Advanced Model Predictive Control — Fall 2022
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Solution 6

Stochastic MPC - Part I

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1 Exercise

Constraint Tightening RMPC

1. Implementation of robust constraint tightening MPC in the `constraint_tightening_RMPC.m` file.

   a. Implement the computation of the disturbance reachable sets (DRS) and the tightenings in the `compute_robust_tightening` method. Then, run the provided code to plot the DRS and tightenings.

   b. Consider the robust constraint tightening MPC problem

$$
\begin{aligned}
\min_{V} \quad & \|z_N\|_P^2 + \sum_{i=0}^{N-1} \|z_i\|_Q^2 + \|v_i\|_R^2 && && \text{(1a)}\\
\text{s.t.} \quad & z_{i+1} = Az_i + Bv_i, && i \in [0, N-1] && \text{(1b)}\\
& z_i \in \mathcal{X} \ominus \mathcal{F}_i, && i \in [0, N-1] && \text{(1c)}\\
& v_i \in \mathcal{U} \ominus K\mathcal{F}_i, && i \in [0, N-1] && \text{(1d)}\\
& z_N \in \mathcal{X}_f \ominus \mathcal{F}_N, && && \text{(1e)}\\
& z_0 = x(k), && && \text{(1f)}
\end{aligned}
$$

   where $\mathcal{F}_i = \bigoplus_{j=0}^{i-1} (A+BK)^j \mathcal{W}$.
   Implement (1) in the provided `constraint_tightening_RMPC.m` file choosing $\mathcal{X}_f$ as the maximal RPI set under the infinite horizon LQR controller K and using $\mathcal{X} = \{x \mid A_x x \leq b_x\}$, $\mathcal{U} = \{u \mid A_u u \leq b_u\}$, $\mathcal{W} = \{w \mid A_w w \leq b_w\}$.

Constraint Tightening SMPC

2. Implementation of stochastic constraint tightening MPC in the `constraint_tightening_SMPC.m` file.

   a. Implement the computation of the DRS, the stochastic backoff term, and the resulting tightenings in the `compute_robust_tightening` and `compute_stochastic_tightening` methods. Then, run the provided code to visualize the computed tightenings.

   b. Consider the stochastic constraint tightening MPC problem

$$
\begin{aligned}
\min_{V} \quad & \|\bar{x}_N\|_P^2 + \sum_{i=0}^{N-1} \|\bar{x}_i\|_Q^2 + \|\bar{u}_i\|_R^2 && && \text{(2a)}\\
\text{s.t.} \quad & \bar{x}_{i+1} = A\bar{x}_i + B\bar{u}_i, && i \in [0, N-1] && \text{(2b)}\\
& \bar{x}_i \in \mathcal{X} \ominus (A+BK)\mathcal{F}_{i-1} \ominus \mathcal{F}_w^x(p), && i \in [1, N-1] && \text{(2c)}\\
& \bar{u}_i \in \mathcal{U} \ominus K(A+BK)\mathcal{F}_{i-1} \ominus \mathcal{F}_w^u(p), && i \in [1, N-1] && \text{(2d)}\\
& \bar{x}_N \in \mathcal{X}_f \ominus \mathcal{F}_N, && && \text{(2e)}\\
& \bar{x}_0 = x(k), && && \text{(2f)}
\end{aligned}
$$

   where $\mathcal{F}_i = \bigoplus_{j=0}^{i-1}(A+BK)^j\mathcal{W}$, $\mathcal{F}_w^x(p) = \sqrt{p}\mathcal{W}$, and $\mathcal{F}_w^u(p) = K\mathcal{F}_w^x(p)$. Implement (2) in the provided `constraint_tightening_SMPC.m` file choosing $\mathcal{X}_f$ as the maximal RPI set under the infinite horizon LQR controller K and using $\mathcal{X} = \{x \mid A_x x \leq b_x\}$, $\mathcal{U} = \{u \mid A_u u \leq b_u\}$, $\mathcal{W} = \{w \mid A_w w \leq b_w\}$, $w \sim \text{Uniform}(\mathcal{W})$.

Comparison

3. Comparison of robust constraint tightening MPC to stochastic constraint tightening MPC.

   a. Run the code labelled Exercise 3a for different values of p and study the produced figure.

   b. Run the code labelled Exercise 3b for different values of p and study the produced figures.

2 Solution

1./2./3. The MATLAB code for these questions can be found on Moodle.