STAT3006:TUTORIAL6

Examples on MH algorithm and Gibbs sampler

MCMC METHOD

- Sample from f(x)
- Construct a Markov chain $\{X_n\}$ with f(x) being its stationary distribution
- If $\{X_n\}$ is irreducible, aperiodic, f(x) is also its limiting distribution
- When n is large, X_n is very like a sample from f(x)

MH ALGORITHM

- Beginning with X_0
- We have a proposal Y from $p(y|x_0)$
  - $p(y|x_0)$ may be independent of x_0
  - $p(y|x_0)$ may be equal to $p(x_0|y)$
- Accept Y as X_1 with probability $\min(\frac{f(y)}{f(x_0)} \frac{p(x_0|y)}{p(y|x_0)}, 1)$
- So on so forth until thousands of (possibly more) samples obtained
- If trace plot becomes stable after B iterations, the period of producing first B samples are called burn-in.

GIBBS SAMPLER

- A special case of MH algorithm, but is more suitable to high dimensional sampling problems.
- f(x) is a multivariate function
- Full conditional functions $f_1(x_1|x_2, \dots, x_p)$, $f_2(x_2|x_1, \dots, x_p)$,…, $f_p(x_p|x_1, \dots, x_{p-1})$
- Given $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_p^{(0)})$
- Sample $x_1^{(1)}$ from $f_1(x_1|x_2^{(0)}, \dots, x_p^{(0)})$
- Sample $x_2^{(1)}$ from $f_2(x_2|x_1^{(1)}, \dots, x_p^{(0)})$
- …
- Sample $x_p^{(1)}$ from $f_p(x_p|x_1^{(1)}, \dots, x_{p-1}^{(1)})$
- So on so forth
- Diagnosis: trace plot

EXAMPLE1: BINOMIAL-BETA

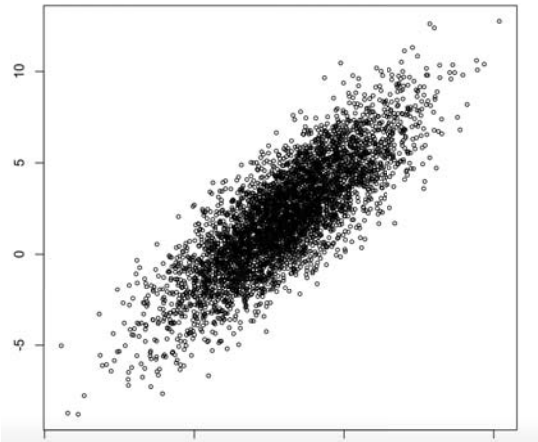
- $X \sim \text{Binomial}(n, \theta)$, $\theta \sim \text{Beta}(a, b)$
- The marginal distribution f(x) is called Binomial-Beta distribution.
- Draw samples from f(x)
- Sample x from f(x) $\Leftrightarrow$ Sample (x, θ) from f(x, θ) and only keep x.

EXAMPLE2: BINOMIAL-BETA-POISSON

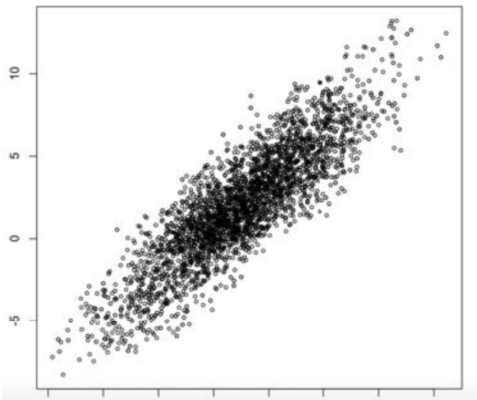
- $X \sim \text{Binomial}(n, \theta)$, $\theta \sim \text{Beta}(a, b)$, $n \sim \text{Poisson}(\boldsymbol{\lambda})$
- The marginal distribution f(x) is called Binomial-Beta-Poisson distribution.
- Draw samples from f(x)
- Sample x from f(x) $\Leftrightarrow$ Sample (x, θ, n) from $f(x, \theta, n)$ and only keep x.

EXAMPLE3: BIVARIATE NORMAL

- Generate samples from bivariate normal distribution $N(\boldsymbol{\mu}, \Sigma)$
- Gibbs sampler:
  - What are the full conditional functions?



- MH algorithm:
  - How to choose the proposal distribution?



EXAMPLE4: POISSON PROCESS WITH A CAHNGE POINT

- $X_i \sim Poisson(\boldsymbol{\mu})\ for\ i = 1,2,\ldots,k; X_i \sim\ Poisson(\boldsymbol{\lambda}) for\ i = k+1,\ldots,m$
- **μ**, k, $\boldsymbol{\lambda}$ are unknown; $X_i\ (i = 1,\ldots,m) and\ m$ are observed data.
- Problem: estimate **μ**, k, $\boldsymbol{\lambda}$.
- Priors on **μ**, k, $\boldsymbol{\lambda}$:
  - k:Uniform prior on {1,2,…,m-1}
  - **μ**: Gamma(a1,b1).
  - $\boldsymbol{\lambda}$: Gamma(a2,b2)
- Draw posterior samples from f(**μ**, k, $\boldsymbol{\lambda}$ |X)
- Use Gibbs sampler