$Q = \int (\vec{v} \cdot \vec{n})\, dA \quad \dot{m} = \rho Q$

• Bernoulli:

$p_1 + \rho g z_1 + \frac{1}{2} \rho V_1^2 = p_2 + \rho g z_2 + \frac{1}{2} \rho V_2^2$

FORMULÁRIO

Forma integral das equações de conservação

Massa: $\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c} \rho \,\mathrm{d}V + \int_{S_c} \rho(\vec{v}_r \cdot \vec{n})\,\mathrm{d}S = 0$

Quantidade de movimento: $\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c} \rho\vec{v} \,\mathrm{d}V + \int_{S_c} \rho\vec{v}(\vec{v}_r \cdot \vec{n})\,\mathrm{d}S = \sum \vec{F}_{ext\_no\_Vc/fluido}$

Energia: $\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c} \rho \left(e + \frac{1}{2}V^2 + gz\right) \mathrm{d}V + \int_{S_c} \rho(e + \frac{1}{2}V^2 + gz)(\vec{v}_r \cdot \vec{n})\,\mathrm{d}S = \dot{Q} - \dot{W}_\mathrm{m} - \dot{W}_\mathrm{p} - \dot{W}_\mathrm{v}$

Momento angular: $\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c} \rho(\vec{r_0} \times \vec{v}) \,\mathrm{d}V + \int_{S_c} \rho(\vec{r_o} \times \vec{v})(\vec{v}_r \cdot \vec{n})\,\mathrm{d}S = \sum \vec{M}_0$

Equações de Navier-Stokes

Equações do movimento para um escoamento incompressível de um fluido newtoniano com propriedades constantes em coordenadas cartesianas:

Continuidade:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Transporte de quantidade de movimento:

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right] = \rho g_x - \frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = \rho g_y - \frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = \rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right]$$

Componentes do tensor das tensões viscosas:

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} \qquad \tau_{xz} = \tau_{zx} = \mu\left[\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right]$$

$$\tau_{yy} = 2\mu\frac{\partial v}{\partial y} \qquad \tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right]$$

$$\tau_{zz} = 2\mu\frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

Continuidade:

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial(v_\theta)}{\partial \theta} + \frac{\partial(v_z)}{\partial z} = 0$$

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{{v_\theta}^2}{r}\right]$$
$$= \rho g_r - \frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right]$$
$$= \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right] =$$
$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\tau_{rr} = 2\mu\frac{\partial v_r}{\partial r} \quad \tau_{\theta\theta} = 2\mu\left[\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right] \quad \tau_{zz} = 2\mu\frac{\partial v_z}{\partial z}$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right] \qquad \tau_{rz} = \tau_{zr} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

$$\frac{1}{\sqrt{f}} = -1{,}8 \log\left[\frac{6{,}9}{Re} + \left[\frac{\left(\frac{\varepsilon}{D}\right)}{3{,}7}\right]^{1{,}11}\right]$$

$$Re = \frac{\bar{v} D}{\nu} \qquad \nu = \frac{\mu}{\rho}$$

$\nu \downarrow m^2/s$

$\varepsilon \rightarrow$ Rugosidade

Curva da Instalação

$$H = \frac{p_B - p_A}{\rho g} + z_B - z_A + \left[\frac{f\left(\frac{l}{d}\right)}{2gA^2} + \frac{\sum k}{2gA^2}\right] Q^2 \qquad H_s = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{p_{vap}}{\rho g}$$

Turbomáquinas

Potência ao veio em máquinas hidráulicas: $P = \rho g Q H \eta$ *ou* $P = \rho g Q H / \eta$

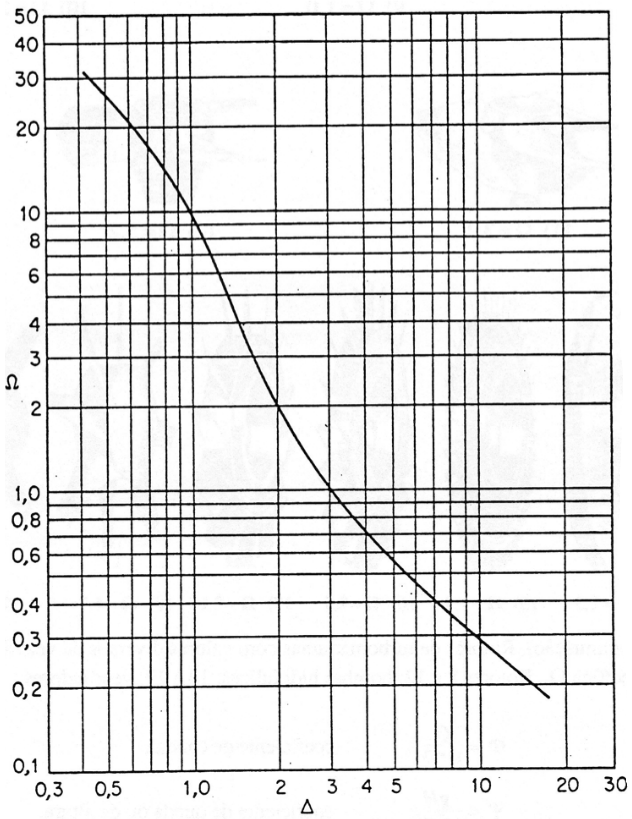
Coeficientes adimensionais em turbomáquinas:

$$C_P = \frac{P}{\rho N^3 D^5} \quad C_Q = \frac{Q}{N D^3} \quad C_H = \frac{gH}{N^2 D^2} \quad \Omega = \frac{N\sqrt{Q}}{(gH)^{\frac{3}{4}}} \quad \Delta = \frac{D(gH)^{\frac{1}{4}}}{\sqrt{Q}} \quad S_i = \frac{N\sqrt{Q}}{(gH_{si})^{\frac{3}{4}}}$$

Pontos dinamicamente semelhantes (mesma máquina) : $\frac{Q_1}{Q_2} \overset{*}{=} \frac{N_1}{N_2} = \left(\frac{H_1}{H_2}\right)^{1/2}$

Para bombas e turbinas temos, respetivamente: $S_{i\eta} = 3{,}0$ *e* $S_{i\eta} = 4{,}0$

Diagrama de Cordier:



- Altura aspiração disponível, bombas e turbinas:

$$H_S = \frac{P_{atm} - P_{vap}}{\rho \gamma} - e_s - z_{asp} \qquad H_S = \frac{P_{atm} - P_{vap}}{\rho g} - e_s$$

$P = N_{\eta max}$

Gráfico $= (Q_{\eta max})_1 \cdot (H_{\eta max})_1$ | $H_{\eta max} = \frac{H_1}{Q_1^2} Q^2$

$H_{\eta max} = H_{inst} \Leftrightarrow Q$ funcionamento | $*$

- Altura max bomba:

H_{Si} pa S_i | $H_S > H_{Si}$ em ordem a e_s

- Diâmetro bomba

$$\Omega = \frac{N\sqrt{Q}}{(gH)^{3/4}} \quad \Big| \quad \Delta = \frac{G}{AL} \quad \Big| \quad D = \frac{\Delta\sqrt{Q_{\eta max}}}{(g H_{\eta max})^{1/4}}$$