Advanced Model Predictive Control — Fall 2022
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Programming Exercise 8

Model Predictive Safety Filter

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1 Exercise

1. **Not Graded.** Simulate the system with the provided learning-based control inputs $u_L(k)$.
2. **Graded.** Compute the largest ellipsoidal invariant set $\mathcal{S} = \{x \in \mathbb{R}^2 |\ V(x) = x^\top P x \leq 1\}$ in `IBSF.m` with the corresponding safe controller $u_S(x(k)) = Kx(k)$, such that invariance, state constraint satisfaction for $\mathcal{X} = \{x | A_x x \leq b_x\}$ and input constraint satisfaction for $\mathcal{U} = \{u | A_u u \leq b_u\}$ using $u_S(k)$ are ensured.
3. **Not Graded.** Implement the following safety filter input in the `solve` method in `IBSF.m` and simulate the system:
$$u(k) = \begin{cases} u_L(k) & \text{if } V(x(k)) \leq 1 \text{ and } u_L(k) \in \mathcal{U} \\ u_S(x(k)) = Kx(k) & \text{otherwise} \end{cases} \tag{1}$$
Is the system safe? Are the constraints on the system satisfied?
4. **Graded.** Instead, use the following safety filter input
$$u(k) = \begin{cases} u_L(k) & \text{if } V(x(k+1)) \leq 1 \text{ and } u_L(k) \in \mathcal{U} \\ u_S(x(k)) = Kx(k) & \text{otherwise} \end{cases} \tag{2}$$
Is the safe set invariant using the given safety filter input? Are the constraints on the system satisfied?
5. **Graded.** Implement the Model Predictive Safety Filter in `MPSF.m` using the ellipsoidal safe set as a terminal safe set in the optimization.
$$\begin{aligned} \min \quad & \|u_0 - u_L\|^2 \\ \text{s.t.} \quad & x_{i+1} = Ax_i + Bu_i \\ & (x_i, u_i) \in \mathcal{X} \times \mathcal{U} \\ & x_N \in \mathcal{S} \\ & x_0 = x(k) \end{aligned}$$
What do you notice? Is recursive feasibility of this safety filter ensured?