

एकाइ (Unit): 6

बेलना र गोला (Cylinder and Sphere)

6.1 बेलनाको सतहको क्षेत्रफल र आयतन (Surface area and Volume of cylinder)

केहि महत्वपूर्ण सुत्रहरू (Some important formulae)

- आधारको परिधि र क्षेत्रफल (circumference of Bases and Area)

यदि बेलनाको अर्धव्यास = r

आधारको परिधि = C र

आधारको क्षेत्रफल = (A) भए

परिधि (C) = $2\pi r$ र आधारको क्षेत्रफल (A) = πr^2

If radius of cylinder = r , circumference of base = c Area of base = A ,

Circumference (C) = $2\pi r$ and area of base (A) = πr^2

- बेलनाको सतहको क्षेत्रफल = (surface area of cylinder)

यदि बेलनाको आधारको अर्धव्यास = r उचाई = h भए

बक्सतहको क्षेत्रफल (CSA) = $2\pi rh = C \times h =$ परिधि \times उचाई

पूरा सतहको क्षेत्रफल (TSA) = $2\pi r(r + h) = c \times (r + h)$

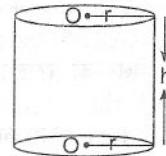
= (परिधि \times अर्धव्यास र उचाईको योगफल)

If radius of bases of cylinder = r height = h

curvesurface area (CSA) = $2\pi rh = C \times h$

= circumference \times height

Total surface area (TSA) = $2\pi r(r + h) = c \times (r + h) =$ circumference (sum of radus and height)



- बेलनाको आयतन (Volume of cylinder):

यदि बेलनाको आधारको अर्धव्यास = r , उचाई = h र आधारको क्षेत्रफल (A) = πr^2 भए

बेलनाको आयतन (V) = $\pi r^2 h = A \times h$ (आधारको क्षेत्रफल \times उचाई)

Area of base \times height

If radius of base = r height = h and Area of base (A) = πr^2

Volume of cylinder (V) = $\pi r^2 h = A \times h$ (Area of base \times height)

- धातुको आयतन (Volume of metal)

बेलनाको बाहीरी अर्धव्यास = r_1

उचाई = h र भिन्नी अर्धव्यास = r_2 भए

बाहीरी आयतन (V_1) = $\pi r_1^2 h$

भिन्नी आयतन (V_2) = $\pi r_2^2 h$

धातुको आयतन (V) = $\pi r_1^2 h - \pi r_2^2 h = \pi h(r_1^2 - r_2^2)$

In a cylinder, outer radius = r_1 inner radius = r_2

and height = h

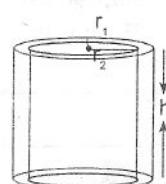
outer volume (V) = $\pi r_1^2 h$, inner volume (V_2) = $\pi r_2^2 h$

Volume of metal (V) = $V_1 - V_2 = \pi r_1^2 h - \pi r_2^2 h = \pi h(r_1^2 - r_2^2)$

- बेलनाको पूरा सतहको क्षेत्रफल = TSA र बक्सतहको क्षेत्रफल = CSA र आधारको क्षेत्रफल = A भए

$\therefore TSA = CSA + 2A$

Total surface area of cylinder = TSA curve surface area CSA Area of base = A



- $\therefore \text{TSA} = \text{CSA} + 2A$
- अर्धबेलना (Hemi-cylinder):—
अर्धबेलनामा आधारको अर्धव्यास = r
र उचाई = h भए

$$\text{परिधि } (c) = \pi r + 2r, \text{ आधारको क्षेत्रफल } (A) = \frac{\pi r^2}{2}$$

$$\text{वक्सतहको क्षेत्रफल } (\text{CSA}) = (\pi r + 2r) \times h$$

$$\text{पूरासतहको क्षेत्रफल } (\text{TSA}) = (\pi r + 2r) \times h + \pi r^2$$

$$\text{आयतन } (V) = \frac{1}{2} \pi r^2 h = \frac{1}{2} A \times h$$

In semi-cylinder, radius of base = r , and height = h , then

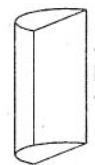
$$\text{circumference } (C) = \pi r + 2r, \text{ Area of base } (A) = \frac{\pi r^2}{2}$$

$$\text{Curve surface area } (\text{CSA}) = (\pi r + 2r) \times h$$

$$\text{Total surface area } (\text{TSA}) = (\pi r + 2r) \times h + \pi r^2$$

$$\text{Volume } (V) = \frac{1}{2} \pi r^2 h = \frac{1}{2} A \times h$$

Note: $1000\text{cm}^3 = 1 \text{ liter}$, $1000 \text{ liter} = 1\text{m}^3$



Very Short Questions:

- एउटा बेलनाको आधारको क्षेत्रफल 82cm^2 र उचाई 10cm भए आयतन पत्ता लगाउनुहोस् ।
Area of base 82cm^2 and height is 10cm of cylinder find the volume.
- बेलनाको पुरासतहको क्षेत्रफल निकाल्ने सुन्न लेख्नुहोस् ।
Write the formula find the total surface area of cylinder.
- 44cm आधारको परिधि र 5cm उचाई भएको बेलनाको वक्सतहको क्षेत्रफल निकाल्नुहोस् ।
What is the curved surface area of cylinder whose circumference of base 44cm and height is 5cm.
- बेलनाको अर्धव्यास र उचाइको योगफल 24cm र परिधि 88cm भए पुरासतहको क्षेत्रफल निकाल्नुहोस् ।
Find the total surface area of cylinder whose sum of radius and height 24cm and circumference is 88cm .
- एउटा बेलना एउटा धातुबाट बनेको छ । यसको बाहीरी अर्धव्यास r_1 भिन्नी अर्धव्यास r_2 र उचाई (h) भए धातुको आयतन कति होला ?
A cylinder formed by metal its outer radius r_1 inner radius r_2 and height h , what is the volume of metal.
- एउटा बेलना भएको समलत सतहको क्षेत्रफल लेख्नुहोस् । Write the area of plane surface of a cylinder.
- यदि एउटा बेलनाको आयतन $\pi xy^2 \text{ cm}^3$ भए अर्धबेलनाको आयतन कति होला ?
If the cylinder has its volume $\pi y^2 x$, what will be the volume of semi – cylinder.
- आधारको परिधि c र उचाई h भएको बेलनको वक्सतहको क्षेत्रफल कति हुन्छ ?
What is the curved surface area of cylinder whose circumference of base c and height is h .

Short Questions:

Model 1:

60cm अग्लो र 14cm व्यास भएको एउटा बेलनालाई ठाडो चिरेर दुई बराबर भाग लगाउँदा एक भागको आयतन कति होला ?

A 60cm hight cylinder with 14cm its diameter is cut vertically in to equal halves. What is its volume.

Solution:

Here, height of cylinder (h) = 60cm

$$\text{diameter } (d) = 14\text{cm}, \text{radius } (r) = \frac{d}{2} = \frac{14\text{cm}}{2} = 7\text{cm}$$

$$\text{Volume of semicylinder (V)} = \frac{1}{2} \pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times (7\text{cm})^2 \times 60\text{cm} = \frac{1 \times 22 \times 49 \times 60\text{cm}^3}{14} \\ = \frac{63680\text{cm}^3}{14} = 4620\text{cm}^3$$

Volume of semi-cylinder (V) = 4620cm³

Model 2:

एउटा बेलनाको लम्बाई र आधारको क्षेत्रफल क्रमशः 14cm र 154cm² छ भने यसको अर्धव्यास र आयतन पत्ता लगाउनुहोस् ।
The length and base area of cylinder are 14cm and 154cm² find its radius and volume.

Solution:

Here, length of cylinder (h) = 14cm

Area of base (A) = 154cm²

$$\therefore A = \pi r^2$$

$$\text{or, } 154\text{cm}^2 = \frac{22}{7} r^2$$

$$\text{or, } \frac{154 \times 7\text{cm}^2}{22} = r^2$$

$$\text{or, } 49\text{cm}^2 = r^2$$

$$\therefore r = 7\text{cm}$$

$$\text{Volume (V)} = \pi r^2 h = \frac{22}{7} \times (7\text{cm})^2 \times 14\text{cm} = \frac{22 \times 49 \times 14\text{cm}^3}{7} = \frac{15092\text{cm}^3}{7} \\ = 2156\text{cm}^3$$

Volume of Cylinder (V) = 2156cm³

Model 3

बेलनाको आधारको परिधि 14cm र अर्धव्यास र उचाइको योगफल 17cm छ भने बेलनाको अर्धव्यास र पुरा सतहको क्षेत्रफल निकाल्नुहोस् ।

The circumference of base is 44cm and the sum of cylinder radius and height is 17cm, find the radius and total and height is 17cm, find the radius and total surface area of cylinder.

Solution:

Here, circumference of base (C) = 44cm

$$\text{or, } C = 2\pi r$$

$$\text{or, } 44\text{cm} = 2 \times \frac{22}{7} \times r$$

$$\text{or, } 44\text{cm} = \frac{44r}{7}$$

$$\therefore r = 7\text{cm}$$

$$\text{Again TSA} = C \times (r + h) = 44\text{cm} \times 17\text{cm} = 748\text{cm}^2$$

Radius of base (r) = 7cm

Total surface area of cylinder (TSA) = 748cm²

Model 4

बेलनाको अर्धव्यास र उचाइको अनुपात 5:7 छ । यसको आयतन 550cm³ भए बेलनाको अर्धव्यास पत्ता लगाउनुहोस् ।
The base of radius and height of cylinder are in the ratio 5:7 and the volume of cylinder 550cm³, find the radius of cylinder.

Solution:

Here, for the cylinder the ratio of radius and height (r:h) = 5:7 r: h = 5:7 then r = 5x and h = 7x

Volume of cylinder (V) = 550cm³

$$\therefore V = \pi r^2 h$$

$$\text{or, } 550 \text{ cm}^3 = \frac{22}{7} \times (5x)^2 \times 7x$$

$$\text{or, } 550 \text{ cm}^3 = \frac{22 \times 25x^2 \times 7x}{7}$$

$$\text{or, } 550 \times 7 \text{ cm}^3 = 22 \times 25x^2 \times 7x$$

$$\text{or, } \frac{550 \times 7 \text{ cm}^3}{22 \times 25 \times 7} = x^3$$

$$\text{or, } 1 \text{ cm}^3 = x^3$$

$$\therefore x = 1 \text{ cm}$$

$$\text{Radius of base (r)} = 5x = 5 \times 1 \text{ cm} = 5 \text{ cm}$$

Model 5

एउटा बेलनाको वक्सतहको क्षेत्रफल र पुरासतहको क्षेत्रफल कमश 104cm² र 412cm² भए अर्धव्यास पत्ता लगाउनुहोस् ।

The curved surface area and total surface area of a cylinder are 104cm² and 412cm² respectively. Find the radius.

Solution:

Here, In a cylinder

$$\text{Curved surface area (CSA)} = 104 \text{ cm}^2$$

$$\text{Total surface area (TSA)} = 412 \text{ cm}^2$$

$$\therefore \text{TSA} = \text{CSA} + 2A$$

$$\text{or, } 412 \text{ cm}^2 = 104 \text{ cm}^2 + 2\pi r^2$$

$$\text{or, } 104 \text{ cm}^2 - 104 \text{ cm}^2 = 2 \times \frac{22}{7} r^2$$

$$\text{or, } 308 \text{ cm}^2 \times 7 = 44r^2$$

$$\text{or, } \frac{308 \times 7 \text{ cm}^2}{44} = r^2$$

$$\text{or, } 49 \text{ cm}^2 = r^2$$

$$\therefore r = 7 \text{ cm}$$

$$\text{Radius of base of cylinder (r)} = 7 \text{ cm}$$

Model 6:

एउटा बेलनाकार बट्टाको आयतन 1.54 लीटर र आधारको क्षेत्रफल 77cm² भए उचाई पत्ता लगाउनुहोस् ।

The volume of cylindrical can is 1.54 liter and area of base is 77cm² find the height of the cylinder.

Solution:

$$\text{Volume of cylinder can (V)} = 1.5 \text{ liter } (\because 1 \text{ liter} = 1000 \text{ cm}^3) = 1.54 \times 1000 \text{ cm}^3 = 1540 \text{ cm}^3$$

$$\text{Area of base (A)} = 77 \text{ cm}^2$$

$$\therefore V = A \times h$$

$$\text{or, } 1540 \text{ cm}^3 = 77 \text{ cm}^2 \times h$$

$$\text{or, } \frac{1540 \text{ cm}^3}{77 \text{ cm}^2} = h$$

$$\text{or, } 20 \text{ cm} = h$$

$$\therefore h = 20 \text{ cm}$$

$$\text{The height of cylindrical can (h)} = 20 \text{ cm}$$

Model 9:

उचाई र अर्धव्यास बराबर भएको बेलनाको वक्सतहको क्षेत्रफल 308cm² छ भने बेलनाको उचाई पत्ता लगाउनुहोस् ।

The height and radius of cylinder are equal and curved surface area is 308cm². find the height of cylinder

Solution:

Here, height of cylinder (h) = radius (r)

curved surface area (CSA) = 308 cm² Using formula

$$\therefore \text{CSA} = 2\pi rh$$

$$\text{or, } 308\text{cm}^2 = 2 \times \frac{22}{7} \times h \times h$$

$$\text{or, } 308\text{cm}^2 = \frac{44\text{cm}^2}{7}$$

$$\text{or, } 308 \times 7\text{cm}^2 = 44h^2$$

$$\text{or, } \frac{308 \times 7\text{cm}^2}{44} = h^2$$

$$\text{or, } 49\text{cm}^2 = h^2$$

$$\text{or, } (7\text{cm})^2 = h^2$$

$$\therefore h = 7\text{cm}$$

Height of cylinder (h) = 7cm

Model 10:

एउटा बेलनाको वक्सतहको क्षेत्रफल पूरासतहको क्षेत्रफल दुई तिहाई छ । यसको आधारको अर्धव्यास 6cm छ । बेलनाको उचाई पत्ता लगाउनुहोस् ।

The Curved surface area of a cylinder is two third of the total surface area of radius of the base is 4cm find the height of cylinder.

Solution:

Here, curved surface area cylinder (CSA) = $2\pi rh$

Total surface area of cylinder (TSA) = $2\pi rh(r + h)$ according to the question

Radius of base (r) = 6cm

Curve surface area (CSA) = $\frac{2}{3}$ of total surface area (TSA)

$$(\text{TSA}) 2\pi rh = \frac{2}{3} \times 2\pi rh(r + h)$$

$$\text{or, } 3h = 2r + 2h$$

$$\text{or, } 3h - 2h = 2r$$

$$\therefore h = 2r = 2 \times 6\text{cm} = 12\text{cm} \text{ height of cylinder (h)} = 12\text{cm}$$

Model 11:

एउटा बेलनाको ट्याकीको आयतन 936m^3 छ । यदि यसको उचाई 6m भए ट्याकीको आधारको क्षेत्रफल पत्ता लगाउनुहोस् ।

The volume of a cylindrical tank 936m^3 of its height is 6m, determine the base area of the tank.

Solution:

Here, volume of cylinder tank (V) = 936m^3

its height (h) = 6m

Area of base (A) = ?

$$\therefore V = A \times h$$

$$\text{or, } 936\text{m}^3 = A \times 6\text{m}$$

$$\text{or, } A = \frac{936\text{m}^3}{6\text{cm}}$$

$$\text{or, } A = 156\text{m}^2$$

$$\therefore \text{Area of base a tank (A)} = 156\text{m}^2$$

Model 12:

10cm लामो खोको बेलनाकार धातुको भित्री अर्धव्यास 24cm र धातुको मोटाई 1cm छ । भने सो धातुको आयतन पत्ता लगाउनुहोस् ।

The internal radius of 10cm long hollow cylindrical metal is 24cm and if the thickness of the metal is 1cm. Find the volume .

Solution:

Here, in hollow cylinder metal

Length of cylinder metal (l) = 10cm

Inner radius (r_2) = 24cm

Outer radius (r_1) = 24cm + 1cm = 25cm

Using formula

$$\begin{aligned}\text{Volume of metal (V)} &= \pi h (r_1^2 - r_2^2) = \frac{22}{7} \times 10 \text{cm} [(25\text{cm})^2 - (24\text{cm})^2] \\ &= \frac{22 \times 10}{7} \text{cm} [(625 - 576) \text{cm}^2] = \frac{22 \times 10 \times 49 \text{cm}^3}{7} = 1540 \text{cm}^3\end{aligned}$$

$$\text{Volume of metal (V)} = 1540 \text{cm}^3$$

Model 13

एउटा बेलनाको अर्धव्यास 7cm र उचाई अर्धव्यास को दोब्बर छ । यदि बेलनालाई उचाइबाट दुई वरावर चिरा हुने गरी काटिएको छ भने प्रत्येकका बक्सतहको क्षेत्रफल निकाल्नुहोस् ।

A cylinder has radius 7cm and the height is twice of the radius. if its cut the halves height, find the curve surface area.

Solution:

Radius of cylinder (r) = 7cm,

Height (h) = $2r = 2 \times 7\text{cm} = 14\text{cm}$

$$\begin{aligned}\text{Curve surface area of half of cylinder (CSA)} &= \pi rh + rh = \frac{22}{7} \times 7\text{cm} \times 14\text{cm} + 14 \times 14\text{cm}^2 \\ &= 308\text{cm}^2 + 196\text{cm}^2 = 504\text{cm}^2\end{aligned}$$

Model 14:

अर्धव्यास दोब्बर उचाई भएको एउटा बेलनाको पुरा सतहको क्षेत्रफल 123.2cm^2 भए अर्धव्यास र उचाई पत्तलगाउनुहोस् ।

The total surface area of cylinder whose height three times of the radius is 1232cm^2 find the radius and height

Solution:

Total surface area of cylinder (TSA) = 1232cm^2

if radius = r , then height (h) = $3r$

TSA = $2\pi r(r + h)$

$$\text{or, } 1232\text{cm}^2 = 2 \times \frac{22}{7} \times r(r + 3r)$$

$$\text{or, } 1232\text{cm}^2 = \frac{44r}{7} \times 4r$$

$$\text{or, } 1232\text{cm}^2 = \frac{176r^2}{7}$$

$$\text{or, } r^2 = \frac{1232 \times 7\text{cm}^2}{176}$$

$$\text{or, } r^2 = 49\text{cm}^2$$

$$\therefore r = 7\text{cm}$$

$$\text{Radius (r)} = 7\text{cm} \text{ and height (h)} = 3 \times 7\text{cm} = 21\text{cm}$$

Model 15

एउटा बेलनाको अर्धव्यास पत्ता लगाउनुहोस् । यसको बक्सतहको क्षेत्रफल 3201cm^2 र आयतन 25608cm^3 छ ।

Find the radius of the cylinder whose curved surface area 3201cm^2 and volume is 25608cm^3 .

Solution:

Here, curved surface area of cylinder (CSA) = 3201cm^2 , its volume (V) = 25608cm^3

$$\therefore \text{CSA} = 2\pi rh$$

$$\text{or, } 3201\text{cm}^2 = 2\pi rh \dots \dots \dots \text{(i)}$$

$$\text{Again } V = \pi r^2 h$$

$$\text{or, } 25608\text{cm}^3 = \pi r^2 h \dots \dots \dots \text{(ii)}$$

According to equation (ii) by (i)

$$\text{or, } \frac{25608 \text{ cm}^3}{3201 \text{ cm}^2} = \frac{\pi r^2 h}{2\pi r h}$$

$$\text{or, } 8 \text{ cm} = \frac{r}{2}$$

$$\therefore r = 16 \text{ cm}$$

Radius of cylinder (r) = 16cm

Model 16

अर्धव्यास दोब्बर गर्दा कुनै बेलनाको आयतनमा कति गुणाले फरक हुन्छ ।

What would be the difference in the volume of a cylinder if its radius is doubled.

Solution:

Let the radius of cylinder be r and height be h.

$$\text{Volume (V)} = \pi r^2 h$$

Again the radius (r) = 2r

$$\text{Volume (V)} = \pi r^2 h^2 = \pi (2r)^2 h = 4\pi r^2 h$$

$$\text{Difference of volume (V)} = 4\pi r^2 h - \pi r^2 h = \pi r^2 h \times (4 - 1) = \pi r^2 h \times 3$$

$$\text{Difference of volume of cylinder} = 3\pi r^2 h$$

Model 17:

लम्बाई भएको बेलनाका काठको मूढालाई लम्बाई बाट दुई बराबर चिरा हुनेगरी काटिएको छ । यदि मुढाको व्यास 1.4m छ भने प्रत्येक चिराको पुरासतहको क्षेत्रफल निकाल्नुहोस् ।

A solid cylindrical wooden hight 5m is cut the two halves along at the length find the total surface area of each part. If the diameter of wooden log is 1.4m.

Solution:

Length of cylindrical wooden (h) = 5m

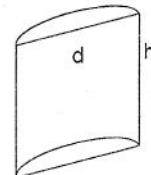
and its diameter (d) = 1.4 m

$$\text{Radius (r)} = \frac{d}{2} = \frac{1.4}{2} \text{ m} = 0.7 \text{ m}$$

$$\text{Total surface area (TSA)} = \frac{1}{2} (2\pi rh) + d \times h + \pi r^2 = \pi rh + d \times h + \pi r^2$$

$$= \frac{22}{7} \times 0.7 \text{ m} \times 5 \text{ m} + 1.4 \text{ m} \times 5 \text{ m} + \frac{22}{7} \times (0.7 \text{ m})^2 = 11 \text{ m}^2 + 7 \text{ m}^2 + 1.54 \text{ m}^2 \\ = 19.54 \text{ m}^2$$

$$\text{Total surface area (TSA)} = 19.54 \text{ m}^2$$



Model 18:

दिइएको ठोस अर्ध बेलनाकार ट्याकीमा कतिलीटर पानी अटाउछ ? जसमा उचाई 1.5m व्यासको लम्बाई 1.2m छ । How many liters of water can be retained in the given hemicylinder tank in which height 1.5m and diameter is 1.2 m.

($1000 \text{ cm}^3 = 1 \text{ liter}, 1000 \text{ liter} = 1 \text{ m}^3$)

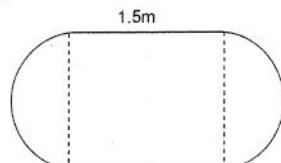
Height of hemicylinder (h) = 1.5m and diameter (d) = 1.2m

$$\text{then radius (r)} = \frac{1.2 \text{ m}}{2} = 0.6 \text{ m}$$

$$\text{Volume (V)} = \frac{1}{2} \pi r^2 h$$

$$= \frac{1}{2} \times \frac{22}{7} \times (0.6 \text{ m})^2 \times 1.5 \text{ m} = \frac{22 \times 0.35 \times 1.5}{14} \text{ m}^3 = 8.4857 \text{ m}^3$$

$$\text{Capacity of water in tank} = 8.485 \times 1000 \text{ liter} \\ = 8485.7 \text{ liter}$$



Practice Yourself

1. 12m अग्लो एउटा बेलनाकार चिम्नीको आधारको क्षेत्रफल 616m^2 भए आयतन पत्ता लगाउनुहोस् ।
Find the volume of cylinder of the area of base cylindrical chimney of 12m high is 616m^2 .
(Ans: 7392m^3)
 2. 60cm उचाई र 14cm व्यास भएको एउटा बेलनालाई ठाडो चिरेर दुई बराबर लगाउँदा एक भागको आयतन कति हुन्छ ? A 60cm height of cylinder and 14cm its diameter is cut vertically in two equal halves. What is the volume of a half part.
(Ans: 4620 cm^3)
 3. आधारको परिधि 44cm र अर्धव्यास र उचाईको योगफल 17cm छ भने बेलनाको पुरा सतहको क्षेत्रफल पत्ता लगाउनुहोस् । Find the total surface area of cylinder the circumference of base 44cm and the sum of height and radius 17cm .
(Ans: 2112cm^2)
 4. उचाई 7cm भएको ठोस बेलनाको पुरासतहको क्षेत्रफल $276\frac{4}{7}\text{ cm}^2$ भए बेलनाको उचाई पत्ता लगाउनुहोस् ।
Find the radius of the base of the cylinder whose height is 7cm and total surface area is $276\frac{4}{7}\text{ cm}^2$.
(Ans: 4cm)
 5. आधारको अर्धव्यास र उचाईको अनुपात $5:7$ छ र आयतन 550cm^3 भए बेलनाको अर्धव्यास पत्ता लगाउनुहोस् ।
Find the radius of cylinder whose the base radius and height are in the ratio $5:7$ and the volume of is 550cm^3 .
(Ans: 5cm)
 6. 1.4m व्यास भएको बेलनाकार ट्याङ्कीमा 770 liter पानी अटाउँछ भने यसको उचाई निकाल्नुहोस् ।
Find the height of cylinder tank whose diameter is 1.4m and hold 770 liter of water.
(Ans: 0.5m)
 7. एउटा बेलनाको आयतन 1.54 लीटर छ । यदि उचाई 20cm छ भने यसको आधारको क्षेत्रफल निकाल्नुहोस् ।
The volume of cylinder is 1.54 liter and the height is 20cm find its base area.
(Ans: 77cm^2)
 8. एउटा अर्धबेलनाकारको व्यास 28cm र उचाई 40cm छ भने त्यसको आयतन पत्ता लगाउनुहोस् ।
Find the volume of semi-cylindrical whose diameter is 28cm and height is 40cm .
(Ans: 123cm^3)
 9. एउटा बेलनाकार ट्याङ्कीको उचाई 2m र आयतन 3080 भए उक्त ट्याङ्कीको आधारको क्षेत्रफल पत्ता लगाउनुहोस् । The capacity of a cylindrical of height 2m , is 3000 . Find the base area of tank.
(Ans: 1540cm^2)
 10. 25cm लामो खोको बेलनाकार धातुको भित्री व्यास 4cm र मोटाई 1cm छ भने धातुको आयतन निकाल्नुहोस् ।
The internal radius of 25cm long hollow cylindrical metal is 4cm . If the thickness of metal is 1cm find the volume of the metal.
(Ans: 707.14cm^3)
 11. एउटा बेलनाको बक्सतहको क्षेत्रफल यसैको पुरासतहको क्षेत्रफल एक तिहाई र उचाई 5cm भए अर्धव्यास पत्ता लगाउनुहोस् ।
The curved surface area of cylinder is one third of the total surface area and the height is 5cm .
Find the radius.
(Ans: 10cm)
 12. दिइएको बेलनाकार भाँडोमा 2200cm^3 पानी भरिएको छ । 10cm व्यास भएको भाँडोमा पानीको सतहको उचाई पत्ता लगाउनुहोस् ।
If the given cylineerical vessel is filled with 2200 cm^3 , water its diameter is 10cm , find the height of water surface.
(Ans: 28cm)
-
13. बराबर आयतन भएका दुईओटा बेलनाका उचाईहरू $1:2$ को अनुपातमा छन् भने तिनीहरूको अर्धव्यास निकाल्नुहोस् ।
Two cylinder whose volume are equal and their heights in the ratio $1:2$ find ratio of their radius.
(Ans: $2:1$)

14. एउटा बेलनाको पूरासतहको क्षेत्रफल 2464cm^2 छ। यसको उचाई र अर्धव्यास बराबर छ भने अर्धव्यास पत्त लगाउनुहोस्।
The total surface area of cylindrical is 2464cm^2 where radius of base and height are equal, find the radius.
(Ans: 14cm)
15. एउटा खम्बाको बक्सतहको क्षेत्रफल 264cm^2 र आयतन 924cm^3 छ भने अर्धव्यास र उचाई पत्तालगाउनुहोस्।
The curved surface area of cylindrical piller is 264cm^2 and its volume is 924cm^3 , find the radius and height.
(Ans: 7cm 6cm)
16. उचाई 1m भएको एउटा बेलनाकार धातुको ट्याङ्कीमा 1500 लिटर पानी अटाउँछ। उक्त ट्याङ्कीको आधावनाउन कति वर्ग मिट्री धातु आवश्यक पर्दछ ? पत्ता लगाउनुहोस्। The capacity of cylinder tank of height 1m is 1500 liter how many square meter of metal sheet whoul be needed to make its base?
(Ans: 1.5m^2)
17. एउटा बेलनाको बक्सतहको क्षेत्रफल 22cm किनारा भएको एउटा धनको पुरा सतहको क्षेत्रफल बराबर छ। यदि बेलनाको उचाई 33cm भए, यसको अर्धव्यास पत्ता लगाउनुहोस्। The curved surface area of a cylinder is equal in the total surface area of a cube whose is 22cm if the height of cylinder is 33cm . Find its radius.
(Ans: 10cm)

Long Questions

Model : 1

एउटा ठोस बेलनाको बक्सतहको क्षेत्रफल यसको पुरासतहको क्षेत्रफलको $\frac{2}{3}$ छ। यदि पुरा सतहको क्षेत्रफल 924cm^2 , त्त भने यसको व्यासार्ध र उचाई पत्ता लगाउनुहोस्।

The area of the curved surface area of a solid cylinder is equal to $\frac{2}{3}$ of total surface area of the same cylinder. If the total surface area of it is 924cm^2 find the radius and height.

Solution:

$$\text{Here, curve surface area (CSA)} = 2\pi rh$$

$$\text{Total surface area (TSA)} = 2\pi r(r + h)$$

$$\text{CSA} = \frac{2}{3} \text{ of TSA}$$

$$\text{or, } 2\pi rh = \frac{2}{3} 2\pi r \times (r + h)$$

$$\text{or, } 3h = 2r + 2h$$

$$\text{or, } 3h - 2h = 2r$$

$$\therefore h = 2r$$

$$\text{Again curve surface of area (CSA)} = \frac{2}{3} \text{ of TSA} = \frac{2}{3} \times 924\text{cm}^2 = 308 \times 2 \text{ cm}^2 = 616\text{cm}^2$$

$$\text{Now, CSA} = 2\pi rh$$

$$\text{or, } 616\text{cm}^2 = 2 \times \frac{22}{7} \times r \times 2r$$

$$\text{or, } 616\text{cm}^2 = \frac{88r^2}{7}$$

$$\text{or, } 616 \times 7\text{cm}^2 = 88r^2$$

$$\text{or, } \frac{616 \times 7\text{cm}^2}{88} = r^2$$

$$\text{or, } 49\text{cm}^2 = r^2$$

$$(7\text{cm})^2 = r^2$$

$$\therefore r = 7\text{cm}$$

$$\text{Now, } h = 2r = 2 \times 7\text{cm} = 14\text{cm}$$

$$\text{Radius of base (r)} = 7\text{cm} \text{ and height (h)} = 14\text{cm}$$

Model 2:

एउटा बेलनाको वक्सतहको क्षेत्रफल 1232 cm^2 छ । यसको आधारको अर्धव्यास र उचाई बराबर छ भने बेलनाको पुरासतहको क्षेत्रफल निकाल्नुहोस् ।

The curved surface area of cylinder whose height is equal to the radius of base is 1232cm^2 , find the total surface area of the cylinder.

Solution:

Here, curved surface area (CSA) = 1232cm^2

Radius of base (r) = height (h)

$$\therefore \text{CSA} = 2\pi rh$$

$$\text{or, } 1232\text{cm}^2 = 2 \times \frac{22}{7} \times h \times h$$

$$\text{or, } 1232 \text{ cm}^2 = \frac{44 \times h^2}{7}$$

$$\text{or, } 1232 \times 7\text{cm}^2 = 44 \times h^2$$

$$\text{or, } 196\text{cm}^2 = h^2$$

$$\therefore h = 14\text{cm}$$

Radius of base (r) = height (h) = 14cm

$$\therefore \text{TSA} = 2\pi r(r + h) = 2 \times \frac{22}{7} \times 14\text{cm} (14 + 14)\text{cm} = 44 \times 2\text{cm} \times 28\text{cm} = 2464\text{cm}^2$$

Total surface area of cylinder (TSA) = 2464cm^2

Model 3:

बेलनाको उचाई र आधारको व्यासर्धको अनुपात $4:3$ छ । यसको पुरासतहको क्षेत्रफल 528cm^2 भए आयतन पत्ता लगाउनुहोस् ।

The ratio of the height and radius of base of a cylinder is $4:3$, If the total surface area is 528cm^2 find the volume.

Solution:

Here, height of cylinder = h radius = r now $h:r = 4:3$, then $h = 4x$ and $r = 3x$

Surface area (CSA) = 528cm^2

$$\therefore \text{CSA} = 2\pi r(r + h)$$

$$\text{or, } 528\text{cm}^2 = 2 \times \frac{22}{7} 3x \times (3x + 4x)$$

$$\text{or, } 528\text{cm}^2 = 132x \times 7$$

$$\text{or, } x^2 = \frac{528 \times 7\text{cm}^2}{532}$$

$$\text{or, } x^2 = 4\text{cm}^2$$

$$\therefore x = 2\text{cm}$$

$$\text{Height (h)} = 4x = 4 \times 2\text{cm} = 8\text{cm}$$

$$\text{Radius (r)} = 3x = 3 \times 2\text{cm} = 6\text{cm}$$

$$\text{Volume (V)} = \pi r^2 h = \frac{22}{7} \times (6\text{cm})^2 \times 8\text{cm} = \frac{22 \times 36 \times 8\text{cm}^3}{7} = 905.14\text{cm}^3$$

$$\text{Volume of cylinder (V)} = 905.14\text{cm}^3$$

Model 4:

एउटा बेलनाको आयतन 3080 cm^3 र उचाई 20cm छ भने यसको वक्सतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

The volume of cylinder is 3080 cm^3 , and its height is 20cm find the curved surface area

Solution:

Here, volume of cylinder (V) = 3080cm^3

$$\begin{aligned}
 & \text{its height (h)} = 20\text{cm} \\
 \therefore & V = \pi r^2 h \\
 \text{or, } & 3080\text{cm}^3 = \frac{22}{7} \times r^2 \times 20\text{cm} \\
 \text{or, } & 3080 \times 7\text{cm}^3 = 440\text{cm} \times r^2 \\
 \text{or, } & \frac{3080 \times 7\text{cm}^3}{440\text{cm}} = r^2 \\
 \text{or, } & 49\text{cm}^2 = r^2 \\
 \text{or, } & (7\text{cm})^2 = r^2 \\
 \therefore & r = 7\text{cm} \\
 & \text{Radius of base (r)} = 7\text{cm} \\
 \text{CSA} & = 2\pi rh = 2 \times \frac{22}{7} \times 7\text{cm} \times 20\text{cm} = 44 \times 20\text{cm}^2 = 880\text{cm}^2 \\
 & \text{Curved surface area (CSA)} = 880\text{cm}^2
 \end{aligned}$$

Model 5:

एउटा बेलनाको उचाई 66cm छ । यसको बक्सतहको क्षेत्रफल र एउटा घनको पुरा सतहको क्षेत्रफल बराबर छ । यदि घनको प्रत्येक भुजाको लम्बाई 44cm भए बेलनाको अर्धव्यास पता लगाउनुहोस् ।

The height of a cylinder is 66cm. The curved surface area is equal to total surface of area, if each edge of the cube is 44cm. Find its radius of cylinder.

Solution: Here, height of cylinder (h) = 66cm

Curved surface area of cylinder (CSA) = Total surface area of cube (TSA)

$$\therefore \text{CSA} = 2\pi rh \quad \text{TSA} = 6l^2$$

Length of each side of curve (C) = 44cm

$$\text{Now, } 2\pi rh = 6l^2$$

$$\text{or, } 2 \times \frac{22}{7} r \times 66\text{cm} = 6 \times (44\text{cm})^2$$

$$\text{or, } \frac{44 \times 66\text{cm}}{7} = 6 \times 44 \times 44\text{cm}^2$$

$$\text{or, } r = \frac{7 \times 6 \times 44 \times 44\text{cm}^2}{44 \times 66\text{cm}}$$

$$\therefore r = 28\text{cm}$$

Radius of base of cylinder (r) = 28cm

Model 6:

एउटा बेलनाकार ट्याकीमा 1000 ली. पानी हाल्दा πm उचाई सम्म पानी चढ़छ । सो ट्याङ्ककी $\frac{1}{2} \pi m$ उचाई सम्म चढेको पानीले ट्याकीको कति क्षेत्रफल भिजाउला ?

In a cylindrical tank water raises upto km level when filled with 1000 liter in the tank how much area of the tank will be made by the water filled upto $\frac{1}{2} \pi m$ in level ?

Solution:

Here, volume of water cylindrical tank = 1000 m³ liter = 1 m³

Height of water (h) = πm

$$\therefore V = \pi r^2 h$$

$$\text{or, } 1\text{m}^3 = \pi \times r^2 \times \pi m$$

$$\text{or, } 1\text{m}^3 = \pi^2 r^2 m$$

$$\text{or, } \frac{1}{\pi^2} \text{m}^3 = r^2$$

$$\therefore r = \frac{1}{\pi} \text{ m.}$$

$$\text{Radius of cylindrical tank (r)} = \frac{1}{\pi} \text{ m.}$$

$$\text{Height of water level (h)} = \frac{1}{2\pi} \text{ m}$$

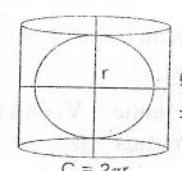
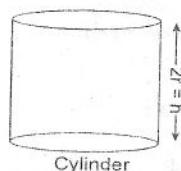
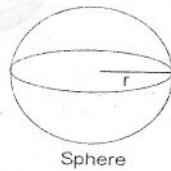
$$\text{TSA} = 2\pi rh + \pi r^2 = 2\pi \times \frac{1}{\pi} \text{ m} \times \left(\frac{1}{2\pi}\right) \text{ m.} + \pi \left(\frac{1}{\pi}\right)^2 = 2\pi \times \frac{1}{2\pi} \text{ m}^2 + \pi \times \frac{1}{\pi^2} \text{ m}^2 = \frac{2}{\pi} \text{ m}^2$$

$$\therefore \text{Total surface area of tank} = \frac{2}{\pi} \text{ m}^2$$

Practice yourself

- उचाई र अर्धव्यास बराबर भएको एउटा बेलनाको बक्सतहको क्षेत्रफल 308cm^2 छ भने आधारको अर्धव्यास र आयतन पत्ता लगाउनुहोस् । The curved surface of a cylinder, whose height is equal to radius of its base is 308cm^2 . Find the radius and volume.
 (Ans: 7cm, 1408cm^3)
- एउटा बेलनाको बक्सतहको क्षेत्रफल 4400cm^2 र आधारको परिधि 110cm छ। भने यसको उचाई र आयतन निकाल्नुहोस् । The area of curved surface area is 4400cm^2 and its circumference 110cm .Find the its height and volume.
 (Ans: 40cm, 38500cm^3)
- एउटा बेलनाको आयतनको संख्या यसकै पुरा सतहको क्षेत्रफलको संख्याको आधा छ । यदि बेलनाको आधारको अर्धव्यास 7cm छ भने आयतन पत्ता लगाउनुहोस् । The number of volume of a cylinder is half of its number of the total surface area, if the radius of base is 7cm , find the volume.
 (Ans: 179.67cm^3)
- पानी पूरै भरिएको एउटा बेलनाकार वालिटनको व्यास 56cm र उचाई 24cm छ । यसको पानीलाई 88cm लामो र 42cm फराकिलो आयतकार ट्याङ्की मा खन्याउदा ट्याकीको कर्ति उचाई सम्म पानीको सतह पुरछ ? A cylindrical bucket 56cm in diameter and 24cm when the water is poured into rectangular tank $88\text{cm} \times 42\text{cm}$, find high to full of water level in this tank.
 (Ans: 16cm)
- आधारको क्षेत्रफल 3850cm^2 भएको बेलनाकार जस्ता पाताले बनेको ट्याकीको आयतन 77000cm^3 , छ भने यसको निर्माण कर्ति cm^2 धातुको प्रयोग गरेको होला ? The volume of cylindrical tank which is made by zinc sheet is 77000cm^3 . If the area of base is 3850cm^2 , what cm^2 of the metal is used in constructing the tank.
 (Ans: 121000cm^2)
- एउटा बेलनाको बक्सतहको क्षेत्रफल यसको पुरा सतहको क्षेत्रफल $\frac{2}{5}$ बराबर छ । यदि पुरासतहको क्षेत्रफल 1732.5cm^2 भए अर्धव्यास र उचाई पत्ता लगाउनुहोस् । The curved surface area of a social cylinder is equal to $\frac{2}{5}$ of the total surface area find the radius and height of it. If its total surface area is 1732cm^2 .
 (Ans: 12.86cm and 8.57cm)
- दुई बेलनाकार भाँडोहरू तेल भरिएका छन् । एउटा बेलनाकार भाँडोको अर्धव्यास 10cm र उचाई 25cm छ । अर्को भाँडोको अर्धव्यास 10cm र उचाई 18cm छन् । एउटा अर्को बेलनाकार भाँडो 30cm उचाई को छ जसमा दुवै बेलनाकार भाँडोको तेल अटाउद्दू भने अर्धव्यास पत्ता लगाउनुहोस् । Two cylindrical vessels are filled with oil. The radius and height of one cylinder vessel are 15cm and 25cm respectively. The radius and height other vessel are 10cm and 18cm respectively. find the radius of a cylindrical vessel 30cm in height which will just contain the oil of two given vessel.
 (Ans: 15.73cm)

6.2 (a) गोलाको क्षेत्रफल र आयतन (Area and Volume of Sphere)



- गोलाको सतहको क्षेत्रफल (Surface area of sphere)

गोलाको अर्धव्यास (Radius of sphere) = r ,

सतहको क्षेत्रफल (Surface Area) = $SA = 4\pi r^2$

$$\therefore \text{Surface area (SA)} = 4\pi r^2 \text{ and diameter (d)} = 2r, \text{ Radius (r)} = \frac{d}{2}$$

$$\therefore \text{Surface Area (SA)} = \pi d^2$$

- ठूलो वृत्तको परिधि (Circumference of great circle):-

अर्धव्यास Radius = r and व्यास diameter = d

$$\therefore C = 2\pi r \text{ or } \therefore C = \pi d$$

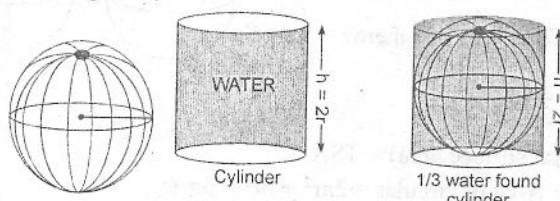
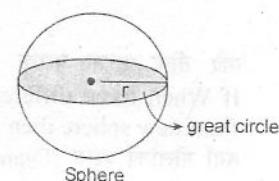
- ठूलो वृत्तको क्षेत्रफल (Area of great circle):

$$\text{Area of great circle (A)} = \pi r^2 \text{ or } \frac{1}{4} \pi d^2$$

अर्धव्यास = r , व्यास = d भए परिधि (C) = πd , $C = 2\pi r$

$$\text{ठूलो वृत्तको क्षेत्रफल (A)} = \pi r^2 \text{ or } \frac{1}{4} \pi d^2$$

- गोलाको आयतन (Volume of sphere):



अर्धव्यास (Radius) = r

व्यास (Diameter) = d

आयतन (volume) = V

$$\text{गोलाको आयतन (V)} = \frac{2}{3} \text{ बेलनाको आयतन (Volume of cylinder)} = \frac{2}{3} \pi r^2 h = \frac{2}{3} \pi r^2 \times 2r = \frac{4}{3} \pi r^3$$

$$\text{गोलाको आयतन (V)} = \frac{1}{6} \pi d^3$$

$$\text{volume of sphere (V)} = \frac{2}{3} \text{ volume of cylinder} = \frac{2}{3} \times \pi r^2 h = \frac{2}{3} \times \pi r^2 \times 2r = \frac{4}{3} \pi r^3$$

$$\text{Volume of sphere (V)} = \frac{1}{6} \pi d^3 = \frac{4}{6} \pi d^3 = \frac{4}{3} \pi r^3$$

- खोको गोलाको प्रयोग भएको धातुको आयतन (Volume of material contained by hollow sphere shell.)
मानौ बाहीरी अर्धव्यास = r_1
भित्री अर्धव्यास = r_2

यदि बाहरी आयतन = V_1 र

भिन्नी आयतन = V_2 भए

ध्रातुको आयतन (V) = बाहरी आयतन (V_1) भिन्नी आयतन (V_2)

$$\text{Let outer radius} = r_1 = \frac{4}{3} \pi r_1^3 - \frac{4}{3} \pi r_2^3 = \frac{4}{3} \pi (r_1^3 - r_2^3)$$

Inner radius = r_2

Let outer volume = V_1 and inner volume = V_2

And outer radius = r_1

$$\text{Volume of metal (V)} = V_1 - V_2 = \frac{4}{3} \pi r_1^3 - \frac{4}{3} \pi r_2^3 = \frac{4}{3} \pi (r_1^3 - r_2^3)$$

If d_1 and d_2 are outer and inner diameter.

$$\therefore \text{Volume of metal (V)} = \frac{1}{6} \pi (d_1^3 - d_2^3)$$

कक्ष

- यदि तीन फरक फरक गोलालाई पगालेर नयाँ गोला बनाउदा ।

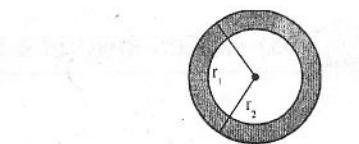
If When three different size of spheres are melted together from new sphere then

नयाँ गोलाको व्यास (Diameter of new sphere) = d

$$d = \sqrt[3]{d_1^3 + d_2^3 + d_3^3}$$

नयाँ गोलाको अर्धव्यास (Radius of new sphere) = r

$$\therefore r = \sqrt[3]{r_1^3 + r_2^3 + r_3^3}$$



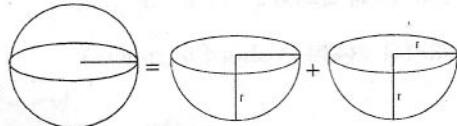
6.2(b) अर्धगोलाको सतहको क्षेत्रफल र आयतन (Surface area and volume of hemisphere):

व्यास (diameter) = d

अर्धव्यास (radius) = r

वक्रसतहको क्षेत्रफल (Curved surface area) = CSA

$$\begin{aligned} \therefore \text{CSA} &= \frac{1}{2} (\text{Surface Area of sphere}) = \frac{1}{2} 4\pi r^2 \\ &= 2\pi r^2 = \frac{1}{2} \pi d^2 \end{aligned}$$



- पुरासतहको क्षेत्रफल (Total surface area) = TSA

$$\therefore \text{TSA} = \text{CSA} + \text{Area of circular} = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

$$\therefore \text{TSA} = 3\pi r^2 = \frac{3}{4} \pi d^2$$

- अर्धगोलाको आयतन (volume of hemisphere)

अर्धव्यास (radius) = r व्यास (diameter) = d

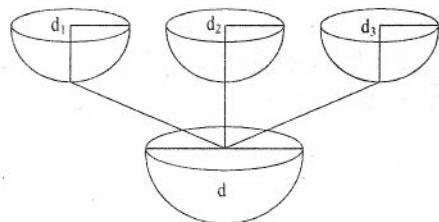
आयतन (Volume) = v

$$\therefore V = \frac{1}{2} (\text{volume sphere}) = \frac{1}{2} \cdot \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$$

$$\therefore V = \frac{1}{2} (\text{volume of sphere}) = \frac{1}{2} \left(\frac{1}{6} \pi d^3 \right) = \frac{1}{12} \pi d^3$$

- यदि तीनओटा फरक फरक साइजका ठोस अर्धगोलाहरू पगालेर नयाँ अर्धगोला बनाउँदा ।

If when the three difference size semispheres are melted together to form new hemisphere then. नयाँ अर्धगोलाको व्यास (diameter of new hemisphere) = d



$$\therefore d = \sqrt[3]{d_1^3 + d_2^3 + d_3^3}$$

नयाँ अर्धगोलाको अर्धव्यास (Radius of new hemisphere) = r

$$\therefore r = \sqrt[3]{r_1^3 + r_2^3 + r_3^3}$$

Very short Questions

1. एउटा बलको अर्धव्यास r भए सतहको क्षेत्रफल कति हुन्छ ?
What the is the total surface area of a ball having radius of ball is r.
2. एउटा ठोस फलामको गोलाको व्यास d भए आयतन कति होला ?
What the is volume of a solid Iron sphere having diameter of sphere is d.
3. एउटा गोलाको ठूलो वृत्तको परिधि π cm भए अर्धव्यास कति होला ?
What is radius of sphere, when the circumference of great circle is π cm.
4. गोला ठूलो वृत्तको अर्धव्यासलाई आधा गर्दा परिधि कति हुन्छ ?
What is circumference of great circle of sphere if its radius is half of original ?
5. व्यास d भएको अर्धगोलाको पुरासतहको क्षेत्रफल कति हुन्छ ?
What is the total surface area of hemisphere having the diameter d?
6. व्यास d भएको अर्धगोलाको आयतन कति हुन्छ ?
What is the volume of hemi-sphere having diameter is d.
7. अर्धगोलाको ठूलो वृत्तको क्षेत्रफल निकाल्ने सुन्न लेखनुहोस् । Write the formula to find the area of great circle of hemisphere.
8. अर्धव्यास r भएको अर्धगोलाको आयतन कति होला ? What is the volume of hemi-sphere having radius r ?
9. अर्धगोलाको वक्रसतहको क्षेत्रफल निकाल्ने सुन्न लेखनुहोस् । Write the formula of to find the curved surface area of hemisphere.
10. r_1, r_2 र r_3 अर्धव्यासहरू भएका तीनवटा गोलाहरू पगालेर बन्ने नयाँ गोला अर्धव्यास r_1 लाई r_2 र r_3 मा व्यक्त गर्नुहोस् । Three spheres with radio r_1, r_2 and r_3 are melted to from ${}^0\text{c}$ new sphere with radius R_1 . Explain terms of r_1, r_2 and r_3 .

Short Questions

Model 1

दिएको चित्रमा अर्धगोलाको आयतन पत्ता लगाउनुहोस् ।

Aind the volume of the adjoning solod hemisphere:

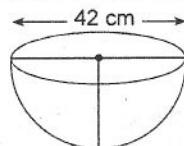
Solution:

In the given figure of hemisphere

diameter (d) = 42cm

$$\therefore V = \frac{1}{12} \pi d^3 = \frac{1}{12} \times \frac{22}{7} \times (42\text{cm})^3 = \frac{22 \times 74088\text{cm}^3}{84} = 19404\text{cm}^3$$

$$\therefore \text{Volume of hemisphere (v)} = 1240.4\text{cm}^3$$



Model 2:

दिइएको ठोस वस्तु अर्धगोलाको पुरा सतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

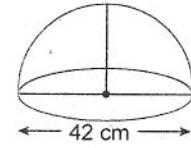
Find the total surface area of solid hemisphere.

Solution:

In the given figure of hemisphere diameter of hemisphere (d) = 42cm

$$\text{TSA} = \frac{3}{4} \pi d^2 = \frac{3}{4} \times \frac{22}{7} \times (42\text{cm})^2 = \frac{3}{4} \times \frac{22}{7} \times 1764\text{cm}^2 = \frac{488980\text{cm}^3}{28} = \frac{116424\text{cm}^2}{28} \\ = 4158\text{cm}^2$$

Total surface area of hemisphere (TSA) = 4158cm²

**Model 3:**

एउटा 21cm अर्धव्यास भएको धातुको गोलालाई दुई बराबर हुने गरी काटदा दुई अर्धगोलाको पुरासतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

A solid metallic sphere radius 21cm is cut into two halves. find the total surface area of the two hemisphere so formed.

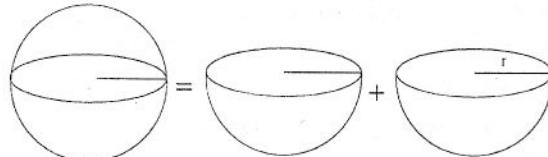
Solution:

Radius of sphere (r) = 21cm

according to the questions

$$\text{Total area of 2 hemisphere (TSA)} = 2 \times 3 \pi r^2 = 6 \times \frac{22}{7} \times (21\text{cm})^2 = \frac{6 \times 22 \times 441\text{cm}^2}{7} \\ = \frac{58212\text{cm}^2}{7} = 8316\text{cm}^2$$

∴ Total surface area of two hemisphere is 8316cm²

**Model 4:**

एउटा अर्धगोलाको पुरासतहको क्षेत्रफल यसको आयतन पत्ता लगाउनुहोस् ।

Total surface area of a hemisphere is $243\pi\text{cm}^2$ find its volume.

Solution:

Total surface area of hemisphere (TSA) =

$$243\pi\text{cm}^2$$

$$\therefore \text{TSA} = 3\pi r^2$$

$$\text{or, } 243\pi\text{cm}^2 = 3\pi r^2$$

$$\text{or, } \frac{243\pi\text{cm}^2}{3\pi} = r^2$$

$$\text{or, } 81\text{cm}^2 = r^2$$

Radius (r) = 9cm

$$V = \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \times 2\pi \times (9\text{cm})^3$$

$$= \frac{2}{3} \pi \times 843\text{cm}^3$$

$$= 162\pi\text{cm}^3$$

Volume of hemisphere

$$(V) = 162\pi\text{cm}^3$$

Model 5:

एउटा अर्धगोलाको ठोस वस्तुको आयतन $18\pi\text{ cm}^3$ छ । यसको पुरा सतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

The volume of the a solid hemisphere object is $18\pi\text{cm}^3$. find its total surface area.

Solution:

Here, volume of hemisphere (V) = $18\pi\text{cm}^3$

$$\therefore V = \frac{2}{3} \pi r^3$$

or, $18\pi \text{cm}^3 = \frac{2}{3} \pi r^3$

or, $\frac{18\pi \times 3}{2\pi} \text{cm}^3 = r^3$

or, $27\text{cm}^3 = r^3$

or, $(3\text{cm})^3 = r^3$

$\therefore r = 3\text{cm}$

Radius (r) = 3cm

$$\text{Total surface area (TSA)} = 3\pi r^2 = 3\pi(3\text{cm})^2 = 3\pi \times 9\text{cm}^2 = 27\pi\text{cm}^2$$

Total surface area of solid hemispherical object is $27\pi\text{cm}^2$.

Model 6: एउटा गोलाको पुरासतहको क्षेत्रफल $196\pi\text{cm}^2$ छ भने त्यसलाई ठीक आधा गर्दा बन्ने दुवै अर्धगोलाको पुरासतहको क्षेत्रफल कति हुन्छ ? If total surface area of sphere is $196\pi\text{cm}^2$, find the total surface area of both the hemisphere formed by making half.

Solution:

Here, total surface area of sphere (TSA) = $196\pi\text{cm}^2$

$$\therefore \text{TSA} = 4\pi r^2$$

or, $196\text{cm}^2 = 4\pi r^2$

or, $\frac{196\pi\text{cm}^2}{4\pi} = r^2$

$$\text{or, } 49\text{cm}^2 = r^2$$

$$\therefore r = 7\text{cm}$$

Radius of sphere (r) = 7cm

According to the question

radius of hemisphere (r) = 7cm

$$\begin{aligned} \text{Total surface area of two hemisphere (TSA)} &= 2 \times 3\pi r^2 = 2 \times 3\pi \times (7\text{cm}) = 6\pi \times 49\text{cm}^2 \\ &= 294\pi\text{cm}^2 \end{aligned}$$

Model 7:

तीनओटा चाँदीका ठोस अर्धगोलाहरूको अर्धव्यासहरू 2cm, 12cm and 16cm छन् । यी तिनै अर्धगोलाहरूलाई पगालेर बनाइएको नयाँ अर्धगोलाको व्यास पत्ता लगाउनुहोस् ।

Three silver hemisphere having radii 2cm, 12cm and 16cm are melted to form new hemisphere find the radius of the new hemisphere.

Solution:

Here, three silver solid hemisphere are 2cm, 12cm and 16cm which radii is denoted by r_1 , r_2 and r_3 respectively

$$\therefore r_1 = 2\text{cm}, r_2 = 12\text{cm} \text{ and } r_3 = 16\text{cm} \text{ if radius of new hemisphere} = r_1 \text{ then}$$

$$\begin{aligned} r &= \sqrt[3]{r_1^3 + r_2^3 + r_3^3} = \sqrt[3]{(2\text{cm})^3 + (12\text{cm})^3 + (16\text{cm})^3} = \sqrt[3]{8\text{cm}^3 + 1728\text{cm}^3 + 4096\text{cm}^3} \\ &= \sqrt[3]{5832\text{cm}^3} \end{aligned}$$

$$\therefore \text{radius of new hemisphere (r)} = 18\text{cm} = 18\text{cm}$$

Model 8:

2 m व्यास भएको अर्धगोलाकार कराहीको क्षमता लिटरमा निकाल्नुहोस् ।

Find the liter of the capacity of a hemispherical container, whose diameter is 2m.

Solution:

Here, diameter of hemispherical container (d) = 2m

$$\text{Volume } (V) = \frac{1}{12} \pi d^3 = \frac{1}{12} \times \frac{22}{7} \times (2m)^3 = \frac{1 \times 22 \times 8m^3}{84} = \frac{44}{21} m^3$$

We know $1m^3 = 1000$ liter

$$\text{Now capacity of water} = \frac{44}{21} \times 1000 \text{ liters} = 2095.23 \text{ liters.}$$

Model 9:

यदि दुई बराबर अर्धगोलाहरूमा एउटा अर्धगोलाकार वस्तुको पुरासतहको क्षेत्रफल 462cm^2 छ भने ती दुई अर्धगोलालाई टाँसेर बनाइएको गोलाकार वस्तुको सतहको क्षेत्रफल निकाल्नुहोस् ।

In two equal hemispherical object one of them total surface area of hemispherical is 462cm^2 , find total surface area of the sphere. Which is formed by atteling these two hemispherical objects.

Solution:

Here, In two equal hemispherical object

total surface area (TSA) = 462cm^2

$$\therefore \text{TSA} = 3\pi r^2$$

$$\text{or, } 462\text{cm}^2 = 3 \times \frac{22}{7} \times r^2$$

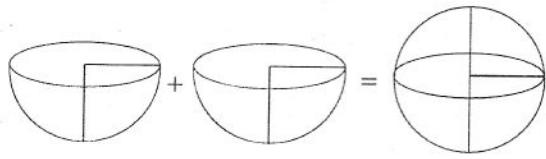
$$\text{or, } 462\text{cm}^2 \times 7 = 66r^2$$

$$\text{or, } r^2 = \frac{462\text{cm}^2 \times 7}{60}$$

$$\text{or, } r^2 = 49\text{cm}^2$$

$$\therefore r = 7\text{cm}$$

According the questions



$$\text{TSA} = 4\pi r^2 = 4 \times \frac{22}{7} \times (7\text{cm}) = \frac{88 \times 49\text{cm}^2}{7} = 616 \text{ cm}^2$$

$$\text{Total surface area of sphere (TSA)} = 616 \text{ cm}^2$$

Model 10:

आयतन $18\pi\text{cm}^3$ भएको एउटा ठोस बेलनालाई पगालेर ठोस अर्धगोला बनाइएको छ भने अर्धगोलाको अर्धव्यास पत्ता लगाउनुहोस् ।

If a solid cylinder having volume of $18\pi\text{cm}^3$ is melted to form a hemisphere find the radius of hemisphere.

Solution:

Here, volume of solid cylinder (V) = $18\pi\text{cm}^3$

According to the question

volume of solid hemisphere (V) = $18\pi\text{cm}^3$

$$\therefore V = \frac{2}{3} \pi r^3$$

$$\text{or, } 18\pi\text{cm}^3 = \frac{2}{3} \pi r^3$$

$$\text{or, } 18\pi\text{cm}^3 = \frac{2\pi r^3}{3}$$

$$\text{or, } 18\pi\text{cm}^3 \times 3 = 2\pi r^3$$

$$\text{or, } r^3 = \frac{18\pi\text{cm}^3 \times 3}{2\pi}$$

$$\text{or, } r^3 = 27\text{cm}^3$$

$$\text{or, } r^3 = (3\text{cm})^3$$

$$\therefore r^3 = 3\text{cm}$$

$$\text{Radius of hemisphere (r)} = 3\text{cm}$$

Practice Yourself

1. २८ सेमी. व्यास भएको एउटा अर्धगोलाकारको पूरा सतहको क्षेत्रफल निकाल्नुहोस् ।
Obtain the total surface area of the hemisphere having the diameter 28cm. (Ans: 7392 cm^2)
2. एउटा २१ cm अर्धव्यास भएको धातुको गोलालाई दुई बराबर भाग हुने गरी काटदा बन्ने दुई अर्धगोलाको पूरा सतहको क्षेत्रफल पत्ता लगाउनुहोस् । A solid metallic sphere of radius 21cm is cut into two halves. find the total surface area if the two hemispheres formed. (Ans: 4158cm^2)
3. यदि एउटा अर्धगोलाको आयतन 19.404 घन सेमी. भए उक्त अर्धगोलाको अर्धव्यास पत्ता लगाउनुहोस् । If the volum of a hemisphere is 19.404 cubic cm, find the radius of the hemisphere. (Ans: 14cm)
4. एउटा अर्धगोलाको वक्ससतहको क्षेत्रफल 1232cm^2 भए, find the radius of the hemisphere.
A hemisphere has curved surface area 1232cm^2 , find the radius of the hemisphere. (Ans: 14cm)
5. एउटा अर्धगोलाको पूरा सतहको क्षेत्रफल 462cm^2 छ । यसको व्यास पत्ता लगाउनुहोस् ।
The total surface area of a hemisphere is 462cm^2 . Find the diameter. (Ans: 14cm)
6. एउटा अर्धगोलाको पूरा सतहको क्षेत्रफल $48\pi\text{cm}^2$ छ । यसको आयतन पत्ता लगाउनुहोस् ।
The total surface area of a hemisphere is 48cm^2 . Find its volume. (Ans: $\frac{128}{3}\pi \text{ cm}^3$)
7. एउटा अर्धगोलाको ठोसको आयतन $18\pi\text{cm}^3$ छ । यसको पूरा सतहको क्षेत्रफल पत्ता लगाउनुहोस् ।
The volume of the a solid hemispherical object is $18\pi \text{ cm}^3$. Find its total surface area. (Ans: $27\pi\text{cm}^2$)
8. यदि आयतन 1540cm^3 भएको एउटा धातुको अर्धगोलालाई पगालेर उचाइ 10cm भएको एउटा वेलना बनाइयो भने स्रो वेलनाको अर्धव्यास पत्ता लगाउनुहोस् ।
If a metallic hemisphere having volume 1540cm^3 is melted to form a cylinder having height 10cm, find the radius of this cylinder.
9. तीनओटा चाँदीका ठोस अर्धगोलाहरूको अर्धव्यास क्रमशः 3cm , 4cm र 5cm छन् । यी तीनै अर्धगोलाहरूलाई पगालेर बनाइएको नयाँ अर्धगोलाको व्यास निकाल्नुहोस् ।
Three silver hemisphere having radii 3cm , 4cm and 5cm are melted to form a new hemisphere. Find the diameter of the new hemisphere. (Ans: 12cm)
10. एउटा ठोस अर्ध गोलाको आयतन 19404 cm^3 पूरा सतहको क्षेत्रफल निकाल्नुहोस् । The volume of solid sphere is 194cm^3 .Find the Total surface area. (Ans: 4158cm^2)
11. एउटा अर्धगोलाको पूरा सतहकामे क्षेत्रफल 1848cm^2 भए अर्धव्यास पत्ता लगाउनुहोस् । If the total surface anrea of a hemisphere is 184cm^2 . Find its radius (Ans: 14cm)

Short Questions

Model 1:

एउटा गोलाको आयतन 38808cm^3 भए उक्त गोलाको अर्धव्यास पत्ता लगाउनुहोस् ।

The volume of sphere is 38808cm^3 . Find the radius of sphere.

Solution:

Here, volume os sphere (V) = 38808cm^3

$$\therefore V = \frac{4}{3} \pi r^3$$

$$\text{or, } 38808\text{cm}^3 = \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$\text{or, } 38808\text{cm}^3 = \frac{88r^3}{21}$$

$$\text{or, } 38808\text{cm}^3 \times 21 = 88r^3$$

$$\text{or, } r^3 = \frac{38808\text{cm}^3 \times 21}{88\text{cm}}$$

$$\text{or, } r^3 = 9261\text{cm}^3$$

$$\text{or, } r^3 = (21\text{cm})^3$$

$$\therefore r = 21 \text{ cm}$$

Radius of sphere (r) = 21 cm

Model 2:

एउटा गुच्चाको आयतन $\frac{\pi}{6} \text{ cm}^3$ छ भने यसको व्यास पत्ता लगाउनुहोस् ।

The volume of a marble is $\frac{\pi}{6} \text{ cm}^3$, find the its diameter.

Solution:

$$\text{Here, volume of marble (V)} = \frac{\pi}{6} \text{ cm}^3$$

$$V = \frac{1}{6} \pi d^3 = \frac{\pi}{6} \text{ cm}^3 = \frac{1}{6} \pi d^3$$

$$\text{or, } \frac{\pi}{6} \text{ cm}^3 = \frac{\pi d^3}{6}$$

$$\text{or, } \frac{\pi \times 6}{5 \times \pi} \text{ cm}^3 = d^3$$

$$\text{or, } 1 \text{ cm}^3 = d^3$$

$$\therefore d = 1 \text{ cm}$$

$$\text{Diameter of marble (d)} = 1 \text{ cm}$$

Model 3:

एउटै गोलाको ठुलो वृत्तको परिधी 66cm छ भने यसको आयतन पत्ता लगाउनुहोस् ।

If the circumference of great circle is 66cm find the its volume.

Solution :

$$\text{Here, circumference of sphere (C)} = 66 \text{ cm}$$

$$\therefore C = 2\pi r$$

$$\text{or, } 66 \text{ cm} \times 7 = 44r$$

$$\text{or, } 66 \text{ cm} \times 7 = 44r$$

$$\text{or, } \frac{66 \text{ cm} \times 7}{44} = r$$

$$\text{or, } \frac{21}{2} \text{ cm} = 4$$

$$\therefore r = 10.5 \text{ cm}$$

$$\text{Radius of sphere (r)} = 10.5 \text{ cm}$$

$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (10.5 \text{ cm})^3 = \frac{4 \times 22 \times 1157.625 \text{ cm}^3}{21} = \frac{101871 \text{ cm}^3}{21} = 4851 \text{ cm}^3$$

$$\therefore \text{Volume of sphere (v)} = 4851 \text{ cm}^3$$

Model 4:

$1372\pi \text{ cm}^3$ आयतन भएको गोलाको पुरासतहको क्षेत्रफल निकाल्नुहोस् ।

Find the total surface of sphere whose volume is 1372 cm^3 .

Solution:

$$\text{Here, volume of sphere (V)} = \frac{1372\pi \text{ cm}^2}{3}$$

$$\therefore V = \frac{4}{3} \pi r^3$$

$$\text{or, } \frac{1372\pi \text{ cm}^3}{3} = r^3$$

$$\text{or, } 323 \text{ cm}^3 = r^3$$

$$\text{or, } (7 \text{ cm})^3 = r^3$$

$$\therefore r = 7\text{cm}$$

Radius (r) = 7cm

$$\text{Total surface area (TSA)} = 4\pi r^2 = 4 \times \frac{22}{7} \times (7\text{cm})^2 = \frac{88 \times 49\text{cm}^2}{7} = \frac{4312}{7} \text{cm}^2 = 616\text{cm}^3$$

Total surface area (TSA) = 616cm³

Model 5:

एउटा गोलाको सतहको क्षेत्रफल $\frac{1}{\pi}$ cm² भए यसको आयतन किए होला ?

What is the volume of sphere? if its surface area is $\frac{1}{\pi}$ cm².

Solution:

$$\text{Here, surface area of sphere (SA)} = \frac{1}{\pi} \text{ cm}^2$$

$$\therefore SA = 4\pi r^2$$

$$\text{or, } \frac{1}{\pi} \text{ cm}^2 = 4\pi r^2$$

$$\text{or, } \frac{1}{4\pi^2} \text{ cm}^2 = r^2$$

$$\text{or, } \left(\frac{1}{2\pi} \text{ cm}\right) = r^2$$

$$\therefore r = \frac{1}{2\pi} \text{ cm}$$

$$\text{Radius of sphere (r)} = \frac{1}{2} \pi \text{ cm}$$

$$\text{Volume (V)} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(\frac{1}{2} \pi \text{ cm}\right)^3 = \frac{4}{3} \pi \times \frac{1}{8\pi^3} \text{ cm}^3 = \frac{1}{6\pi^2} \text{ cm}^3$$

$$\text{Volume sphere (V)} = \frac{1}{6\pi^2} \text{ cm}^3$$

Model 6:

6cm, 8cm र 10cm व्यास भएको तीन ओटा ठोस गोलाहरू पगालेर एउटा नयाँ गोला बनाइएको छ भने उक्त गोलाको व्यास पत्ता लगाउनुहोस् । Three solid spheres of diameters are 6cm 8cm and 10cm are melted and formed a single new sphere. Find the diameter of the sphere.

Solution: Here, diameters of three solid spheres are 6cm, 8cm and 10cm, which denotes by d₁, d₂ and d₃ respectively, then do is the diameter using formula.

$$\begin{aligned} d &= \sqrt[3]{d_1^3 + d_2^3 + d_3^3} = \sqrt[3]{(6\text{cm})^3 + (8\text{cm})^3 + (10\text{cm})^3} \\ &= \sqrt[3]{(216\text{cm}^3) + (512\text{cm}^3) + (1000\text{cm}^3)} = \sqrt[3]{1728\text{cm}^3} = 12\text{cm} \end{aligned}$$

Diameter of new sphere (d) = 12cm

Model 7:

यदि 2464cm³ आयतन भएको धातुको बेलनालाई पगालेर गोला बनाएको छ भने उक्त गोलाको अर्धव्यास निकाल्नुहोस् । If a metallic cylinder having volume 2464cm³ melted into a sphere then find the radius of the sphere.

Solution:

Here,

$$\text{Volume of cylinder (V)} = 2464\text{cm}^3$$

According to the question

$$\text{Volume of sphere (V)} = 2464 \text{ cm}^3$$

$$\therefore V = \frac{4}{3} \pi r^3$$

$$\text{Volume of cylinder (V)} = \text{Volume of Sphere (V)}$$

$$\text{or, } 2464 \text{ cm}^3 = \frac{4}{3} \pi r^3$$

$$\text{or, } 2464 \text{ cm}^3 = \frac{4}{3} \times \frac{22}{7} r^3$$

$$\text{or, } 2464 \text{ cm}^3 = \frac{88}{21} r^3$$

$$\text{or, } \frac{2464 \text{ cm}^3 \times 21}{88} \text{ cm}^3 = r^3$$

$$\text{or, } \frac{54744}{88} \text{ cm}^3 = r^3$$

$$\text{or, } 618 \text{ cm}^3 = r^3$$

$$\therefore r = 8.37 \text{ cm}$$

$$\text{Radius of sphere (r)} = 8.37 \text{ cm}$$

Model 8

2cm अर्धव्यास भएको एउटा गोलालाई 4cm अर्धव्यास भएको बेलनाकार पानीको भाँडोमा ढुवाउदा वृद्धि भएको पानीको उचाई कति होला ? A sphere having 2cm radius its dropped into a cylinder having 4cm radius. find the height of waterlevel.

Solution:

$$\text{Here, radius of sphere (r)} = 2 \text{ cm}$$

$$\text{Volume of sphere (V)} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2 \text{ cm})^3 = \frac{88 \times 8 \text{ cm}^3}{21} = \frac{104}{21} \text{ cm}^3$$

$$\text{Volume water (V)} = \frac{704}{21} \text{ cm}^3$$

$$\text{Radius of cylinder (r)} = 4 \text{ cm}$$

$$V = \pi r^2 h$$

$$\text{or, } \frac{704}{21} \text{ cm}^3 = \frac{22}{7} \times (4 \text{ cm})^2 \times h$$

$$\text{or, } \frac{704}{21} \text{ cm}^3 = \frac{22 \times 16 \text{ cm}^3}{7} \times h$$

$$\text{or, } \frac{704 \text{ cm}^3}{3 \times 22 \times 16 \text{ cm}^2} = h$$

$$\text{or, } \frac{2}{3} \text{ cm} = h$$

$$\therefore h = \frac{2}{3} \text{ cm}$$

$$\therefore \text{height of water level (h)} = \frac{2}{3} \text{ cm}$$

Model 9:

यदि $45\pi \text{ cm}^3$ आयतन भएको ठोस गोलालाई पगालेर 5cm उचाई भएको एउटा बेलना बनाइयो भने बेलनाको अर्धव्यास पता लगाउनुहोस् ।

If a solid sphere having $45\pi \text{ cm}^3$ is melted to form a cylinder of height 5cm, then find the radius of cylinder.

Solution: Here, volume of solid sphere (V) = $45\pi \text{ cm}^3$

According to the question

$$\text{Volume of cylinder (V)} = 45\pi \text{cm}^3$$

$$\text{Height of cylinder (h)} = 5\text{cm}$$

Now,

$$\therefore V = \pi r^2 h$$

$$\text{or, } 45\pi \text{cm}^3 = \pi r^2 \times 5\text{cm}$$

$$\text{or, } \frac{45\pi \text{cm}^3}{\pi \times 5\text{cm}} = r^2$$

$$\text{or, } 9\text{cm}^2 = r^2$$

$$\therefore r = 3\text{cm}$$

$$\text{Radius of cylinder (r)} = 3\text{cm}$$

Model 10:

एउटा गोलाको क्षेत्रफल πm^2 यसको अर्धव्यास दोब्बर गर्दा क्षेत्रफलको फरक पता लगाउनुहोस् ।

What will be the difference of in the area if its radius is doubled?

Solution:

$$\text{Here, surface area of sphere (SA)} = \pi m^2$$

Now,

$$\therefore SA = 4\pi r^2$$

$$\text{or, } \pi m^2 = 4\pi r^2$$

$$\text{or, } r^2 = \frac{1}{4} m^2$$

$$\text{or, } r^2 = \left(\frac{1}{2} m\right)^2$$

$$\therefore r = \frac{1}{2} m$$

According to the question

$$\text{Radius (r)} = \frac{1}{2} \times 2 = 1\text{cm} \quad SA = 4\pi r^2 = 4\pi (1\text{m})^2 = 4\pi m^2$$

$$\text{Difference of area} = 4\pi m^2 - \pi m^2 = 3\pi m^2$$

Practice Yourself

1. $36\pi \text{cm}^3$ आयतन भएको गोलाको व्यास पता लगाउनुहोस् ।
Find the diameter of sphere having value $36\pi \text{cm}^3$. (Ans: 6cm)
2. एउटा गोलाको ठूलो वृत्तको परिधि 44m छ । यसको सतहको क्षेत्रफल पता लगाउनुहोस् ।
The circumference of great circle of sphere is 44cm , find its surface area. (Ans: 6m^2)
3. एउटा गोलाकार वस्तुको आयतन $\frac{9\pi}{2} \text{ cm}^3$, भए उक्त गोलाको सतहको क्षेत्रफल कति होला ?
If the volume of the spherical object is $\frac{9\pi}{2} \text{ cm}^3$, find its diameter. (Ans: 2cm)
4. एउटा गोलाको आयतन $\frac{4}{3\pi^2} \text{ cm}^3$, भए उक्त गोलाको सतहको क्षेत्रफल कति होला ?
If the volume of a sphere is $\frac{4}{3\pi^2}$, find the surface area of the sphere. (Ans: $\frac{4}{\pi} \text{ cm}^2$)
5. एउटा गोलाकार वस्तुको आयतन $\left(\frac{3}{4\pi} \text{ cm}\right)^2 \text{ cm}^3$, भए अर्धव्यास पता लगाउनुहोस् ।
The volume of a spherical object is $\left(\frac{3}{4\pi}\right)^2 \text{ cm}^2$, find its radius. (Ans: $\frac{3}{4} \pi \text{cm}$)
6. एउटा गोलाको सतहको क्षेत्रफल 5544 cm^2 भए आयतन पता लगाउनुहोस् ।
The surface area of a sphere is 5544cm^2 . Find its volume. (Ans: 38808cm^3)

7. एउटा गोलाको आयतन $\frac{1375}{21} \text{ cm}^3$ छ भने यसको ठूलो वृत्तको परिधी निकाल्नुहोस् ।
 The volume of spherical object is $\frac{1375}{21} \text{ cm}^3$, find the its circumference of great circle.(Ans: 15.71cm)
8. 6cm अर्धव्यास भएको ठोस गोला बाट 2cm अर्धव्यास भए करि बटा ठोस गोला बनाउन सकिन्छ ?
 How many spheres of radius 2cm can be formed by a solid sphere of radius 6cm. (Ans: 27)
9. तीनओटा फ्लामका ठोस गोलाहरूको अर्धव्यास क्रमशः 1cm 6cm र 8cm छन् । यदि तीनै गोलालाई पगालेर बनाइएको नयाँ गोलाको अर्धव्यास निकाल्नुहोस् ।
 Three Iron solid spheres having radii 1cm 6cm and 8cm are melted to form a new solid sphere.
 find the radius of the new sphere. (Ans: 9cm)
10. यदि $45\pi\text{cm}^3$ आयतन भएको एउटा ठोस गोलालाई पगालेर 5cm उचाई भएको बेलनाबनाइएको छ भने बेलनाको अर्धव्यास पत्ता लगाउनुहोस् ।
 If a solid sphere having $45\pi\text{cm}^3$ melted to form a cylinder of height 5cm. The find the radius of cylinder. (Ans: 3cm)
11. 27 ओटा प्रत्येक $x \text{ cm}$ अर्धव्यास भएको गोलाहरू पगालेर $y \text{ cm}$ अर्धव्यास भएको गोला बनाइएको छ । भने $x:y$ को अनुपात पत्ता लगाउनुहोस् । 27 solid iron spheres each of radius $x \text{ cm}$ melted to form with $y \text{ cm}$. Find the ratio of $x:y$. (Ans: 3 : 1)

Long Questions

Model 1:

एउटा गोलाको सतहको क्षेत्रफल र बेलनाको वक्रसतहको क्षेत्रफल बराबर छ । यदि बेलनाको उचाई र व्यास प्रत्येक 16cm भए गोलाको व्यास पत्ता लगाउनुहोस् ।

The surface area of a sphere to equal the area of the curves surface area of a right circular cylinder. If the height and diameter of the cylinder is 16cm each, find the diameter of sphere.

Solution:

$$\text{Here, Surface area of sphere (SA)} = \pi d^2$$

$$\text{Curve surface area of cylinder (CSA)} = \pi dh$$

$$\text{Height of cylinder (h)} = \text{diameter (d)} = 16\text{cm}$$

According to the question

$$\text{Surface area of sphere (SA)} = \text{curved surface area of cylinder (CSA)}$$

$$\text{or, } \pi d^2 = \pi dh$$

$$\text{or, } d^2 = 16\text{cm} \times 16\text{cm}$$

$$\text{or, } d^2 = 16\text{cm} \times 16\text{cm}$$

$$\therefore d = 16\text{cm}$$

$$\text{Diameter of sphere (d)} = 16\text{cm}$$

Model 2:

यदि 8cm अर्धव्यास भएको बेलनाको पुरासतहको क्षेत्रफल 12cm अर्धव्यास भएको गोलाको सतहको क्षेत्रफलसँग बराबर छ भने बेलनाको आयतन पत्ता लगाउनुहोस् ।

If the total surface area of a cylinder of radius 8cm is equal to surface area of a sphere of radius 12cm, find the volume of cylinder.

Solution:

$$\text{Here, Radius of cylinder (r)} = 8\text{cm}$$

$$\text{Total surface area of cylinder (TSA)} = 2\pi r(r + h)$$

$$\text{Radius of sphere (r)} = 12\text{cm}$$

$$\text{Surface area of sphere (SA)} = 4\pi r^2$$

According to the questions

$$\text{Total surface area of cylinder (TSA)} = \text{surface area of sphere (SA)}$$

$$\text{or, } 2\pi r(r + h) = 4\pi r^2$$

$$\begin{aligned}
 \text{or, } & 2 \times 8\text{cm} (8\text{cm} + h) = 4 \times (12\text{cm})^2 \\
 \text{or, } & 16\text{cm} (8\text{cm} + h) = 4 \times 144\text{cm}^2 \\
 \text{or, } & 8\text{cm} + h = \frac{4 \times 144\text{cm}^2}{16\text{cm}} \\
 \text{or, } & 8\text{cm} + h = 16\text{cm} \\
 \text{or, } & h = 16\text{cm} - 8\text{cm} \\
 \therefore & h = 8\text{cm} \\
 \text{Height of cylinder (h)} &= 8\text{cm} \\
 V &= \pi r^2 h = \frac{22}{7} \times (8\text{cm})^2 \times 8\text{cm} = \frac{22 \times 64 \times 8\text{cm}^3}{7} = 1609.14\text{cm}^3 \\
 \text{Volume of cylinder (r)} &= 1609.14\text{m}^3
 \end{aligned}$$

Model 3:

कुनै गोलाको आयतनमा हुने भिन्नता फरक पत्ता लगाउनुहोस् । जसमा गोलाको व्यासलाई एक तिहाइले घटाइएको छ । What will be the difference in volume of sphere if its diameter is reduced to one-third.

Solution:

Here, diameter of sphere (d) be x

$$\text{Volume of sphere (V)} = \frac{1}{6} \pi d^3 = \frac{1}{6} \pi x^3$$

According to the question

$$\text{No diameter of sphere (d)} = \frac{1}{3} \text{ of } x = \frac{x}{3}$$

$$\text{Volume (V)} = \frac{1}{6} \pi d^3 = \frac{1}{6} \pi \left(\frac{x}{3}\right)^3 = \frac{1}{6} \pi x^3 \cdot \frac{1}{27}$$

$$\text{Difference of volume (V)} = \frac{1}{6} \pi x^3 - \frac{1}{6} \pi x^3 \cdot \frac{1}{27} = \frac{1}{6} \pi x^3 \left(1 - \frac{1}{27}\right) = \frac{1}{6} \pi x^3 \cdot \frac{26}{27}$$

$$\therefore \frac{26}{27} \text{ times difference of volume of sphere.}$$

Model 4:

तीनओटा ठोस गोलाका अर्धव्यासहरू 5:4:5 का अनुपातमा छन् । तिनीहरूलाई पगाली एउटै नयाँ गोलाबनाउँदा गोलाट अर्धव्यासहरू पत्ता लगाउनुहोस् ।

The radius of three solid spheres is in the ratio of 3:4:5. If these spheres are melted to form a new sphere whose volume is $2304\pi\text{cm}^3$, find radii of three spheres.

Solution:

Here, the ratio of three radii of spheres 3:4:5 if r_1 , r_2 and r_3 are radii of three spheres.

$$r_1 = 3x, r_2 = 4x \text{ and } r_3 = 5x$$

$$\text{Volume of new sphere (V)} = 2304\pi\text{cm}^3$$

According to the questions of the V_1 , V_2 and V_3 are the volume of three spheres respectively.

$$V_1 = \frac{4}{3} \pi r_1^3 \quad V_2 = \frac{4}{3} \pi r_2^3 \quad \text{and } V_3 = \frac{4}{3} \pi r_3^3$$

$$\therefore V = V_1 + V_2 + V_3$$

$$\text{or, } 2304\pi\text{cm}^3 = \frac{4}{3} \pi r_1^3 + \frac{4}{3} \pi r_2^3 + \frac{4}{3} \pi r_3^3$$

$$\text{or, } 2304\pi\text{cm}^3 = \frac{4}{3} \pi(r_1^3 + r_2^3 + r_3^3)$$

$$\text{or, } 2304\text{cm}^3 = \frac{4}{3} [(3x)^3 + (4x)^3 + (5x)^3]$$

$$\text{or, } 2304\text{cm}^3 = \frac{4}{3} [27x^3 + 64x^3 + 125x^3]$$

$$\text{or, } 2304 \text{ cm}^3 = \frac{4}{3} 216x^3$$

$$\text{or, } 2304 \text{ cm}^3 = 4 \times 72x^3$$

$$\text{or, } \frac{2304}{4 \times 72} \text{ cm}^3 = x^3$$

$$\text{or, } \frac{2304}{288} \text{ cm}^3 = x^3$$

$$\text{or, } 8 \text{ cm}^3 = x^3$$

$$\therefore x = 2 \text{ cm}$$

Now radius of first sphere (r_1) = $3 \times 2 \text{ cm} = 6 \text{ cm}$

Radius of 2nd sphere (r_2) = $4 \times 2 \text{ cm} = 8 \text{ cm}$

Radius of 3rd sphere (r_3) = $5 \times 2 \text{ cm} = 10 \text{ cm}$

Model 5:

1cm अर्धव्यास भएको केहि गोलाहरू 6cm व्यास भएको बेलनाकार भाँडाको पानीमा पूर्ण रूपमा डुबाउँदा पानीको सतह 4cm माथि उढ्छ भने कति वटा गोलालाई डुवाएको होला ?

A number of sphere of radius 1cm are dropped into a cylindrical vessel which contains water of diameter 6cm. If the sphere are completely and the rise in water level is 4cm find the number of sphere immersed.

Solution:

Here, diameter of cylinder (d) = 6cm

$$\text{Radius } (r) = \frac{d}{2} = \frac{6\text{cm}}{2} = 3\text{cm}$$

Height of cylinder (h) = 4cm

$$\text{In cylinder volume of water } (V) = \pi r^2 h = \frac{22}{7} \times (3\text{cm})^2 \times 4\text{cm} = \frac{22 \times 9 \times 4\text{cm}^3}{7}$$

Again radius of each sphere (r) = 1cm

$$\text{volume of each sphere } (V_1) = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (1\text{cm})^3 = \frac{4 \times 22 \times 1}{21} \text{ cm}^3$$

$$\text{Now, Number of sphere } (N) = \frac{V}{V_1} = \frac{2}{\frac{4 \times 22 \times 1}{21} \text{ cm}^3} = \frac{22 \times 9 \times 4}{7} \times \frac{21}{4 \times 22 \times 1} = 27$$

\therefore The required number of sphere is 27.

Model 6:

दुई ओटा गोलाको सतहको क्षेत्रफलको अनुपात 1:4 छ भने तिनीहरूको आयतनको अनुपात निकाल्नुहोस् ।

The surface area of two sphere are in the ratio 1:4 find the ratio of their volumes.

Solution: Here if r_1 and r_2 are the ratio of two spheres respectively.

Then SA_1 and SA_2 are also surface area of two spheres

$$SA_1 = 4\pi r_1^2 \text{ and } SA_2 = 4\pi r_2^2$$

According to the questions

$$\frac{SA_1}{SA_2} = \frac{1}{4}$$

$$\text{or, } \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{1}{4}$$

$$\text{or, } \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$\therefore \frac{r_1}{r_2} = \frac{1}{2}$$

Again V_1 and V_2 are the volume of two spheres

$$\text{Now, } V_1 = \frac{4}{3} \pi r_1^3 \text{ and } V_2 = \frac{4}{3} \pi r_2^3$$

according to the question

$$\text{or, } \frac{V_1}{V_2} = \frac{\frac{4}{3} \pi r_1^3}{\frac{4}{3} \pi r_2^3}$$

$$\text{or, } \frac{V_1}{V_2} = \left(\frac{r_1^3}{r_2^3} \right)^3$$

$$\text{or, } \frac{V_1}{V_2} = \left(\frac{1}{2} \right)^3$$

$$\therefore \frac{V_1}{V_2} = \frac{1}{8}$$

∴ Hence, ratio of volumes is 1:8

Model 7

एउटा गोलाको व्यास 6cm छ। यसलाई पगालेर 2mm व्यास भएको तार बनाइएको छ भने तारको लम्बाई पत्ता लगाउनुहोस्।

The diameter of a sphere is 6cm. If it is melted and drawn into a cylindrical wire of diameter 2mm, find the length of wire.

Solution:

Here, diameter solid sphere (d) = 6cm

$$\text{Volume of sphere (V)} = \frac{1}{6} \pi d^3 = \frac{1}{6} \pi (6\text{cm})^3 = \frac{1}{6} \pi 216\text{cm}^3 = 36 \text{ cm}^3$$

$$\text{Again diameter of wire (d)} = 2\text{mm} = \frac{2}{10} = \frac{1}{5} \text{ cm}$$

$$\text{Volume of wire (V)} = \frac{1}{4} \pi d^2 h = \frac{1}{4} \pi d^2 h = \frac{1}{4} \pi \left(\frac{1}{5} \text{ cm}\right)^2 \times h$$

$$\text{of the length of wire} = 1 \times h = \frac{1}{4} \pi \times \frac{1}{25} \text{ cm}^2$$

According to the question

$$\text{Volume of sphere (V)} = \text{Volume of wire (V)}$$

$$\text{or, } \frac{1}{6} \pi 216\text{cm}^3 = \frac{1}{4} \pi \frac{1}{25} h \text{ cm}^2$$

$$\text{or, } \frac{1}{6} \times 216\text{cm}^3 = \frac{1}{4} \times \frac{1}{25} h \text{ cm}^2$$

$$\text{or, } 36\text{cm}^3 = \frac{1}{100} \times h \text{ cm}^2$$

$$\text{or, } 36 \times 100\text{cm} = h$$

$$\therefore h = 3600\text{cm}$$

$$\text{Length of wire (h)} = 3600\text{cm}$$

Practice Yourself

- अर्धव्यास 8cm भएको एउटा ठोस गोलाबाट प्रत्येकको अर्धव्यासको 1cm भएको किति गोलाकार बलहरू बनाउन सकिन्छ? How many spherical balls of each radius 1cm. Can be made from a solid sphere of radius 8cm.
(Ans:512)
- अर्धव्यासहरू 6cm 8cm र xcm भएका तीनओटा गोलाहरू पगालेर एक ठोस बनाउला अर्धव्यास 12cm हुन्छ भने x को मान किति होला?

Three spherical metal balls of radii 6cm 8cm and xcm are melted into a solid sphere of radius 12cm, find the volume of x.

(Ans: 10cm)

3. 1.4cm व्यास भएको गुच्छाहरू 7cm व्यास भएको केहि पानी भएको एउटा बेलनाकार विकरमा खानालिएका छन् । यदि पानीको सतह 5.6cm माथि उठाउन खसाल्नु पर्ने गुच्छाको संख्या निकाल्नुहोस् । (Ans: 150)

Marbles of diameter 1.4cm are dropped into a cylindrical beaker of diameter 7cm. containing some water. find the number of marbles that should be dropped into beaker so that the water level rise by 5.6cm.

4. एउटा 6cm अर्धव्यास भएको ठोस गोलालाई पगालेर 3cm अर्धव्यास भएको ठोस बेलनाकार बनाइएको छ भने उक्त बेलनाको उचाई किति पत्ता लगाउनुहोस् ।

A solid sphere of radius 6cm is melted and a solid cylinrical of radius 3cm is formed find is height of cylinder. (Ans: 32cm)

5. दुईओटा गोलाहरूको अर्धव्यासको अनुपात 3:2 छ । भने वक्सतहको क्षेत्रफलको अनुपात निकाल्नुहोस् । The radii of two spheres are in the ratio 3:2 final the ratio of their surface area. (Ans: 9:4)

6. कुनै गोलाको अर्धव्यासलाई दोब्बर गर्दा आयतन किति गुणाले फरक हुन्छ ? पत्ता लगाउनुहोस् । Find the what will be the times in volume of sphere, if its radius is in doubled. (Ans: 7times)

7. एउटा गोलाको अर्धव्यास 9cm छ । यदि यसलाई पगालेर 2m व्यास भएको पाइप बनाइएको छ भने पाइपको लम्बाई पत्ता लगाउनुहोस् ।

The radius of a sphere is 9m it is melted and drown into pipe of dameter 2m. Find the length of pipe. (Ans: 972m)

8. 6cm व्यास भएको कम्ती किति वटा ठोस गोलालाई पगालेर 48cm उचाई र 4cm व्यास भएको ठोस बेलनाकार वस्तु बनाउन सकिन्द्य ? पत्ता लगाउनुहोस् ।

Find the what is the least number of solid metallic solid spheres each of 6cm diameter that should be melted and vessel to from a solid metal cylinder object whose height is 4cm and diameter 4cm. (Ans: 5)

9. एउटा गुच्छाको सतहको क्षेत्रफल $2\pi\text{cm}^2$ छ । यसको अर्धव्यासलाई $\frac{2}{3}$ गर्दा सतहको क्षेत्रफल र आयतनमा किति फरक हुन्छ ? पत्ता लगाउनुहोस् । Surface area of marble is $2\pi\text{cm}^2$ of its radius is made $\frac{2}{3}$ of the original, find the what will be difference of surface area and volume. (Ans. $\frac{8\pi}{9}, \frac{11\sqrt{2}}{81}$)

10. उचाई 18cm र अर्धव्यास 4cm भएको बेलनाको वक्सतहको क्षेत्रफलसँग एउटा गोलाको पुरासतहको क्षेत्रफलसँग बराबर छ । गोलाको अर्धव्यास र आयतन पत्ता लगाउनुहोस् ।

If the curve surface area is of cylinder whose height is 18cm and radius is 4cm equal in the surface area of a sphere, find the radius and volume of sphere. (Ans: 6cm)

6.3 संयुक्त ठोस वस्तुको क्षेत्रफल र आयतन

(Surface Area and Volume of combined Solid Object)

संयुक्त ठोस वस्तुको केहि जानकारी

(Some Information of Combined Solid object)

1. ठोस वस्तुको आयतन (Volume of combined solid objects)

बेलनाको उचाई (Hight of cylinder) = h

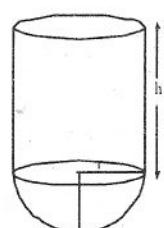
सामान्य अर्धव्यास (Common radius) = r

बेलनाको आयतन (Volume of cylinder) = V_1

$\therefore V = \pi r^2 h$

अर्धगोलाको आयतन (Volume of cylinder) = V_2

$$\therefore V_2 = \frac{2}{3} \pi r^3$$



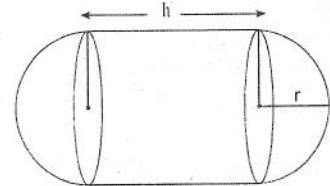
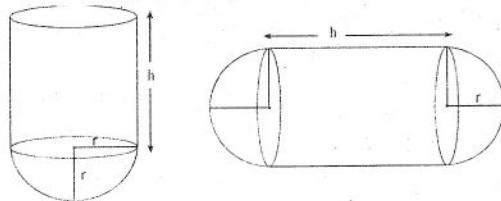
$$\begin{aligned} \text{संयुक्त ठोस वस्तुको आयतन (V)} &= V_1 + V_2, \\ V = V_1 + V_2 &= \pi r^2 h + \frac{2}{3} \pi r^3 = \pi r^2 \left(h + \frac{2r}{3} \right) \\ \therefore V &= \pi r^2 \left(h + \frac{2r}{3} \right) \end{aligned}$$

- बेलनाको उचाई (height of cylinder) = h , साफा अर्धव्यास (Common radius) = r
बेलनाको अयतन (Volume of cylinder) = V_1 दुई अर्धगोलाको आयतन (Volume of two hemispheres) = V_2

$$\begin{aligned} \therefore V_1 &= \pi r^2 h, V_2 = 2 \left(\frac{2}{3} \pi r^3 \right) = \frac{4\pi}{3} r^3 \\ \text{ठोस वस्तुको आयतन (Volume of solid object)} &= V \\ V = V_1 + V_2 &= \pi r^2 h + \frac{4\pi}{3} r^3 = \pi r^2 \left(h + \frac{4r}{3} \right) \end{aligned}$$

2. संयुक्त ठोस वस्तुको वक्सतहको क्षेत्रफल

(Curved surface Area of combined solid object)



$$\begin{aligned} \text{साफा अर्धव्यास (Common radius)} &= r \text{ उचाई (height)} = h \\ \text{बेलनाको वक्सतहको क्षेत्रफल (Curved surface Area)} &= CSA, \\ \text{दुई अर्धगोलाको वक्सतहको क्षेत्रफल (Curved surface Area)} &= CSA_2 \\ \therefore CSA &= 2\pi rh \quad CSA_2 = \pi r^2 \\ \text{संयुक्त ठोस वस्तुको वक्सतहको क्षेत्रफल (Curved surface area of combined solid object)} &= CSA \\ \therefore CSA &= CSA_1 + CSA_2 \\ \therefore CSA &= CSA_1 + CSA_2 = 2\pi rh + \pi r^2 = 2\pi r(h + r) \end{aligned}$$

$$\begin{aligned} \text{बेलनाको वक्सतहको क्षेत्रफल (Curved surface area)} &= CSA \\ \text{दुई अर्धगोलाको वक्सतहको क्षेत्रफल (Curved surface area)} &= CSA_2 \\ \therefore CSA_1 &= 2\pi rh \quad CSA_2 = 4\pi r^2 \\ \text{ठोसवस्तुको वक्सतहको क्षेत्रफल (Curved surface area)} & \\ \therefore CSA &= CSA_1 + CSA_2 \\ CSA &= CSA_1 + CSA_2 = 2\pi rh + 4\pi r^2 = 2\pi r(h + 2r) \end{aligned}$$

3. संयुक्त ठोस वस्तुको पुरासतहको क्षेत्रफल

(Total surface area of Combined solid object)

$$\begin{aligned} \text{पुरासतहको क्षेत्रफल (Total surface area)} &= TSA \\ \text{दुई अर्धगोलाको वक्सतहको क्षेत्रफल} &= CSA = 2\pi r^2 \\ \text{बेलनाको वक्सतहको क्षेत्रफल (Curve surface area of cylinder)} &= CSA = 2\pi rh \\ \text{वृत्ताकार सतहको क्षेत्रफल (Surface area of circle)} &= \pi r^2 \\ TSA &= CSA_1 + CSA_2 + A \\ TSA &= 2\pi^2 + 2\pi rh + \pi r^2 = 2\pi rh + (3\pi r^2) \\ \therefore TSA &= \pi(2h + 3r) \\ \text{दुई अर्धगोलाको सतहको क्षेत्रफल (Surface area of two hemispheres)} &= CSA_1 = 4\pi r^2 \\ \text{बेलनाको वक्सतहको क्षेत्रफल (Curve surface area of cylinder)} &= 2\pi rh \\ TSA &= CSA_1 + CSA_2 \end{aligned}$$

$$\therefore \text{TSA} = 4\pi r^2 + 2\pi rh = 2\pi r(2r + h)$$

Long Questions

Model 1

दिइएको चित्रबाट ठोसवस्तुको आयतन पता लगाउनुहोस्।

From given figure, find the volume combined solid object

Solution:

In the figure of combined solid objects.

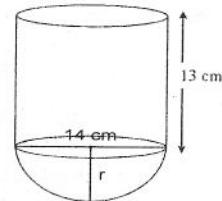
diameter (d) = 14cm

$$\text{radius (r)} = \frac{d}{2} = \frac{14}{2} \text{ cm} = 7\text{cm}$$

Total height of combined solid (h) = 13cm

height of cylinder (h_1) = 13cm - 7cm = 6cm

$$\begin{aligned} \text{Volume of cylinder (V}_1\text{)} &= \pi r^2 h_1 = \frac{22}{7} \times (7\text{cm})^2 \times 6\text{cm} = \frac{22 \times 49 \times 6\text{cm}^3}{7} = 22 \times 7 \times 6\text{cm}^3 \\ &= 924\text{cm}^3 \end{aligned}$$

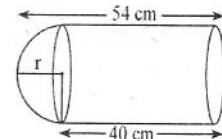


$$\begin{aligned} \text{Volume of hemisphere (V}_2\text{)} &= \frac{2}{3} \pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (7\text{cm})^3 = \frac{2 \times 22 \times 343\text{cm}^3}{21} = \frac{15092\text{cm}^3}{21} \\ &= 418.66\text{cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of combined splid(V)} &= V_1 + V_2 = 924\text{cm}^3 + 718.66\text{cm}^3 \\ &= 1642.66\text{cm}^3 \end{aligned}$$

Model 2 दिइएको चित्रबाट ठोस वस्तुको पुरा सतहको क्षेत्रफल पता लगाउनुहोस्।

Find the total surface area of given combined solid objects.



Solution:

In the given figure combined solid object total height of solid objects (h) = 54cm

height of cylinder (h_1) = 40cm

common radius (r) = (54 - 40)cm = 14cm

$$\begin{aligned} \text{curve surface of cylinder (CSA}_1\text{)} &= 2\pi rh_1 = 2 \times \frac{22}{7} \times 14\text{cm} \times 40\text{cm} = 2 \times 22 \times 2 \times 40\text{cm}^2 \\ &= 3520\text{cm}^2 \end{aligned}$$

$$\begin{aligned} \text{curve surface area of hemisphere (CSA}_1\text{)} &= 2\pi r^2 = 2 \times \frac{22}{7} \times (14\text{cm})^2 \\ &= \frac{2 \times 22 \times 196\text{cm}^2}{7} = \frac{864\text{cm}^2}{7} = \frac{704\text{cm}^2}{7} = 1232\text{cm}^2 \end{aligned}$$

$$\text{Area of circular surface (A)} = \pi r^2 = \frac{22}{7} \times (14\text{cm})^2 = \frac{22 \times 196\text{cm}^2}{7} = \frac{4312\text{cm}^2}{7} = 616\text{cm}^2$$

Total surface area of solid object

$$(\text{TSA}) = \text{CSA}_1 + \text{CSA}_2 + A = 3520\text{cm}^2 + 1232\text{cm}^2 + 616\text{cm}^2 = 5368\text{cm}^2$$

Model 3:

दिएको चित्रमा बेलना र अर्धगोलाका मिली ठोस वस्तु बनेको छ । यदि उक्त ठोस वस्तुको वक्सतहको क्षेत्रफल र पुराउचाई क्रमशः 528cm^2 र 14cm भए बेलनाको उचाई पत्ता लगाउनुहोस् । Given figure is solid composed of a cylinder and hemisphere at one end. If curved surface area and whole height of solid are 528cm^2 and 14cm respectively, find the height of cylinder.

Solution:

In the given figure of combined solid object,

$$\text{curve surface area of solid object (CSA)} = 528\text{cm}^2$$

$$\text{Total height of solid object (h)} = 14\text{cm} \text{ or } h_1 + r = 14\text{cm}$$

$$\text{curve surface area of cylinder (CSA}_1) = 2\pi rh_1$$

$$\text{curve surface area of hemisphere (CSA}_2) = 2\pi r^2$$

$$\text{Now, CSA} = \text{CSA}_1 + \text{CSA}_2$$

$$\text{or, } 528\text{cm}^2 = 2\pi rh_1 + 2\pi r^2$$

$$\text{or, } 528\text{cm}^2 = 2\pi r(h_1 + r)$$

$$\text{or, } 528\text{cm}^2 = 2 \times \frac{22}{7} \times r \times 14\text{cm}$$

$$\text{or, } 528\text{cm}^2 = 44 \times r \times 2\text{cm}$$

$$\text{or, } r = \frac{528\text{cm}^2}{88\text{cm}}$$

$$\text{or, } r = 6\text{cm}$$

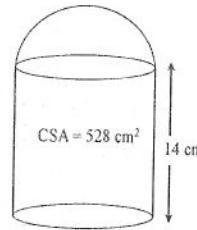
$$\text{Again, } h_1 + r = 14\text{cm}$$

$$\text{or, } h_1 + 6\text{cm} = 14\text{cm}$$

$$\text{or, } h_1 = (14 - 6)\text{cm}$$

$$\therefore h_1 = 8\text{cm}$$

$$\text{Height of cylinder (h}_1) = 8\text{cm}$$



Model 4:

दिएको वस्तु बेलना र अर्धगोलाकार वस्तु मिली बनेको ठोस वस्तु हो । उक्त ठोस वस्तुको पुरासतहको क्षेत्रफल 770cm^2 र उचाई 14cm भए बेलनाको उचाई निकाल्नुहोस् ।

Given figure is a solid composed of a cylinder with hemisphere at one if total surface area is 770cm^2 and its height 14cm find the height of cylinder.

Solution:

In the given figure of combined solid object,

$$\text{Total surface area of solid object (TSA)} = 770\text{cm}^2$$

$$\text{total height (h)} = 14\text{cm} = h_1 + r$$

$$\text{curved surface area of cylinder (CSA}_1) = 2\pi rh_1$$

$$\text{curved surface area of hemisphere (CSA}_2) = 2\pi r^2$$

$$\text{and circular area of cylinder (A)} = \pi r^2$$

$$\text{Now TSA} = \text{CSA}_1 + \text{CSA}_2 + A$$

$$\text{or, } 770\text{cm}^2 = 2\pi rh_1 + 2\pi r^2 + \pi r^2$$

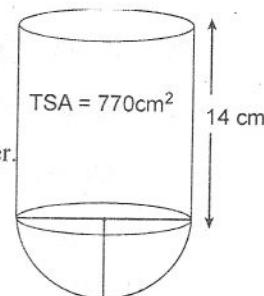
$$\text{or, } 770\text{cm}^2 = 2\pi r(h_1 + r) + \pi r^2$$

$$\text{or, } 770\text{cm}^2 = 2 \times \frac{22}{7} r (14\text{cm}) + \frac{22}{7} r^2 (h_1 + r = h = 14\text{cm})$$

$$\text{or, } 770\text{cm}^2 = 88\text{cmr} + \frac{22r^2}{7}$$

$$\text{or, } 770\text{cm}^2 = \frac{88\text{cm} \times 7 \times r + 22r^2}{7}$$

$$\text{or, } 770\text{cm}^2 \times 7 = 616\text{cmr} + 22r^2$$



$$\text{or, } 770\text{cm}^2 \times 7 = 22(28\text{rcm} + r^2)$$

$$\text{or, } \frac{770\text{cm}^2 \times 7}{22} = 28\text{rcm} + r^2$$

$$\text{or, } 245 = 28r + r^2$$

$$\text{or, } r^2 + 28r - 245 = 0$$

$$\text{or, } r^2 + 35r - 7r - 245 = 0$$

$$\text{or, } r(r+35) - 7(r+35) = 0$$

$$\text{or, } (r+35)(r-7) = 0$$

either $r+35 = 0$

$\therefore r = -35$ (impossible)

$$\text{or, } r-7 = 0$$

$$\therefore r = 7$$

Radius (r) = 7cm, height of cylinder (h_1) = $(14 - 7)\text{cm} = 7\text{cm}$

Model 5:

hem लम्बाई h cm अर्धव्यास भएको एउटा बेलना र उहि अर्धव्यास अर्धगोला मिली बनेको ठोस वस्तुको आयतन 792cm^3 भए h को मान पत्ता लगाउनुहोस् ।

A combined solid made up of a cylinder of radius 3cm and length $h\text{cm}$ and a hemisphere of same radius at end of the cylinder has volume of 792cm^3 find the volume of h .

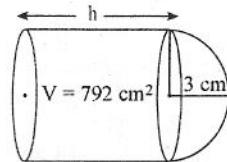
Solution:

In the given figure combined solid figure,

$$\text{common radius (r)} = 3\text{cm}$$

$$\text{length of cylinder (h)} = h\text{cm}$$

$$\text{volume of combined solid object (V)} = 792\text{cm}^3$$



$$\text{Now, volume of cylinder (V}_1\text{)} = \pi r^2 h \text{ volume of hemisphere (V}_2\text{)} = \frac{2}{3} \pi r^3$$

$$\therefore V = V_1 + V_2$$

$$\text{or, } 792\text{cm}^3 = \pi r^2 h + \frac{2}{3} \pi r^3$$

$$\text{or, } 792\text{cm}^3 = \frac{22}{7} (3\text{cm})^2 \times h + \frac{2}{3} \times \frac{22}{7} \times (3\text{cm})^3$$

$$\text{or, } 792\text{cm}^3 = \frac{22}{7} \times 9\text{cm}^2 h + \frac{44 \times 27}{3 \times 7} \text{cm}^3$$

$$\text{or, } 782\text{cm}^3 = \frac{198\text{cm}^2 \times h}{7} = \frac{396}{7} \text{cm}^3$$

$$\text{or, } 792\text{cm}^3 - \frac{396\text{cm}^3}{7} = \frac{198\text{cm}^2 \times h}{7}$$

$$\text{or, } \frac{(5544 - 396)}{7} \text{cm}^3 = \frac{198\text{cm}^2 \times h}{7}$$

$$\text{or, } 5148\text{cm}^3 = 198\text{cm}^2 \times h$$

$$\text{or, } h = \frac{5148\text{cm}^3}{198\text{cm}^2}$$

$$\therefore h = 6\text{cm}$$

Length of cylinder (h) = 26cm

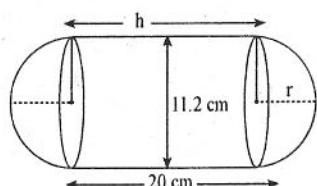
Model 6:

दिइएको ठोस वस्तुको आयतन र पुरासतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

Find the volume and total surface area of given solid object cylinder with hemisphere.

Solution:

In the given figure of solid object.



height of cylinder (h_1) = 20cm and

$$\text{diameter (d)} = 11.2\text{cm} \text{ common radius (r)} = \frac{d}{2} = \frac{11.2}{2} \text{ cm} = 5.6\text{cm}$$

curved surface area of cylinder = CSA_1

$$\text{CSA}_1 = 2\pi rh_1$$

$$\text{or, } \text{CSA}_1 = 2 \times \frac{22}{7} \times 5.6\text{cm} \times 20\text{cm}$$

$$\text{or, } \text{CSA}_1 = \frac{44 \times 5.6 \times 20\text{cm}^2}{7}$$

$$\therefore \text{CSA}_1 = 44 \times 0.8 \times 20\text{cm}^2 \\ = 704\text{cm}^2$$

curved surface area of two spheres = CSA_2

$$\text{CSA}_2 = 4\pi r^2 = 4 \times \frac{22}{7} \times (5.6\text{cm})^2 = \frac{88 \times 31.36\text{cm}^2}{7} = \frac{2759.24\text{cm}^2}{7} = 394.24\text{cm}^2$$

Now total surface of combined solid object = TSA

$$\therefore \text{TSA} = \text{CSA}_1 + \text{CSA}_2 = 704\text{cm}^2 + 294.24\text{cm}^2 = 1098.24\text{cm}^2$$

Total surface Area (TSA) = 1098.24cm²

Again, volume of cylinder = V_1

$$\therefore V_1 = \pi r^2 h_1 = \frac{22}{7} \times (5.6\text{cm})^2 \times 20\text{cm} = \frac{22 \times 31.36 \times 20\text{cm}^3}{7} \\ = \frac{13798.40\text{cm}^3}{7} = 1971.20\text{cm}^3$$

Volume of two hemisphere = V_2

$$V_2 = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (5.6\text{cm})^3 = \frac{4 \times 22 \times 175.6 \text{ cm}^3}{3 \times 7} = \frac{15454.208\text{cm}^3}{21} = 735.91\text{cm}^3$$

Total volume of combined solid object = V

$$\therefore V = V_1 + V_2 = 1971.20\text{cm}^3 + 735.91\text{cm}^3 = 2707.11\text{cm}^3$$

$$\therefore \text{Total volume of solid object (V)} = 2707.11\text{cm}^3$$

Model 7:

दिएको चित्रमा अर्धगोलाकार छेउहरू भएको ठोस वस्तुको पुरा सतहको क्षेत्रफल कति हुन्छ ? प्रति cm² रु. 10 का दरले यसमा चाँदीको लेपन गर्न कति खर्च लाग्छ ?

What is total surface area of the given figure of cylindrical solid with hemispherical ends what is the painting its surface with silver at the rate of Rs 10cm².

Solution:

In the figure os solid combined solid objects

Height of cylinder (h_1) = 60cm, its total height (h) = 70cm

The common radius (r) = 70cm - 60cm = 10cm

curved surface area of cylinder

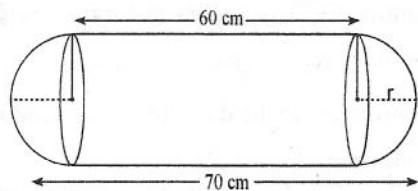
$$\therefore \text{CSA}_1 = 2\pi rh_1$$

$$\text{or, } \text{CSA}_1 = 2 \times \frac{22}{7} \times 60\text{cm}$$

$$\text{or, } \text{CSA}_1 = \frac{46.40\text{cm}^2}{7}$$

$$\therefore \text{CSA}_1 = 377.144\text{cm}^2$$

Curved surface area of two of hemisphere = CSA_2



$$\therefore \text{CSA}_2 = 4\pi r^2 = 4 \times \frac{22}{7} \times (10\text{cm})^2 = \frac{4 \times 22 \times 100\text{cm}^2}{7} = \frac{8800\text{cm}^2}{7} = 1257.14\text{cm}^2$$

Now total surface area of solid object = TSA

$$\text{TSA} = \text{CSA}_1 + \text{CSA}_2 = 377.14\text{cm}^2 + 1257.14\text{cm}^2 = 1634.28\text{cm}^2$$

Again rate of painting (c) = Rs 10/cm²

Total cost of painting (T) = Rs 10/cm² × 1634.28cm² = Rs 16342.80

Model 8

एउटा 15cm अर्धव्यास भएको खाली अर्धगोलाकार कचौरामा चिनी भर्नलाई 5cm व्यास र 6cm, उचाई भएको बेलनाकार भाँडोले कति पटक भर्दा सो भाँडोमा चिनी भर्न आवश्यक छ ?

A hemispherical empty bowl with 15cm radius is full of sugar from the suya is powered cylindrical vessel of diameter 5cm and length 6cm. How many cylinder bowl.

Solution:

Here, radius of spherical bowl (r) = 15cm

$$\text{Then volume of spherical bowl (V)} = \frac{2}{3} \pi r^3 = \frac{2}{3} \pi (15\text{cm})^3 = \frac{6750\pi\text{cm}^3}{3}$$

Diameter of cylinder (d) = 5cm

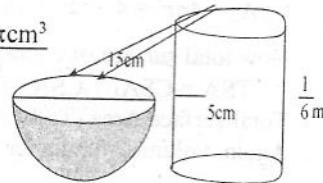
$$\text{and radius (r)} = \frac{d}{2} = \frac{5\text{cm}}{2} = 2.5\text{cm}$$

Height of cylinder (h) = 6cm

$$\text{Volume of cylindrical vessel (V)} = \pi r^2 h = \pi \times (2.5\text{cm})^2 \times 6\text{cm} = 37.5\pi\text{cm}^3$$

$$\text{Number of cylindrical vessel (N)} = \frac{\text{volume of spherical bowl}}{\text{volume of cylindrical}}$$

$$= \frac{6750\pi\text{cm}^3}{37.5\pi\text{cm}^3} = \frac{6750\pi\text{cm}^3}{37.5\pi\text{cm}^3 \times 3} = \frac{6750}{3 \times 37.5} = 60$$



Model 9

दिइएको संयुक्त ठोस वस्तु बेलना र अर्धगोला मिली बनेको छ । यदि पुरा ठोसको उचाई आधारको व्याससँग $\frac{320\pi}{3}\text{ cm}^3$ छ भने ठोस वस्तुको उचाई निकाल्नुहोस् ।

In the given figure of combined solid object is made up of cylinder and hemisphere to the diameter of the solid is equil to $\frac{320\pi}{3}\text{ cm}^3$, find the total height.

Solution:

In the given figure of combined solid object figure

Total height (h) = diameter of base (d) ($\therefore h = 2r$)

$$\text{volume of solid object (V)} = \frac{320\pi}{3}\text{ cm}^3$$

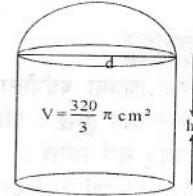
$$\text{volume of cylinder (V}_1\text{)} = \pi r^2 h_1 \text{ and volume hemisphere} = \frac{2}{3} \pi r^3$$

$$\therefore V = V_1 + V_2$$

$$\text{or, } \frac{320\pi}{3}\text{ cm}^3 = \pi r^2 h_1 + \frac{2}{3} \pi r^3$$

$$\text{or, } \frac{320\pi}{3}\text{ cm}^3 = \pi r^2 (h_1 + \frac{2}{3} r)$$

$$\text{or, } \frac{320\pi}{3}\text{ cm}^3 = r^2 (h - r + \frac{2r}{3})$$



$$\text{or, } \frac{320\pi}{3} \text{ cm}^3 = \frac{d^2}{4} \left(d - \frac{d}{2} + \frac{d}{3}\right)$$

$$\text{or, } \frac{320\pi}{3} \text{ cm}^3 = \frac{d^2}{4} \times \frac{5d}{6}$$

$$(h_1 + r = h \therefore h_1 = h - r)$$

$$\text{or, } \frac{320\pi}{3} \text{ cm}^3 = \frac{5d^3}{24}$$

$$\text{or, } 320 \times 8 \text{ cm}^3 = 5d^3$$

$$\text{or, } \frac{320 \times 8}{5} \text{ cm}^3 = d^3$$

$$\text{or, } 64 \times 8 \text{ cm}^3 = d^3$$

$$\therefore d = 8 \text{ cm}$$

$$\text{Height (h)} = 8 \text{ cm}$$

Model 10

एउटा पानी टायाइको तल्लो भाग बेलना त्यसमाथि अर्धगोला छुनेगरी बसेको छ। सो टायाइको पुरा उचाई 15m र आधारको क्षेत्रफल 154 cm^2 भए उक्त टायाइकीकमा प्रति ली. रु. 0.35 दरले पानी भर्न जस्ता किंतु खर्च लाग्दै ? पता लगाउनुहोस् ।

A water tank is formed with the combination of cylinder in the lower part and above hemisphere if the total height of the tank is 15m and base is 154 m^2 . If the tank is filled with water at the rate of Rs. 0.35 per liter what is the total cost for the water find it.

Solution:

Area of base (A) = 154 cm^2 , total height (h) = 15m

$$\therefore A = \pi r^2$$

$$\text{or, } 154 \text{ m}^2 = \frac{22}{7} r^2$$

$$\text{or, } 154 \times 7 = 22r^2$$

$$\text{or, } r^2 = \frac{154 \times 7 \text{ cm}^2}{22} \text{ m}^2$$

$$\text{or, } r^2 = 49 \text{ m}^2$$

$$\therefore r = 7 \text{ m.}$$

$$\text{Common radius (R)} = 7 \text{ m}$$

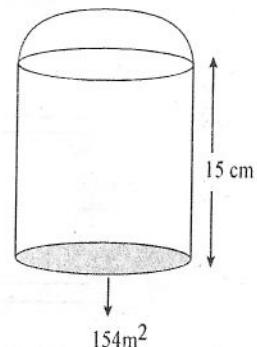
$$\text{Height of cylinder (h)} = 15 \text{ m} - 7 \text{ m}$$

$$\begin{aligned} \text{Volume of combined tank (V)} &= A \times h_1 + \frac{2}{3} \pi r^3 = 154 \text{ m}^2 \times 8 \text{ m} + \frac{2}{3} \times \frac{22}{7} \times (7 \text{ m})^3 \\ &= 1232 \text{ m}^3 + \frac{15092}{21} \text{ m}^3 = 1950.667 \text{ m}^3 \end{aligned}$$

$$\therefore \text{Volume of water (V)} = 1950.667 \times 1000 = 1950667 \text{ liter}$$

$$\text{Rate of water} = \text{Rs. } 0.35 \text{ per liter}$$

$$\therefore \text{Cost of water} = \text{Rs. } 0.35 \times 1950667 = \text{Rs. } 682733.45$$

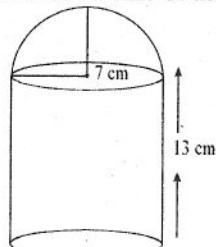


Practice yourself

1. चित्रमा दिइएको ठोस वस्तुको आयतन पत्ता लगाउनुहोस् ।

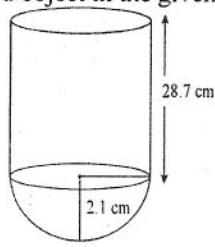
Find the volume of the given solid object in the given figures.

(a)



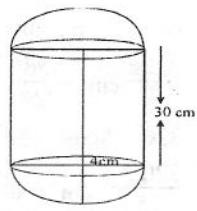
Ans: 1642.66cm^3

(b)



Ans: 1388.08cm^3

(c)



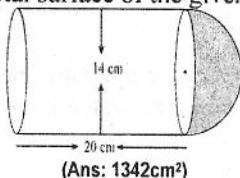
Ans: 6057.33cm^3

(Ans: 6057.33cm^2)

2. दिइएको चित्रमा दिएको ठोसवस्तुको पुरासतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

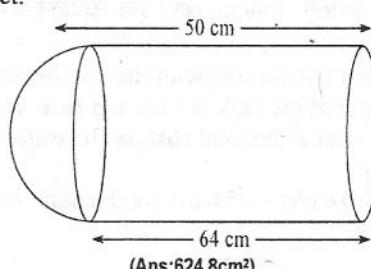
Find the total surface of the given combined solid object.

(a)



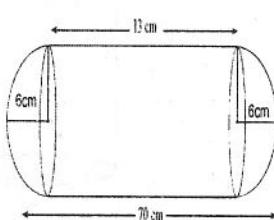
(Ans: 1342cm^2)

(b)



(Ans: 624.8cm^2)

(c)

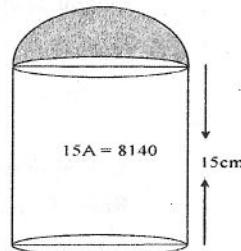


(Ans: 942.85cm^2)

3. दिएको चित्र एउटा बेलना र एक छेउमा अर्धगोला मिलेर बनेको संयुक्त ठोस वस्तु हो । यदि यसको पुरा सतहको क्षेत्रफल र उचाई क्रमशः 814cm^2 र 15cm भए उचाई पत्ता लगाउनुहोस् ।

In the adjoining figure, a solid object is composed of cylinder with hemisphere at the end. if the total surface area of and height of the are 814cm^2 and 15cm respechvely. find the height of cylinder.

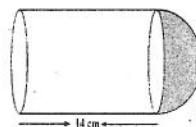
(Ans: 8cm)



4. चित्रमा दिइएको ठोस वस्तुको वक्सतहको 924cm^2 भए ठोस वस्तुको अर्धव्यास र पुरा सतहको क्षेत्रफल पत्ता लगाउनुहोस् ।

(Ans: Q = 7cm TSA = 1048cm^2)

The curved surface area of the combined solid given figure is 924cm^2 , then find the radius and total surface area of the combined solid objects.



5. दिइएका चित्रमा बेलनाका दुवातर अवशालालू जाडेखा का उचाई 20cm पुरासतहको क्षेत्रफल र आयतन पत्ता लगाउनुहोस् ।

The given figure is the cylinder with two hemispheres at the ends, find the total surface area and volume of given solid objects.

(Ans: TSA = 749.76cm^2 1429.26cm^3)

6. दिएको ठोस वस्तु एक अर्धगोला र एक बेलना मिलि बनेको छ । अर्धगोलाको अर्धव्यास र बेलनाको अर्धव्यास बराबर छ । बेलनाको उचाई 80cm र अर्धगोलाको आयतन $144\pi\text{cm}^3$ छ । सो वस्तुको पुरा सतहको क्षेत्रफल निकाल्नुहोस् ।

Given solid object is made up of hemisphere and a cylinder. The radius of hemisphere is equal to the radius of cylinder. The height of cylinder is 80cm and volume of hemisphere is $144\pi\text{cm}^3$, find the total surface area of the solid object.

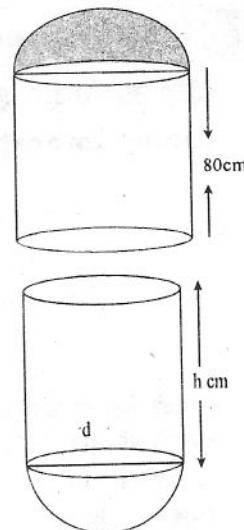
(Ans: 3356.57cm^2)



7. दिइएको संयुक्त ठोस वस्तु बेलना र अर्धगोलाबाट बनेको छ । पुरा ठोसको उचाई आधारको व्यास सँगबराबर छ । यदि पुरा ठोस वस्तुको आयतन $5625\pi\text{cm}^3$ छ भने ठोस वस्तुको उचाई पत्ता लगाउनुहोस् ।

In the given combined solid object is made up of a cylinder and hemisphere. The height of solid is equal to the diameter of the base and volume of the $5625\pi\text{cm}^3$ find the height of solid object.

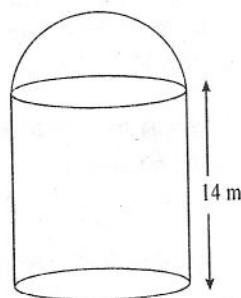
(Ans: 30cm)



8. एउटा पानीको बेलनाकार र अर्धगोलाकार भाग मिलेर बनेको छ । सो ट्याइकीको पुरा उचाई 14m छ । आधारको क्षेत्रफल 38.5m^2 भए उक ट्याइकीको प्रति लि. रु. 0.24 दरले पानीभन्न जस्मा कति अर्च लाग्न्न ? पत्ता लगाउनुहोस् ।

A water tank is formed with the combination of cylinder and hemisphere. The total height of the tank is 14m area of base is 38.5cm^2 . If the tank is filled with water at the rate of Rs. .24 per liter what is the total cost for the water. Find it.

(Ans: Rs. 118579.20)



- 9.s एक जना मानिसले आफ्नो घरको प्रयोजनका लागि 1.05 m अर्धव्यास र 3.5m उचाई भएको एउटा वृत्ताकार आधार भएको मायिल्लो भाग अर्ध गोलाकार छ भने सो ट्याइकीको कति ली. पानीअटाउँन सकिन्छ होला ?

A person bought a water tank of greater base having the radius 1.05 m and height 3.5m for the use of own from the shop. IF the upper part of the tank is hemispherical. How many liter water will be contained in the tank. Find it.

(Ans: 10914.75liter)

