pCT Collaboration Documentation

Data Acquisition/Processing and Image Reconstruction Software: Theory and Implementation Details

Blake Edward Schultze

October 18, 2018

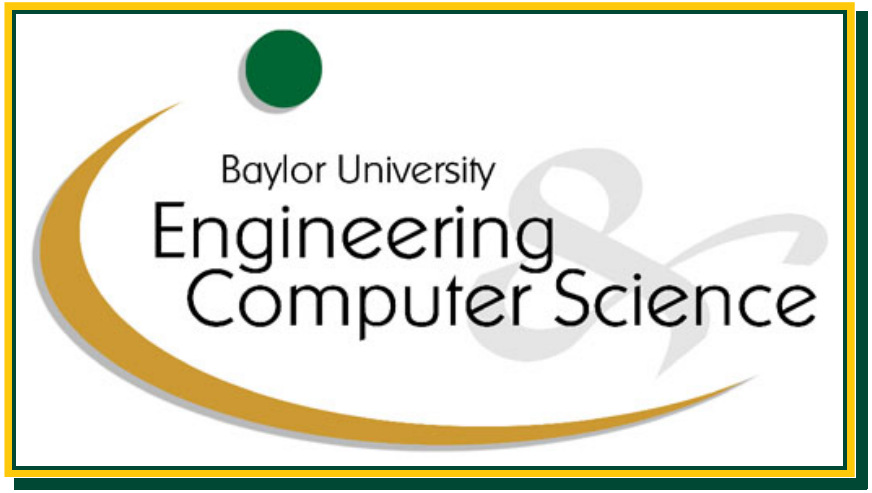

Table of Contents

Page

Part I : Image Reconstruction 1

1 Image Reconstruction Algorithms . . . 2

1.1 Initial Iterate . . . 2

1.2 Iterative Projection Algorithms . . . 2

1.2.1 Sequential Projection Algorithms . . . 2

1.2.2 Block-Iterative Projection Algorithms . . . 3

Part I

Image Reconstruction

1.1 : Initial Iterate

1.2 : Iterative Projection Algorithms

1.2.1 Sequential Projection Algorithms

1.2.2 Block-Iterative Projection Algorithms

Mathematical Notation

Block-Iterative Algorithms: Notation

- m : total # of proton histories
- n : total # of image vector voxels
- k : iteration # of iterative projection algorithm
- $\square_{(k)}$: denotes $\square$ at iteration k
- $B_{(k)}$: total # of BIP blocks at iteration k
- $\{a:b\} = \{\, i \mid i \in [a,b]\}$: interval of indices
- $A_{*,j}$: Column vector composed of all $* = 1:m$ rows of the j-th column of the matrix A
- $A_{i,*}$: Row vector composed of all $* = 1:n$ columns of the i-th row of the matrix A
- $A_{\{a:b\},j}$: Column vector composed of the interval $a:b$ of rows of the j-th column of the matrix A
- $A_{i,\{a:b\}}$: Row vector composed of the interval $a:b$ of columns of the i-th row of the matrix A
- $x_{(k)}$: Image vector x at iteration k
- $\mathcal{I} = \{1,2,3,\cdots,m\}$: the (sequentially ordered) index set of all proton history indices
- $\mathcal{J}_{(k)} = \left\{1,2,3,\cdots,B_{(k)}\right\}$: the (sequentially ordered) index set of all BIP block indices, where the # of BIP blocks $B_{(k)}$ may vary as a function of iteration k.
- $f_{(k)} : \mathcal{I} \to \mathcal{J}_{(k)} = \left\{ f_{(k)}(1), f_{(k)}(2), f_{(k)}(3), \cdots, f_{(k)}(m) \,\middle|\, f_{(k)}(i) = j \in \mathcal{J}_{(k)}, i \in \mathcal{I} \right\}$: surjective function $f_{(k)}$ mapping each of the m proton histories to one of the $B_{(k)}$ BIP blocks, which may vary as a function of k, thereby establishing the # of histories in each block and the order they are processed.
- $\mathcal{M}_{j(k)} = \left\{ i \in \mathcal{I} \,\middle|\, f_{(k)}(i) = j \in \mathcal{J}_{(k)} \right\}$: the ordered set of proton history indices assigned to the j-th BIP block by the function $f_{(k)}$
- $\mathcal{M}_{(k)} = \bigcup_{j \in \mathcal{J}_{(k)}} \mathcal{M}_{j(k)} = \left\{ \mathcal{M}_{1(k)}, \mathcal{M}_{2(k)}, \cdots, \mathcal{M}_{B(k)} \right\}$: the family of sets (BIP blocks) of proton history indices for iteration k, assigned according to the function $f_{(k)}$
- $g_{(k)} : \mathcal{J}_{(k)} \to \mathcal{J}_{(k)} = \left\{ g_{(k)}(1), g_{(k)}(2), g_{(k)}(3), \cdots, g_{(k)}\left(B_{(k)}\right) \,\middle|\, g_{(k)}(i) = j \wedge\ i, j \in \mathcal{J}_{(k)} \right\}$: bijective function $g_{(k)}$ imposing an order on the sets (BIP blocks) of proton histories to define the sequence that the BIP blocks are procecessed at iteration k.
- $\mathcal{B}_{(k)} = \left\{ \left(\mathcal{M}_{j(k)}\right)_{j \in \mathcal{J}_{(k)}} \,\middle|\, \mathcal{M}_{j(k)} < \mathcal{M}_{i(k)} \Leftrightarrow g(j) < g(i) \right\}$: the ordered family of sets (BIP blocks) of proton

histories generated by the bijective mapping function $g_{(k)}$.

- $H_i = \left\{ x \in \mathbb{R}^n \,\middle|\, \langle A_{i,*}, x \rangle = b_i, i \in \mathcal{I} \right\}$ **: the hyperplanes corresponding to the ith row of the $m \times n$ linear system $Ax = b$ upon which the image vector $x_{(k)}$ is projected**