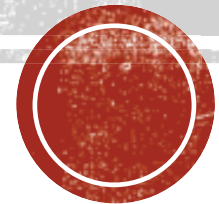


TUTORIAL7

1. Hybrid Gibbs Sampler
2. Examples on MCMC



HYBRID GIBBS SAMPLER

- We are now familiar with Gibbs sampler
- Target: sample from $f(x_1, x_2, \dots, x_p)$
- Gibbs sampler proceeds as follows:
 - Given initial values $x_1^{(0)}, x_2^{(0)}, \dots, x_p^{(0)}$
 - Sample from $f_i(x_i | x_1^{(t+1)}, x_2^{(t+1)}, \dots, x_{i+1}^{(t)}, \dots, x_p^{(t)})$ iteratively
 - After burn-in, collect samples $x^{(B+1)}, x^{(B+2)}, \dots$
- If one conditional distribution $f_i(x_i | x_1, x_2, \dots, x_p)$ is not standard (not common distributions, e.g. normal, Poisson, ...), how do we do?
- We can use MH algorithm to sample from $f_i(x_i | x_1, x_2, \dots, x_p)$!
 - MH step in Gibbs sampler
- How many steps do we need in the MH step?



HYBRID GIBBS SAMPLER

- It turns out ONLY **ONE** step is enough.
- Since MH step(s) is(are) present in Gibbs sampler, we call this type of algorithm hybrid Gibbs sampler.
- We assume conditional functions f_1, \dots, f_{p-1} are standard, but directly sampling from f_p is hard.
- In each iteration, **steps 1 to p-1** in hybrid Gibbs sampler are the **same** as those in Gibbs sampler. However, for step **p**, we adopt the MH,
- Generate a proposal from $q(y|x_p^{(t)})$
- Calculate $r = \min\left(\frac{f_p(y|x_1^{(t+1)}, x_2^{(t+1)}, \dots, x_{p-1}^{(t+1)})}{f_p(x_p^{(t)}|x_1^{(t+1)}, x_2^{(t+1)}, \dots, x_{p-1}^{(t+1)})} \frac{q(x_p^{(t)}|y)}{q(y|x_p^{(t)})}, 1\right)$,
- Accept $x_p^{(t+1)}$ as y with probability r . Otherwise we let $x_p^{(t+1)}$ be $x_p^{(t)}$.



EXAMPLE 1: HYBRID GIBBS SAMPLER

- There are 360 Poisson distributed with parameter λ items X_1, X_2, \dots, X_{360} .
- 139 items are zero.
- 128 items are one.
- 55 items are two
- 25 items are three.
- The rest of 13 items are only known to be more than or equal to four.
- How to use **ALL** the observations to estimate λ .
 - What is the observed likelihood function ? (Difficult to calculate MLE)
 - Hybrid Gibbs Sampler (MH step is in sampling unobserved 13 items).
 - Other ways? (EM, only estimates, it cannot provide the information about posterior distribution)



EXAMPLE 2: DIRICHLET DISTRIBUTION

- Consider the multinomial distribution: $(Y_1, Y_2, Y_3) \sim \text{Multinomial}(1000; p_1, p_2, p_3)$
- Y_1, Y_2, Y_3 are observed
- What is the conjugate distribution of (p_1, p_2, p_3) ?
- How to sample from Dirichlet distribution?



EXAMPLE 3(CONT.)

- $X_i \sim \text{Poisson}(\mu)$ for $i = 1, 2, \dots, k$; $X_i \sim \text{Poisson}(\lambda)$ for $i = k + 1, \dots, m$
- μ, k, λ are unknown; X_i ($i = 1, \dots, m$) and m are observed data.
- Problem: estimate μ, k, λ .
- Priors on μ, k, λ :
 - k : Uniform prior on $\{1, 2, \dots, m-1\}$
 - μ : $\text{Gamma}(a_1, b_1)$.
 - λ : $\text{Gamma}(a_2, b_2)$
- Draw posterior samples from $f(\mu, k, \lambda \mid X)$
- Use Gibbs sampler

