RMSC4002 Tutorial 4

Chapter 3

October 18, 2017

1 Historical Simulation (nonparametric) Approach

```
d<-read.csv("stock.csv")      # read in data
x<-as.matrix(d)               # change to matrix
n<-nrow(x)                    # no. of obs

xn<-as.vector(x[n,])          # select the last obs
w<-c(40000,30000,30000)       # amount on each stock
p0<-sum(w)                    # total amount
ws<-w/xn                      # no. of shares bought at day n
ns<-n-1                       # no.of scernarios
hsim<-NULL                    # initialize hsim
for (i in 1:ns) {
  t<-xn*(x[i+1,]/x[i,])       # scenario i
  hsim<-rbind(hsim,t)         # append t to hsim
}

hsim<-as.matrix(hsim)         # change to matrix
ws<-as.matrix(ws)
ps<-as.vector(hsim%*%ws)      # compute portfolio value
loss<-p0-ps                   # compute loss
(VaRs<-quantile(loss,0.99))   # compute and display 1-day 99% VaR
3535.733
```

2 Model Building Approach

2.1 Normal Distribution

```
t1<-as.ts(d$HSBC)            # change to time series
t2<-as.ts(d$CLP)
t3<-as.ts(d$CK)
u1<-(lag(t1)-t1)/t1          # compute u
u2<-(lag(t2)-t2)/t2
u3<-(lag(t3)-t3)/t3
u<-cbind(u1,u2,u3)           # form matrix u
S<-var(u)                    # sample cov. matrix
dp<-as.vector(u%*%w)         # Delta P
sdp<-sd(dp)                  # sd of portfolio (same as sqrt(w%*%S%*%w))
(VaRn<-qnorm(0.99)*sdp)      # compute and display 1-day 99% VaR
3062.165
```

2.2 Student's t(v) Distribution

```
ku<-sum((dp/sdp)^4)/length(dp)-3        # sample excess kurtosis
v<-round(6/ku+4)                        # degree of freedom
VaRt<-qt(0.99,v)*sdp                    # 1-day 99% VaR
VaRt
4136.686
```

3 Approach via Extreme Value Theory

```
# EVT
u<-3.2            # threshold value
m<-mean(loss)     # mean loss
s<-sd(loss)       # sd loss
z<-(loss-m)/s     # standardize loss
zx<-z[z>u]        # select z>u
nu<-length(zx)    # no. of zx
```

```
# define -log_likelihood function
log_lik<-function(p,dat) {          # parameter vector p=(xi,beta)
  length(dat)*log(p[2])+(1/p[1]+1)*sum(log((1+p[1]*dat/p[2])))
}

p0<-c(0.2,0.01)                     # initial p0=(xi,beta)
res<-optim(p0,log_lik,dat=(zx-u))   # min -log_lik
(p<-res$par)                        # MLE p=(xi,beta)
[1] 0.6755755 0.3117039
> -res$value                        # max value
[1] -3.058536

q<-0.99
(VaR<-u+(p[2]/p[1])*((length(z)*(1-q)/nu)^(-p[1])-1))
[1] 3.056386

(VaRe<-m+VaR*s)                     # 1-day 99% VaR using EVT
[1] 4000.848
```

4 Back Testing

```
n<-nrow(d)-1                # no. of obs. of u
n1<-n-250+1                 # starting index for 250 days before n
x<-as.matrix(d[n1:n,])      # select the most recent 250 days
ps<-as.vector(x%*%ws)       # compute portfolio value
ps<-c(ps,sum(w))            # add total amount at the end
loss<-ps[1:250]-ps[2:251]   # compute daily loss
sum(loss>VaRs)              # count the no of exceptions
0
sum(loss>VaRn)
0
sum(loss>VaRt)
0
sum(loss>VaRe)
0
```

5 VaR using EXCEL

In EXCEL, computing VaRs, VaRn and VaRt are rather simple and straight forward. EXCEL has built-in function NORMSINV() and TINV() to return the percentile points of normal and t distribution respectively.

- In the **hist-sim tab**, A2:C1044 are the closing prices of HSBC, CLP and CK. Column E2:E1043 is the scenario number.
- Enter the formula =A$1044*A3/A2 in F2 for computing the future closing price of HSBC based on the first scenario. Copy this formula to F2:H1043 for other stocks and scenarios.
- Enter 40000, 30000 and 30000 in N3:P3 to represent the amount of money spent on each stock and Q3 is the total amount. Enter =N3/A$1044 in N4 to compute the number of shares for HSBC. Copy it to O4:P4 for other stocks.
- Enter the formula =SUMPRODUCT(F2:H2,$N$4:$P$4) in I2 and =$Q$3-I2 in J2 to compute the portfolio value and loss based on the first scenario. Copy them downward to I1043:J1043 for other scenarios.
- K2:K1043 contains the loss sorted in descending order. N6 is the 1-day 99% VaRs and is obtained by entering =PERCENTILE(K2:K1043,0.99).
- In the **model tab**, Columns A, B and C contains the closing prices and Columns E, F and G contains the corresponding returns. K2:M2 contains the amount spent on each stock.
- Enter =SUMPRODUCT(E2:G2,$K$2:$M$2) in H2 to compute the change in portfolio value ΔP. Copy it downward to H1043 to form the distribution of ΔP.
- Use the built-in covariance function to compute the Sample covariance matrix of u and store it in K5:M7.
- Enter the formula =SUMPRODUCT(MMULT(K2:M2,K5:M7),K2:M2) in K10 to compute the sample variance of ΔP and =STDEV(H2:H1043) to compute the sample s.d. of ΔP.
- Enter =NORMSINV(0.99) in K14 to return the 99% percentile of standard normal and =K14*K12 in K15 to give 1-day 99% VaRn.
- For the student's t model, enter =KURT(H2:H1043) in K17 to compute the excess kurtosis of ΔP and =ROUND(6/K17+4,0) in K18 to estimate the degree of freedom. Finally enter =TINV(0.02,6) in K20 to return the 99% percentile of t distribution and enter =K20*K12 in K21 to give 1-day 99% VaRt.

- In the **EVT tab**, $u1, u2, u3, \Delta P$ and Loss($=-\Delta P$) are stored in columns E to I respectively. The mean and sd of Loss are computed and stored in P3 and P4. Then we compute the standardized score of Loss by entering =(I2-$P$3)/$P$4 in J2 and copy it down to J1043. Column K are the sorted value of Z.
- Suppose we choose $u = 3.2$ in Q4. Then we can create the Loss> u in L2 to L7. Enter some initial values for ξ and β in O7 and P7 , say 0.5 and 0.2.
- Enter the formula in M2 ==(-1/$O$7-1)*LN(1+$O$7*(L2-$R$4)/$P$7)-LN($P$7) which used to compute the log-likelihood value.
- We compute the log-likelihood value in P9 by =SUM(M2:M7). Use solver() to maximize P9 with variable cells O7 and P7 to obtain MLE.
- VaR of the Z in computed in P11 and the VaRe is computed in P12.
- In the **back-test tab**, columns A, B and C contain the closing prices. K2:M2 contain the amount spent on each stock. K3:M3 contains the number of shares of each stock.
- Enter =SUMPRODUCT(A2:C2,$K$3:$M$3) in D2 to compute the portfolio value and enter =D2-D3 in E2 to compute the loss. Copy it downward to E1044 to give the portfolio value and loss on each day. Copy VaRs, VaRn, VaRt and VaRe in K6:K9.
- Finally we can find out the number of exceptions in the past 250 days by entering =($E794>$K$6)+0 in F794, copy it down to F1043 and =SUM(F794:F1043) in F1096. Similarly for columns G and I.
- In the **ES tab**, the setup is similar. ΔP, Loss and sorted Loss are in columns H, I and J respectively. M4:6 are the $n, \omega, n\omega$. In M8, enter =(SUM(J2:J11)/M4+(M5-10/M4)*J12)/M5 to obtain the ES without the normality assumption.
- M10 and M11 are the mean and sd of Loss in column I. In M14, enter =M11*NORMDIST(NORMINV(1-M5,0,1),0,1,FALSE)/M5+M10 to obtain the ES under the normal model.

6 Exercises

Exercise 2013-14 final Q3 Let u_t be the relative return of the daily closing stock price X at day t, $t = 1, ..., n$. Suppose that the closing prices of these three stocks on Dec 30th, 2012 are $(79.9, 63, 1, 119.9)'$. We compute the sample mean $\bar{u} = (0.00050, 0.00087, 0.00138)'$, and covariance matrix of stock A, B, and C from Jan 4, 2012 to Dec 30, 2012.

$$S = \begin{bmatrix} 0.00022 & 0.00002 & 0.00009 \\ 0.00002 & 0.00005 & 0.00003 \\ 0.00009 & 0.00003 & 0.00020 \end{bmatrix}$$

We formed a portfolio P by buying 200 shares of stock A, 600 shares of stock B and 200 shares of stock C on Dec 30, 2012.

- Assume that $u_t \sim N(\bar{u}, S)$ and denote the daily change in the portfolio value of P by ΔP. Find the mean and standard deviation of ΔP. What is the expected value of the portfolio P on Dec 31, 2012?
- Find the 1-day 99% VaR of ΔP.
- Let the portfolio loss $L \sim N(\mu, \sigma^2)$. Let q_ϵ be the $(1-\epsilon)100\%$ 1-day VaR. Prove that the expected shortfall is $ES(q_\epsilon) = \sigma\phi(q_\epsilon)/\epsilon + \mu$, where $\phi(.)$ is the probability density function of the standard normal distribution. Hence find the expected shortfall for the 1-day 99% VaR.