Computer Exercise 3
EL2520 Control Theory and Practice

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Suppression of disturbances

The weight is

$$W_S(s) = \frac{1}{(s + 0.1 + \sqrt{(100\pi)^2 - 0.1^2})(s + 0.1 - \sqrt{(100\pi)^2 - 0.1^2})}$$

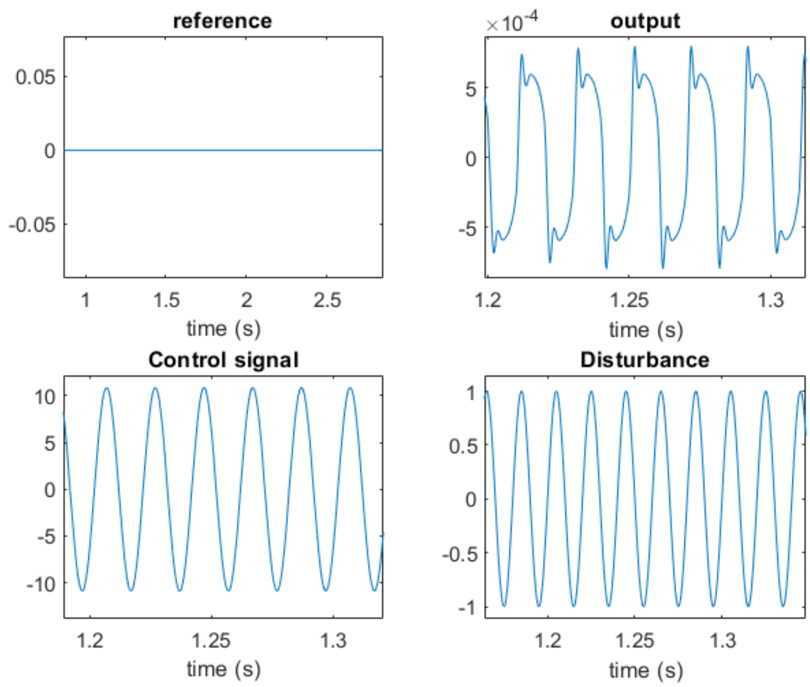

Figure 1: Simulation results with system G, using W_S.

How much is the disturbance damped on the output? What amplification is required for a P-controller to get the same performance, and what are the disadvantages of such a controller?

The disturbance damped on the output is 7.478×10^{-4}. To design P-controller to achieve the same performance is to use gain $K_p = 2.4863 \times 10^5$. The main disadvantage is that P-controller will attenuate the disturbance in all frequency, not only the desired frequency (50 Hz). Moreover, we can observe that the gain is very large, which may not be feasible in the real world application.

Robustness

What is the condition on T to guarantee stability according to the small gain theorem, and how can it be used to choose the weight W_T?

From $G_0(s) = G(s)(1 + \Delta_G(s))$, we get $\Delta_G(s) = \frac{-3}{s+2}$. Therefore, from the small gain theorem, $|T(i\omega)| < \frac{1}{|\Delta_G(i\omega)|}$ and $|T(i\omega)| \leq |W_T^{-1}(i\omega)|$, we can choose the weight such that this inequality is fulfilled.
The weights are

$$
\begin{aligned}
W_S(s) &= \frac{1}{(s + 0.1 + \sqrt{(100\pi)^2 - 0.1^2})(s + 0.1 - \sqrt{(100\pi)^2 - 0.1^2})} \\
W_T(s) &= \frac{0.0001}{s + 2}
\end{aligned}
$$

Is the small gain theorem fulfilled?

From Figure 2., we can observe that $|T|$ is smaller than $|\Delta G^{-1}|$ for all frequency. Thus, the small gain theorem is fulfilled.

Compare the results to the previous simulation

From the result we can observe that there is slightly less attenuation of the disturbance compared to the previous simulation. More specifically, the previous controller reduces the disturbance amplitude from 1 to approximately 5×10^{-4} while this controller decreases the amplitude to 4×10^{-3}.

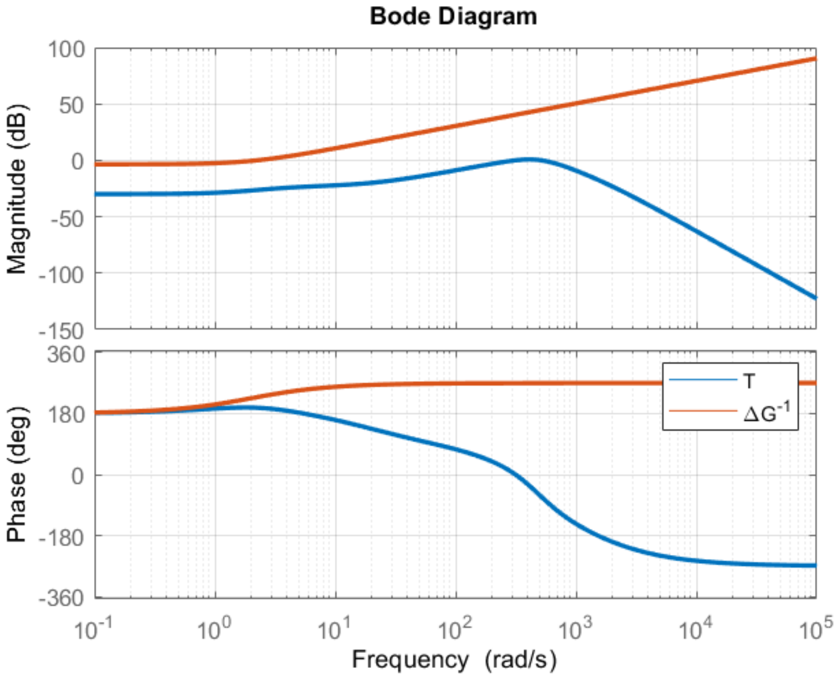

Figure 2: Bode diagram showing that the small gain theorem is satisfied.

Control signal

The weights are

$$W_S(s) = \frac{1}{(s + 0.1 + \sqrt{(100\pi)^2 - 0.1^2})(s + 0.1 - \sqrt{(100\pi)^2 - 0.1^2})}$$

$$W_T(s) = \frac{0.0001}{s + 2}$$

$$W_U(s) = \frac{2}{s + 2}$$

Compare the results to the previous simulations

This simulation result shows that this controller can reduce the control signal by half; however, it has the worst performance in attenuation of the output disturbance and is only able to reduce the disturbance amplitude to approximately 0.75. In other words, the disturbance is almost entirely propagated through to the output.

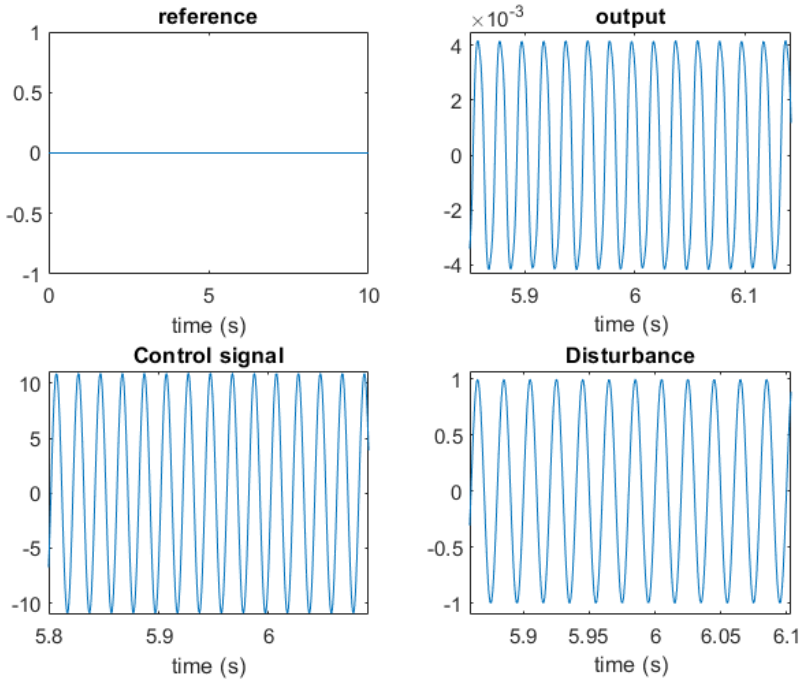

Figure 3: Simulation results with system G_0, using W_S and W_T.

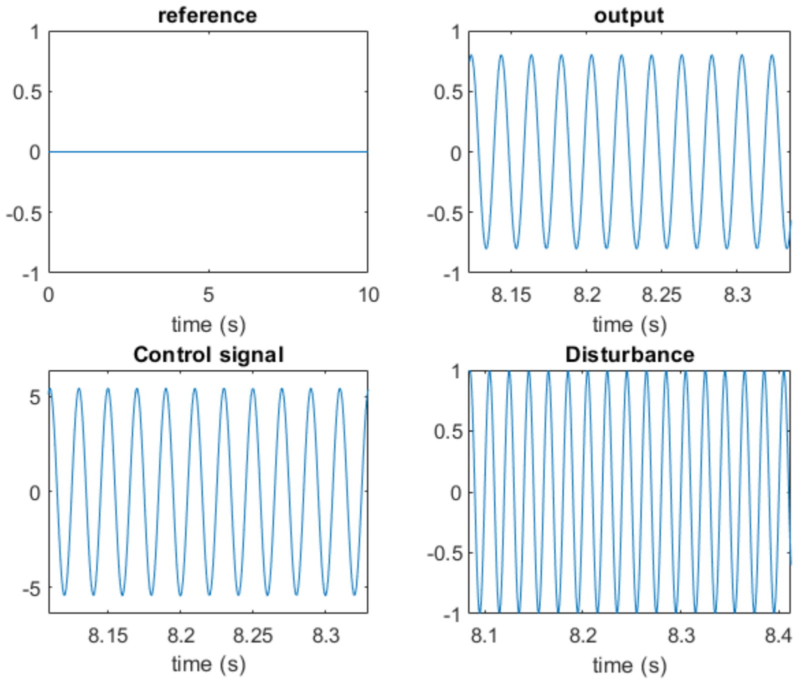

Figure 4: Simulation results with system G_0, using W_S, W_T and W_U.