_3(x_1, x_2, x_3, u) \end{bmatrix} = \begin{bmatrix} x_1 + x_2 x_3 + cosu \\ x_2 + x_1 x_3 + u \\ x_3 + x_2 x_3 + sinu \end{bmatrix} \quad (1)$$

(a) Linearize the system around equilibrium point x_{eq} = $(0,0,0)^T$ and u_{eq} = 0. Find the state space representation of the linearized system (i.e., find A, B, C and D matrices.)

(b) Derive transfer functions of this system. Are these transfer functions proper rational functions?

(c) Is the system BIBO stable?

(d) Evaluate the Jordan form of the matrix A and corresponding transformation matrix Q.

(e) Derive state transition matrix from Jordan form.

(f) Given the initial state x_{ini} = $(0.1,-0.1,0.2)^T$, please evaluate zero-input responses of the system in time span: **tspan = 0:0.01:10**. Plot system responses after evaluation.

(g) Following previous step, evaluate impulse response of this system.

(h) Check if the system is asymptotically stable by solving the Lyapunov equation:

$$A^T P + PA = -Q$$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where Q is a symmetric positive-definite matrix.

Problem 2 Consider two systems as (2).

$$\dot{x} = \begin{bmatrix} 3 & 4 & -5 \\ 1 & -7 & -2 \\ 8 & -8 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} u \qquad \dot{x} = \begin{bmatrix} -30 & 97.5 & -94 \\ -22 & 64 & -60 \\ -15.75 & 46.875 & -45 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} + \begin{bmatrix} 12 \\ 8 \\ 6 \end{bmatrix} u \tag{2}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} \qquad y = \begin{bmatrix} -0.25 & 1.125 & -1 \\ 0.125 & -0.8125 & 1 \\ 0.25 & -0.125 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix}$$

(a) Please derive the transfer functions of these two systems.

(b) Are these dynamic systems zero-state equivalent?

Problem 3 Consider a matrix A.

$$A = \begin{bmatrix} 2 & -8 & 4 & -2 & -7 \\ -3 & -10 & -7 & -2 & -3 \\ -10 & 1 & -7 & -5 & 9 \\ 7 & -8 & -6 & -6 & -5 \\ 1 & 8 & -10 & 8 & -9 \end{bmatrix} \tag{3}$$

a) Evaluate eigenvalues and characteristic polynomial of matrix A.

b) Evaluate Jordon form and transformation matrix Q of matrix A.

c) Please evaluate a function f(A) = A^{10} by the Jordan Form method.