STAT 3006 Assignment 3

Due date: 5:00 pm on 6 April

(30%)**Q**1: There are 30 independent Poisson distributed random variables $\{X_1, X_2, \ldots, X_{30}\}$. $X_t (1 \le t \le 30)$ denotes the value we observed at time t. There exists a *change point* k such that $\{X_1, \ldots, X_k\} \sim Poi(\lambda)$ but $\{X_{k+1}, \ldots, X_{30}\} \sim Pois(\mu)$. We are interested in finding the change point k and estimating the unkown parameters λ and μ. Please derived a Gibbs sampler and implement it in R using Q1 dataset to find the values of k, λ and μ. **Note**: assign $Gamma(2, 2)$ priors to both of λ and μ, and assign uniform prior $\frac{1}{29} I(1 \le k \le 29)$ to k; 1000 iterations are needed, and collect posterior samples in the last 800 iterations; use posterior mean to estimate λ, μ, and use posterior mode to estimate the change point k.

(30%)**Q**2: There are 500 i.i.d. Poisson distributed random variables (r.v.s) $Pois(\lambda)$. You know 27 r.v.s take values at zero, 79 r.v.s at one, 117 r.v.s at two, 113 r.v.s at three, 77 r.v.s at four, and 87 r.v.s are more than or equal to five. Based on the information, please carry out a hybrid Gibbs sampler to estimate λ. **Note**: assign the prior $\pi(\lambda) \propto \frac{1}{\lambda}$ to λ; 10,000 iterations are needed, and collect posterior samples in the last 8000 iterations; use posterior mean to estimate π.

(40%)**Q**3: There are 100 samples $\{X_1, X_2, \ldots, X_{100}\}$. For each sample i, $X_i = (X_{i1}, X_{i2})$. You know that these samples are from three clusters. For sample i, we use Z_i to denote the cluster number to which sample i belongs. The proportion of the three clusters is denoted by π_1, π_2, π_3. Specifically, for each sample i, $P(Z_i = k) = \pi_k (1 \le k \le 3)$. Given $Z_i = k$, $X_{i1} \sim N(\mu_{1k}, 1)$ and $X_{i2} \sim N(\mu_{2k}, 1)$. Use Q3 dataset to estimate parameters $(\mu_{1k}, \mu_{2k})(k = 1, 2, 3)$, (π_1, π_2, π_3) and $Z_i (1 \le i \le 100)$. **Note**: assign $Dirichlet(1, 1, 1)$ prior to (π_1, π_2, π_3), and assign uniform prior $p(\mu_{jk}) \propto 1$ to $\mu_{jk} (j = 1, 2; k = 1, 2, 3)$; implement Gibbs sampler 10,000 iterations, and only samples in the last 8000 iterations are kept; use posterior means to estimate $\boldsymbol{\mu}$, $\boldsymbol{\pi}$ and use posterior modes to estimate $\mathbf{Z}$.

Requirements:

-	in the paper report	in the R code file
Q1	Detailed derivation of Gibbs sampler estimates for k, λ, μ	R code
Q2	Detailed derivation of hybrid Gibbs samlper the estimate for λ	R code
Q3	Detailed derivation of Gibbs sampler Trace plots for μ_{11}, Z_1 and π_1 all estimates for $\boldsymbol{\mu}$, $\boldsymbol{\pi}$,$\mathbf{Z}$	R code