College Algebra - Core Concept Cheat Sheet

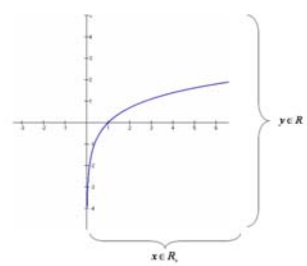
15: Logarithmic Functions

Key Terms

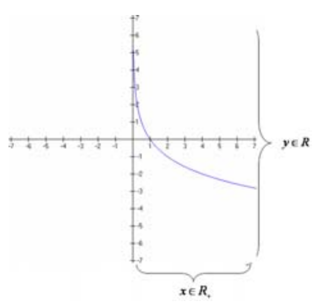
- **Logarithm:** the number of times a base must be multiplied by itself to reach a given number.
- **Logarithmic equation:** the inverse of an exponential equation with base b.
- **Exponential counterpart:** a logarithmic function $y = \log_b x$ has an exponential counterpart of $x = b^y$.
- **Common logarithm:** a logarithm with a base of 10. Its notation is $y = \log x$.
- **Natural logarithm:** a logarithm whose base is Euler's number e. Its notation is $y = \ln x$.

Graphs of Logarithms

- Given $f(x) = \log_b x$ and $b > 1$:
  - The domain of $f(x)$ consists of all positive real numbers.
  - The range of $f(x)$ is the collection of all real numbers.
  - $f(x)$ is a monotonously increasing function.
  - The graph of $f(x)$ goes through the point (1, 0).
  - The graph of $f(x)$ is the reflection of $y = b^x$ over the line $y = x$.



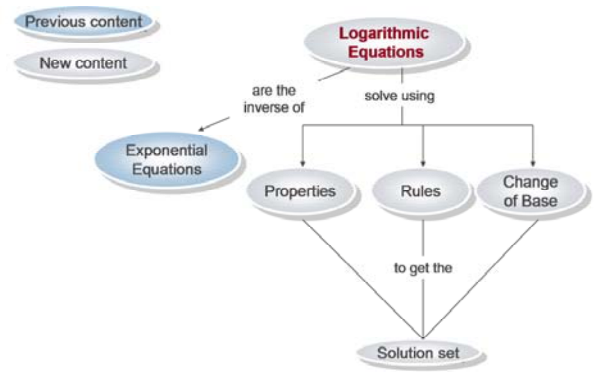
- Given $f(x) = \log_b x$ and $0 < b < 1$
  - The domain of $f(x)$ consists of all positive real numbers.
  - The range of $f(x)$ is the collection of all real numbers.
  - $f(x)$ is a monotonously decreasing function.
  - The graph of $f(x)$ goes through the point (1, 0).
  - The graph of $f(x)$ is the reflection of $y = b^x$ over the line $y = x$.



Methodology to Determine Logarithms

- **Method one:** you can transform it to the common or natural logarithms by the law of changing bases and then use calculator.
- **Method two:** you can write the exponential counterpart of the given logarithms and setup an equivalent exponential function. Then, find the approximate value of the power exponent to balance the equation. The power exponent is the solution of the logarithms.
- **Method Three:** Graph the logarithmic function with the same base of the given logarithm and measure the y-value when the x-value is just the number after the logarithmic sign of the logarithm.

Concept Map



Properties of Logarithms

Where $b > 0$ and $b \neq 0$,

- **Identity Property:** $\log_b b = 1$
- **Zero Property:** $\log_b 1 = 0$
- **Inverse Property 1:** $\log_b b^x = x$
- **Inverse Property 2:** $b^{\log_b x} = x$
- **Product Rule:** $\log_b MN = \log_b M + \log_b N$
- **Quotient Rule:** $\log_b \frac{M}{N} = \log_b M - \log_b N$
- **Power Rule:** $\log_b N^p = p \log_b N$
- **Change of Base:** $\log_b N = \frac{\log N}{\log b}$ OR $\log_b N = \frac{\ln N}{\ln b}$

How-to: Solve a Logarithmic Equation

To solve a logarithmic equation:

- Reduce the equation to a single logarithm using the properties of logarithms.
- Transform the equation by using the relation of exponential functions and logarithms functions (they are pairs of inverse functions).

How-to: Solve an Exponential Equation

To solve an exponential equation:

- Isolate the expression containing the exponent.
- Take the log (or natural log) of both sides of the equation.
- Simplify using the power rule: $\log_b N^p = p \log_b N$.

Example: Logarithmic Expression

Evaluate $\log_7 26$.

Solution: Use the Change of Base property to rewrite this expression as the quotient of two logarithms with the same base.

$$\log_7 26 = \frac{\log 26}{\log 7} \quad \text{OR} \quad \log_7 26 = \frac{\ln 26}{\ln 7}$$

Use the LOG or LN key on your calculator to solve.

$$\log_7 26 = \frac{\log 26}{\log 7} \approx 1.674$$

How to Use This Cheat Sheet: These are the key concepts related this topic. Try to read through it carefully twice then write it out on a blank sheet of paper. Review it again before the exam.

