hw2_problem4_starter_code

May 4, 2024

[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
import cvxpy as cp
```

[ ]:
```python
dt = 0.1
goal_state = np.zeros(4)
initial_state = np.array([-4, 0., -1., 2.])
u_max = 2.

### put your code here ###

A_dynamics = np.array([[1., 0., dt, 0.],
                        [0., 1., 0., dt],
                        [0., 0., 1., 0.],
                        [0., 0., 0., 1.]])

B_dynamics = np.array([[0.5*dt**2, 0.],
                        [0., 0.5*dt**2],
                        [dt, 0.],
                        [0.,  dt]])




T = 50  # time steps
n = 4   # state dimension
m = 2   # control dimension

us = cp.Variable([T,m])   # optimization variable.
xs = cp.Variable([T,n])

goal_state = np.zeros(n)
# quadratic cost matrices
Q = np.diag([1., 1., 1., 1.])
R = np.diag([1., 1.])
```

```
objective =  0
constraints = []


state = initial_state
constraints += [xs[0] == initial_state]
for t in range(T):
    objective += (cp.quad_form(state, Q) + cp.quad_form(us[t], R))
    state = A_dynamics @ state + B_dynamics @ us[t]
    constraints += [cp.norm(us[t], 2) <= u_max]

constraints += [state == goal_state]


problem = cp.Problem(cp.Minimize(objective), constraints)
problem.solve()
```

```
/home/p8410077/.local/lib/python3.10/site-
packages/cvxpy/reductions/solvers/solving_chain.py:336: FutureWarning:
    Your problem is being solved with the ECOS solver by default. Starting in
    CVXPY 1.5.0, Clarabel will be used as the default solver instead. To
continue
    using ECOS, specify the ECOS solver explicitly using the ``solver=cp.ECOS``
    argument to the ``problem.solve`` method.

  warnings.warn(ECOS_DEPRECATION_MSG, FutureWarning)
```

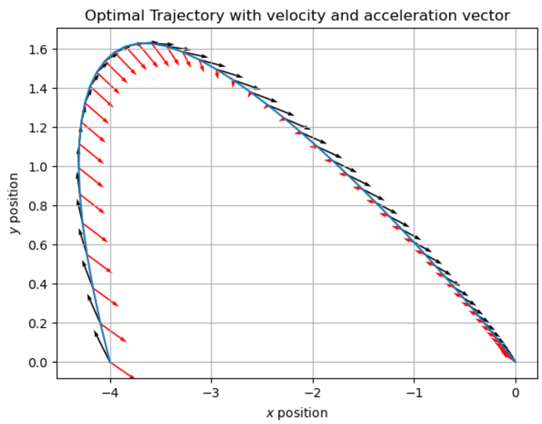
[ ]: 604.1460607145349

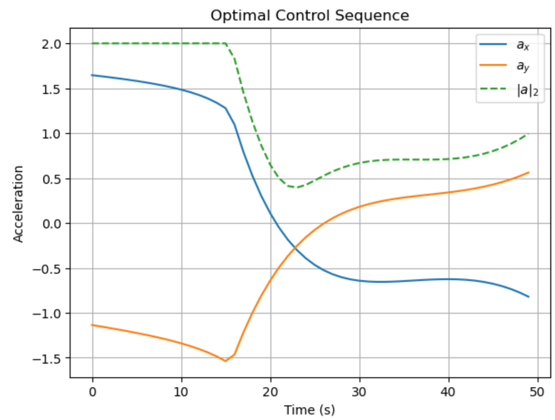
[ ]:
```
states = [initial_state]
for t in range(T):
    states.append(A_dynamics @ states[t] + B_dynamics @ us.value[t])
states = np.stack(states)

controls = us.value
```

0.1 Plotting reults

[ ]:
```
plt.figure(figsize=(15,5))
plt.subplot(1,2,1)
plt.plot(states[:,0], states[:,1])
plt.quiver(states[:,0], states[:,1], states[:,2], states[:,3], scale=30, width=.
 ↪003)  # show velocity vector
plt.quiver(states[:-1,0], states[:-1,1], controls[:,0], controls[:,1],␣
 ↪scale=30, width=.003, color='red')  # show acceleration vector
plt.title("Optimal Trajectory with velocity and acceleration vector")
plt.xlabel("$x$ position")
```

```
plt.ylabel("$y$ position")
plt.grid()

plt.subplot(1,2,2)
plt.plot(controls)
plt.plot(np.linalg.norm(controls, 2, axis=1), '--')
plt.title("Optimal Control Sequence")
plt.xlabel("Time (s)")
plt.ylabel("Acceleration")
plt.legend(["$a_x$", "$a_y$", "$\|a\|_2$"])
plt.grid()
```

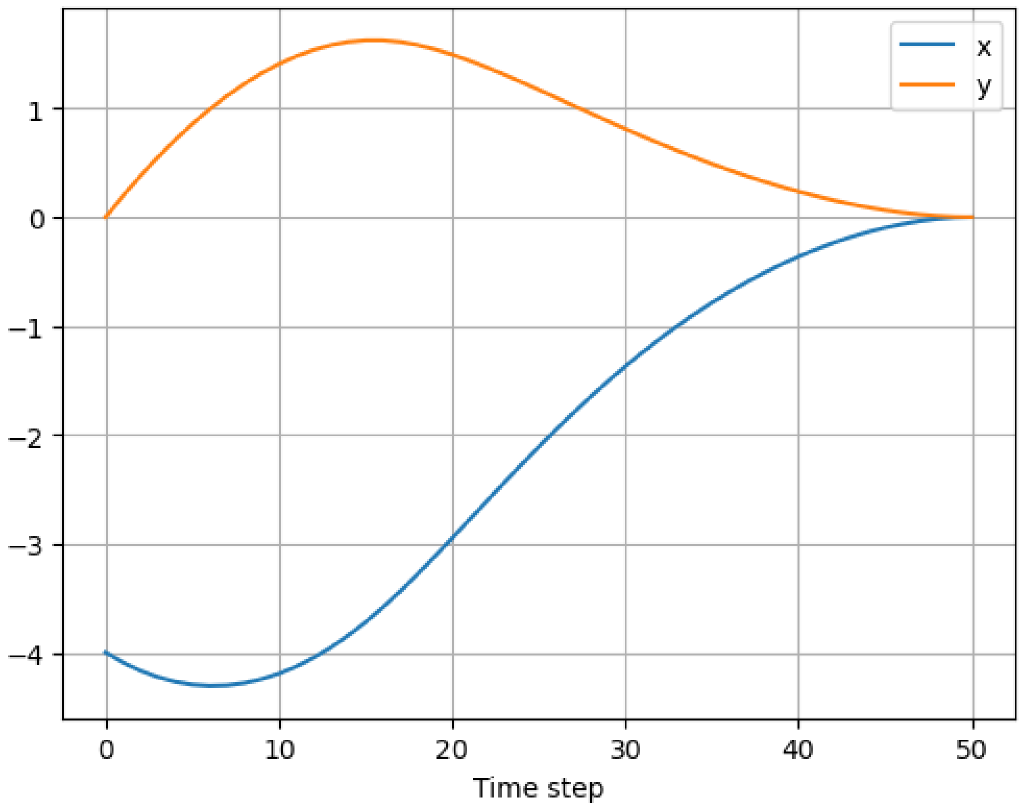




0.1.1 plotting x and y, and computing which time step the state reaches within 10E-1 of goal state

[ ]:
```
plt.plot(states[:,0], label="x")
plt.plot(states[:,1], label="y")
plt.xlabel("Time step")
plt.legend()
plt.grid()
k_close = np.where(np.linalg.norm(states - goal_state, 2, axis=1) < 1E-1)[0][0]
print("Time step when within 1E-1 to goal state: %i"%k_close)
```

```
Time step when within 1E-1 to goal state: 49
```



[ ]:
```
dt = 0.1
goal_state = np.zeros(4)
initial_state = np.array([-4, 0., -1., 2.])
u_max = 2.

### put your code here ###

A_dynamics = np.array([[1., 0., dt, 0.],
                        [0., 1., 0., dt],
                        [0., 0., 1., 0.],
                        [0., 0., 0., 1.]])

B_dynamics = np.array([[0.5*dt**2, 0.],
                        [0., 0.5*dt**2],
                        [dt, 0.],
                        [0.,  dt]])




T = 50  # time steps
```

```python
n = 4   # state dimension
m = 2   # control dimension

us2 = cp.Variable([T,m])   # optimization variable.
xs2 = cp.Variable([T,n])

goal_state = np.zeros(n)
# quadratic cost matrices
Q = np.diag([50., 50., 50., 50.])
R = np.diag([1., 1.])


objective2 =  0
constraints2 = []


state2 = initial_state
constraints2 += [xs2[0] == initial_state]
for t in range(T):
    objective2 += (cp.quad_form(state2, Q) + cp.quad_form(us2[t], R))
    state2 = A_dynamics @ state2 + B_dynamics @ us2[t]
    constraints2 += [cp.norm(us2[t], 2) <= u_max]

constraints2 += [state2 == goal_state]


problem = cp.Problem(cp.Minimize(objective2), constraints2)
problem.solve()
```

```
/home/p8410077/.local/lib/python3.10/site-
packages/cvxpy/reductions/solvers/solving_chain.py:336: FutureWarning:
    Your problem is being solved with the ECOS solver by default. Starting in
    CVXPY 1.5.0, Clarabel will be used as the default solver instead. To
continue
    using ECOS, specify the ECOS solver explicitly using the ``solver=cp.ECOS``
    argument to the ``problem.solve`` method.

  warnings.warn(ECOS_DEPRECATION_MSG, FutureWarning)
```

[ ]: 25612.230258706346

[ ]:
```python
states2 = [initial_state]
for t in range(T):
    states2.append(A_dynamics @ states2[t] + B_dynamics @ us2.value[t])
states2 = np.stack(states2)

controls2 = us2.value
```

[ ]:

```python
dt = 0.1
goal_state = np.zeros(4)
initial_state = np.array([-4, 0., -1., 2.])
u_max = 2.

### put your code here ###

A_dynamics = np.array([[1., 0., dt, 0.],
                        [0., 1., 0., dt],
                        [0., 0., 1., 0.],
                        [0., 0., 0., 1.]])

B_dynamics = np.array([[0.5*dt**2, 0.],
                        [0., 0.5*dt**2],
                        [dt, 0.],
                        [0.,  dt]])




T = 50  # time steps
n = 4   # state dimension
m = 2   # control dimension

us3 = cp.Variable([T,m])   # optimization variable.
xs3 = cp.Variable([T,n])

goal_state = np.zeros(n)
# quadratic cost matrices
Q = np.diag([1., 1., 0., 0.])
R = np.diag([50., 50.])


objective3 =  0
constraints3 = []


state3 = initial_state
constraints3 += [xs3[0] == initial_state]
for t in range(T):
    objective3 += (cp.quad_form(state3, Q) + cp.quad_form(us3[t], R))
    state3 = A_dynamics @ state3 + B_dynamics @ us3[t]
    constraints3 += [cp.norm(us3[t], 2) <= u_max]

constraints3 += [state3 == goal_state]
```

```python
problem = cp.Problem(cp.Minimize(objective3), constraints3)
problem.solve()
```

[ ]: 4225.855725956044

[ ]:
```python
states3 = [initial_state]
for t in range(T):
    states3.append(A_dynamics @ states3[t] + B_dynamics @ us3.value[t])
states3 = np.stack(states3)

controls3 = us3.value
```

[ ]:
```python
dt = 0.1
goal_state = np.zeros(4)
initial_state = np.array([-4, 0., -1., 2.])
u_max = 20.

### put your code here ###

A_dynamics = np.array([[1., 0., dt, 0.],
                        [0., 1., 0., dt],
                        [0., 0., 1., 0.],
                        [0., 0., 0., 1.]])

B_dynamics = np.array([[0.5*dt**2, 0.],
                        [0., 0.5*dt**2],
                        [dt, 0.],
                        [0.,  dt]])




T = 50  # time steps
n = 4   # state dimension
m = 2   # control dimension

us4 = cp.Variable([T,m])   # optimization variable.
xs4 = cp.Variable([T,n])

goal_state = np.zeros(n)
# quadratic cost matrices
Q = np.diag([50., 50., 50., 50.])
R = np.diag([1., 1.])


objective4 =  0
constraints4 = []
```

```python
state4 = initial_state
constraints4 += [xs4[0] == initial_state]
for t in range(T):
    objective4 += (cp.quad_form(state4, Q) + cp.quad_form(us4[t], R))
    state4 = A_dynamics @ state4 + B_dynamics @ us4[t]
    constraints4 += [cp.norm(us4[t], 2) <= u_max]

constraints4 += [state4 == goal_state]


problem = cp.Problem(cp.Minimize(objective4), constraints4)
problem.solve()
```

[ ]: 10800.148006298885

[ ]:
```python
states4 = [initial_state]
for t in range(T):
    states4.append(A_dynamics @ states4[t] + B_dynamics @ us4.value[t])
states4 = np.stack(states4)

controls4 = us4.value
```

[ ]:
```python
dt = 0.1
goal_state = np.zeros(4)
initial_state = np.array([-4, 0., -1., 2.])
u_max = 2.

### put your code here ###

A_dynamics = np.array([[1., 0., dt, 0.],
                        [0., 1., 0., dt],
                        [0., 0., 1., 0.],
                        [0., 0., 0., 1.]])

B_dynamics = np.array([[0.5*dt**2, 0.],
                        [0., 0.5*dt**2],
                        [dt, 0.],
                        [0.,  dt]])




T = 50  # time steps
n = 4   # state dimension
m = 2   # control dimension
```

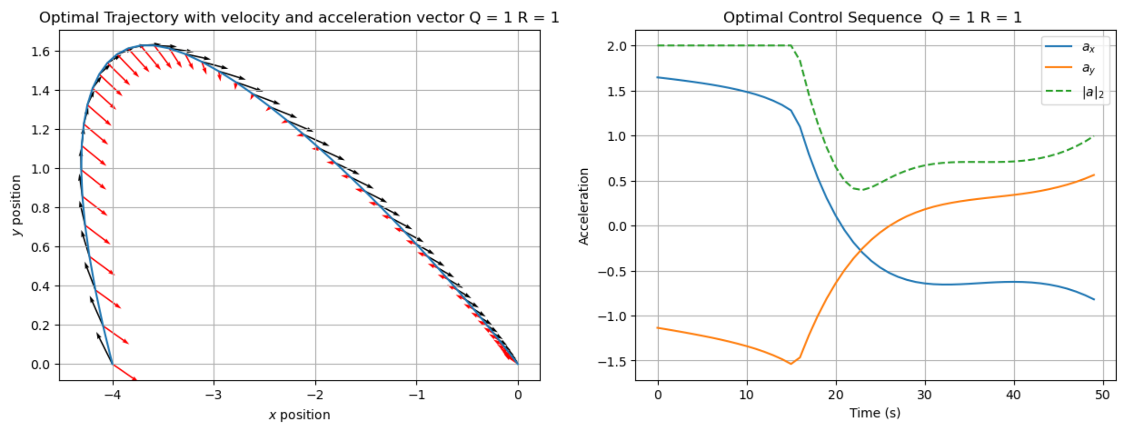
```python
us5 = cp.Variable([T,m])   # optimization variable.
xs5 = cp.Variable([T,n])

goal_state = np.zeros(n)
# quadratic cost matrices
Q = np.diag([1., 1., 0., 0.])
R = np.diag([0., 0.])


objective5 =  0
constraints5 = []


state5 = initial_state
constraints5 += [xs5[0] == initial_state]
for t in range(T):
    objective5 += (cp.quad_form(state5, Q) + cp.quad_form(us5[t], R))
    state5 = A_dynamics @ state5 + B_dynamics @ us5[t]
    constraints5 += [cp.norm(us5[t], 2) <= u_max]

constraints5 += [state5 == goal_state]


problem = cp.Problem(cp.Minimize(objective5), constraints5)
problem.solve()
```

[ ]: 398.7867727543614

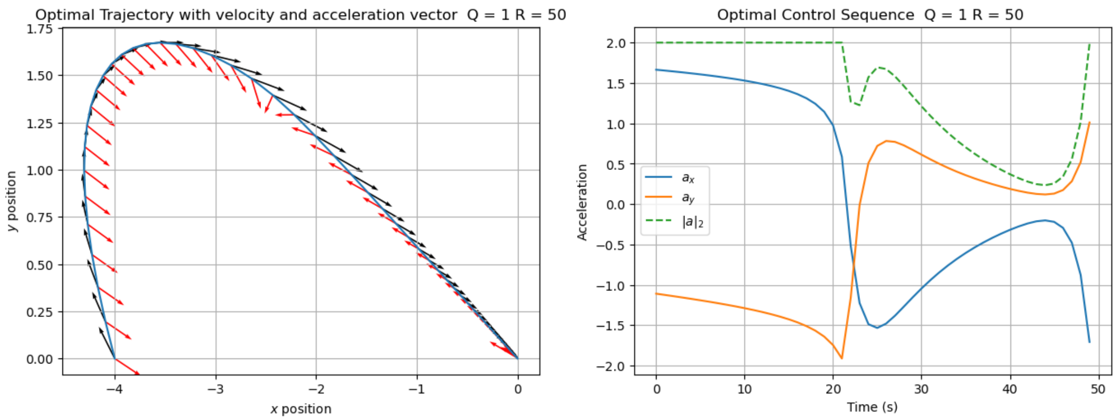
[ ]:
```python
states5 = [initial_state]
for t in range(T):
    states5.append(A_dynamics @ states5[t] + B_dynamics @ us5.value[t])
states5 = np.stack(states5)

controls5 = us5.value
```

[ ]:
```python
plt.figure(figsize=(15,5))
plt.subplot(1,2,1)
plt.plot(states[:,0], states[:,1])
plt.quiver(states[:,0], states[:,1], states[:,2], states[:,3], scale=30, width=.
 ↪003)  # show velocity vector
plt.quiver(states[:-1,0], states[:-1,1], controls[:,0], controls[:,1],␣
 ↪scale=30, width=.003, color='red')  # show acceleration vector
plt.title("Optimal Trajectory with velocity and acceleration vector Q = 1 R =␣
 ↪1")
plt.xlabel("$x$ position")
plt.ylabel("$y$ position")
```

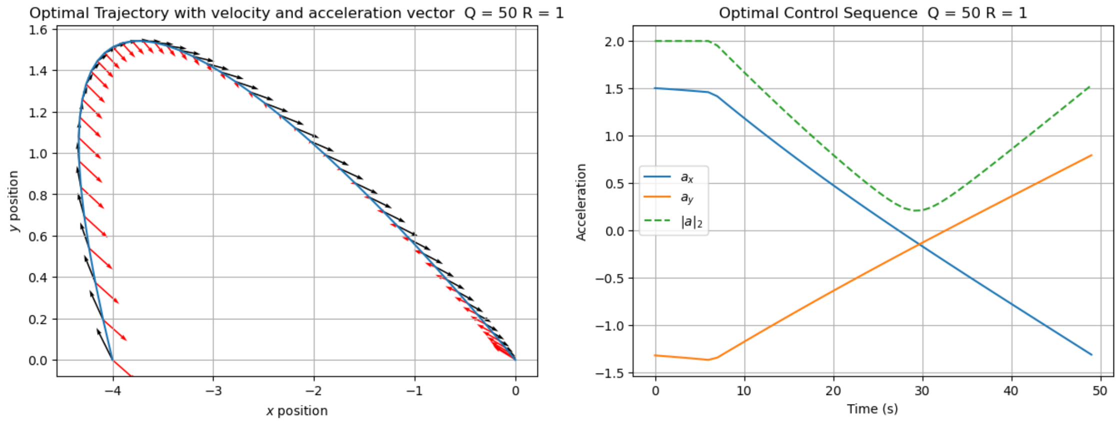
```
plt.grid()

plt.subplot(1,2,2)
plt.plot(controls)
plt.plot(np.linalg.norm(controls, 2, axis=1), '--')
plt.title("Optimal Control Sequence  Q = 1 R = 1 ")
plt.xlabel("Time (s)")
plt.ylabel("Acceleration")
plt.legend(["$a_x$", "$a_y$", "$\|a\|_2$"])
plt.grid()
```



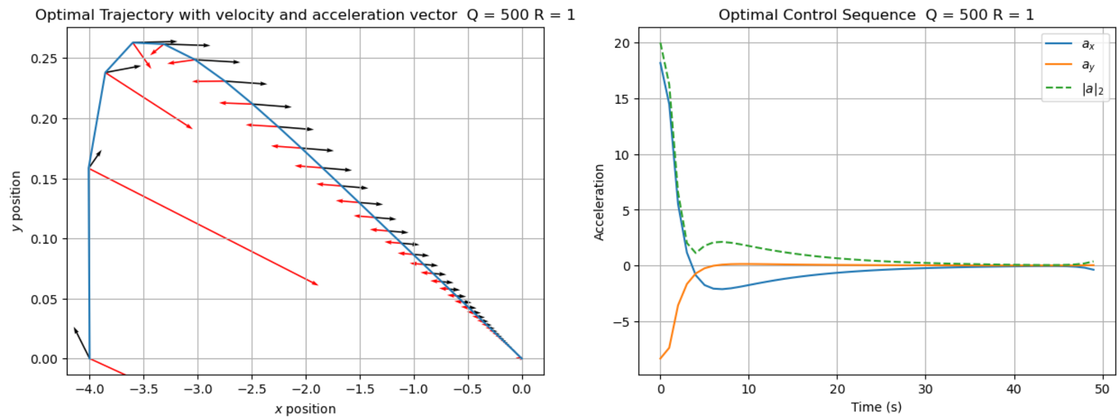

```
[ ]: plt.figure(figsize=(15,5))
plt.subplot(1,2,1)
plt.plot(states2[:,0], states2[:,1])
plt.quiver(states2[:,0], states2[:,1], states2[:,2], states2[:,3], scale=30,␣
 ↪width=.003)  # show velocity vector
plt.quiver(states2[:-1,0], states2[:-1,1], controls2[:,0], controls2[:,1],␣
 ↪scale=30, width=.003, color='red')  # show acceleration vector
plt.title("Optimal Trajectory with velocity and acceleration vector  Q = 1 R =␣
 ↪50")
plt.xlabel("$x$ position")
plt.ylabel("$y$ position")
plt.grid()

plt.subplot(1,2,2)
plt.plot(controls2)
plt.plot(np.linalg.norm(controls2, 2, axis=1), '--')
plt.title("Optimal Control Sequence  Q = 1 R = 50 ")
plt.xlabel("Time (s)")
plt.ylabel("Acceleration")
plt.legend(["$a_x$", "$a_y$", "$\|a\|_2$"])
plt.grid()
```



[ ]:
```
plt.figure(figsize=(15,5))
plt.subplot(1,2,1)
plt.plot(states3[:,0], states3[:,1])
plt.quiver(states3[:,0], states3[:,1], states3[:,2], states3[:,3], scale=30,␣
 ↪width=.003)  # show velocity vector
plt.quiver(states3[:-1,0], states3[:-1,1], controls3[:,0], controls3[:,1],␣
 ↪scale=30, width=.003, color='red')  # show acceleration vector
plt.title("Optimal Trajectory with velocity and acceleration vector  Q = 50 R =␣
 ↪1")
plt.xlabel("$x$ position")
plt.ylabel("$y$ position")
plt.grid()

plt.subplot(1,2,2)
plt.plot(controls3)
plt.plot(np.linalg.norm(controls3, 2, axis=1), '--')
plt.title("Optimal Control Sequence  Q = 50 R = 1")
plt.xlabel("Time (s)")
plt.ylabel("Acceleration")
plt.legend(["$a_x$", "$a_y$", "$\|a\|_2$"])
plt.grid()
```




```
[ ]: plt.figure(figsize=(15,5))
plt.subplot(1,2,1)
plt.plot(states4[:,0], states4[:,1])
plt.quiver(states4[:,0], states4[:,1], states4[:,2], states4[:,3], scale=30,␣
 ↪width=.003)  # show velocity vector
plt.quiver(states4[:-1,0], states4[:-1,1], controls4[:,0], controls4[:,1],␣
 ↪scale=30, width=.003, color='red')  # show acceleration vector
plt.title("Optimal Trajectory with velocity and acceleration vector  Q = 500 R␣
 ↪= 1")
plt.xlabel("$x$ position")
plt.ylabel("$y$ position")
plt.grid()

plt.subplot(1,2,2)
plt.plot(controls4)
plt.plot(np.linalg.norm(controls4, 2, axis=1), '--')
plt.title("Optimal Control Sequence  Q = 500 R = 1")
plt.xlabel("Time (s)")
plt.ylabel("Acceleration")
plt.legend(["$a_x$", "$a_y$", "$\|a\|_2$"])
plt.grid()
```




```
[ ]: plt.plot(states[:,0], label="x")
plt.plot(states[:,1], label="y")
plt.plot(states2[:,0], label="x = Q=50 R=1")
plt.plot(states2[:,1], label="y = Q=50 R=1")
plt.plot(states3[:,0], label="x = Q=1 R=50")
plt.plot(states3[:,1], label="y = Q=1 R=50")
plt.plot(states4[:,0], label="x = Q=50 R=1 umax = 20")
plt.plot(states4[:,1], label="y = Q=00 R=1 umax = 20")
plt.plot(states5[:,0], label="x = Q=[1, 1, 0, 0] R=0 umax = 2")
plt.plot(states5[:,1], label="y = Q=[1, 1, 0, 0] R=0 umax = 2")
plt.xlabel("Time step")
plt.legend(bbox_to_anchor=(1.1, 1.05))
plt.grid()
k_close = np.where(np.linalg.norm(states - goal_state, 2, axis=1) < 1E-1)[0][0]
print("Time step when within 1E-1 to goal state: %i"%k_close)

k_close = np.where(np.linalg.norm(states2 - goal_state, 2, axis=1) < 1E-1)[0][0]
print("Time step when within 1E-1 to goal state: %i"%k_close)

k_close = np.where(np.linalg.norm(states3 - goal_state, 2, axis=1) < 1E-1)[0][0]
print("Time step when within 1E-1 to goal state: %i"%k_close)

k_close = np.where(np.linalg.norm(states4 - goal_state, 2, axis=1) < 1E-1)[0][0]
print("Case: Q=50 R=1 umax = 20 Time step when within 1E-1 to goal state:␣
↪%i"%k_close)

k_close = np.where(np.linalg.norm(states5 - goal_state, 2, axis=1) < 1E-1)[0][0]
print("Case: Q=[1, 1, 0, 0] R=0 umax = 2 Time step when within 1E-1 to goal␣
↪state: %i"%k_close)
```

```
Time step when within 1E-1 to goal state: 49
Time step when within 1E-1 to goal state: 50
```

```
Time step when within 1E-1 to goal state: 50
Case: Q=50 R=1 umax = 20 Time step when within 1E-1 to goal state: 43
Case: Q=[1, 1, 0, 0] R=0 umax = 2 Time step when within 1E-1 to goal state: 47
```