एकाइ (Unit): 15

वृत (Circle)

साध्य (Theorem) 5

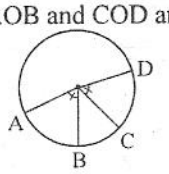
कुनै वृत बराबर केन्द्रिय कोणहरुले बनाउने चापहरु बराबर हुन्छन् ।

If two arc s of a circle subtend equal angles at the centre of the circle, the arcs are equal.

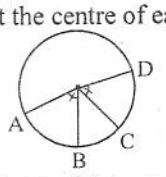
प्रयोगात्मक परीक्षण (Experimental verification)

Step: 1 Two circles with cent re O and different radii are drawn.

Step: 2 Two equal angles AOB and COD are drawn at the centre of each circle.



(i)



(ii)

Step: 3 The lengths of $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are measured with the help of thread and ruler and the result are tabulated.

Figure	arc AB	arc CD	Result
(i)			$\overset{\frown}{AB} = \overset{\frown}{CD}$
(ii)			$\overset{\frown}{AB} = \overset{\frown}{CD}$

Conclusion: If two arcs of a circle subtend equals at the centre of the circle, the circle, the arcs are equal.

Converse of Theorem 5

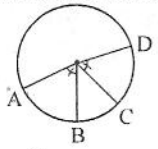
कुनै वुतमा बराबर चापको बनाउने केद्रिय कोणहरु बराबर हुन्छ ।

Equal arcs of a circle substend equal angles at the centre of the circle.

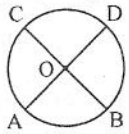
प्रयोगात्मक परीक्षण (Experimental verification)

Step:1 two circle with centre O and different radii are drawn.

Step:2 Two equal arcs AB and CD are drawn with the help of pencil and compasses.



(i)



(ii)

Step:3 ∠AOB and ∠COD are measured and the result are tabulated.

Figure	∠AOB	∠COD	Result
(i)			∠AOB = ∠COD
(ii)			∠AOB = ∠COD

Conclusion: Equal arcs of a circle subtend equal angles at the centre of the circle.

साध्य (Theorem) 6

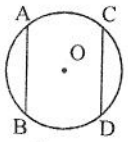
बराबर जिवाहरुले काटेका चापको लम्बाई बराबर हुन्छ ।

Equal chords of a circle from equal arcs in the circle.

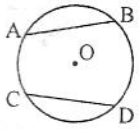
प्रयोगात्मक परीक्षण (Experimental verification)

Step:1 Two circle with centre O and different radii are drawn.

Step: 2 Two equal chords AB and CD are drawn in each circle.



(i)



(ii)

Step:3 The lenth of arcs AB and CD are measured with the help of thread and rular and the results are tabulated.

Figure	arc AB	arc CD	Result
(i)			$\widehat{AB} = \widehat{CD}$
(ii)			$\widehat{AB} = \widehat{CD}$

Conclusion: Equal chords of a circle from equal arcs in the circle.

Converse of Theorem 6

वृत्तका बराबर चापहरुले बनाउने जीवाहरुको लम्बाई बराबर हुन् ।

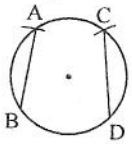
If the arcs formed by chords of a circle are equal, the chords are equal.

प्रयोगात्मक परीक्षण (Experimental verification)

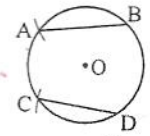
Step:1 Two circles with centre O and different radii are drawn.

Step:2 Two equal arcs AB and CD are drawn with the help of pencil and compasses in each circle.

Step:3 A, B and C and dare join ned to from chords AB and CD.



(i)



(i)

Step:4 The lengths of chords AB and CD are measured and the results are tabulated.

Figure	arc AB	arc CD	Result
(i)			AB = CD
(ii)			AB = CD

Conclusion: If the arcs formed by chords of a circle are equal, the chords are equal.

साध्य (Theorem)7

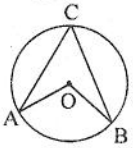
कुनै वृत्तको एउटै चापमा आधारित केन्द्रिय कोण परिधीमा बनेको कोणको दुई गुणा हुन्छ ।

The angle at the centre of a circle is twice the angle at its circumf erence standing on the same arc.

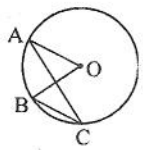
प्रयोगात्मक परीक्षण (Experimental verification)

Step:1 Two circles with centre O and different radii are drawn.

Step:2 $\angle AOB$ is drawn at the centre and $\angle ACD$ is drawn at the circumference of each circle. $\angle AOB$ and $\angle ACB$ are standing on the same arc AB.



(i)



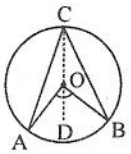
(ii)

Step:3 $\angle AOB$ and $\angle ACB$ are measured and the result are tabulated.

Figure	∠AOB	∠ACB	Result
(i)			∠AOB = 2 ∠ACB
(ii)			∠AOB = 2 ∠ACB

Conclusion: The angle at the centre of a circle is twice the angle at its circumference standing on the same arc.

सैद्धान्तिक प्रमाण (Theoretical proof)



Given: O is the centre of a circle. ∠AOB is the angle at the centre and ∠ACB is the angle at the circumference of the circle. They are standing on the same arc AB.

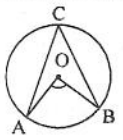
Tp prove: ∠AOB = 2 ∠ACB

Construction: C, O are joined and the line is product to D.

Proof

	Statements		Reasons
1.	In ΔOAC, ∠OAC = ∠OCA	1.	OA = OC (radii of the same circle), so abse angles of isosceles ΔOAC.
2.	∠AOD = ∠OAC + ∠OCA	2.	Exterior and opposite interior angles of ΔOAC.
3.	∠AOD = ∠OCA + ∠OCA = 2 ∠OCA	3.	From statement (1)
4.	Similarly, ∠BOD = 2 ∠OCA	4.	Same as above facts and reasons.
5.	∠AOD + ∠BOD = 2 ∠OCA + 2 ∠OCB.	5.	Adding statements (3) and (4)
6.	∠AOB = 2(∠OCA + ∠ OCA) = 2 ∠ACB	6.	Whole part axiom.

Proved.



वैकल्पिक तरीका (Alternative process)

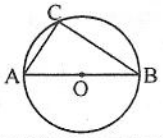
	Statements		Reasons
1.	∠OAC ≐ $\widehat{AB}$	1.	Relation between angle at the centre of a circle and its opposite arc.
2.	∠ACB ≐ $\frac{1}{2}\widehat{AB}$ i.e $\widehat{AB}$ ≐2ACB	2.	Relation between inscribed angle and its opposite arc.
3.	∠AOB = 2 ∠ACB	3.	From the statements (1) and (2)

Proved.

साध्य (Theorem)8

अर्धवृत्तमा बन्ने परिधिकोण एक समकोण हुन्छ ।

The angle in the circumference if a semi – circle is one right angle.



Given: O is the centre of a circle and AB is a diameter of the circle.

To prove: ∴∠ACB = 90°

Proof:

	Statements		Reasons
1.	∠AOB = 180°	1.	∠AOB is a straight angle.
2.	∠ACB = $\frac{1}{2}$ ∠AOB	2.	Inscribed angle is half of the angle at the centre of a circle standing on the same arc.
3.	∠AOB = $\frac{1}{2}$ × 180° = 90°	3.	From the statements (10 and (2)

Note: *A diameter divides a circle into two halves and each half is called a semi – circle. Angle in the same – circle is always a right angle (90°).*

साध्य (Theorem) 9

एउटै वृतखण्डमा बनेका परिधि कोणहरु बराबर हुन्छन ।

Angles in the same segment of a circle are equal.

वा Or,

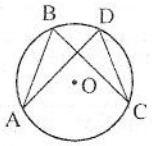
वृत्तको एउटै चापमा बनेको परिधि कोणहरु बराबर हुन्छन ।

The angles at the circumference of a circle (inscribed angles) standing on the same arc are equal.

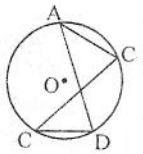
प्रयोगात्मक परीक्षण (Experimental verification)

Step:1 Two circles with centre O and different radii are drawn.

Step:2 Inscribed angles ABC and ADC are drawn in the same segment AB and DC and on the same arc AC.



(i)



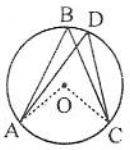
(ii)

Step:3 ∠ABC and ∠ADC measured and the result are tabulated.

Figure	∠ABC	∠ADC	Result
(i)			∠ABC = ∠ADC
(ii)			∠ABC = ∠ADC

Conclusion: Angles in the same segment of a circle are equal.

सैद्धान्तिक प्रमाण (Theoretical proof)



Given: O is the centre of a circle. ∠ABC and ∠ADC are the inscribed angles standing on the same arc AC and in the same segment ABDC.

To prove: ∠ABC = ∠ADC

Construction: O, A and O, C are joined. So, ∠AOC is the angle at the centre of the circle.

	Statements		**Reasons**
1.	$\angle ABC = \frac{1}{2} \angle ADC$	1.	Inscribed angle is half of the angle at the centre of a circle standing on the same arc.
2.	$\angle ADC = \frac{1}{2} \angle AOC$	2.	Same reason as above
3.	∠ABC = ∠ADC	3.	From statements (1) and (2)

साध्य (Theorem) 10

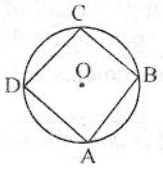
चक्रिय चतुर्भुजको सम्मुख कोणहरु परिपुरक हुन्छन् ।

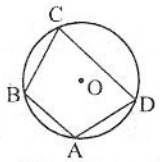
The opposite angles of a cyclic quadrilateral are supplementary.

प्रयोगात्मक परीक्षण (Experimental verification)

Step:1 Three circles with centre O and different radii are drawn.

Step:2 Cyclic quadrilate rals ABCD are drawn in each circle.





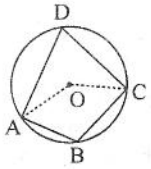
(i) (ii)

Step:3 ∠A, ∠B, ∠C and ∠D of each cyclic quadrilateral are measured and the result are tabulated.

Figure	∠A	∠C	∠A + ∠C	∠B	∠D	∠B+∠D	Result
(i)							∠A + ∠C = 180° ∠B + ∠D = 180°
(ii)							∠A + ∠C = 180° ∠B + ∠D = 180°

Conclusion: The opposite angels of a cyclic quadrilateral are supplementary.

सैद्धान्तिक प्रमाण (Theoretical proof)



Given: O is the centre of a circle. ABCD is the cyclic quadrilateral.

To prove: ∠ABC + ∠ADC = 180° and ∠BAD + ∠BCD = 180°

Construction: O, A and O, C are joined.

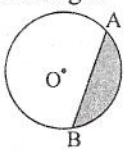
Proof:

	Statements		Reasons
1.	$\angle ABC = \frac{1}{2}$ Ref ∠AOC	1.	Inscribed angle is half of the angle at the centre of a circle standing on the same arc.
2.	$\angle ADC = \frac{1}{2}$ obt. ∠AOC	2.	Same reasons as above.
3.	$\angle ABC + \angle ADC = \frac{1}{2}$ [Ref. ∠AOC + obt. ∠AOC)	3.	Adding the statements (1) nd (2).
4.	$\angle ABC + \angle ADC = \frac{1}{2} \times 360°$ i.e. ∠ABC + ∠ADC = 180°	4.	The sum of reflex and obtuse abgles AOC forms a complete turn.
5.	Similarly, ∠BAD + ∠BCD = 180°	5.	By joining O, B and O, D and following the same facts abd reasons.

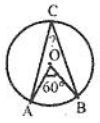
Proved

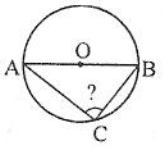
Very Short Questions:

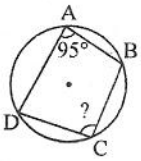
1. एउटा वृतको जीवाहरुका संगत चापहरु बराबर भए जीवाको सम्बन्ध के हुन्छ ? लेख्नुहोस् ।
   Write the relation of circle. if the chords of a circle of corresponding arcs are equal.
2. एउटा अर्ध वृतमा बनेको परिधि कोण कति हुन्छ ? लेख्नुहोस् ।
   Write the, what is the circumference in semi - circle ?
3. एउटै चापमा आधारित परिधि कोण कति हुन्छ ? लेख्नुहोस् ।
   Write the relation between the circumference angles standing on the same arc.
4. चक्रिय चतुर्भुजको सम्मुख कोणहरुको सम्बन्ध लेख्नुहोस् ।
   Write the relation of the opposite angle of a circle quadrilateral.
5. एउटा वृतको बराबर चापको केन्द्रमा बनाएको केन्द्रिय कोणहरुको सम्बन्ध लेख्नुहोस् ।
   If the equal area substebd the centre angle the circle write the relation of the centre angle.
6. दिइएको चित्रमा AB वृतको जिवा हो । छायाँ पारेको भागको नाम लेख्नुहोस् ।
   In the given figure, AB is chord of circle. Write the name of shaded region.

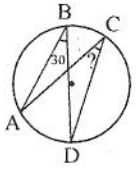


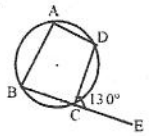
7. एउटै चापमा आधारित केन्द्रिय कोण र परिधि कोणको सम्बन्ध के छ ?
   What is the relation between central angle and inscribed angle on the same arc ?
8. दिइएको चित्रबाट नभएको कोण को मान निकाल्नुहोस् ।
   Find the value of unkown angle of the following.

i. 

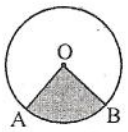
ii. 

iii. 

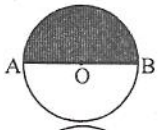
iv. 

v. 

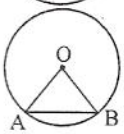
9. दिइएको चित्रमा, o वृतको केन्द्र हो छायाँ पारेको नाम लेख्नुहोस् ।
In the given figure o is the centre of circle, write the name of shaded part of circle.



10. दिइएको चित्रमा AB वृतको व्यास हो छायाँ पारेको भागको नाम लेख्नुहोस् ।
In the given figure, AB is diameter of circle, write the name of shaded region.

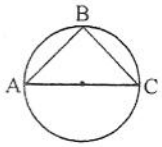


11. दिइएको चित्रमा, O वृतको केन्द्र हो । Δ OAB कुन प्रकार चिभुज हो ? लेख्नुहोस् ।
In the given figure, o is the centre of circle write the name, what kind of Δ OAB.

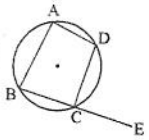


12. दिइएको चित्रमा ∠BAC+ ∠ACB = 90° भए AC भुजाको नाम लेख्नुहोस् ।
In the given figure, if ∠BAC + ∠ACB = 90° write the name of AC side.

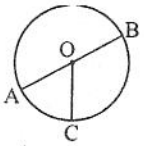
13. दिएको चित्रमा ∠BAD संग कुन कोण बराबर हुन्छ ?
In the given figure write the name which angle will be equal to the ∠ BAD.



14. दिइएको चित्रमा ABC अर्धव्यास हो । AB = 16 cm, BC = 12 cm OB को लम्बाई कति होला ।
In the given figure, ABC is semi - circle of AB = 16 cm, BC = 12 cm what is the length of OB.



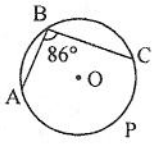
15. दिइएको चित्रमा AB वृत्तको व्यास हो । यदि OC = 6 cm भए AB को मान पत्ता लगाउनुहोस् ।
In the given figure AB is diameter of circle of OC = 6 cm, find the length of AB.



Short Questions

Model 1:

सँगैको चित्रमा, ∠ABC = 86°, $\overset{\frown}{ABC}$ को नाप पत्ता लगाउनुहोस् ।



In the adjoining diagram, ∠ABC = 86°, $\overset{\frown}{ABC}$

Solution: To find: $\overset{\frown}{ABC}$ = ? $\angle ABC \equiv \frac{1}{2} \overset{\frown}{APC}$

[In a circle, inscribed angle $\equiv \frac{1}{2}$ (its opposite arc)]

or, $2 \times \angle ABC \equiv \overset{\frown}{APC}$ or $2 \times 86° = \overset{\frown}{APC}$

$\therefore \overset{\frown}{APC} \equiv 172°$

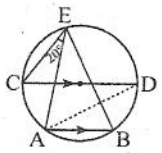
Now, $\overset{\frown}{ABC} \equiv 360° - \overset{\frown}{APC} \equiv 360° - 172° \equiv 188°$

Hence, $\overset{\frown}{ABC} \equiv 188°$s

Model 2

दिइएको चित्रमा **AB \\ CD, ∠ABC = 20° ∠BED** को नाप पत्ता लगाउनुहोस् ।

In the given figure, AB \\ CD, ∠ABC = 20°, then find ∠BED.



Solution:

To find: ∠BED = ?

Construction: Join AD

∠CDA = ∠BAD (Alternate angles are equal; CD \\ AB)

$\overset{\frown}{AC} = \overset{\frown}{BD}$ (∴ ∠CAD = ∠BAD; Arc subtended angels are equal at circumferences, then arcs are also equal)

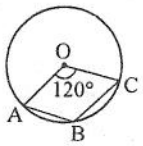
∴ ∠BED = ∠AEC = 20° (∴ $\overset{\frown}{AC} = \overset{\frown}{BD}$; equal arcs subtended angles at circumferences are equal)

Model 3

दिइएको चित्रमा, **O** वृतको केन्द्र हो । **∠AOC = 120° ∠ADC** को नाप कति हुन्छ ।

In the given figure, O is the centre of the circle and ∠AOC = 120°.

What is the measure of ∠ABC.



Solution: To find : ∠ABC = ?

Reflex ∠AOC + obtuse ∠AOC = 360°(Sum of angle around at point)

or, Reflex ∠AOC + 120° = 360° (∴∠AOC = 120°)

∴ Reflex ∠AOC = 360° – 120° = 240°

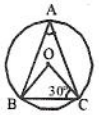
Again,

∴ $\angle ABC = \frac{1}{2}$ × reflex ∠AOC

(In a circle, inscribed angle = $\frac{1}{2}$ of the centre angle standing on same arc) = $\frac{1}{2} \times 240° = 120°$.

Model 4

दिएको चित्रमा, O वृतको केन्द्र हो ।भए यदि ∠OCB = 30° ∠BAC पत्ता लगाउनुहोस् । In the given figure, O is the centre of the circle, ∠OCB = 30°, find the ∠BAC.



Solution:

To find: ∠BAC = ?

∠OBC = ∠OCB = 30° (in ΔOBC, OB = OC)

Again,

∠BOC + ∠OBC + ∠OCB = 100 (Sum of angle of a trangle)

or, ∠BOC + 30° + 30° = 180°

or, ∠BOC + 60° = 180°

∴ ∠BOC = 180° – 60° = 120°

Lastly, $\angle BAC = \frac{1}{2} \times \angle BOC$ (in a circle, inscribed angle = $\frac{1}{2}$ × Centre angle standing on same arcs)

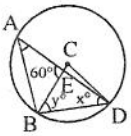
$= \frac{1}{2} \times 120° = 60°$

Hence, ∠BAC = 60°

Model 5

दिइएको चित्रमा, **C** वृतको केन्द्र **∠DAB = 40°, ∠AED = 60° ∠CBD = y° ∠ADB= x° x र y** को मान पत्ता लगाउनुहोस्।

In the given figure, C is the centre of the circle, ∠DAB = 40°, ∠AEB = 60°, ∠CBD= y° and ∠ADB = x°, find the values of x and y.



Solution:

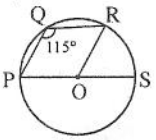
Given: In the given, C is the centre of the circle,
$\angle DAB = 40°$, $\angle AEB = 60°$ $\angle CBD = y°$ and $\angle ADB = x°$
To find: $x°$ and $y°$
Here, $\angle BCD = 2\angle BAD = 2 \times 40° = 80°$
In a circle, Central angle = 2 × inscribed standing on same arc BD
Again,
In ΔBCD, $\angle CBD = \angle CDB = y$ [$\therefore \Delta BCD$, BC = CD]
$\angle BCD + \angle CBD + \angle CDB = 180°$ [$\therefore$ Sum of angles of a triuangle = 180°]
or, $80° + y + y = 180°$
or, $2y = 180° - 80°$
$\therefore y = \frac{100°}{2} = 50°$

Listly, in ΔEBD
$\angle AEB = \angle EDB + \angle EBD$ [In ΔEBD, ecterior anglea = sum its opposite interior angle]
or, $60° = x + y$
or, $60° = x + 50°$
or, $x = 60° - 50°$
$\therefore$ $x = 10°$

Model 6

दिएको चित्रमा, O वृतको केन्द्र यदि $\angle PQR = 115°$ भए $\angle ROS$ को मान पत्ता लगाउनुहोस् ।

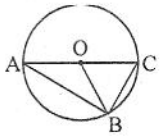
In the given figure, O is the centre of the circle. If $\angle PQR = 115°$, find the value of $\angle ROS$.



Solution: To find: $\angle ROS$
Here, Reflex $\angle POR = 2 \times \angle PQR$
[In a circle, cental angle = 2 × inscribed angle standing on same arc]
$= 2 \times 115° = 230°$
Again, Obtuse $\angle POR$ + Reflex $\angle POR = 360°$ [Sum od angle at a point = 360°]
Obtuse $\angle POR + 230° = 360°$
$\therefore$ Obtuse $\angle POR + \angle ROS = 180°$ [Straight angles = 180°]
or, $130° + \angle ROS = 180°$
$\therefore$ $\angle ROS = 180° - 130° = 50°$
Hence, $\angle ROS = 50°$

Model 7

दिइएको चित्रमा, O वृतको केन्द्र हो । यदि $2\angle A = \angle C = x°$ भए पत्तन लगाउनुहोस् ।



In the given figure, O is the centre of circle,
If $2\angle A = \angle C = x°$, find the value of $\angle OBC$.

Solution:

To find: $\angle OBC$
Here, $\angle ABC = 90°$ (Angle in semicircle)
$\angle A + \angle C + \angle ABC = 180°$ (Sum of angels of a ΔABC)
or, $\frac{x}{2} + x + 90° = 180°$ [$\therefore 2\angle A = \angle C = x$, $\therefore \angle A = \frac{x}{2}$]
or, $\frac{x + 2x}{2} = 180° = 90°$
or, $\frac{3x}{2} = 90°$

$\therefore \quad x = 90 \times \frac{2}{3}$

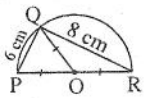
Lastly,

$\therefore \quad \angle OBC = \angle OCB = x = 60°$ [In ΔOBC, OB = OC]

$\therefore$ Hence, $\angle OBC = 60°$

Model 8

दिइएको चित्रमा केन्द्र O भएको PQR अर्धवृत्त PQ = 6cm QR = 8cm भए OQ को मान पत्ता लगाउनुहोस् ।



In the semi – circle PQR of centre O. PQ = 6cm, QR = 8cm. Find the length of OQ?

Solution:

To find = OQ

Here, $\angle PQR = 90°$ (Angle in semicircle) = 90°

Now, $PR^2 + QR^2$ (In right angled ΔPQR, $h^2 = p^2 + b^2$)

or, $PR^2 = 6^2 + 8^2 = 36 + 84 = 100$

$\therefore \quad PR = \sqrt{100} = 10cm$

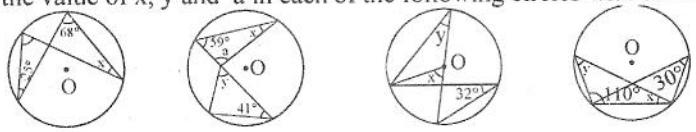
$\therefore \quad OQ = \text{radius} = \frac{\text{Diameter}}{2} = \frac{QR}{2} = \frac{10}{2} = 5 \text{ cm}$

Hence, OQ = 5 cm

Model 9

केन्द्र भएको वृतहरुको x, y र a को मान पत्ता लगाउनुहोस् ।

Calcualte the value of x, y and a in each of the following circles with centre O.



Solution:

(i) here, x = 25° (angle subtended by same arc)
Again, y = 68° (angle subtended by same arc)
Hence x = 25°, y = 68°

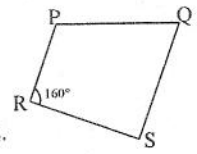
(ii) Here, x = 41° (angles subtended by same arc)
Again a + 59° + x = 180° (Sum of angle of a triangle)
or, a + 59° + 41° = 180°
$\therefore$ a = 180° – 100° = 80°
$\therefore$ y = a = 80° (vertically opposite angles are equal)
Hence, x = 41°, y = a = 80°

(iii) Hence, y = 32° (angles subtended by same arc)
Again, x = 2 × 32° (In a circle, centre angle 2 × inscribed angl;e standing on same arc) = 64°
Hence, x = 64° and y = 32°

(iv) Here, y = 30 (angles subtended by same arc)
Again x + y + 100° = 180° (Sum of angle of a Δ)
x + 30° + 100° = 180°
$\therefore$ x = 180° – 130° = 50°
Hence, x = 50° and y = 30°

Model 10

दिइएको चित्रमा यदि y = 160° ∠P = 20 भए PQRS चक्रिय विन्दुहरु हुन ।



In the given figure, If y = 160°, ∠P = 20°, then P, Q R and S are concyclic points.

Solution:

To prove: P Q R and S are concyclic points.

Here,

$\angle QSR + \angle RST = 180°$ [Straight angles = 180°]

or, $x + y = 180°$

or, $x + 160° = 180°$ [$\because$ 160° (given)]

or, $x = 180° - 160°$

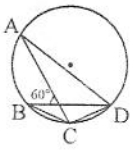
$\therefore \quad x = 20°$

Thus, $\angle QPR (\angle P) = \angle QSR = 20°$ [$\because \angle P = 20°$]

Hence, P, Q R and S are concyclic points, (If a line joining two points subtendes equal angles at other two points on the same side of the line then the tour points are concylic)

Model 11

सँगै रहेको चित्रमा $\angle DAC$ र $\angle ACB$ को मान पत्ता लगाउनुहोस् ।



From the adjoining diagram calculate $\angle DCA$ and $\angle ACB$.

Solution:

To find: $\angle DAC$ and $\angle ACB$.

Now,

$\angle AEB = \angle EAD + \angle ADE$

[In a triangle AED, exterior angle = sum of its opposite interior angles]

$60° = 25° + \angle ADE$

$\therefore \quad \angle ADE = 60 - 25 = 35°$

Again, $\angle ACB = \angle ADE$ (i.e. $\angle ADE$) = 35° (Angles in same segment)

Now, $\angle DCB + \angle DAB$ 180° [Sum of opposite angles of cyclic quadrilateral)

$\angle DCB + (25° + 50°) = 180°$

$\therefore \quad \angle DCB = 180° - 75° = 105°$

and $\angle DCB = \angle DCA + \angle ACB$ (Whole parts axiom)

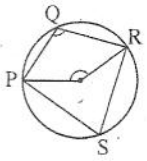
$105° = \angle DCA + 35°$

$\therefore \quad \angle DCA = 105° - 35° = 70°$

Hence, $\angle DCA = 70°$ and $\angle ACB = 35°$

Model 12

दिएको चित्रमा O वृतको केन्द्र हो यदि $\angle PQR = 95°$ भए $\angle PSR$ र अर्धकोण $\angle POR$ मान पत्ता लगाउनुहोस् ।



In the given figure, O is the centre of the circle. If $\angle PQR = 95°$. Find the measures of $\angle PSR$ and $\angle POR$.

Solution:

To find : $\angle PSR$ and $\angle POR$.

Here, $\angle PSR + \angle POR = 180°$ [Sum of opposite angles of cyclic quadrilateral = 180°]

or, $\angle PSR + 95° = 180°$

$\therefore \angle PSR = 180° - 95° = 85°$

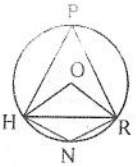
Again,

$\angle POR = 2 \angle PSR$ [In a circle, centre angles = 2 × inscribed angles standing on same arc]

Hence, $\angle PSR = 85°$ and $\angle POR = 170°$

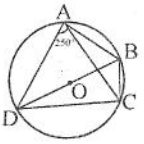
Model 13

सँगैको चित्रमा O वृत्तको केन्द्र हो। र MNव्यास हो। यदि MN || RP र $\angle PQR$ को मान पत्ता लगउनुहोस्। In the adjoining figure O is the cantre of circle the and MN diameter of circle Find the value of $\angle PQR$ if MN || RP $\angle NPR = 25°$



Model 17

संगैको चित्रमा O वृत्तको केन्द्रविन्दु हो। यदि $\angle DAC = 25^0$ $\angle BOC$ र $\angle BAC$ को मान पत्ता लगाउनुहोस्। In adjoining figure o is the centre of circle of $\angle DAC = 25^0$, find the $\angle BDC$ and $\angle BAC$ (SEE2074)



Solution: To find : BDC = ? $\angle BAC$ = ?

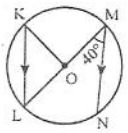
(i) $\angle BAD = 90^0 \rightarrow$ inscribed angleof based on the diameter

(ii) $\angle BAC = 90^0 - 25^0 \rightarrow$ Substraction
$= 65^0$

(iii) $\angle BAC = \angle BDC = 65^0 \rightarrow$ inscribed anlges on the same BC

Model 18

चित्रमा O वृत्तको केन्द्र विन्दु हो। MN \\KL छ। यदि $\angle NML = 40^0$ भए $\angle KOM$ को मान पत्ता लगाउनुहोस्। In the figure O is the centre of circle and MN//KL If $\angle NML = 40^0$, fidn $\angle KOM$



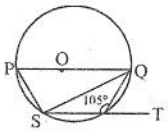
Solution: To find : $\angle KOM$ = ?

(i) $\angle NML = \angle KLM = 40^0 \rightarrow$ alternate angles between KL//MN

(ii) $\angle KOM = 2\ \angle KLM \rightarrow$ Relation between centre angle and inscribed angle on the same arc KM
$= 2 \times 40^0 = 80$

Model 19

दिइएको चित्रमा O वृत्तको केन्द्र हो। यदि $\angle QRS = 105^0$ भए $\angle PQS$ को मान कति हुन्छ ? पत्ता लगाउनुहोस् In the given figure, O is the entre of circle if $\angle QRs = 105^0$ find the $\angle PQS$.



Solution: To find : $\angle PQS$ = ?

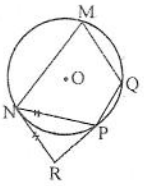
(i) $105^0 + \angle SPQ = 180 \rightarrow$sum of opposite angle of cyclic quad^r PQRS
$\angle SPQ = 180^0 - 105^0 = 75^0$

(ii) $\angle PSQ = 90^0 \rightarrow$ inscribed angle on the diameter PQ

(iiii) $90^0 + 75 + \angle PQS = 180^0 \rightarrow$ Sum of three angle of $\Delta PQS = 15^0$

Model 20

दिइएको चित्रमा O वृत्तको केन्द्र र MNPQ चक्रिय चतुर्भुज हो। यदि NR -= NP र $\angle PRR = 50^0$ भए $\angle NMQ$ को मान पत्ता लगाउनुहोस्। In the given figure O is the centreof crcle and MN PQ is cycleic quadr. If NR = NP and $\angle PNR = 550^0$ find $\angle NMQ$.



Solution: To find $\angle NMQ$ = ?

(i) $50 + 2\angle NRP = 180 \rightarrow$ sun of three angle ΔNRP

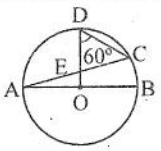
or, $2\angle NRP = 180^0 - 50^0$

or, $\angle NRP = \frac{130^0}{2} = 65 = \angle NPR$

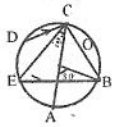
(ii) $\angle NMQ = \angle NPR = 65^0 \rightarrow$ extieror angle opposite angle of adjoining interior angle of quad^r.

Practice Yourself

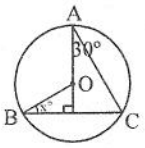
1. दिइएको चित्रमा AB वृतको व्यास हो । DO ⊥ AB र ∠ COD = 60° भए ∠ CEO को मान पत्ता लगाउनुहोस् ।
In the given figure AB is a diameter of circle of DO ⊥ AB and ∠ CDO = 60°, find ∠ CEO. (Ans: 105°)



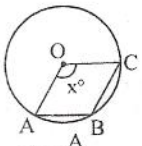
2. चित्रमा AC वृतको व्यास हो । यदि DC // EB
∠ ECA = 10° ∠AOB = 80° भए
∠DCB को मान पत्ता लगाउनुहोस् ।
In the given figure AC is a diameter. (Ans: 100°)
If DC // EB ∠ ECA = 10° ∠AOB = 80°, find the value of ∠DCB.



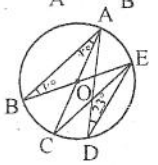
3. दिएको चित्रमा AD ⊥ BC र O वृतको केन्द्र विन्दु हो । यदि ∠ DAC = 30° भए x को मान पत्ता लगाउनुहोस् ।
In the given figure AD ⊥ BC O is centre of circle if ∠ DAC = 30° find the value of x. (Ans: 30°)



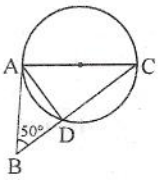
4. दिएको चित्रमा O वृतको केन्द्र हो । OABC एउटा स.च. भए x को मान पत्ता लगाउनुहोस् ।
In the given figure, o is the centre of circle. If OABC is parm. Find the value of x.(Ans. 120°)



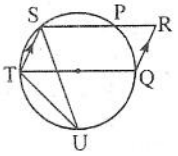
5. दिइएको चित्रमा O वृतको केन्द्र हो । x र y को मान पत्ता लगाउनुहोस् ।
In the given figure, O is the centre of circle them find the value of x and y. (Ans: 23°, 23°)



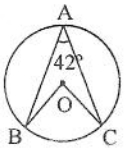
6. सँगै चित्रमा AC वृत्तको व्यास BC को मध्य विन्दु D र ∠ABC=50° भए ∠ACB को मान पत्ता लगाउनुहोस् ।
In the adjoining figure, AC is a diameter of the circle D is mi point of BC are ∠ ABC = 50° find thje value of ∠ ACB. (Ans: 50)



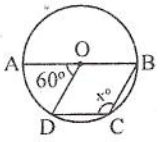
7. चित्रमा O वृत्तको केन्द्र विन्दु र SPQR स.च. हो । यदि ∠SRQ = 42° भए ∠SPQ ⊥ ∠SOT को मान पत्ता लगाउनुहोस् ।
In the figure, O is the centre circle and circle and STQR is parm. If ∠ SRQ = 42°, find ∠SPQ and ∠SOT. (Ans: 138°)



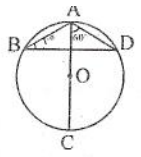
8. दिएको चित्रमा, O वृतको केन्द्र विन्दु हो । यदि ∠BAC = 42° भए ∠OBC को मान पत्ता लगाउनुहोस् ।
In the given figure, o is the centre of circle if ∠ BAC = 42°, find ∠ OBC. (Ans: 36°)



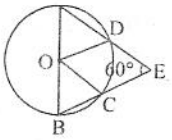
9. दिएको चित्रमा x को मान पत्ता लगाउनुहोस् ।
Find the value of x, from the given figure. (Ans: 35°)



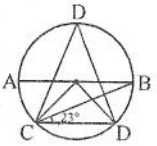
10. दिइएको चित्रमा O वृतको केन्द्र विन्दु हो । x को मान पत्ता लगाउनुहोस् ।
In the given figure O is the the centre of circle, find the value of x. **(Ans: 120°)**



11. दिइएको चित्रमा वृत्तको केन्द्र विन्दु O हो । y को मान पत्ता लगाउनुहोस् ।
In the given figure O is the centre of circle, find the value of y. **(Ans: 30°)**



12. दिइएको चित्रमा p वृत्तको केन्द्र र AB व्यास हो । यदि ∠AEB = 60° भए ∠COD को मान पत्ता लगाउनुहोस् ।
In the given figure, O is the centre of circle and AB diameter. If ∠AEB = 60° find ∠COD. **(Ans: 30°)**



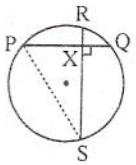
13. दिइएको चित्रमा AB व्यास र O वृतको केन्द्र हो । यदि ∠BCD = 23° , भए ∠ CPD को मान पत्ता लगाउनुहोस् ।
In the given figure AB diameter and o is centre of circle. If ∠BCD = 23°, find ∠ CPD. **Ans: 67°**

14. दिइएको चित्रमा O वृतको केन्द्र हो । b र y को मान पत्ता लगाउनुहोस् ।
In the given figurem O is the centre of ciecle, find the value of b and y.

Long Questions

Model 1

दिइएको चित्रमा, PQ र RS जीवाहरु विन्दु X मा समकोण हुने गरि काटिएका छन । प्रमाणित गर्नुहोस् ।
$(\overline{PQ} - \overline{PS}) = (\overline{PR} - \overline{PQ})$



In the given, two chords PQ and Rs intersect at right angle at the point X, Prove that arc [PS – QS] = arc [PR – QR]

Solution:

Given: Chords PQ and RS intersect at right angles at the point X.

To prove: arc [PS – QS) = arc [PR – QR]

Construction: P and S joined.

Proof:

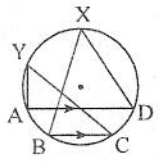
	Statements		Reasons
1.	$\angle SPQ = \frac{1}{2}\ \overset{\frown}{QS}$	1.	Relation between inscribed angle and its opposite arc.
2.	$\angle PSR = \frac{1}{2}\ \overset{\frown}{PR}$	2.	Same reason as above.
3.	$\angle SPQ + \angle PSR \overset{o}{=} \frac{1}{2}(\overset{\frown}{QS} + \overset{\frown}{PR})$	3.	Adding statement (1) and (2)
4.	$90° = \frac{1}{2}(\overset{\frown}{QS} + \overset{\frown}{PR})$ i.e. $\overset{\frown}{QS} + \overset{\frown}{PR} = 180°$	4.	ΔPXS is a right angled triangle and ∠X is a right angle and the sum of two acute angles of a right angled Δ
5.	$\overset{\frown}{PS} + \overset{\frown}{QR} = 180°$	5.	$\overset{\frown}{PS} + \overset{\frown}{SQ} + \overset{\frown}{QR} + \overset{\frown}{RP} = 360°$ and from statement (4)
6.	$\overset{\frown}{PS} + \overset{\frown}{QR} = \overset{\frown}{QS} + \overset{\frown}{PR}$ i.e. arc (PS – QS) = arc (PR – QR)	6.	From sta tement (4) and (5)

Proved.

Model 2

संगैको वृत्तकोण, AD//BC भए प्रमाणित गर्नुहोस् ।

In the adjoining circle, AD//BC Prove that $\angle AYC = \angle BXD$.



Given: In the given circle, AD//BC.

To prove: $\angle AYC = \angle BXD$

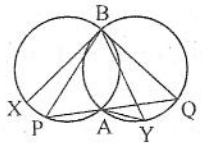
Proof

	Statements		Reasons
1.	$\overset{\frown}{AB} = \overset{\frown}{CD}$	1.	AD//BC and they subtend equal arcs.
2.	$\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{BC} + \overset{\frown}{CD}$	2.	Adding $\overset{\frown}{BC}$ to both sides of statement 1.
3.	$\overset{\frown}{ABC} = \overset{\frown}{BCD}$	3.	Circumference angles of the circle standing on the equal arcs.
4.	$\angle AYC = \angle BXD$	4.	From (3)

Proved

Model 3

दिएको चित्रमा दुई वृत्तहरू A र B प्रतिच्छेदन भएका छन । यदि PAQ र XAY दुई सीधा रेखाहरु परिधि सम्म खिचिएका छन भने प्रमाणित गर्नुहोस् । $\angle PBX = \angle QBY$



In the given figure, two circles intersect at A and B. Through A two straight lines PAQ and XAY are drawn terminated by the circumference.

$\angle PBX = \angle QBY$

Solution:

Given: A and B are the points of in tersection of two circles. Two straight lines PAQ and XAY are passing through A.

To prove: $\angle PBX = \angle QBY$

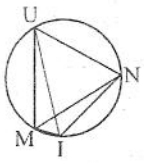
Proof:

	Statements		Reasons
1.	$\angle PBX = \angle PAX$	1.	Circumference angles standing on the same $\overset{\frown}{XP}$
2.	$\angle PAX = \angle QAY$	2.	Vertically opposite angles.
3.	$\angle QAY = \angle QBY$	3.	Circumference angles standing on the same $\overset{\frown}{QY}$
4.	$\angle PBX = \angle QBY$	4.	From statement (1), (2) and (3).

Proved

Model 4

दिइएको चित्रमा, यदि **MN = TU** भए प्रमाणित गर्नुहोस् ।



In the given figure, If MN = IU, ptove that :

(i) MU = IN (ii) MI//UN

Solution:

Given: MN and IU are two chords of a circle and MN = IU.

To prove: (i) MU = IN and (ii) MI//UN

Proof

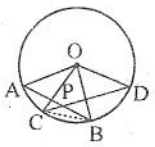
	Statements		Reasons
1.	$\overset{\frown}{UMI} = \overset{\frown}{NIM}$	1.	Arcs made by two equal chords MN and IU.
2.	$\overset{\frown}{UM} + \overset{\frown}{MI} = \overset{\frown}{NI} + \overset{\frown}{MI}$	2.	Whole parst axiom
3.	$\overset{\frown}{UM} = \overset{\frown}{NI}$	3.	$\overset{\frown}{MI}$ is cancelled from both sides of statement (2).

4.	MU = IN	4.	From statement (3), corresponding arcs are equal.
5.	$\angle MNU = \angle IMN$	5.	The in scribed angled standing on the equal arcs ($\overset{\frown}{UM} = \overset{\frown}{NI}$)
6.	MI//UM	6.	From statement (5), being the alternate angles equal.

Proved

Model 5

दिइएको चित्रमा, O वृत्तको केन्द्र हो । यदि दुई जिवाहरुमा AB र CD विन्दु P मा प्रतिच्छेदन भएका छन भने प्रमाणित गर्नुहोस् । $2\angle APC = \angle AOC + \angle BOD$



In the given figure, O is the centre of the circle. If two chords AB and CD intersect at a point P.

Prove that: $2\angle APC = \angle AOC + \angle BOD$

Solution:

Given: O is the centre of a circle. Chords AB and CD intersect at the point P.

To prove: $2\angle APC = \angle AOC + \angle BOD$

Construction: B and C are joined

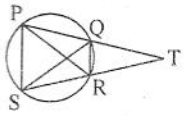
Proof:

	Statements		**Reasons**
1.	$\angle ABC = \frac{1}{2}\angle AOC$	1.	Relation between inscribed angle and centr e angle standing on the same arc.
2.	$\angle BCD = \frac{1}{2}\angle BOD$	2.	Same reason as in (1)
3.	$\angle ABC + \angle BDC = \frac{1}{2}(\angle AOC + \angle BOD)$	3.	Adding statement (1) and (2)
4.	$\angle APC = \frac{1}{2}(\angle AOC + \angle BOD)$	4.	In ΔAPC is the exterioe angle.

Proved.

Model 6

दिइएको चित्रमा, PQRS चक्रिय चतुर्भुज हो । PQ र RS लाई T सम्म लम्बाउँदा यदि QT = RT भए प्रमाणित गर्नुहोस् ।



In the given figure, PQRS is a cyclic quadrilateral; PQ and SR are producded to meet at T. If QT = RT, prove that:

(i) PS//QR (ii) PR = QS.

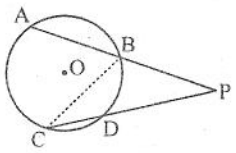
Proof:

	Statement		**Reasons**
1.	$\angle TQR = \angle TRQ$	1.	QT = RT and base angles of isosceles $\angle TQR$
2.	$\angle TQR = \angle PSR$	2.	The exterior angle of a cyclic quadrilateral is equal to the opposite in terior angle.
3.	$\angle TQR = \angle SPQ$	3.	Same rasons as above
4.	$\angle PSR = \angle TRQ$ and $\angle SPQ = \angle TQR$	4.	From statements (1), (2) amd (3)
5.	PS//QR	5.	From statement (4), corresponding amgles are equal.
6.	In Δ^s PRT and SQT (i) $\angle TPR = \angle TSQ$ (ii) $\angle PTR = \angle STQ$ (iii) $\angle RT = QT$ (iv) $\therefore \Delta PRT \cong \Delta SQT$	6.	 (i) Inscribed abgles standing on the same arc (ii) Common angle (iii) Given (iv) A.S.A axiom
7.	PR = QS	7.	Corresponding sides of congruent triangles

Proved

Model 7

यदि दुई जिवाहरु AB र CD लम्बाउदा बाह्य विन्दु P मा प्रतिच्छेदन भएका छन भने प्रमाणित गर्नुहोस् ।



$\angle APC = \frac{1}{2}(\overset{\frown}{AC} - \overset{\frown}{BD})$

Solution: Here,

Given: (i) Chor ds AB and CD of a circle inter sect at an ex ternal point P.

To prove: $\angle APC = \frac{1}{2}(\overset{\frown}{AC} - \overset{\frown}{BD})$

Construction: Join B and C.

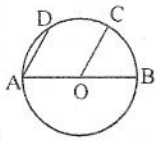
Proof

	Statements		Reasons
1.	$\angle APC = \frac{1}{2}\overset{\frown}{AC}$	1.	Relation between inscribed angle ABC and its opposite arc AC.
2.	$\angle APC = \frac{1}{2}\overset{\frown}{BD}$	2.	Relation between inscribed angle BCD and its opposite arc BD.
3.	$\angle BPO = \angle ABC - \angle BCD$	3.	Exterior and opposite interior angle of a triangle.
4.	$\angle BPO = \frac{1}{2}(\overset{\frown}{AC} - \overset{\frown}{BD})$	4.	From statement 1 and 2
5.	$\therefore \angle BPD = \angle APC = \frac{1}{2}(\overset{\frown}{AC} - \overset{\frown}{BD})$	5.	Same angle in the figure .

Proved

Model 8

दिइएको चित्रमा, O वृत्तको केन्द्र र AB ब्यास र $\overset{\frown}{BC} = \overset{\frown}{CD}$ भए प्रमाणित गर्नुहोस् AD//OC



In the given figure, O is the centre of the circle, AB the diameter and $\overset{\frown}{BC} = \overset{\frown}{CD}$. prove that: AD//OC.

Solution: Here,

Given: O is the center of the circle, AB be the diameter and $\overset{\frown}{BC} = \overset{\frown}{CD}$

To prove: AD//OC

Proof

	Statements		Reasons
1.	$\angle BOC = \overset{\frown}{BC}$	1.	Angle at the center is equal to the degree measure of its opposite arc.
2.	$\angle DAB = \frac{1}{2}\overset{\frown}{BD}$	2.	Angle at the circumference is equal to half of degree measure of its opposite arc.
3.	$\angle DAB = \frac{1}{2}2\overset{\frown}{BC} = \overset{\frown}{BC}$	3.	$\overset{\frown}{BC} = \overset{\frown}{CD}$ (given)
4.	$\angle BOC = \angle DAB$	4.	Statements (10 and (3)
5.	AD//OC	5.	From 94), corresponding angles are equal.

Proved

Model 9

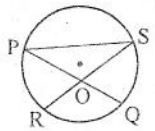
यदि PQ र RS दुई जीवाहरु वृत्तको चित्र X विन्दुको प्रतिच्छेदन हुन्छ प्रमाणित गर्नुहोस् ।

If two chords PQ and RS of a circle intersect at as internal point A, prove that:

(i) $\angle PXR = \angle QXS = \frac{1}{2}(\overset{\frown}{PR} + \overset{\frown}{SQ})$

(ii) $\angle PXQ = \angle PXS = \frac{1}{2}(\overset{\frown}{PS} + \overset{\frown}{RQ})$

Solution: Here,
Given: The chords PQ and RS of a circle meet at an internal point X.



To prove: (i) $\angle PXR = \angle QXS = \frac{1}{2}(\overset{\frown}{PR} + \overset{\frown}{SQ})$

(ii) $\angle PXQ = \angle PXS = \frac{1}{2}(\overset{\frown}{PS} + \overset{\frown}{RQ})$

Construction: join P and S.
Proof:

	Statements		Reasons
1.	$\angle PXR = \angle XSP + \angle XPS$	1.	In atriangle, the exterior angle = sum of the inter opposite angles.
2.	$\angle PXR = \angle RSP + \angle QPS$	2.	Comparing with the figure.
3.	$\angle PXR = \frac{1}{2}\overset{\frown}{PR} + \frac{1}{2}\overset{\frown}{SQ}$	3.	Relation between the inscribed angle and the arc on which it stand.
4.	$\angle PXR = \angle QXS = \frac{1}{2}(\overset{\frown}{PR} + \overset{\frown}{SQ})$	4.	Vetically opposite angle and from (3).
5.	$\angle RXQ = \angle PXS = \frac{1}{2}(\overset{\frown}{PS} + \overset{\frown}{RQ})$	5.	Similar to reason (1).

Proved

Model 10

दिइएको चित्रमा **AB//CD** $\angle$**AQC** र $\angle$BPD अन्तर्गत कोणहरु $\overset{\frown}{AC}$ र $\overset{\frown}{BD}$ क्रमश उभिएका छन । प्रमाणित गर्नुहोस् । $\angle ACQ = \angle BPD$

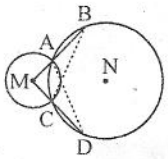
In the given figure, AB // CD $\angle$AQC and $\angle$BPD are inscriced angles standing on the arcs AC and BD respectively, prove that: $\angle ACQ = \angle BPD$.
Solution: Here,
Given: (i) AB//CD
(ii) $\angle$AQC and $\angle$BPD are inscribed angle standing on arcs AC and BD respectively.
To prove: $\angle ACQ = \angle BPD$.
Proof:

	Statements		Reasons
1.	AB // CD	1.	Given
2.	arx AC = arc BD	2.	AB//CD
3.	$2\angle AQC = 2\angle BPD$	3.	Relation between the inscribed angle and the arc on which it stands.
4.	$\angle ACQ = \angle BPD$	4.	Dividing both the sides by 2.

Proved

Model 11

दिएको चित्रमा M र N केन्द्र भएको दुई वृतहरु A र B मा प्रतिच्छेदन भएका छन । यदि MA र MC लाई क्रमश B र D प्रमाणित गर्नुहोस् AB = CD



In the given diagram M and N are centres of two circle, which intersect at A and C, MA and MC are produced to meet the other circle at B and D, prove that AB = CD.
Solution: Here,
Given: (i) M and N are centres of two circles intersecting at A and C respectively.
To prove: AB = CD
Construction: Join AD and BC

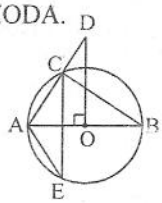
Proof:

	Statements		Reasons
1.	In ΔBCM and ΔADM	1.	
	(i) ∠CBM = ∠ADM (A) (ii) ∠BMC = ∠DMA (A) (iii) MC = MA (S)		(i) Angles in the same segmenmt (ii) Common angle (iii) Radii of same circle
2.	∴ ΔBCM ≅ ΔDAM	2.	AAS fact
3.	MB = MD	3.	Corresponding sides of the congruent triangles
4.	MB – MA = MD – MC	4.	Subtracting 1 (iii) from the statement (3)
5.	AB = CD	5.	Remaining facts

Proved

Model 12

दिइएको चित्रमा, O वृत्तको केन्द्र र AB व्यास हो । DO ⊥ AD भए प्रमाणित गर्नुहोस् । ∠AEC = ∠ODA.



In the given gifure, O is the centre of the circle, AB the diameter and DO ⊥ AD, prove that ∠AEC = ∠AOD.

Solution: Here,

Given: O is the center and Ab is diameter of circle AEBC.

To prove: ∠AEC = ∠ODA

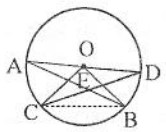
Proof:

	Statements		Reasons
1.	In ΔACB and ΔADO	1.	
	(i) ∠ACB = ∠AOD (ii) ∠BAC = ∠DAO (iii) ∠ODA = ∠ABC		(i) Both are right angle (ii) Common angle (iii) Remaining angles in a triangle
2.	∠AEC = ∠ABC	2.	Angles standing on the same arc
3.	∠AEC = ∠ODA	3.	From the statements (2) and 1 (iii)

Proved

Model 13

दिएको चित्रमा O दिइएको वृत्तको केन्द्र हो । AB र CD जीवाहरु विन्दु E मा प्रतिच्छेदन भएका छन । प्रमाणित गर्नुहोस् । ∠AOC + ∠BOD = 2∠AEC



In the given figure, O is the centre of the given circle, and chords AB and CD intersect at a point E inside the circle.

Prove that: ∠AOC + ∠BOD = 2 ∠AEC.

Solution:

Given: (i) O is the centre of a circle

(ii) Chords AB and CD interse ct at point E .

To prove: ∠AOC + ∠BOD = 2 ∠AEC.

Construction: Join BC.

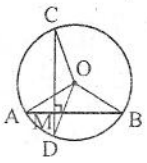
Proof:

	Statements		Reasons
1.	∠AOC = 2 ∠ABC	1.	Angle of the centre of a circle is twice the inscribed angle, standing on the same arc AC.
2.	∠BOD= 2 ∠BCD	2.	Angle at the centre of a circle is twice the inscribed angle, standing on the same arc BD.
3.	∠AOC + ∠BOD = (∠ABC + ∠BCD)	3.	Adding the statement (1) and (2)
4.	∠AEC = ∠EBC + ∠BCE	4.	Exterior angle of a triangle is equal to the sum of interior opposite angles.
5.	∠AOC + ∠BOD = 2∠AEC	5.	From the statement s (3) and (4).

Proved

Model 14

सँगैको चित्रमा AB र CD जीवाहरु लम्ब हुने गरि काटिएका छन् भने प्रमाणित गर्नुहोस् ।

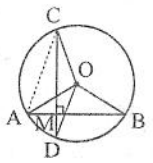


$\angle AOD + \angle BOC = 180°$

In the adjoining fig ure, the chords AD and CD are perpendiculars to each other then prove that:

$\angle AOD + \angle BOC = 180°$

Solution: Here,



Given: (i) O is the centre of a circle.
(ii) AB ⊥ CD.
(iii) Join OA, OB, OD and OC

To prove: $\angle AOD + \angle BOC = 180°$

Construction:

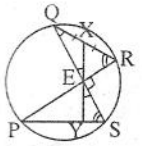
Proof:

	Statements		Reasons
1.	$\angle AMC = 90°$	1.	Being AB ⊥ CD.
2.	$\angle ACM + \angle MAC = 90°$	2.	Sum of a cute angles of a right angled triangle.
3.	$\angle AOD = 2\angle ACM$	3.	Angle at the centre is twice the angle at the circumference.
4.	$\angle BOC = 2\angle MAC$	4.	Same as the above the reason (3)
5.	$\angle AOD + \angle BOC = 2(\angle ACM + \angle MAC)$	5.	Adding the statements (3) and (4)
6.	$\angle AOD + \angle BOC = 2 \times 90°$ $\therefore \angle AOD + \angle BOC = 180°$	6.	From the statements (2) and (5)

Proved

Model 15

दिएको चित्रमा वृत्तको PR र QS जीवाहरु विन्दु E मा समकोण हुनेगरी प्रतिच्छेदन भएका छन । यदि X विन्दु QR को मध्य बिन्दु XE लाई PS को Y सम्म लम्बाईको छ भने प्रमाणित गर्नुहोस् ।



EY ⊥ PS

In the given figure, chord PR and QS of a circle intersect point E at right angles, X is the mid point of QR and XE produced meets PS at Y. Prove that: EY ⊥ PS.

Solution: Here,

Given: (i) Chord PR and QS of a circle interest at point E at a right angle.
(ii) X is mid point of QR. (i.e QX = XR)
(iii) XE produced meets PS in Y.

To prove: EY ⊥ PS

Proof:

	Statements		Reasons
1.	QX = XR = XE	1.	In right angled triangle a line joining the mid point of hypotenuse to its opposite vertex.
2.	$\angle EQX = \angle QEX$	2.	Being QX = Ex
3.	$\angle QEX = \angle YES$	3.	Vertically opposite angles.
4.	$\angle QRP = \angle QSP$	4.	Angles
5.	$\angle R + \angle EQX = 90°$	5.	The sum of acute angles of right angled triangle QER.
6.	$\angle S + \angle YES = 90°$	6.	From the statements (2), (3), (4) and (5)
7.	$\angle EYS + \angle S + \angle YES = 180°$	7.	The sum of interior angles of triangles.
8.	$\therefore \angle EYS = 90°$	8.	From the statements (6) and (7)
9.	$\therefore$ EY ⊥ PS	9.	Being $\angle EYS = 90°$

Proved.

Model 16

दिएको चित्रमा, MN वृत्तको व्यास हो । यदि PR = PQ भए प्रमाणित गर्नुहोस् ।

In the given figure MN is the diameter of a circle with centre O. If PR = PQ. Prove That ∠OSP = ∠ORP.

Solution: Here,

Given: (i) O is the centre of a circle.
(ii) MN is a diameter.
(iii) PR = PQ.

To prove: ∠OSP = ∠ORP.

Construction: Join OQ.

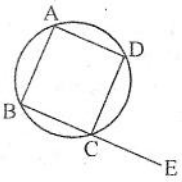
Proof:

	Statements		Reasons
1.	In ΔOPR and ΔOPQ (i) OR = OQ (ii) OP = OP (iii) PR = PQ	1.	 (i) Radii of same circle (ii) Common sides. (iii) From the given.
2.	∴ ΔOPR ≅ ΔOPQ	2.	By SSS axiom.
3.	∴ ∠ORP = ∠OQP	3.	Corresponding axiom of the congru cent triangles.
4.	∠OQP = ∠OSP	4.	Being OS = OQ i.e. radius of same circle.
5.	∠ORP = ∠OSP	5.	From the statements (3) and (4)

Proved

Model 17

चक्रिय चर्तुभुजको एउटा भुजा लम्बाउदा बनेको वाह्यकोण आशन्नकोण को सम्मुखकोण संग बराबर हुन्छन ।



In one side of a cyclic quadrilateral is produced, prove that the exterior angles, so formed is equal to the opposite interior angles of the cyclic quadrilateral.

Solution: Here,

Given: One side BC the cyclic ABCD is produced up to at a point E.

To prove: ∠DCE = ∠BAD

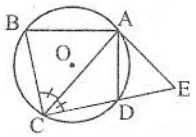
Proof:

	Statements		Reasons
1.	∠DCE + ∠BCD = 180°	1.	Being the adjacent angles formed on the same side of the straight line BCE.
2.	∠BAD + ∠BCD = 180°	2.	The opposite angles of a cyclic quadrilateral are supplementary.
3.	∠DCE + ∠BCD = ∠BAD + ∠BCD ∴ ∠DCE = ∠BAD	3.	By equal axiom from 1 and 2.

Proved

Model 18

सँगै चित्रमा ABCD चक्रिय चर्तुभुज हो । CD लाई E सम्म लम्बाइ A र E जोडिएको छ । जहाँ BC = DE र ∠BCD को अर्धक भए प्रमाणित गर्नुहोस् । ΔACE समवाहु त्रिभुज हो ।



In the adjoining diagram ABCD is a cyclic quadrilateral, the side CD produced to the point E, where BC = DE. If CA bisects ∠BCD prove that ΔACE is an isosceles triangle.

Solution: Here,

Given: (i) ABCD is a cyclic quadrilateral with cent re O.

To prove: (ii) CA is an angular bisector of ∠BCD and BC = DE.

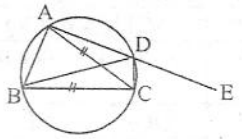
Proof:

	Statements		Reasons
1.	$\angle ACB = \angle ACD$	1.	Given
2.	$\overset{\frown}{AB} = \overset{\frown}{AD}$	2.	Arcs subtended by equal angles at the circumference are also equal.
3.	$AB = AD$	3.	Equal chords cut off equal arcs in a circle.
4.	In ΔABC and ΔADE	4.	
	(i) $AB = AD$ (S) (ii) $\angle ABC = \angle ADE$ (A) (iii) $BC = DE$ (S)		(i) From the statement 3 (ii) The exterior angles of a cyclic quadrilateral and opposite interior angle. (iii) Given
5.	$\therefore \Delta ABC \cong \Delta ADE$	5.	By SAS fact
6.	$AC = AE$	6.	Corresponding sides of the congruent triangles.
7.	$\therefore \Delta ACE$ is an isosceles	7.	Being $AC = AE$

Proved

Model 19

दिइएको चित्रमा AC = BC र ABCD चक्रिय चतुर्भुज हो। प्रमाणित गर्नुहोस्।∠ BDE को अर्धक DC हो।



In given figure, AC = BC and ABCD is a cyclic quadrilateral. Prove that DC bisects ∠BDE.

Solution: Here,

Given: In circle ABCD

(i) AC = BC. (ii) ABCD is a cyclic quadrilat eral.

To prove: DC bisects ∠BDE. i.e. ∠BDC = ∠CDE.

Proof:

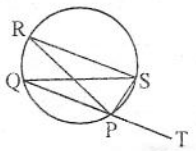
	Statements		Reasons
1.	$\angle BAC = \angle ABC$	1.	In ΔABC, $AC = BC$
2.	$\angle BAC = \angle BDC$	2.	Angles at the circumference standing on the same arc BC.
3.	$\angle ABC = \angle CDE$	3.	Exterior angle of a cyclic quadrilateral is equal to its interior opposite angles.
4.	$\angle BDC = \angle CDE$	4.	From the statement (1), (2) and (3).

Proved

Model 20

दिइएको चित्रमा ∠RPT को अर्धक PS हो। PQRS चक्रिय चतुर्भुज हो प्रमाणित गर्नुहोस । SQ = SR

In the adjoining figure, SP is the bisector of ∠RPT and PQRS is a cyclic quadrilateral. Prove that SQ = SR



Solution:

Given: (i) SP is the bisector of ∠RPT.i.e ∠RPS = ∠SPT.

(ii) PQRS is a cyclic quadrilat eral.

To prove: SQ = SR.

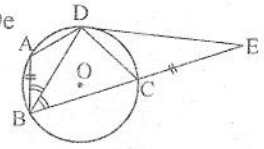
Proof:

	Statements		Reasons
1.	$\angle SPT = \angle RPS$	1.	From given.
2.	$\angle SPT = \angle QRS$	2.	Exterior angles of a cyclic quadrilateral is equal to its interior opposite angle.
3.	$\angle SQR = \angle RPS$	3.	Angles at the circumference standing on the sma earc RS.
4.	$\angle SQR = \angle QRs$	4.	From the statement (1) and (2) amnd (3)
5.	In ΔSQR, $SQ = SR$	5.	Being base abgles equal.

Proved

Model 21 दिइएको चित्रमा AB = CE ∠ABD = ∠DBC प्रमाणित गर्नुहोस्। BD = De

In the fiven figure, AB = CE, ∠ABD = ∠DBC, Prove that BD = DE



Given: AB = CE and ∠ABD = ∠DBC

To prove: BD = DE

Proof:

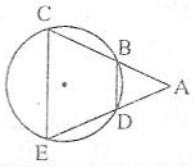
	Statements		Reasons
1.	In ΔABD and ΔDCE (i) AB = CE (S) (ii) ∠BAD = ∠DCE (A) (iii) AD = CD (S)	1.	(i) From the given (ii) Exterior angles of a cy cyclic quadrilateral equal to its interior opposite angles. (iii) If the angles at the circumference of a circle are equal then its oppsite chords are equal.
2.	ΔABD ≅ ΔDCE	2.	By SAS fact.
3.	BD = DE	3.	Corresponding sides of the congruent triangles.

Proved

Model 22

चित्रमा यदि AC = AF, भए प्रमाणित गर्नुहोस् । BD // CE र AB = AD

In the figure, AC = AE, prove that BD // CE and AB = AD



Given: BCED is a cyclic quadrilateral and AC = AE.

To prove: BD // CE and AB = AD

Proof:

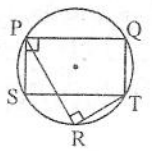
	Statements		Reasons
1.	AC = AE	1.	From the given
2.	∠BCE = ∠DEC	2.	Being in DAEC; AC = AE.
3.	∠BCE = ∠ADB	3.	Exterior angle of a cyclic quadrilateral is equal to its interior opposite angle.
4.	∠ADB = ∠DEC	4.	From the statements (2) and (3).
5.	∴ BD \\ CE	5.	Being corresponding angles anges, equal statement (4).
6.	∠ABD = ∠BCD	6.	Being corresponding angles in BD // CE.
7.	∠ABD = ∠ADB	7.	From the statements (3) and (6).
8.	AB = AD	8.	Being base angles of ΔABD are equal.

Proved

Model 23

बिन्दुहरु P, Q र R वृतको परिधि मा पर्दछ ।जसमा PS ⊥ PQ TR ⊥ PR भए प्रमाणित गर्नुहोस् ।

Points P, Q, R are taken on the circumference of a circle such that PS drawn at right angles to PQ meets the circle at S and RT drawn at right angles to PR meets the circle at T, Prove that PQ = ST.



Given: (i) Points P, Q and R are on the circumference of a circle.
(ii) PS ⊥ PQ and RT ⊥ PR
(iii) Joined ST.

To prove: PQ = ST

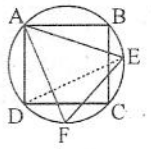
Proof:

	Statements		Reasons
1.	∠PRT = 90°	1.	Being RT ⊥ PR.
2.	∠PST = ∠PRT = 90°	2.	Angles at the circumference standing on the same arc and form the statement (1).
3.	∠PST + ∠PQT = 180°	3.	Sum of opposite angles of cyclic quadrilateral.
4.	∠PQT = 90°	4.	From statements (2) and (3)

5.	$\angle SPQ = 90°$	5.	Being PS ⊥ PQ.
6.	$\angle SPQ + \angle QTS = 180°$	6.	Sum of opposite anglers of cyclic quadriteral.
7.	$\angle QTS = 90°$	7.	From the statements (5) and (6).
8.	$\therefore$ PQTS is rectangle	8.	From the statements (2), (4), (5) and (7)
9.	$\therefore$ PQ = ST	9.	Opposite sides of rectangle are equal.

Proved

Model 24



ABCD वर्ग र ΔAFE एउटा त्रिभुज हो । जसमा AF = AE र कुनै वृतमा अन्तर्गत छ । प्रमाणित गर्नुहोस् । EF // BD.

ABCD is a square and AFE is triangle, in which AF = AE both inscribed in a circle, prove that EF//BD.

Given: (i) ABCD is a cyclic square.
(ii) AFE is a trangle.
(iii) AF = AE

To prove: EF \\ BD

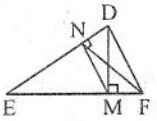
Construction: Join DE.

Proof:

	Statements		**Reasons**
1.	$\widehat{AF} = \widehat{AE}$	1.	Being AF= AE.
2.	AD = AB	2.	Sides of square.
3.	$\widehat{AD} = \widehat{AB}$	3.	Being AD = AB
4.	$\widehat{AF} - \widehat{AD} = \widehat{AE} - \widehat{AB}$	4.	Statement (3) subtract from the statements (1)
5.	$\widehat{DF} = \widehat{BE}$	5.	From thye statement equal angle.
6.	$\angle DEF = \angle BDE$	6.	Equal arc subtends equal angle.
7.	$\therefore$ EF \\ BD	7.	Being alternate angles are equal.

Proved

Model 25



संगैको चित्रमा DM ⊥ EF र FN ⊥ ED भए प्रमाणित गर्नुहोस् । $\angle EMN = \angle EDF$

In the figure alongside DM ⊥ EF and EN ⊥ ED. Prove that $\angle EMN = \angle EDF$

Solution:

Given: (i) In DDEF, DM ⊥ EF and FN ⊥ ED.

TO prove: $\angle EMN = \angle EDF$

Construction: Join QT.

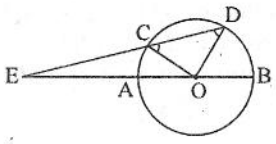
	Statements		**Reasons**
1.	$\angle FND = \angle DMF = 90°$	1.	FN ⊥ ED and DM ⊥ EF.
2.	D, N, M, F are conyclic point.	2.	The line joioning the pint DF subtends equal angles at other two points on the same side of it.
3.	DNMF is a cyclic Quadrilateral.	3.	All vertices of DNMF are concylcic.
4.	$\therefore \angle EMN = \angle EDF$	4.	Exterior angles of a cyclic quadrilater4al is equal to its interior opposite angle.

Proved

Model 26

वृतको बाहय विन्दुबाट EAB र ECD रेखाहरू खिचेको छ। EAB रेखा केन्द्र विन्दुबाट गएको छ । EC अर्धव्यास बराबर भए प्रमाणित गर्नुहोस् । चाप BD = $\frac{1}{3}$ चाप CA

From a point E outside a circle, two lines EAB and ECD are drawn cutting the circle. EAB passes through the centre and EC is equal to the radius of a circle. Join CO and OD, prove that are BD = 3 arc CA. or, arc CA = $\frac{1}{3}$ arc, BD



Given:

(i) O is the centre of the circle.

(ii) Straight lines EAB and ECD such that EC = OC = OD

To prove: arc CA = $\frac{1}{3}$ arc BD

Proof:

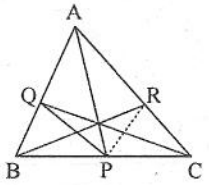
	Statements		Reasons
1.	$\angle BOD = \angle ODE + \angle OED$ i.e. $\angle BOD = \angle ODC + \angle OEC$	1.	In ΔEOD, ext angle = sum of its opposite interior angles and $\angle ODE = \angle ODE$ and $\angle OED = \angle OEC$ (same angle)
2.	$\therefore \angle ODC = \angle OCD$	2.	In ΔCOD, OC = OD
3.	$\angle BOD = \angle OCD + \angle OEC$	3.	from (1) and (2)
4.	$\angle OEC = \angle COE$ i.e. $\angle OEC = \angle COA$	4.	In ΔEOC, EC = CO 9given) and $\angle COE = \angle COA$ (same angle)
5.	$\angle BOD = \angle OCD + \angle COA$	5.	From (30 and (4)
6.	$\angle OCD = \angle OEC + \angle COA$	6.	In ΔEOC, exterior angle = sum of its opposite interior angles.
7.	$\angle OCD = \angle COA + \angle COA = 2\angle COA$	7.	From (5) and (7)
8.	$\angle BOD = 2\angle COA + \angle COA = 3\angle COA$	8.	Form (5) and (7)
9.	arcBD = 3 arc CA i.e arc CA = $\frac{1}{3}$ arc BD	9.	From (8), if arcs of a circle subtend equal angles at the centre, they are equal.

Proved.

Model 27

दिइएको चित्रमा AP⊥BC BR ⊥AC र CQ ⊥ AB छन्। भनी प्रमाणित गर्नुहोस्।

$\angle OPQ = \angle OPR$ in the given figure AP ⊥ BC, BR ⊥Ac and CD ⊥AB, prove that : $\angle OPQ = \angle OPR$



Given :- In a ΔABC, AP ⊥BC BR ⊥AC and CQ⊥AB

TD prove : $\angle OPQ = \angle OPR$

Proof:

Statements	Reasons
1. PORC and BPOQ is cyclic quad's.	1. Beigh sum opposite angle of 180^0
2. Then B, C, R, and Q are cyclic points	2. Being inscribed angle are equal on the same arh BC
3. $\angle OPR = \angle OCR$ $\angle CPQ = \angle OBQ$	3. Being inscribdded angle on same arc
4. $\angle OBQ = \angle QCR$	4. Being in scribed angle on same arc
5. $\angle QPQ = \angle OPR$	5. For (3) and (4)

Proved

Model 28

PQRS एउटा चक्रिय चतुर्भुज हो। यदि $\angle QPS$ र $\angle QRsS$ अर्धकहरूले वृत्तलाई क्रमश A र B मा भेट्छन्। भने AB व्यास हो भने प्रमाणित गनुहोस्। PQRS is a cyclic quadr. If the base

other of ∠QPS and ∠QRS meet the circle at the points A and B, respectively prove that AB is diameter of the circle.

Solution: Given (i) PQRS is a cyelic quard^r.(ii) PA and RB are sisector of ∠QPS and ∠QRS.

To prove AB is diameter . $\overset{\frown}{QRA}$

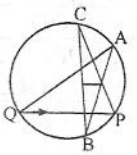
Proof:

Statements	Reasons
1. $\overset{\frown}{QRA} = \overset{\frown}{AS}$	1. Opposite the arcs of equal angles
2. $\overset{\frown}{BPQ} = \overset{\frown}{RS}$	2. Same as reason (1)
3. $\overset{\frown}{QRA} + \overset{\frown}{BPQ} = \overset{\frown}{AS} + \overset{\frown}{RS}$	3. addition (1) and (2)
4. $\overset{\frown}{BPQRA} = \overset{\frown}{BSA}$	4. Whole part axian
5. AB is dimeter	5. both of AB are equal.

proved

Model 29

दिइएको चित्रमा MN//PQ छ। जीवा PB = जीवा QB हुन्छ भनि प्रमाणित गर्नुहोस्।



In the adjoining figure MN//PQ prove that chord PB = chord QB

Solution: Given : In the given figure MN//PQ

To prove: chord PB= chord QB

constract, join A and C

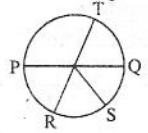
Proof:

Statements	**Reasons**
1. ∠CAQ = ∠CPQ	1. Inscribed angles stading on the angles
2. ∠CMN = ∠CPQ	2. corresponding angles
3. ∠CAQ = ∠CMN	3. From statements (1) and (2)
4. A,M ,N and C are concylin	4. Being equal angles standing o same arc
5. ∠NCM = ∠NAM	5. Inscribed angles on the same segment.
6. PB = QB	6. Corresponding chords of equil angles

Proved

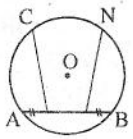
Practice Yourself

1. दिएको चित्रमा केन्द्र विन्दु O भएको वृतमा दुई व्यासहरु POQ र ROT हुन । यदि चाप TQS मा मध्य विन्दु त्त र ∠QOR अधिक कोण भए PQ // RS हुन्छ भनि सिद्ध गर्नुहोस् ।



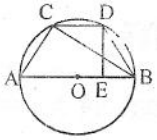
In the given figure, POQ and ROT are two diameter of a circle with centre at o if Q is mid point of arc TQS and ∠ QOR is obtuse angle proved that. PQ // RS

2. दिइको चित्रमा O वृतको केन्द्र विन्दु हो । जीवा AB मा कुनै दुई विन्दुहरु C र D छन् । यदि AC = BD र $\overset{\frown}{AM} = \overset{\frown}{BN}$ भए ∠ ACM = ∠ BDN हुन्छ भनि प्रमाणित गर्नुहोस् ।



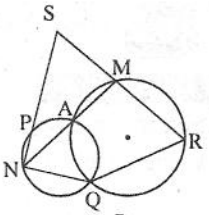
In the given figure O is the centre of circle C and D are two points on the chord. if AC = BD spand $\overset{\frown}{AM} = \overset{\frown}{BN}$ prove that ∠ ACM = ∠ BDN.

3. दिइएको चित्रमा X वृतको केन्द्र हो । AB वृतको व्यास हो । यदि DE ⊥ AB and $\overset{\frown}{AC} = \overset{\frown}{BD}$ छन् भने ∠ ABC = ∠BDE हुन्छ भनि प्रमाणित गर्नुहोस् ।

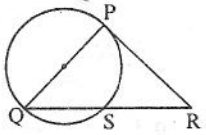


In the given figure, X is the centre of a circle AB is a diameter of circle DE ⊥ AB and $\overset{\frown}{AC} = \overset{\frown}{BD}$ prove that ∠ABC = ∠ BDE.

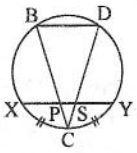
4. दिइएको चित्रमा NPS, MAN र SMS सरल रेखाहरु हुन भने PQRS एउटा चक्रिय चतुर्भुज हुन्छ भनि प्रमाणित गर्नुहोस् ।
In the given figure NPS, MAN and SMS are straight lines prove that PQRS is cyclic quadrilateral.



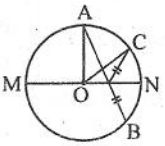
5. चित्रमा समद्विवाहु ΔPQR मा PQ = PR छन् यदि PQ व्यास हुने गरि एउटा वृत खिचिएको छ भने QS = RS हुन्छ भनि प्रमाणित गर्नुहोस् ।
In the figure, Δ PQR Isosceles triangle with PQ = PR. A circle is drawn with PQ is a diameter. Prove that QS = RS



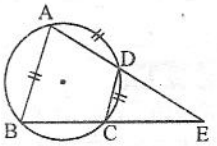
6. दिएको चित्रमा BCD एउटा समद्विवाहु त्रिभुज र चाप XC = चाप YC चए ΔPCS एउटा समद्विवाहु त्रिभुज हो भनी प्रमाणित गर्नुहोस् ।
In the given figure, BCD is a Isosceles triangle and arc XC = arc YC, prove that Δ PCS is Isosceles triangle.



7. दिइएको चित्रमा O वृतको केन्द्र विन्दु हो । यदि BD = CD प्रमाणित गर्नुहोस् ∠OAD = ∠OCD
In the given figure, o is centre of circle of BD = CD, prove that ∠OAD = ∠OCD.
(**Hints:** Join O and B)

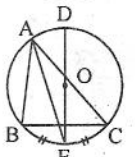


8. दिइएको चित्रमा चक्रिय चतुर्भुज ABCD भुजाहरु AD र BC लाई E मा भेटिने गरि लम्बाइएको छ । यदि AB = CE र $\overset{\frown}{AD} = \overset{\frown}{DC}$ भए प्रमाणित गर्नुहोस् । ∠ACB = 2 ∠DEC
In the given figure, the sides AD and BC of a cyclic quadrilateral ABCD are produced to meet at the point E. AB = CE and $\overset{\frown}{AD} = \overset{\frown}{DC}$, then prove that ∠ACB = 2 ∠DEC. (**Hints**: join BD).

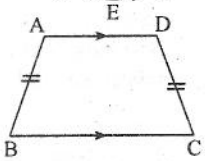


9. दिइएको चित्रमा वृत्तको केन्द्र विन्दु O हो । यदि AB//CD भए ∠ AOC = 2 ∠BED हुन्छ भनि प्रमाणित गर्नुहोस् ।
In the given figure O is the centre of circle of AB//CD prove that ∠ AOC = 2 ∠BED.

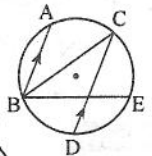
10. दिइएको चित्रमा, DE व्यास हो । यदि $\overset{\frown}{BE} = \overset{\frown}{CE}$ भए $\angle AFD = \frac{1}{2}$ (∠ABC – ∠ACB) हुन्छ भनि प्रमाणित गर्नुहोस् ।
In the given figure, DE is diameter of circle. If $\overset{\frown}{BE} = \overset{\frown}{CE}$ prove that $\angle AFD = \frac{1}{2}(\angle ABC - \angle ACB)$.



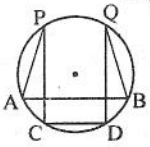
11. दिइएको चित्रमा ABCD समलम्ब चतुर्भुज हो यदि AB = CD भए ABCD चक्रिय समलम्ब चतुर्भुज हुन्छ भनि प्रमाणित गर्नुहोस् ।
In the given figure ABCD is a trapezium of AD = CD, prove that ABCD is a cyclic trapezium.



12. दिएको चित्रमा AB//CD र ∠ABC = ∠ CBE भए प्रमाणित गर्नुहोस् BE = CD
In the given figure AB//CD and ∠ABC = ∠CBE, prove that BE = CD.



13. दिएको चित्रमा ∠APC = ∠BQD छ भने AB//CD हुन्छ भनि प्रमाणित गर्नुहोस् ।
In the given figure, ∠APC = ∠BQD, prove that BC//CD.

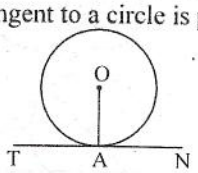


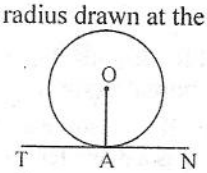
15.2 स्पर्स रेखा (Tangent)

Experimental:1

अर्धव्यास र स्पर्श रेखा बिचको सम्बन्ध **(Related between of radius and tangent)**

A tangent to a circle is porpendiculer to the radius drawn at the point of contact.





Step:1 Draw the circle of different radii of with centre O and each diagram.
Draw tangent TAN of circle at point A and join OA from centre as shown in the fig No.1 of fig No. 2.

To verify : OA ⊥ TAN (∠ OAT = ∠ OAN = 90°)

Verification table:

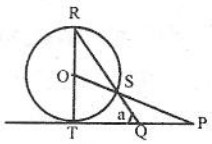
Figure No	∠OAT	∠OAN	Result
(i)	90°	90°	∠ OAT = ∠ OAN
(ii)	90°	90°	∠ OAT = ∠ OAN

Conclusion: From the above verification table shows that a tengent to a circle is perpendicular to the radius draqn at the point of centact.

Short Questions

Model 1

दिइएको चित्रमा केन्द्र वृत O केन्द्र भएको वृतका TP स्पर्श रेखा ∠ OPT = 38° भए a को मान पत्ता लगाउनुहोस् ।



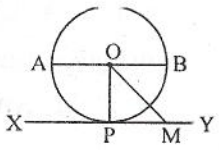
In the figure, TP is lenget of the circle O and ∠PRT = 38° find the value of a.

Solution: To find : a = ?

1.	38° + 90° + ∠ TOP = 180° ∠ TOP = 180° –128° = 52°	1.	Sum of three angles of Δ TOP
2.	$\angle TRS = \frac{1}{2}52° = 26°$	2.	Relation of inseribed angle and cetral angle on the same arc.
3.	26° + a + 90° = 180° a = 180° – 116° = 64°	3.	Sum of three angle of ΔTQR

Model 2

दिइएको चित्रमा O वृतको केन्द्र हो । यदि AB = 8cm र ON = 5cm भए PN को लम्बाई पत्ता लगाउनुहोस् ।



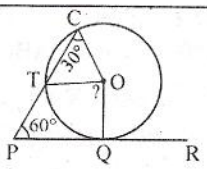
In the given figure, O is the centre of the circle If AB = 8cm and ON = 5cm find the length of PN.

Solution: To find : PN = ?

1.	$OP = OB = \frac{1}{2}AB = \frac{1}{2} \times 8\text{ cm} = 4\text{ cm}$	1.	Radius is half of diameter.
2.	$PN = \sqrt{ON^2 - OP^2} = \sqrt{(5cm)^2 - (4cm)^2}$ $= \sqrt{(25 - 16)cm^2} = \sqrt{9\ cm^2} = 3cm$	2.	From Pythagoras theorem.

Model 3

दिइएको चित्रमा ∠QOT मान पत्ता लगाउनुहोस् ।



In the given figure, Find the valume of ∠ QOT.

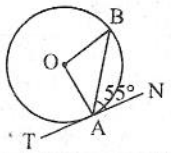
Solution: To find : ∠ QOT = ?

1.	$\angle OCR = \angle OTC = 31°$	1.	Base of angles of Δ ORT
2.	$\angle OTP = 180° - 31° = 149°$	2.	Sum of adjacent angles
3.	$149° + 60° + 90° + \angle QOT$ $= 360\ \angle QOT = 360° - 299° = 61°$	3.	Sum of angles quadriletral.

Model 4

दिइएको चित्रमा $\angle$ OBA को मान निकाल्नुहोस् ।

In the given figure find the $\angle OBA$.

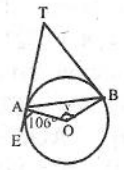


Solution:

1.	$\angle OAN = 90°$	1.	Being OA $\perp$ TAN
2.	$\angle OAB = 90° - 55°$ $= 35°$	2.	Substraction.
3.	$\angle OBA = \angle OAB = 35°$	3.	Base angles of $\angle$ OAB

Model 5

दिइएको चित्रमा O वृतको केन्द्रको केन्द्र विन्दु हो TA र TB स्पर्श रेखाहरु भए x र y को मान पत्ता लगाउनुहोस् ।



In the given figure O is the centre of circle, TA amd TB are tangent find the value of x^0 am y^0.

Solution: To find : m = ?

(i) $x = (106^0 - 90^0) = 16^0 \rightarrow \angle OAB = \angle BAE - \angle OAE$

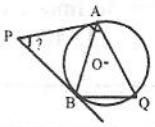
(ii) $16^0 + 16^0 + y^0 = 180^0 \rightarrow$ sum of three angles ΔOA is

$\therefore y^0 = 180^0 - 32^0$

$\therefore y^0 = 148^0$

Model 7

दिइएको चित्रमा PA र PB दुई स्पर्श रेखाहरु हुन यदि $\angle AQB = 62°$ छ भने मान निकाल्नुहोस् ।



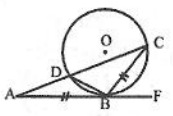
In the given figure PA anf PB are the two tangetnt lines of $\angle$ AQB = 62° find the $\angle APB$.

Solution: To find $\angle APB$ = ?

1.	$\angle PAB = \angle AQB = 62°$	1.	alternate seqment angles
2.	$\angle PAB = \angle APB = 62°$	2.	Base angles of Δ PAB
3.	$62° + 62° + \angle PAB = 180°$ $\angle APB = 180° - 124°$ $= 56°$	3.	Sum of three angles ΔPAB

Model 8

दिइएको चित्रमा ABE स्पर्श रेखा जसमा B स्पर्श विन्दु छ । यदि AB = BC र $\angle ABD = 23°$ भए $\angle$ DBC को मान पत्ता लगाउनुहोस् ।

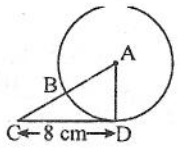


In the given figure, ABE is a langent where B is the point of contact. If AB = BC and $\angle ABD = 25°$ find the $\angle DBC$. Solution: To find : $\angle DBC$ = ?

1.	$\angle BCD = \angle ABD = 25°$	1.	alternate segment angles.
2.	$\angle BAD = \angle BCD = 25°$	2.	Base angles of ΔABC
3.	$25° + 25° = \angle CBF\ \angle BDC = \angle CBF = 50$	3.	exterior angle = sum of Interior angles. of $\angle$ABC
4.	$50° + 25° + \angle DBC = 180°$ $\angle DBC = 180° - 75° = 105°$	4.	sum of three angles of Δ BCD

Model 9

दिइएको चित्रमा O वृतको केन्द्र र CD स्पर्श रेखा हो यदि CD = 8cm BC = 4cm भए AB मान पत्ता लगाउनुहोस् ।



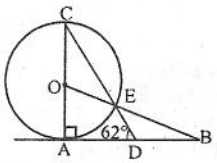
In the given figure O is the centre of circle and CD is lengent line. If CD = 8cm BC = 4cm, find AB.

Solution: To find : AB = ?

1.	AB = AD	1.	Radii of circle
2.	$AC^2 = CD^2 + AD^2$ or, $(AB + BC)^2 = (8\ cm)^2 + AD^2$ or, $(AB + 4\ cm)^2 = 64\ cm^2 + AD^2$ or, $AB^2 + 8AB + 16 = 64 + AD^2$ or, $8AB = 64 - 16$ or, $8AB = 48 \quad \therefore \quad AB = 6cm$	2.	From the phthagoras theorem

Model 10

संगैको चित्रमा O वृत्तको केन्द्र हो। व्यास र ADB स्पर्श रेखा हो। यदि $\angle ADC = 62^0$ रेखा हो र यदि $\angle ADC = 62^0$ भए $\angle ABD$ को मान पत्ता लगाउनुहोस। In the adjoining figure O is the centre of circle, AC is diameter and $\angle ADB$ is a tangent. Find $\angle AB$ of $\angle ADC = 62^0$



Solution: To find $\angle ABO$ = ?

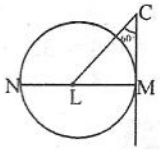
(i) $90^0 + 62^0 + \angle ACD = 180°$ $\quad \angle ACD = 180 - 152^0 \rightarrow$ Sum of three angles of $\Delta = 28^0$

(ii) $\angle AOE = 2 \times \angle ACD \rightarrow$ Relation between center $= 2 \times 18^0 = 56^0$

(iii) $56^0 + 90_0 + \angle ABO = 180$ $\quad \angle ABO = 180 - 145 = 34^0$

Model 11

दिइएको चित्रमा L वृत्रको केन्द्र र cm स्पर्श रेखा हो। यदि KMCL = 55^0 भए $\angle CLM$ को मान कति होला ? In the given figure , L is centre of circle and CM tangent . If $\angle MCL = 55^0$ find the $\angle$ CLM.



Solution:

Here, to find: $\angle CLM$ = ?

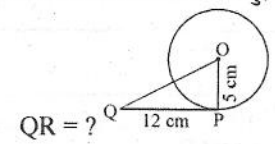
(i) $\angle LMC = 90^0 \rightarrow LM \perp CM$ (ii) $90^0 + KCLM = 180^0 \rightarrow$ sum of three angles of ΔLMC

$\angle CLM = 180 - 145 = 35^0$

Practice Yourself

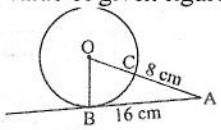
दिइएको चित्रहरुमा मान पत्ता लगाउनुहोस् । (Find the value of given figures.)

(i)



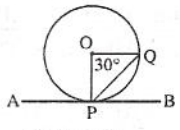
QR = ?

(ii)



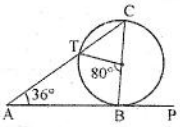
OB = ?

(iii)



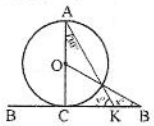
$\angle BPQ$ = ?

(iv)



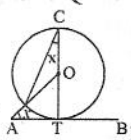
$\angle OCT$ = ?

(v)



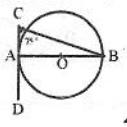
x = ? y = ?

(vi)



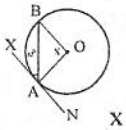
x = ?

(vii)



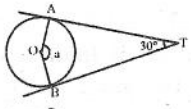
$\angle ABC$ = ?

(viii)



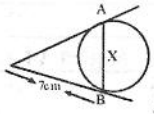
x = ?

(ix)



a = ?

(x)



AB = >?