EE 547 (PMP)Midterm

Professor Linda Bushnell

Electrical Engineering, University of Washington

EE 547 (PMP): Linear Systems Control Theory

---

1) Read the questions carefully before solving them.
2) Attempt all questions.
3) For each question, **show all steps**.

Determinant of 3×3 matrix A

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \tag{1}$$

is given as

$$\det(A) = a \cdot \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \cdot \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot \det \begin{bmatrix} d & e \\ g & h \end{bmatrix} \tag{2}$$

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
1	$\frac{1}{s}$	(1)
$e^{at} f(t)$	$F(s-a)$	(2)
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$	(3)
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$	(4)
$\delta(t)$	1	(5)
$\delta(t-t_0)$	e^{-st_0}	(6)
$f'(t)$	$sF(s) - f(0)$	(7)
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	(8)
$t^n \ (n \in \mathbb{Z})$	$\frac{n!}{s^{n+1}}$	(9)
$\sin kt$	$\frac{k}{s^2+k^2}$	(10)
$\cos kt$	$\frac{s}{s^2+k^2}$	(11)
e^{at}	$\frac{1}{s-a}$	(12)
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(13)

Problem 1

Consider the system dynamics given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} (x_1 + x_2)^2 + x_2 \cos x_1 + u_1 \\ (x_1 - x_2)^2 - \sin x_2 - u_2 \end{bmatrix} \tag{3}$$

$$y = x_1 + x_2 \tag{4}$$

Linearize the system around $\mathbf{x}_{eq} = \mathbf{0}$. Find the state space representation of the linearized dynamics (i.e., find A,B,C,D matrices.)

Problem 2

2.1. Given the dynamics

$$\dot{x_1} = x_2 + u \tag{5}$$

$$\dot{x_2} = x_1 - u \tag{6}$$

$$y(t) = x_1(t) \tag{7}$$

Find the state space representation of the dynamics (i.e., find A,B,C,D matrices).

2.2. Find the transfer function $G(s)$.

2.3. Find the output $y(t)$ when initial state $x_1 = x_2 = 0$ (zero state response) and impulse is given as the input $u(t)$.

2.4. Consider the linear transformation $\bar{\mathrm{x}} = P\mathrm{x}$, where

$$P = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \tag{8}$$

Find the state space representation of $\bar{\mathrm{x}}$.

Problem 3

Given the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \mathbf{x} \tag{9}$$

Find the state transition matrix of the system. Let $t_0 = 0$.

Problem 4

4.1. Given the matrix A_1,

$$A_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \tag{10}$$

Find the characteristic polynomial of A.

4.2. Given the matrix A_2,

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{11}$$

Compute A_2^{51}. (Hint: A_2 is a permutation matrix).

Problem 5. Consider the matrix A given as

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \tag{12}$$

Compute A^{50}. Show your steps.

Problem 6. Let A be

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} \tag{13}$$

Find the Jordan normal form of A.