Hints and partial answers for Homework Assignment 1

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1 Exercise 1

In this exercise one has to answer questions concerning the Bayesian network given in Figure 1. This network defines the independencies and dependencies for the global probability function $P(C, D, I, G, S)$. For instance the global joint probability value

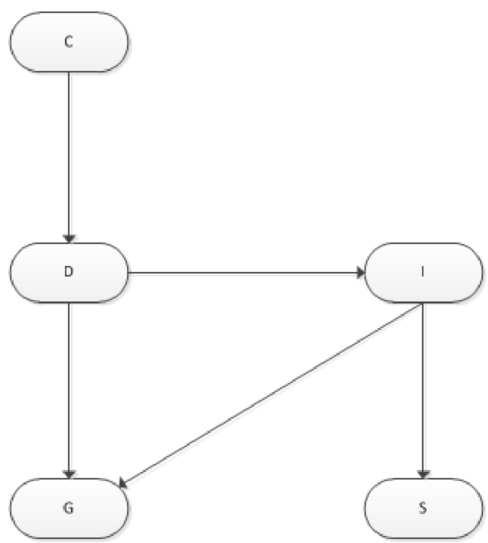

Figure 1: Bayesian network of exercise 1.

$P(C = c_0, D = d_0, I = i_0, G = g_0, S = s_0)$ can be factorized as:

$$P(C = c_0, D = d_0, I = i_0, G = g_0, S = s_0) =$$
$$P(G = g_0|D = d_0, I = i_0)P(S = s_0|I = i_0)P(I = i_0|D = d_0)P(D = d_0|C = c_0)P(C = c_0)$$

Now the right hand side can be computed using the CPDs given in the assignment, the answer is $0.1 * 0.3 * 0.5 * 0.2 * 0.4$.

1.1 Part a

In order to compute $P(S)$ one has to compute the global probability function as above and marginalize over all variables except S.

$$\begin{aligned}
P(S) &= \sum_{c,d,i,g} P(C=c, D=d, I=i, G=g, S) \\
&= \sum_{c,d,i,g} P(S|I=i)P(G=g|D=d, I=i)P(I=i|D=d)P(D=d|C=c)P(C=c) \\
&= \sum_i P(S|I=i)\sum_d [P(I=i|D=d)\sum_c [P(D=d|C=c)P(C=c)\sum_g [P(G=g|D=d, I=i)]]]
\end{aligned}$$

Now $\sum_g [P(G=g|D=d, I=i)] = 1$ because it is a probability distribution. this could also be deduced from the Bayesian network using independency relations.

$$\sum_c [P(D=d|C=c)P(C=c)]] = (0.32, 0.68)$$

which is a factor multiplication and summation and it defines a probability distribution over $D = (d_0, d_1)$, lets call it $\phi_1(D)$. Hence left to calculate

$$\sum_i P(S|I=i)\sum_d [P(I=i|D=d)\phi_1(D=d)$$

Now wrap it up yourself. Result is $P(S) = (0.516, 0.484)$.

1.2 Part b

In order to compute the probability distribution $P(S|C=c_0)$ on can compute the unnormalized factor $P(G, I=i_0)$ using the same procedure as in Part a but no marginalization (summing) over I but setting the value for I equal to i_0. After computing $P(G, I=i_0)$ one just normalizes the distribution to get $P(G|I=i_0)$. Check or prove this!

1.3 Part c

Same procedure as in Part b.

2 Exercise 2

Let's number the factors defined in the exercise by 1 up to 5. The overall factor is defined as

$$\Phi(C, D, I, G, S) = \Phi_1(C)\Phi_2(C, D)\Phi_3(D, I)\Phi_4(D, I, G)\Phi_5(I, S)$$

To compute $\Phi(S)$ marginalize over all the other variables Same procedure as in Exercise 1.

3 Exercise 3

I will only explain Part a. Other parts are still left as an exercise. First we construct the factor graph. Observe that in the above factor graph the factor between D and I is missing. This factor is included by a factor product in f_5. The reasons is that the communication structure for calculating $\phi(G)$ should be a tree, which is not the

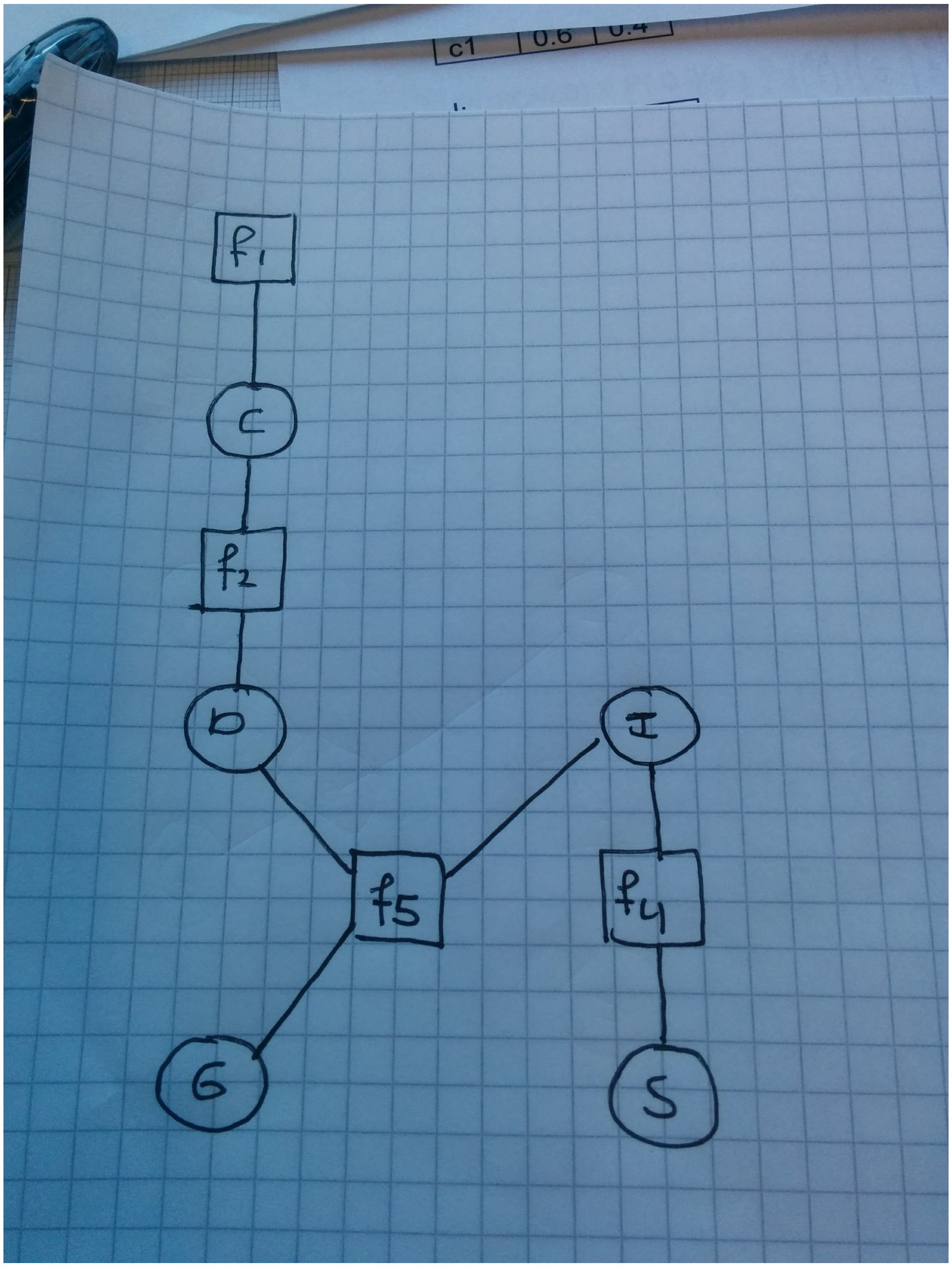

Figure 2: Factor graph for Exercise 3

case when there is a factor node between D and I.
In order to compute $\phi(G)$ the node G is considered the root of the tree and messages are send upwards to G. Moreover we have to marginalize over all variables except G. G receives a message from f_5 which is the requested marginalized factor. D and I sends a marginalized factors (messages) to f_5. D should receive a message from f_2, etcetera.
Message from f_1 to C is the factor $[3, 7]$, from S to f_4 is $[1, 1]$ (see book). Now wrap it up yourself.