Homework 5 Basic Machine Learning
For the deadline see Canvas
Version: Thu 27$^{\text{th}}$ Sept, 2018 at 09:53.

Introduction

1. This is a group assignment, so sign up in groups of two students.
2. Each group has to submit a **pdf** with their answers and explanation. **Please put your names and group number at the top of the hand in**.
3. For questions about this homework assignment use the Discussion Board on Canvas.
4. Of course you may use a calculator or a programming environment such as Matlab or Python. **But your report should not contain any code. Explain your computations and results in English!**
5. It is allowed to incorporate handwritten notes or derivations or drawings in your submission as long as these are readable!
6. Explain your answers!

Exercise 1: Backpropagation 1-D output (15 points)

Consider the neural network as depicted in Figure 5.1 of the course book. Assume that the input is 2 dimensional and that there are 3 neurons in the hidden layer and that there is 1 output neuron. The activation for all neurons is the sigmoid function σ. Moreover assume that the error function is given by formula 5.21 of the course book. The weights of the NN are given by

$$\begin{aligned}
\text{Hidden layer:} \quad & w_{1,0}^{(1)} = 1, w_{1,1}^{(1)} = 1, w_{1,2}^{(1)} = -1, \\
& w_{2,0}^{(1)} = 0, w_{2,1}^{(1)} = -1, w_{2,2}^{(1)} = -1, \\
& w_{3,0}^{(1)} = -1, w_{3,1}^{(1)} = 0, w_{3,2}^{(1)} = 1 \\
\text{Output layer:} \quad & w_{1,0}^{(2)} = 1, w_{1,1}^{(2)} = -1, w_{1,2}^{(2)} = 1, w_{1,3}^{(2)} = -1
\end{aligned}$$

Assume that the input is $\mathbf{x} = (x_1, x_2) = (1, 1)$ and the target output is 1.

Part a

Draw the network including weights and calculate the output of the above NN on the given input $\mathbf{x}$.

Part b

Calculate δ for the output neuron, i.e. $\delta_1^{(2)}$.

Part c

How will the weight $w_{1,2}^{(2)}$ be adapted, based on the above x and corresponding target output, if one applies stochastic backpropagation (stochastic gradient descent) only once with learning rate 0.7?

Part d

Give the back propagation formula for $\delta_1^{(1)}$ of the hidden neuron 1 (the formula which calculates $\delta_1^{(1)}$ in terms of the δ of the output neuron). Calculate for hidden neuron 1 (neuron 1 in the hidden layer) the corresponding $\delta_1^{(1)}$.

Part e

What will be the new weights of neuron 1 in the hidden layer?

Exercise 2: Backpropagation 2-D output (15 points)

Once again consider the neural network as depicted in Figure 5.1 of the course book. Assume that the input is 2 dimensional and that there are 3 neurons in the hidden layer and but now there are 2 output neurons. The activation for all the hidden neurons is the sigmoid function σ, but the activation function for the output neurons is linear ($h(x) = x$). Moreover the error function is given by formula 5.14. The weights of the NN are given by

$$\begin{aligned}
\text{Hidden layer:} \quad & w_{1,0}^{(1)} = 1, w_{1,1}^{(1)} = 1, w_{1,2}^{(1)} = -1, \\
& w_{2,0}^{(1)} = 0, w_{2,1}^{(1)} = -1, w_{2,2}^{(1)} = -1, \\
& w_{3,0}^{(1)} = -1, w_{3,1}^{(1)} = 0, w_{3,2}^{(1)} = 1 \\
\text{Output layer:} \quad & w_{1,0}^{(2)} = 1, w_{1,1}^{(2)} = -1, w_{1,2}^{(2)} = 1, w_{1,3}^{(2)} = -1 \\
& w_{2,0}^{(2)} = -1, w_{2,1}^{(2)} = 1, w_{2,2}^{(2)} = -1, w_{2,3}^{(2)} = 1
\end{aligned}$$

Assume that the input is $\mathbf{x} = (x_1, x_2) = (1, 1)$ and the target output is (1,-1).

Part a

Draw the network including weights and calculate the output of the above NN on the given input $\mathbf{x}$.

Part b

Calculate δ's for the output neurons, i.e. $\delta_1^{(2)}$ and $\delta_2^{(2)}$.

Part c

How will the weight $w_{2,1}^{(2)}$ be adapted, based on the above x and corresponding target output, if one applies stochastic backpropagation (stochastic gradient descent) only once with learning rate 0.5?

Part d

Give the back propagation formula for $\delta_2^{(1)}$ of the hidden neuron 2 (the formula which calculates $\delta_2^{(1)}$ in terms of the δ's of the output neurons). Calculate for hidden neuron 2 (neuron 2 in the hidden layer) the corresponding $\delta_2^{(1)}$.

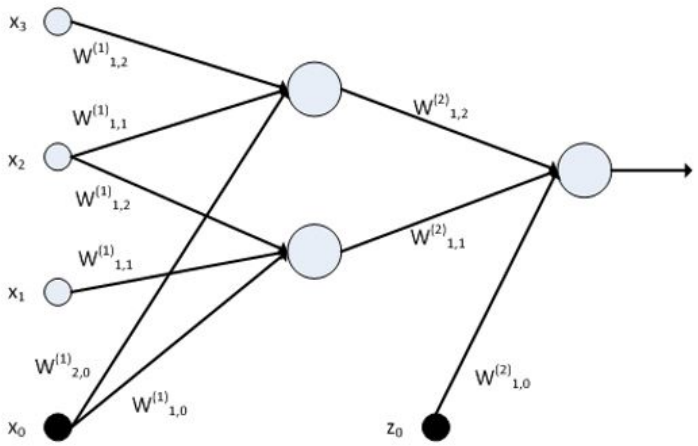
Part e

What will be the new weights of neuron 2 in the hidden layer?

Exercise 3: Backpropagation and weight sharing (15 points)

In this exercise we will consider weight sharing, an important concept in for instance convolutional neural networks (CNN). Once again consider the neural network as depicted in Figure 5.1 of the course book. Assume that the input is 3 dimensional and that there are 2 neurons in the hidden layer and there is 1 output neuron. The activation (transfer) function for all the hidden neurons is the sigmoid function σ, but the activation function for the output neuron is linear ($h(x) = x$). Moreover the error function is given by formula 5.14.
Moreover we assume that:

1. There is no connection between hidden neuron 1 and input 3.
2. There is no connection between hidden neuron 2 and input 1.
3. Hidden neuron 1 and hidden neuron 2 share some weights: in this case $w^{(1)}_{1,1} = w^{(1)}_{2,2}$ and $w^{(1)}_{1,2} = w^{(1)}_{2,3}$. Meaning that these weights are identical and that the hidden layer has only 4 weights instead of 8. See figure below for a graphical representation.



Part a

Assume that we apply stochastic gradient descent. Give the analytic formula for the update of $w^{(1)}_{1,2}$, in terms of δ's and inputs, i.e a standard backpropagation formula. Do not forget that the weight appears two times in the network, it is a weight of hidden neuron 1 and 2.

Part b

The weights of the network are given by:

$$\text{Hidden layer:} \quad w^{(1)}_{1,0} = 1, w^{(1)}_{1,1} = 1, w^{(1)}_{1,2} = -1, w^{(1)}_{2,0} = 0$$
$$\text{Output layer:} \quad w^{(2)}_{1,0} = -1, w^{(2)}_{1,1} = 2, w^{(2)}_{1,2} = -1$$

Assume that the input is $\mathbf{x} = (x_1, x_2, x_3) = (1, 2, 3)$ and the target output is 0. Compute the output of the network and the update for weight vector $\mathbf{w}^{(1)}_1$ (the weight vector for neuron 1 in the hidden layer) for a learning parameter of 0.4.

Exercise 4: Multiple choice questions (3 bonus points)

Design two multiple choice (MC) questions concerning the material of week 5. Clearly indicate what knowledge or skill you want to test with the MC questions and what the correct answers are.