2014年全国硕士研究生入学统一考试数学二试题答案

1.B

$$\lim_{x\to0^+}\frac{\ln^{\alpha}(1+2x)}{x}=\lim_{x\to0^+}\frac{(2x)^{\alpha}}{x}=2^{\alpha}\lim_{x\to0^+}x^{\alpha-1}=0$$

$$\lim_{x\to0^+}\frac{(1-\cos x)^{\frac{1}{2}}}{x}=\lim_{x\to0^+}\frac{(\frac{1}{2}x^2)^{\frac{1}{\alpha}}}{x}=(\frac{1}{2})^{\frac{1}{\alpha}}\lim_{x\to0^+}x^{\frac{2}{a}-1}=0$$

$$\therefore\frac{2}{\alpha}-1>0\therefore\alpha<2$$

2、C

$$y=x+\sin\frac{1}{x}$$

$$k=\lim_{x\to\infty}\frac{y}{x}=\lim_{x\to\infty}\frac{x+\sin\frac{1}{x}}{x}=1$$

$$\lim_{x\to\infty}\left(y-x\right)=\lim_{x\to\infty}\sin\frac{1}{x}=0$$

$$\therefore y=x+\sin\frac{1}{x}\text{存在斜渐近线}y=x$$

3、D

举例：

$$\text{令}f\left(x\right)=x^2,\text{则在}[0,1]\text{区间}$$

$$f(0)=0$$

$$f(1)=1$$

$$\therefore g\left(x\right)=0\cdot(1-x)+1\cdot x=x$$

$$\therefore f\left(x\right)\le g\left(x\right)$$

$$\text{又}f''\left(x\right)=2\ge0\therefore D$$

4．C

$$\frac{dy}{dx}=\frac{2t+4}{2t}$$

$$\frac{dy}{dx}\Big|_{t=1}=3$$

$$\frac{d^2y}{dx^2}=\frac{\frac{2\cdot 2t-2(2t+4)}{(2t)^2}}{2t}=\frac{-8}{(2t)^3}$$

$$\therefore \frac{d^2y}{dx^2}\Big|_{t=1}=-1$$

$$k=\frac{|y''|}{(1+y'^2)^{\frac{3}{2}}}=\frac{1}{(1+3^2)^{\frac{3}{2}}}$$

$$\therefore R=\frac{1}{k}=(1+3^2)^{\frac{3}{2}}=10^{\frac{3}{2}}=10\sqrt{10}$$

5、

$\frac{f(x)}{x}=\frac{\arctan x}{x}=\frac{1}{1+\xi^2}$. 故 $\xi^2=\frac{x-\arctan x}{\arctan x}$.

$$\lim_{x\to 0}\frac{\xi^2}{x^2}=\lim_{x\to 0}\frac{x-\arctan x}{x^2\cdot\arctan x}=\lim_{x\to 0}\frac{x-\arctan x}{x^3}$$

$$=\lim_{x\to 0}\frac{1-\frac{1}{1+x^2}}{3x^2}=\lim_{x\to 0}\frac{x^2}{3x^2(1+x^2)}=\frac{1}{3}.$$

6、

排除法当 $B=\frac{\partial^2 u}{\partial x\partial y}>0$，因为 $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$，故 $A=\frac{\partial^2 u}{\partial x^2}$ 与 $B=\frac{\partial^2 u}{\partial y^2}$ 异号.

$AC-B^2<0$，函数 $u(x,y)$ 在区域 D 内没有极值.

连续函数在有界闭区域内有最大值和最小值，故最大值和最小值在 D 的边界点取到.

7、B

解析：

$$\begin{vmatrix} 0 & \mathrm{a} & b & 0 \\ a & 0 & 0 & b \\ 0 & c & d & 0 \\ c & 0 & 0 & d \end{vmatrix}$$

$$= a\times(-1)^{2+1}\begin{vmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & d \end{vmatrix} + c\times(-1)^{4+1}\begin{vmatrix} a & b & 0 \\ 0 & 0 & b \\ c & d & 0 \end{vmatrix}$$

$$= -a\times d\times(-1)^{3+3}\begin{vmatrix} a & b \\ c & d \end{vmatrix} - c\times b\times(-1)^{2+3}\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= -ad\begin{vmatrix} a & b \\ c & d \end{vmatrix} + bc\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= (bc-ad)\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= -(ad-bc)^2$$

8、A

解析：

已知α_1，α_2，α_3无关

设λ_1（$\alpha_1+\mathrm{k}\alpha_3)+\lambda_2(\alpha_2+l\alpha_3)=0$

即$\lambda_1\alpha_1+\lambda_2\alpha_2+$（$\mathrm{k}\lambda_1+l\lambda_2)\alpha_3=0$

$\Rightarrow \lambda_1=\lambda_2=k\lambda_1+l\lambda_2=0$

从而$\alpha_1+\mathrm{k}\alpha_3$，$\alpha_2+l\alpha_3$无关

反之，若$\alpha_1+\mathrm{k}\alpha_3$，$\alpha_2+l\alpha_3$无关，不一定有$\alpha_1$，$\alpha_2$，$\alpha_3$无关

例如，$\alpha_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$，$\alpha_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$，$\alpha_3=\begin{pmatrix}0\\0\\0\end{pmatrix}$

9. $\int_{-\infty}^{1}\frac{1}{x^2+2x+5}dx=\int_{-\infty}^{1}\frac{1}{(x+1)^2+4}dx=\frac{1}{2}\arctan\frac{x+1}{2}|_{-\infty}^{1}=\frac{1}{2}[\frac{\pi}{4}-(-\frac{\pi}{2})]=\frac{3}{8}\pi$

10.

$f'(x)=2(x-1)x\in[0,2]$

$\therefore f(x)=x^2-2x+c$

又$f(x)$是奇函数

$\therefore f(0)=0\therefore c=0$

$\therefore f(x)=x^2-2x$

$x\in[0,2]$

$f(x)$的周期为4

$\therefore f(7)=f(3)=f(-1)=-f(1)=-(1-2)=1$

11、解：方程两边对 x 求偏导：

$$e^{2yz}(2y\cdot\frac{\partial z}{\partial x})+2x+\frac{\partial z}{\partial x}=0$$

代入$x=\frac{1}{2},y=\frac{1}{2}$解得：

$$\frac{\partial z}{\partial x}=\frac{1}{e^{z(\frac{1}{2},\frac{1}{2})}+1}$$

两边对y求偏导

$$e^{2yz}(2z+2y\frac{\partial z}{\partial y})+2y+\frac{\partial z}{\partial y}=0$$

代入$x=\frac{1}{2},y=\frac{1}{2}$解得：

$$\frac{\partial z}{\partial y}=\frac{1-z(\frac{1}{2},\frac{1}{2})e^{z(\frac{1}{2},\frac{1}{2})}}{e^{z(\frac{1}{2},\frac{1}{2})}+1}$$

12. 解：把极坐标方程化为直角坐标方程

令

$$\begin{cases} x = r\cos\theta = \theta\cos\theta \\ y = r\sin\theta = \theta\sin\theta \end{cases}$$

则 $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{\sin\theta + \theta\cos\theta}{\cos\theta - \theta\sin\theta}$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{1+\frac{\pi}{2}\cdot 0}{0-\frac{\pi}{2}\cdot 1} = -\frac{2}{\pi}$$

当 $\theta = \dfrac{\pi}{2}$ 时，$\begin{cases} x = \theta\cos\theta = 0 \\ y = \theta\sin\theta = \dfrac{\pi}{2} \end{cases}$

则切线方程为

$$(y-\frac{\pi}{2}) = -\frac{2}{\pi}(x-0)$$

化简为

$$y = -\frac{2}{\pi}x + \frac{\pi}{2}$$

13、质心的横坐标：

$$\frac{\int_0^1 x f(x)\,dx}{\int_0^1 f(x)\,dx} = \frac{\int_0^1 x(-x^2+2x+1)\,dx}{\int_0^1 (-x^2+2x+1)\,dx} = \frac{\left.(-\frac{1}{4}x^4+\frac{2}{3}x^3+\frac{1}{2}x^2)\right|_0^1}{\left.(-\frac{1}{3}x^3+x^2+x)\right|_0^1} = \frac{11}{20}$$

14、

$$\begin{aligned} f(x_1, x_2, x_3) &= x_1^2 - x_2^2 + 2a\,x_1x_3 + 4\,x_2x_3 \\ &= (x_1 + a\,x_3)^2 - (x_2 - 2\,x_3)^2 + 4\,x_3^2 - a^2\,x_3^2 \end{aligned}$$

$\because$ f的负惯性指数为1

$\therefore$ $4-a^2 \geq 0$

$\therefore$ $-2 \leq a \leq 2$

15.
解：

$$\lim_{x\to\infty}\frac{\int_1^x (t^2(e^{\frac{1}{t}}-1)-t)\,dt}{x^2\ln(1+\frac{1}{x})}=\lim_{x\to\infty}\frac{\int_1^x (t^2(e^{\frac{1}{t}}-1)-t)\,dt}{x^2\cdot\frac{1}{x}}=\lim_{x\to\infty}\frac{x^2(e^{\frac{1}{x}}-1)-x}{1}=\lim_{x\to\infty}x^2(e^{\frac{1}{x}}-1-\frac{1}{x})$$

$$\underline{\underline{\text{令}\frac{1}{x}=t}}\lim_{x\to\infty}\frac{e^t-1-t}{t^2}=\lim_{x\to\infty}\frac{1+t+\frac{1}{2}t^2+O(t^2)-1-t}{t^2}=\frac{1}{2}$$

16、
解：

$\because x^2+y^2y'=1-y'$

$\therefore y'=\dfrac{1-x^2}{y^2+1}$

令$y'=0,\therefore x=\pm 1$

$\therefore y''=\dfrac{-2x(y^2+1)-(1-x^2)\cdot 2yy'}{(y^2+1)^2}$

又$\because y'(1)=y'(-1)=0$

$\therefore y''(1)=\dfrac{-2}{y^2(1)+1}\langle 0,\therefore y(1)$为极大值

$y''(-1)=\dfrac{2}{y^2(1)+1}\rangle 0, y(-1)$为极小值

下求极值

$\because y'=\dfrac{1-x^2}{y^2+1},\therefore (y^2+1)dy=(1-x^2)dx,\therefore \int(y^2+1)dy=\int(1-x^2)dx$

$\therefore \dfrac{1}{3}y^3+y=x-\dfrac{1}{3}x^3+c$

又　$y(2)=0$

$\therefore c=\dfrac{2}{3}$

$\therefore \dfrac{1}{3}y^3+y=x-\dfrac{1}{3}x^3+\dfrac{2}{3}$

代入 $x=1$

$$\therefore \frac{1}{3}y^3(1)+y(1)=1-\frac{1}{3}+\frac{2}{3}$$

$$\therefore y(1)=1$$

代入 $x=-1,$

$$\therefore \frac{1}{3}y^3(-1)+y(-1)=-1+\frac{1}{3}+\frac{2}{3}=0$$

$$\therefore y(-1)=0$$

17、

解：积分区域D关于$y=x$对称，利用轮对称行，

$$\iint\limits_D \frac{x\sin(\pi\sqrt{x^2+y^2})}{x+y}dxdy=\iint\limits_D \frac{\mathrm{y}\sin(\pi\sqrt{x^2+y^2})}{x+y}dxdy$$

$$=\frac{1}{2}\iint\limits_D \frac{x\sin(\pi\sqrt{x^2+y^2})}{x+y}+\frac{\mathrm{y}\sin(\pi\sqrt{x^2+y^2})}{x+y}dxdy$$

$$=\frac{1}{2}\iint\limits_D \sin(\pi\sqrt{x^2+y^2})dxdy$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}d\theta\int_1^2\sin(\pi r)r\,\mathrm{d}\,r=-\frac{1}{4}\int_1^2 rd\cos(\pi r)$$

$$=-\frac{1}{4}r\cos(\pi r)\big|_1^2+\frac{1}{4}\int_1^2\cos(\pi r)\,\mathrm{d}\,r$$

$$=-\frac{1}{2}-\frac{1}{4}=-\frac{3}{4}$$

18、

解 ：

$$\frac{\partial z}{\partial x} = f' \cdot e^x \cdot \cos y,$$

$$\frac{\partial^2 z}{\partial x^2} = \cos y \cdot (f'' \cdot e^x \cdot \cos y \cdot e^x + f' \cdot e^x) = f'' \cdot (e^x \cdot \cos y)^2 + f' \cdot e^x \cdot \cos y$$

$$\frac{\partial z}{\partial y} = f' \cdot e^x \cdot (-\sin y),$$

$$\frac{\partial^2 z}{\partial y^2} = -e^x[f'' \cdot e^x \cdot (-\sin y) + f' \cdot \cos y] = (e^x)^2 \sin y^2 f'' - f' \cdot \cos y \cdot e^x$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f'' \cdot e^{2x} = (4z + e^x \cdot \cos y)e^{2x}$$

$\therefore f''(e^x \cdot \cos y) = 4f(e^x \cdot \cos y) + e^x \cdot \cos y$

令$t = e^x \cdot \cos y, \therefore f''(t) = 4f(t) + t$

$\therefore y'' - 4y = x$

求特征值：

$\lambda^2 - 4 = 0 \quad \therefore x = \pm 2 \quad \therefore y(x) = C_1 e^{2x} + C_2 e^{-2x}$

再求非其次特征值。

$y^* = (ax + b)$ 代入 $\therefore y^* = -\frac{1}{4}x$

$\therefore y = y(x) + y^* = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{4}x$

$y(0)=0=C_1 + C_2$

$y'(0)=0=x_1 - x_2 - \frac{1}{4}$

$$\therefore \begin{cases} C_1 + C_2 = 0 \\ 2C_1 - 2C_2 = \frac{1}{4} \end{cases} \Rightarrow \begin{cases} C_1 = \frac{1}{16} \\ C_2 = -\frac{1}{16} \end{cases}$$

$$\therefore f(\mu) = \frac{1}{16}e^{2\mu} - \frac{1}{16}e^{-2\mu} - \frac{1}{4}\mu$$

19.

解：（I）

$h_1(x)=\int_a^x g(t)dt$

$h_1(a)=0$

$h_1'(x)=g(x)\ge 0$

$\therefore h_1(x)$单调不减

$\therefore$ 当$x\in[a,b]$时，$h_1(x)\ge 0$

$h_2(x)=\int_a^x g(t)dt-x+a$

$h_2'(x)=g(x)-1$

$\because 0\le g(x)\le 1 \therefore h_2'(x)\le 0$

$\therefore h_2(x)$单调不增又$h_2(a)=0$

$\therefore$ 当$x\in[a,b]$时，$h_2(x)\le 0$

$$p(x)=\int_a^x f(u)g(u)du-\int_a^{a+\int_a^x g(t)dt} f(u)du$$

$$p'(x)=f(x)g(x)-f[a+\int_a^x g(t)dt]\cdot g(x)=\left[f(x)-f[a+\int_a^x g(t)dt]\right]g(x)$$

$\because 0\le g(x)\le 1$

$\therefore \int_a^x g(t)dt\le \int_a^x dt=x-a \therefore a+\int_a^x g(t)dt\le x$

又$f(x)$单调增加

（Ⅱ）$\therefore f(x)\ge f[a+\int_a^x g(t)dt] \therefore p'(x)\ge 0$

$\therefore p(x)$单调不减

又$p(a)=0 \therefore p(b)\ge 0$

即$\int_a^b f(x)g(x)dx\ge \int_a^{a+\int_a^b g(t)dt} f(x)dx$

20、

解：

$$f(x)=\frac{x}{1+x}, f_1(x)=f(x)$$

$$f_2(x)=f(f_1(x))=\frac{\frac{x}{1+x}}{1+\frac{x}{1+x}}=\frac{x}{1+2x}$$

$$f_3(x)=f(f_2(x))=\frac{\frac{x}{1+2x}}{1+\frac{x}{1+2x}}=\frac{x}{1+3x}$$

用归纳法知：$f_n(x)=\dfrac{x}{1+nx}, x\in[0,1]$

$$S_n=\int_0^1\frac{x}{1+nx}dx=\frac{1}{n}\int_0^1\frac{nx+1-1}{1+nx}dx$$

$$=\frac{1}{n}\int_0^1(1-\frac{1}{1+nx})dx$$

$$=\frac{1}{n}-\frac{1}{n^2}\ln(1+n)$$

$$\lim_{n\to\infty}nS_n=\lim_{n\to\infty}n[\frac{1}{n}-\frac{1}{n^2}\ln(1+n)]=1-\lim_{n\to\infty}\frac{\ln(1+n)}{n}$$

$$=1$$

21.

解：

因$\dfrac{\partial f}{\partial y}=2(y+1)$ 则

$$f(x,y)=y^2+2y+\varphi(x)$$

$$\begin{cases}f(y,y)=(y+1)^2-(2-y)\\ f(y,y)=y^2+2y+\varphi(y)\end{cases}$$

则$\varphi(y)=y-1$

故$f(x,y)=y^2+2y+x-1$

$$f(x,y)=0\Rightarrow x=-y^2-2y+1$$

$$V=\int_0^2\pi\left(f(x)+1\right)^2dx=\int_0^2\pi\left[f^2(x)+2f(x)+1\right]dx=\int_0^2\pi(2-x)dx=\pi\left(2x-\frac{x^2}{2}\right)\Bigg|_0^2=2\pi$$

22、

解：

$$(A)=\begin{pmatrix}1 & -2 & 3 & -4\\ 0 & 1 & -1 & 1\\ 1 & 2 & 0 & -3\end{pmatrix}\xrightarrow{-r_1+r_3}\begin{pmatrix}1 & -2 & 3 & -4\\ 0 & 1 & -1 & 1\\ 0 & 4 & -3 & 1\end{pmatrix}\xrightarrow{-4r_2+r_3}\begin{pmatrix}1 & -2 & 3 & -4\\ 0 & 1 & -1 & 1\\ 0 & 0 & 1 & -3\end{pmatrix}$$

$$\xrightarrow[-3r_3+r_1]{r_3+r_2}\begin{pmatrix}1 & -2 & 0 & 5\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & -3\end{pmatrix}\xrightarrow{2r_2+r_1}\begin{pmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & -3\end{pmatrix}$$

$$\begin{aligned}x_1&=-x_4\\ x_2&=2x_4\\ x_3&=3x_4\\ x_4&=x_4\end{aligned}\qquad \begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}=c\begin{pmatrix}-1\\ 2\\ 3\\ 1\end{pmatrix}$$ c 为任意常数

设 $B=\begin{pmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{pmatrix}$

$$A\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}\Rightarrow\begin{pmatrix}1 & -2 & 3 & -4 & \vdots & 1\\ 0 & 1 & -1 & 1 & \vdots & 0\\ 1 & 2 & 0 & -3 & \vdots & 0\end{pmatrix}$$

$$A\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}=\begin{pmatrix}0\\ 1\\ 0\end{pmatrix}\Rightarrow\begin{pmatrix}1 & -2 & 3 & -4 & \vdots & 0\\ 0 & 1 & -1 & 1 & \vdots & 1\\ 1 & 2 & 0 & -3 & \vdots & 0\end{pmatrix}$$

$$A\begin{pmatrix}z_1\\ z_2\\ z_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}\Rightarrow\begin{pmatrix}1 & -2 & 3 & -4 & \vdots & 0\\ 0 & 1 & -1 & 1 & \vdots & 0\\ 1 & 2 & 0 & -3 & \vdots & 1\end{pmatrix}$$

即

$$\begin{pmatrix}1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0\\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0\\ 1 & 2 & 0 & -3 & \vdots & 0 & 0 & 1\end{pmatrix}\to\begin{pmatrix}1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0\\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0\\ 0 & 4 & -3 & 1 & \vdots & 0 & 0 & 1\end{pmatrix}$$

$$\to\begin{pmatrix}1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0\\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0\\ 0 & 0 & 1 & -3 & \vdots & 0 & 0 & 1\end{pmatrix}\to\begin{pmatrix}1 & -2 & 0 & 5 & \vdots & 4 & 12 & -3\\ 0 & 1 & 0 & -2 & \vdots & -1 & -3 & 1\\ 0 & 0 & 1 & 3 & \vdots & -1 & -4 & 1\end{pmatrix}$$

$$\to\begin{pmatrix}1 & 0 & 0 & 1 & \vdots & 2 & 6 & -1\\ 0 & 1 & 0 & -2 & \vdots & -1 & -3 & 1\\ 0 & 0 & 1 & -3 & \vdots & -1 & -4 & 1\end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ -1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = c_2 \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ -3 \\ -4 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = c_3 \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} -c_1+2 & -c_2+6 & -c_3-1 \\ 2c_1-1 & 2c_2-3 & 2c_3+1 \\ 3c_1-1 & 3c_2-4 & 3c_3+1 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

c_1, c_2, c_3为任意常数

23、

解：

$$设 A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & n \end{bmatrix}$$

$$|\lambda E - A| = \begin{vmatrix} \lambda-1 & -1 & \cdots & -1 \\ -1 & \lambda-1 & \cdots & -1 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & \cdots & \lambda-1 \end{vmatrix} = (\lambda - n)\lambda^{n-1}$$

所以A 的 n 个特征值为λ_1=n, $\lambda_2 = \cdots = \lambda_n$=0

又因为A是一个实对称矩阵，所以A可以相似对角化，且

$$A \sim \begin{bmatrix} n & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}, \quad |\lambda E - B| = \begin{vmatrix} \lambda & 0 & \cdots & 0 & -1 \\ 0 & \lambda & \cdots & 0 & -2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \lambda - N \end{vmatrix} = (\lambda - n)\lambda^{n-1}$$

所以B 的 n 个特征值为λ_1'=n, $\lambda_2' = \cdots = \lambda_n'$=0

$$又 |0E - B| = \begin{vmatrix} 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 0 & -2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & -n \end{vmatrix}$$

所以 r(0 E− B) = 1

故B 的 n−1 重特征值 0 有 n−1 个线性无关的特征向量

所以 B 也可以相似对角化，且 $B \sim \begin{bmatrix} n & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$

所以 A 与 B 相似。