Step 3 Continue this labeling process until the sink has been labeled or until no more vertices can be labeled.

If the labeling process results in the sink being labeled, then there will be a chain of labeled arcs (call it C) leading from the source to the sink. By adjusting the flow of the arcs in C, we can maintain a feasible flow and increase the total flow from source to sink. To see this, observe that C must consist of one of the following:

Case 1 C consists entirely of forward arcs.

Case 2 C contains both forward and backward arcs.

In each case, we can obtain a new feasible flow that has a larger flow from source to sink than the current feasible flow. In Case 1, the chain C consists entirely of forward arcs. For each forward arc in C, let i(x, y) be the amount by which the flow in arc (x, y) can be increased without violating the capacity constraint for arc (x, y) . Let

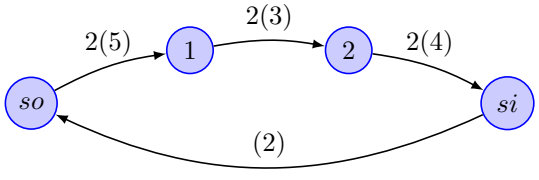
$$k = min(x, y)$$
$$(x, y) \in C$$

Then k > 0. To create a new flow, increase the flow through each arc in C by k units. No capacity constraints are violated, and conservation of flow is still maintained. Thus, the new flow is feasible, and the new feasible flow will transport k more units from source to sink than does the current feasible flow.

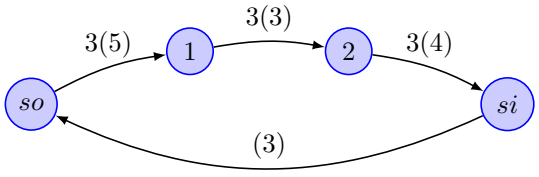
We use Figure 10 to illustrate Case 1. Currently, 2 units are being transported from source to sink. The labeling procedure results in the sink being labeled by the chain C= (so,1)-(1,2)-(2,si). Each arc is in I, and i(so,1) = 5 - 2 = 3; i(1, 2) = 3 - 2 = 1; and i(2,si) = 4 - 2 = 2. Hence, k = min(3,1,2) = 1. Thus, an improved feasible flow can be obtained by increasing the flow on each arc in C by 1 unit. The resulting flow transports 3 units from source to sink (see Figure 11).

In Case 2, the chain C leading from the source to the sink contains both backward and forward arcs. For each backward arc in C, let r(x, y) be the amount by which the flow through arc (x, y) can be reduced. Also define

$$k_1 = minr(x, y) and k_2 mini(x, y)$$
$$x, y \in C \cap Rx, y \in C \cap I$$



Flow from source to sink = 2



Flow from source to sink = 3

Because we exclude arc a_0 from the labeling procedure, no chain made entirely of backward arcs can lead from source to sink.