Advanced Model Predictive Control — Fall 2022

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Exercise 9

Gaussian Processes and Learning-based MPC

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Exercise

Gaussian Processes.

1. Let x and y be jointly Gaussian random vectors

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}, \begin{bmatrix} A & C \\ C^\top & B \end{bmatrix}\right)$$

derive the conditional distribution of x given y

$$x|y \sim \mathcal{N}\left(\mu_x + CB^{-1}(y-\mu_y), A - CB^{-1}C^T\right)$$

2. Given the following noisy data set with noise $w \sim \mathcal{N}(0,2)$ for the chocolate consumption per year per capita and the Nobel prize winners per 10 million population

$$D = \{(x_{USA} = 4.4, y_{USA} = 11.7),\ (x_{UK} = 7.6, y_{UK} = 19.4),\ (x_{SW} = 6.6, y_{SW} = 30.0)\},$$

compute the mean and variance of the Gaussian Process predictive distribution for $x_{CH} = 8.8$ with the kernel function

$$k(x, x') = x^2 x'^2.$$

Then, try a different kernel function, e.g., the squared exponential kernel. Plot the mean and variance of the predictive distribution.

Learning-based MPC.

3. Consider the robust performance learning-based MPC problem

$$\begin{aligned} J_k^\star(x(k)) = \min_u\ & \|x_N\|_P^2 + \sum_{i=0}^{N-1} \|x_i\|_Q^2 + \|u_i\|_R^2 \\ \text{s.t. }\ & x_0 = x(k), \quad z_0 = x(k) \\ & x_{i+1} = Ax_i + Bu_i + \mathcal{O}(x_i, u_i) \\ & z_{i+1} = Az_i + Bu_i \\ & z_i \in \mathcal{X} \ominus \mathcal{F}_i, \quad u_i \in \mathcal{U} \ominus K\mathcal{F}_i \\ & z_{i+N} \in \mathcal{X}_f \ominus \mathcal{F}_N \end{aligned} \tag{1}$$

where $\mathcal{W}$ was derived in the recitation and $\mathcal{F}_i = \bigoplus_{j=0}^{i-1}(A+BK)^j\mathcal{W}$.
Implement (1) in the provided `RPLBMPC.m` file.