Homework Assignment N°2

BML36
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Contents

1 Exercise 2: Logistic classification & discrimination

From now we will use $\sigma(x) = \frac{1}{1+e^{-x}}$

1.1 Part a

- ☐ Initialize w_0 ?

  1. Some fixed w_0 like $\begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}$
  2. The result of computation around the dataset like the mean: $w_0 = \frac{1}{N} \sum_{i=1}^{N} x_i$, concatenated with a constant.
  3. A random vector

  Any vector except the null vector and the multiples of $\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$ is suitable to initialize w_0,

- ☐ How to learn: for batch learning use this equation at each step

$$w_{n+1} = w_n - \eta \nabla E(w_n) = w_n - \eta \sum_{n=1}^{N} (y(n) - t_n)\, x_n$$

- ☐ How to stop the iterative process ?

  1. Stop when the norm of the difference vector is low: $\Delta_n = \frac{\|w_{n+1} - w_n\|}{\|w_n\|} < \epsilon$
     This is a commonly used criterion that stops the process when the steps we take are getting small compared to our current result.
  2. Stop after fixed number of iteration
     This ensures we won't enter in a infinite non-convergent process.
  3. Stop when a threshold error is reached: $E(w_n) < \epsilon$
     This is actually a bad idea because most of the time we can't be certain it is possible to reach such threshold on the error. It would result in an infinite process.

  In a batch version, we can use criteria 1 and 2 together and stop whenever one of the criteria is reached.
  In a stochastic version, criterium 1 is not applicable because it would stop the learning process whenever a well classified data is picked for an iteration.

Our algorithm goes as follows:

1. Chose ϵ, N and η respectivelly for precision, maximum number of iterations and speed convergency.
2. Set current error Δ to $+\infty$ and n to 0
3. Chose the initial discriminant: $w_{current} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}$.
4. While $\Delta > \epsilon \wedge n < N$ do

(a) Compute and store next discriminant w_{next}:

$$w_{next} = w_{current} - \eta \sum_{n=1}^{N} \left(\sigma({w_{current}}^\top x_n) - t_n\right) x_n$$

(b) Compute and store the new error Δ:

$$\Delta = \frac{\|w_{next} - w_{current}\|}{\|w_{current}\|}$$

(c) Prepare for next iteration: store w_{next} in place of $w_{current}$ and increment n

5. If $\Delta > \epsilon$, it means we have not converged enough towards the limit. We should consider increasing N OR using another algorithm for convergence (eg. Newton-Raphson)

6. Result is stored in $w_{current}$, number of steps in n.

1.2 Part b

First important thing to notice is that the point $x = \begin{bmatrix}-1 & 1\end{bmatrix}$ is missclassified. We define $\bar{x} = \begin{bmatrix}1 & -1 & 1\end{bmatrix}$ thus it means that when we compute $w^\top \bar{x}$ the sign of the result is incorrect.
In our case, $w^\top \bar{x} = 1 > 0$, thus the real class of x is 0.
The formula to update the weights is the following:

$$w_{new} = w - \eta\sigma(w^\top \bar{x})\bar{x}$$

We get the following result:

$$w_{new} \approx \begin{bmatrix}0.5614 & 2.4386 & 1.5614\end{bmatrix}$$

2 Exercise 4: MCQ

2.1 First MCQ

This first question tests if the candidate knows how to compute the new discriminant from the previous one, using gradient descent.
Q: Which of the following equations can be used to update the discriminant w using the method of gradient descent ?

1. $w_{n+1} - w_n = \eta \nabla E(w_n)$
2. $w_{n+1} = w_n - \eta \nabla E(w_n)$
3. $w_n + \eta \nabla E(w_{n-1}) = w_{n-1}$
4. $w_n = \eta w_n - \nabla E(w_{n+1})$

2.2 Second MCQ

The second question verifies the student understood well what is implied behind the omnipresent formula $w^\top x$ and each of its components.
Q: When learning in a k-dimensions space, knowing that we compute the class of x by the following formula $C(x) = h(w^\top x + w_0)$ where h is the heaviside function. What are the ranges of w and x ?

1. $range(w) = k \quad range(x) = k$
2. $range(w) = k + 1 \quad range(x) = k$
3. $range(w) = k \quad range(x) = k + 1$
4. $range(w) = k + 1 \quad range(x) = k + 1$