TUTORIAL7

1. Hybrid Gibbs Sampler
2. Examples on MCMC

HYBRID GIBBS SAMPLER

- We are now familiar with Gibbs sampler
- Target: sample from $f\left(x_1, x_2, \dots, x_p\right)$
- Gibbs sampler proceeds as follows:
  - Given initial values $x_1^{(0)}, x_2^{(0)}, \dots, x_p^{(0)}$
  - Sample from $f_i\left(x_i \mid x_1^{(t+1)}, x_2^{(t+1)}, \dots, x_{i+1}^{(t)}, \dots, x_p^{(t)}\right)$ iteratively
  - After burn-in, collect samples $x^{(B+1)}, x^{(B+2)}, \dots$
- If one conditional distribution $f_i\left(x_i \mid x_1, x_2, \dots, x_p\right)$ is not standard (not common distributions, e.g. normal, Poisson, …), how do we do?
- We can use MH algorithm to sample from $f_i\left(x_i \mid x_1, x_2, \dots, x_p\right)$!
  - MH step in Gibbs sampler
- How many steps do we need in the MH step?

HYBRID GIBBS SAMPLER

- It turns out ONLY ONE step is enough.
- Since MH step(s) is(are) present in Gibbs sampler, we call this type of algorithm hybrid Gibbs sampler.
- We assume conditional functions $f_1, \dots, f_{p-1}$ are standard, but directly sampling from f_p is hard.
- In each iteration, steps 1 to p-1 in hybrid Gibbs sampler are the same as those in Gibbs sampler. However, for step p, we adopt the MH,
- Generate a proposal from $q\left(y \middle| x_p^{(t)}\right)$
- Calculate $r = \min\left(\frac{f_p\left(y \middle| x_1^{(t+1)}, x_2^{(t+1)}, \dots, x_{p-1}^{(t+1)}\right)}{f_p\left(x_p^{(t)} \middle| x_1^{(t+1)}, x_2^{(t+1)}, \dots, x_{p-1}^{(t+1)}\right)} \frac{q\left(x_p^{(t)} \middle| y\right)}{q\left(y \middle| x_p^{(t)}\right)}, 1\right),$
- Accept $x_p^{(t+1)}$ as y with probability r . Otherwise we let $x_p^{(t+1)}$ be $x_p^{(t)}$.

EXAMPLE 1: HYBRID GIBBS SAMPLER

- There are 360 Poisson distributed with parameter $\boldsymbol{\lambda}$ items $X_1, X_2, \ldots, X_{360}$.
- 139 items are zero.
- 128 items are one.
- 55 items are two
- 25 items are three.
- The rest of 13 items are only known to be more than or equal to four.
- How to use ALL the observations to estimate $\boldsymbol{\lambda}$.
  - What is the observed likelihood function ? (Difficult to calculate MLE)
  - Hybrid Gibbs Sampler (MH step is in sampling unobserved 13 items).
  - Other ways? (EM, only estimates, it cannot provide the information about posterior distribution)

EXAMPLE 2: DIRICHLET DISTRIBUTION

- Consider the multinomial distribution: $(Y_1, Y_2, Y_3) \sim Multinomial(1000; p_1, p_2, p_3)$
- Y1 Y2 Y3 are observed
- What is the conjugate distribution of (p1,p2,p3)?
- How to sample from Dirichlet distribution?

EXAMPLE 3(CONT.)

- $X_i \sim Poisson(\boldsymbol{\mu})\ for\ i = 1,2, \ldots, k; X_i \sim Poisson(\boldsymbol{\lambda}) for\ i = k + 1, \ldots, m$
- **μ**, k, **λ** are unknown; $X_i\ (i = 1, \ldots, m) and\ m$ are observed data.
- Problem: estimate **μ**, k, **λ** .
- Priors on **μ**, k, **λ** :
  - k:Uniform prior on {1,2,…,m-1}
  - **μ**: Gamma(a1,b1).
  - **λ**: Gamma(a2,b2)
- Draw posterior samples from f(**μ**, k, **λ** |X)
- Use Gibbs sampler