Assignment 1 - Tabular Methods

April 18, 2018

1 Assignment 1: Tabular Methods

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This exercise requires you to solve a simple grid-world problem called 'FrozenLake-v0' in OpenAI Gym. We will solve the problem in two different ways. First we will solve the problem using dynamic programming, thus requiring a model of the system. Second we will do it using model-free temporal difference (Q-Learning). Finally, as a bonus you may also show it learning using a naive approach called hill-climbing.

1.0.1 Required for all

Set up environment

```
In [2]: %matplotlib inline
        import gym
        import numpy as np
        from matplotlib import pyplot as plt
        env = gym.make('FrozenLake-v0')
```

Pre. Test Policy Function Write a function to test a policy. Return the average rate of successful episodes over 100 trials.

```
In [3]: def testPolicy(policy): #where policy is a 4*4 matrix or 1*16 array
            #%% ---> start of code
            countSuccess = 0
            for i_episode in range(100):
                observation_t = env.reset()
                for timeStep in range(100):
                    action_t = policy[observation_t] #where action_t is the index
                    observation_t, reward, done, info = env.step(action_t) #here observation_t =
                    if observation_t == 15:
                        countSuccess += 1
                        #print("Episode finished after {} timesteps, successful".format(timeStep
                        break
                    elif done:
                        #print("Episode finished after {} timesteps, failed".format(timeStep+1))
```

```
                break
        percentSuccess = countSuccess / 100.0
        #%% <--- end of code
        return percentSuccess
```

1.1 Model-based Learning

1.1.1 1. Policy Iteration

Perform policy iteration on the Frozenlake example.

1.1 Find the system model First, model $T(s, a, s')$ and $R(s, a, s')$ over 100000 randomly initializations.

```
In [4]: def learnModel():
            #%% ---> start of code
            env = gym.make('FrozenLake-v0')
            num_observations = env.env.nS
            num_actions = env.env.nA
            T = np.zeros((num_observations, num_actions, num_observations))
            R = np.zeros((num_observations, num_actions, num_observations))

            for _ in range (100000):
                s = np.random.choice(num_observations)
                a_t = np.random.choice(num_actions)
                transitions = env.env.P[s][a_t] # three to one transition
                idx = int(np.random.choice(len(transitions)))
                pr, s_t, reward, done = transitions[idx]
                T [s, a_t, s_t] += 1

                if s_t == 15:
                    R[s, a_t, s_t] += reward

            normalPlane = T.sum(axis =2)
            for i in range(T.shape[2]):
                T[:,:,i] = T[:,:,i]/normalPlane
            #%% <--- end of code
            return R,T


        #first learn the model
        [R,T] = learnModel()
```

1.2 What does the transition model tell you about the stochastic behavior of actions? What does it tell you about the stochasticity of the rewards? What would you expect an optimal agent's policy to do? ans: 1. The transition model tells me, the probability of the agent going to next stage state, given the current stage state if this action takes. For example, if the current satge of the agent is at state S1, there are some probabilities that agent may go to one, two, three etc states of the total state space(say S0, S1, S4). Generally, this transition model can minic the agent operating in real word (having noise).

2. The reward table collected from transition model tells me, the collected reward for an agent going to next stage state given the current stage state if this action is taken.

3. Therefore, optimal agent's policy is chosen based on these two tables; the optimal policy can choose a trajectory that maximaize the long term rewards, and avoid hole places.

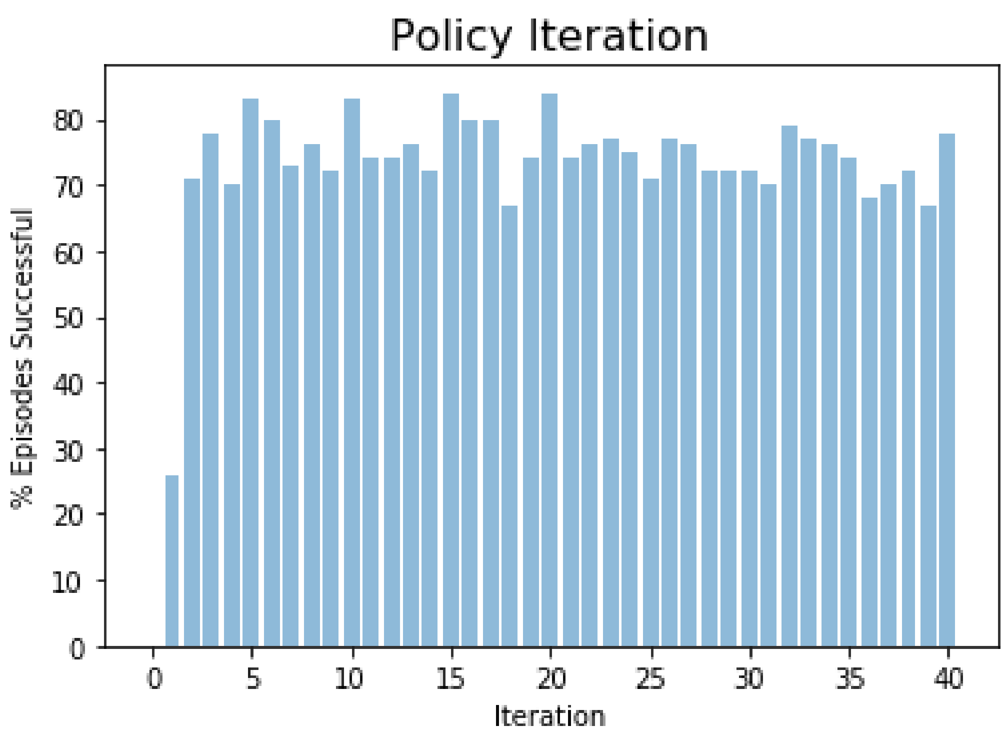
1.3 Write a function for Policy Evaluation

```
In [5]: #runPolicyEvaluation:
        def runPolicyEvaluation(policy,V,R,T,discount_factor):
            #%% ---> start of code
            while True:
                delta = 0
                for s in range (env.env.nS):
                    temp = np.copy(V[s])
                    # where [:] is for s' which is the next stage states.
                    V[s] = T[s, policy[s], :].dot(R[s, policy[s], :] + discount_factor*V[:])
        #             RVs_prime = [T[s, policy[s], i]*(R[s, policy[s], i] + discount_factor*V[i]
        #             V[s] = sum(RVs_prime)
                    delta = max(delta, np.abs(temp-V[s]))
                if delta< 1e-8:
                    Vnew = V
                    break
            #%% <--- end of code
            return Vnew


        def runPolicyImprovement(policy, V, R, T, discount_factor):
            policy_stable = True
            for s in range (env.env.nS):
                temp = np.copy(policy[s])
                Q = np.zeros(env.env.nA)
                for aa in range(env.env.nA):
                    Q[aa] = T[s, aa, :].dot(R[s, aa, :] + discount_factor*V[:])
        #             Q[aa] = sum([T[s, aa, i]*(R[s, aa, i] + discount_factor*V[i]) for i in ran
                policy[s] = Q.argmax()

                if temp != policy[s]:
                    policy_stable = False
            return policy
```

1.4 Run Policy iteration. and show a bar graph of successful runs vs iteration on the policy. Use a discount factor of 0.98, and terminate policy after 40 iterations of policy updates. Plot the percentSuccesses at every iteration (i.e. the return of the testPolicy function).

```
In [6]: #%% ---> start of code
        policy = np.random.choice(env.env.nA, env.env.nS)
        V = np.zeros((env.env.nS)) # 1*16
        discount_factor = 0.98
        percentSuccesses = [testPolicy(policy)]

        for i in range (40):
            V = runPolicyEvaluation(policy,V,R,T,discount_factor)
            policy = runPolicyImprovement(policy, V, R, T, discount_factor)
            percentSuccesses.append(testPolicy(policy))

        #%% <--- end of code


        # plot improvement over time
        plt.figure()
        plt.bar(np.arange(len(percentSuccesses)), 100*np.array(percentSuccesses), align='center'
        plt.ylabel('% Episodes Successful')
        plt.xlabel('Iteration')
        plt.title('Policy Iteration',fontsize=16)
        print('Policy iteration policy:', policy)
```

```
Policy iteration policy: [0 3 3 3 0 0 0 0 3 1 0 0 0 2 1 0]
```



1.2 Model-Free Learning

1.2.1 2 Q Value-Iteration (Q-Learning)

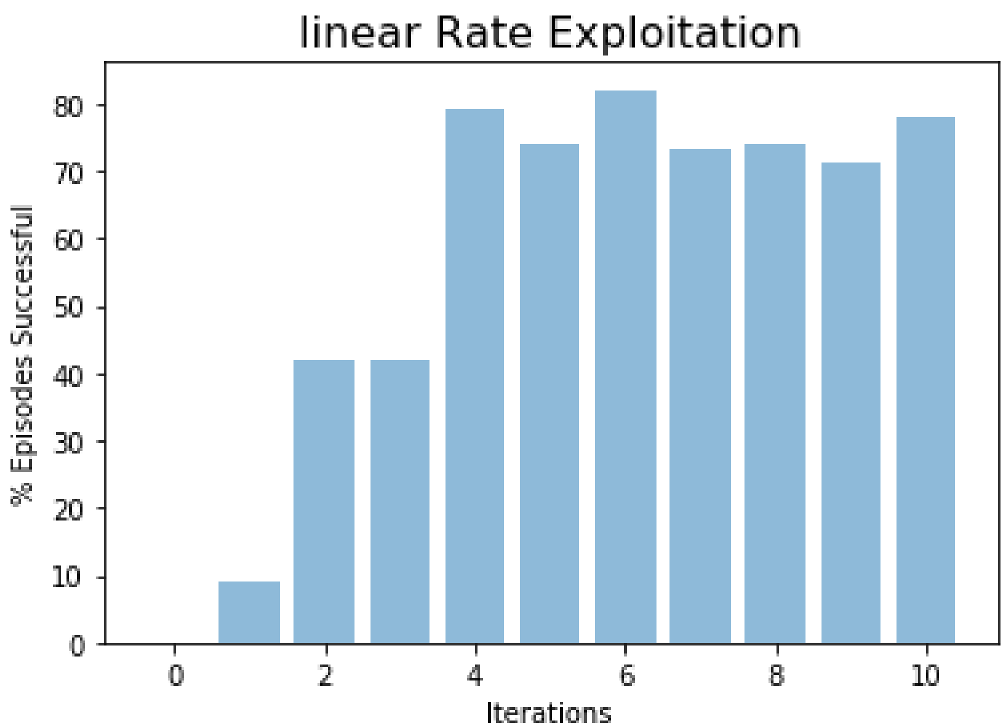
2.1 Set up a Q learning function Set your exploration rate to 1-episode_num/total_num_of_episodes for linear convergence from completely random action selection to a greedy policy. Return a set of policies (at 5%, 10%,...100% of the total number of episodes) so that in the later section you can perform policy evaluation on intermediate Q-tables and show progress.

```
In [20]: def runQLearning(learning_rate,discount_factor,num_of_episodes,Q0,explore_type='linear'
             #%% code starts here
             Q = Q0 #Q is 16 * 4 matrix
             Q_saved = []
             #myLR = learning_rate
             print_episodes = int(0.05* num_of_episodes)
             for episode in range(num_of_episodes):
                 S = env.reset()
                 myLR = learning_rate/(episode/num_of_episodes+1)
                 if explore_type=='linear':
                     epsilon = 1 - episode/num_of_episodes
                 elif explore_type =='log':
                     epsilon = 1000/(1000+episode)
                 for step in range(100):
                     A = getEpsilonGreedyAction (Q, S, epsilon) #choose A from S using policy de
                     S_t, R, done, info = env.step(A) # here S is actually S'
                     Q[S, A] = Q[S, A] + myLR* (R + discount_factor*Q[S_t,:].max() - Q[S, A])
                     #update Q matrix
                     S = S_t
                     if S == 15:
                         break
                     elif done:
                         break
                 if (episode+1) % print_episodes == 0:
                     Q_saved.append(Q.copy()) #save last Q
             #%% code ends here
             return Q_saved

         def getEpsilonGreedyAction(Q, S, epsilon):
             action_idx = int(Q[S].argmax())
             greedy_prob = 1 - epsilon + epsilon/env.env.nA
             not_greedy_prob = epsilon/env.env.nA
             weights = np.full(env.env.nA, not_greedy_prob)
             weights[action_idx] = greedy_prob
             selected_action = np.random.choice(env.env.nA,1,p=weights)
             return int(selected_action)
```

2.2 Perform Q-learning. Show policies during intermediate phases of Q-learning, at 0, 10%, 20%,...,100% of the total episodes during training. Set a learning rate of 0.98 and a discount factor of 0.95. Start with a zero-filled Q-table. Run 10000 episodes. Plot the bar graph of the success rate over time to visualize the improvements to success rate the new policies are providing.

```
In [19]: ## %% ---> start of code
         learning_rate = 0.04
         discount_factor = 0.95
         num_of_episodes = 10000
         percentSuccesses= [0.0]
         Q0 = np.zeros((env.env.nS, env.env.nA))
         lst_Q = runQLearning(learning_rate,discount_factor,num_of_episodes,Q0,explore_type='lin
         for Q in lst_Q:
             policy = Q.argmax(axis = 1)
             percentSuccesses.append(testPolicy(policy))
         percentSuccesses = percentSuccesses[0::2]
         #%% <--- end of code

         plt.figure()
         plt.bar(np.arange(len(percentSuccesses)), 100*np.array(percentSuccesses), align='center
         plt.ylabel('% Episodes Successful')
         plt.xlabel('Iterations')
         plt.title('linear Rate Exploitation', fontsize=16)
         print('Q-learning (linear) policy:', policy)
```

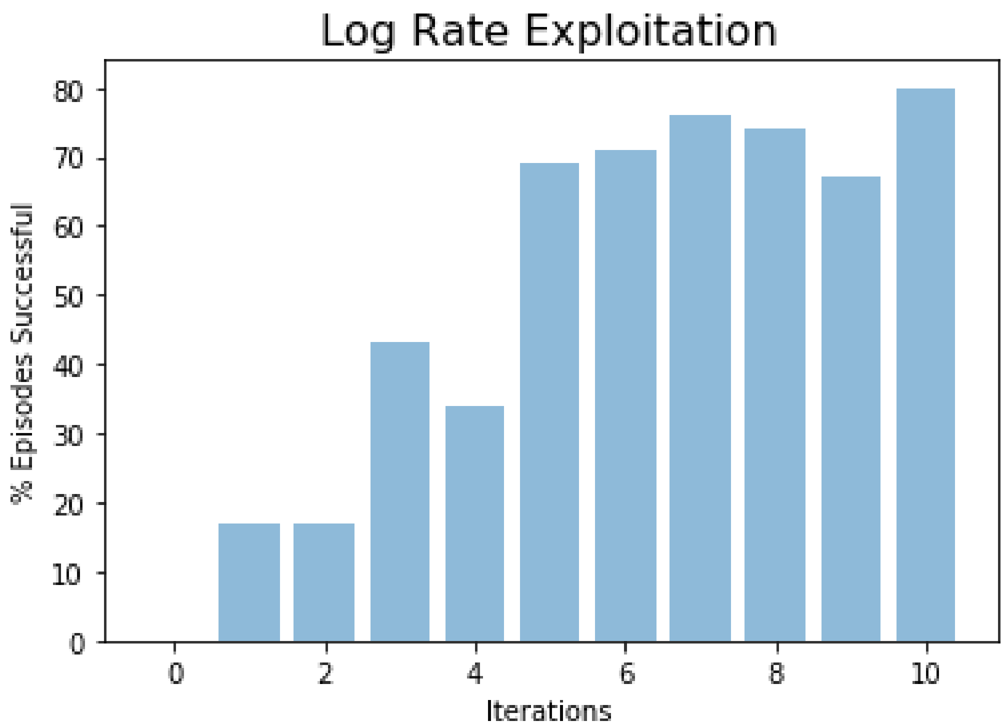
```
Q-learning (linear) policy: [0 3 0 3 0 0 0 0 3 1 0 0 0 2 1 0]
```



2.3 Log Rate Exploration Run Q-learning for a log exploration rate, $\frac{1000}{1000+episodenum}$, for 10,000 episodes. Perform policy evaluation and plot the success rate over time. You may find setting Q0 to a random number initialization helps (set it to something very small, i.e. 0.000001*rand(), since setting it to zero sets a fixed seed).

```
In [27]: #%% ---> start of code
         learning_rate = 0.03
         discount_factor = 0.95
         num_of_episodes = 10000
         percentSuccesses= [0.0]
         #Q0 = np.zeros((env.env.nS, env.env.nA))
         Q0 = 0.000001*np.random.rand(env.env.nS,env.env.nA)
         lst_Q = runQLearning(learning_rate,discount_factor,num_of_episodes,Q0,explore_type='log
         for Q in lst_Q:
             policy = Q.argmax(axis = 1)
             percentSuccesses.append(testPolicy(policy))
         percentSuccesses = percentSuccesses[0::2]
         #%% <--- end of code

         plt.figure()
         plt.bar(np.arange(len(percentSuccesses)),100*np.array(percentSuccesses), align='center'
         plt.ylabel('% Episodes Successful')
         plt.xlabel('Iterations')
         plt.title('Log Rate Exploitation', fontsize=16)
         print('Q-learning (log) policy:', policy)
```

```
Q-learning (log) policy: [0 3 1 3 0 0 2 3 3 1 0 1 3 2 1 2]
```



1.2.2 BONUS: Hill Climbing (25%, granted only if Parts 1 and 2 are complete)

Demonstrate hill climbing, where your Q values are chosen randomly, and you save improvements, with new Q values to try as

$$Q_{test} \leftarrow Q_{best} + rand(S, A)$$

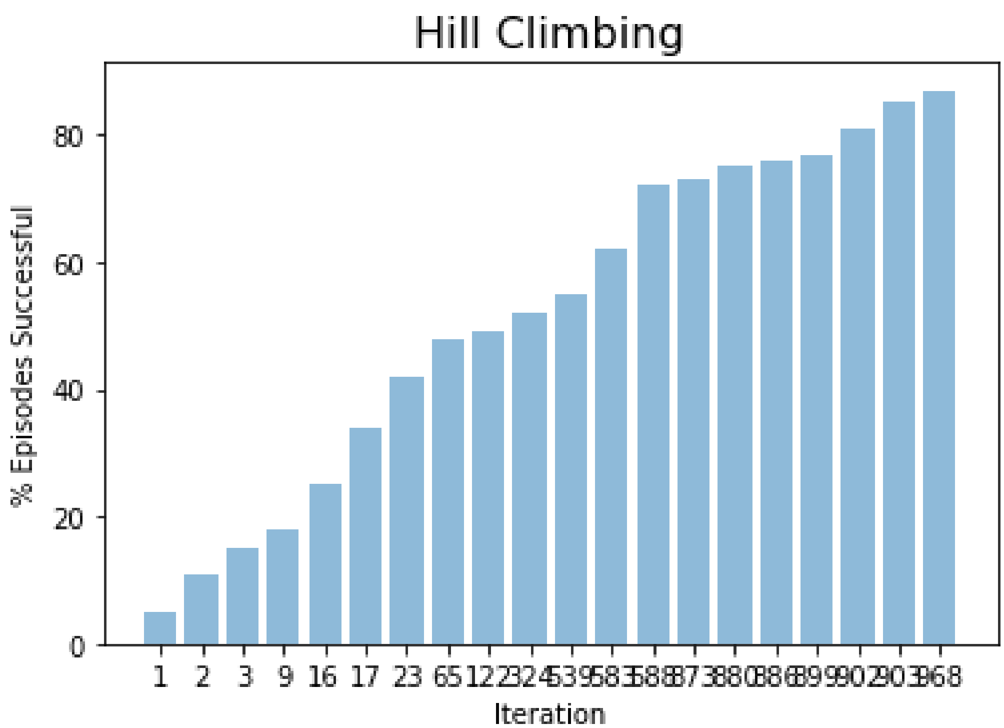
Plot the a bar graph with x-axis labelling the iteration number when an improvement occurred, and y axis as the % of successful episodes. Run on N = 1000 iterations of hill climbing, with 100 episodes per iteration.

```
In [34]: #%% ---> start of code
         import copy
         def runQLearning(learning_rate,discount_factor,num_of_episodes,Q0,explore_type='linear'
             Q = Q0 #Q is 16 * 4 matrix
             Q_saved = []
             #myLR = learning_rate
             print_episodes = int(0.05* num_of_episodes)
             for episode in range(num_of_episodes):
                 S = env.reset()
                 myLR = learning_rate/(episode/num_of_episodes+1)
                 if explore_type=='linear':
                     epsilon = 1 - episode/num_of_episodes
                 elif explore_type =='log':
```

```
            epsilon = 1000/(1000+episode)
        for step in range(100):
            A = getEpsilonGreedyAction (Q, S, epsilon) #choose A from S using policy de
            S_t, R, done, info = env.step(A) # here S is actually S'
            Q[S, A] = Q[S, A] + myLR * (R + discount_factor*Q[S_t,:].max() - Q[S, A]) #
            S = S_t
            if S == 15:
                break
            elif done:
                break
        if (episode+1) % print_episodes == 0:
            Q_saved.append(Q.copy()) #save last Q
    return Q_saved

def getEpsilonGreedyAction(Q, S, epsilon):
    action_idx = int(Q[S].argmax())
    greedy_prob = 1 - epsilon + epsilon/env.env.nA
    not_greedy_prob = epsilon/env.env.nA
    weights = np.full(env.env.nA, not_greedy_prob)
    weights[action_idx] = greedy_prob
    selected_action = np.random.choice(env.env.nA,1,p=weights)
    return int(selected_action)

learning_rate = 0.03
discount_factor = 0.95
num_of_episodes = 100

#Qbest = 0.000001*np.random.rand(env.env.nS,env.env.nA)
Qbest = np.zeros((env.env.nS, env.env.nA))
Qbest_policy = Qbest.argmax(axis = 1)
Qbest_accu = 0.0
#testPolicy(Qbest_policy)
improvementsIndex = []
percentSuccesses = []


for climbing_idx in range(1000):
    Qtest = Qbest +  0.0000001*(np.random.rand(env.env.nS,env.env.nA)-0.5)
    lst_Qtested = runQLearning(learning_rate,discount_factor,num_of_episodes,Qtest,expl

    Qtested = lst_Qtested[-1]
    Qtested_policy = Qtested.argmax(axis = 1)
    Qtested_accu = testPolicy(Qtested_policy)

    if Qtested_accu > Qbest_accu:
        #here I get the improvement
        Qbest = copy.deepcopy(Qtested)
        Qbest_accu = copy.deepcopy(Qtested_accu)
```

```
            print(Qbest_accu, end = '\r')
            improvementsIndex.append(climbing_idx)
            percentSuccesses.append(Qbest_accu)

    policy = Qbest.argmax(axis = 1)
    #%% <--- end of code
    plt.figure()
    plt.bar(np.arange(len(improvementsIndex)), 100*np.array(percentSuccesses), align='cente
    plt.xticks(np.arange(len(improvementsIndex)),improvementsIndex)
    plt.ylabel('% Episodes Successful')
    plt.xlabel('Iteration')
    plt.title('Hill Climbing',fontsize=16)
    print(policy,': Hill-climbing policy')
```

```
[0 3 1 3 0 2 0 2 3 1 0 0 3 2 1 0] : Hill-climbing policy
```




```
In [35]: percentSuccesses[-1]
```

```
Out[35]: 0.87
```