

H1: the origins of quantum theory	
1.1 black body radiation	
def: absorption coefficient	= the fraction of the radiant energy, incident on the surface, which is absorbed at a given wavelength
def: thermal equilibrium	= body at a constant temperature > must emit and absorb the same amount of radiant energy per unit time > the radiation emitted/absorbed under these circumstances is the <i>thermal radiation</i>
def: black body	= a body which absorbs all the radiant energy falling upon it > absorption coefficient is equal to unity at all wavelengths
Kirchhoff's law	= the ratio of emissive power/spectral emittance to the absorption coefficient is the same for all bodies at the same temperature and is equal to the emissive power of a black body at that temperature
def: total emissive power R	= total power emitted per unit area of the black body now: $R(T) = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ > Stefan-Boltzmann law  Now let R be dependent on wavelength $\lambda$ : $R(\lambda, T)$ then: $R(\lambda, T)d\lambda$ = the power emitted per unit area from a black body at temp. T with wavelengths between $\lambda$ and $\lambda+d\lambda$ > then: $R(T) = \int_0^{\infty} R(\lambda, T)d\lambda$
Wiens displacement law	For each temp. T, there is a wavelength $\lambda_{\max}$ for which $R(\lambda, T)$ reaches a peak value > inverse relation with temp: $\lambda_{\max} T = b$ $b = 2.898 \times 10^{-3} \text{ m K.}$
radiation inside a black body	- radiation is isotropic (same in all directions) - radiation is homogeneous (same at every point in the body)
def: monochromatic energy density / spectral distribution function	= $\rho(\lambda, T) d\lambda$ = the energy density of the radiation in the wavelength interval $(\lambda, \lambda+d\lambda)$ at temp T
R relation to $\rho$	There applies: $\rho(\lambda, T) = \frac{4}{c} R(\lambda, T).$
$\rho$ relation to $\lambda$	there can be found through classical reasoning: $\rho(\lambda, T) = \frac{8\pi}{\lambda^4} kT$ however in the limit of short wavelengths this doesn't hold to reality  Let us consider Planck's quantum theory: the energy of an oscillator of a given frequency $\nu$ cannot take arbitrary values between zero and infinity > however it can only take on discrete values $n\varepsilon_0$ where n is a positive integer $\varepsilon_0$ is a finite amount of energy, a quantum, that depends on frequency  Now we can find: $\rho(\lambda, T) = \frac{8\pi}{\lambda^4} \frac{\varepsilon_0}{\exp(\varepsilon_0/kT) - 1}.$  with: $\varepsilon_0 = h\nu = hc/\lambda$  thus: $\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}$

### 1.1.1 Planck's quantum theory

analysis of  $\rho$ -function by Planck

We found:

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}$$

For long wavelengths this can be simplified to:  $\rho(\lambda, T) \rightarrow 8\pi kT/\lambda^4$ , which is in agreement with the classical view

> namely, the quantum steps are so small, that they can be seen as continuous

$$\varepsilon_0 = h\nu = hc/\lambda$$

However at small wavelengths we cannot simply say that the energy is continuous

We can now find at which Planck's law peaks:

$$\lambda_{\max} T = \frac{hc}{4.965k} = b$$

which corresponds with Wiens constant

theory of photons

Einstein assumed that the entire electromagnetic field was quantised

> light consist of corpuscles, called light quanta or *photons*

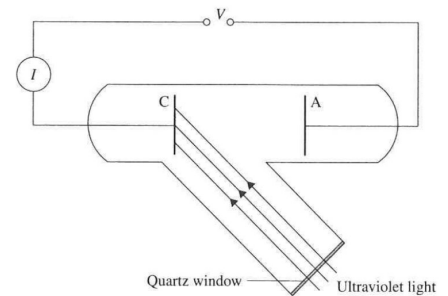
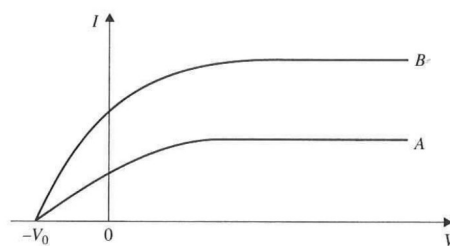
> travelling at the speed of light and having an energy:

$$E = h\nu = hc/\lambda.$$

### 1.2 the photoelectric effect

photoelectric effect: experiment

take a vacuum tube with two electrodes connected by a battery at a certain voltage  
> beam light on cathode and measure the current through the wire



observation:

- when V is positive, the electrons are attracted towards the anode
  - as V increases the current I increases until it saturates
    - > when V is large enough, the electrons can reach the anode by itself
  - when V is negative, there is a  $-V_0$  at which the photoelectric effect ceases
    - > emission of electrons from the cathode stops
    - > =stopping potential
- and is independent from the light intensity

>> explanation: at  $-V_0$  the voltage is just sufficient to repel the fastest electrons

> energies match up:

$$eV_0 = \frac{1}{2}mv_{\max}^2.$$

photoelectric effect: problems

1: there is a threshold frequency  $\nu_t$  of the radiation

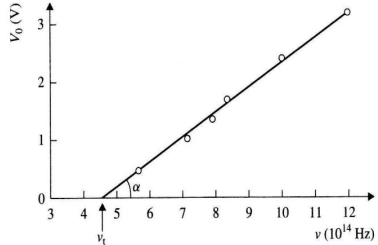
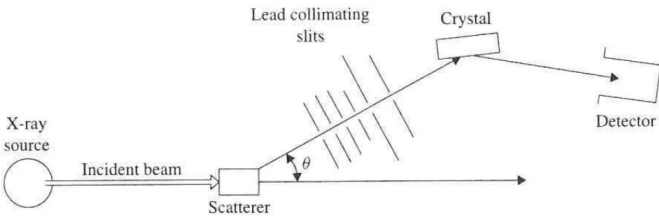
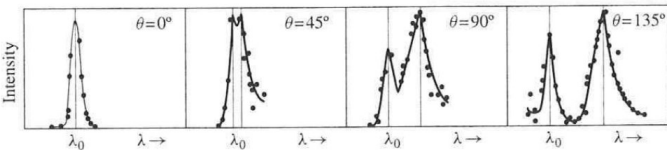
- > no emission of electrons take place, no matter the intensity if incident radiation
- > according to classical theory the effect should occur at any frequency given enough time

2: stopping potential  $V_0$  is linearly dependent only on light frequency

- > according to classical theory this should also increase with intensity

3: electron emission takes place instantly, with no time delay

- > according to classical theory, the energy from the light is spread uniformly over the wavefront thus to reach an energy threshold to eject an electron, there should be time delay

photoelectric effect: solution	<p>Expand Planck's quantum theory to the electromagnetic field</p> <ul style="list-style-type: none"> <li>&gt; light comes in packets of energy, called photons</li> <li>&gt; the energy of these photons depend on its frequency:</li> </ul> $E = h\nu = hc/\lambda$ <p>When a photon falls on the cathode, one atom of the cathode absorbs the photon</p> <ul style="list-style-type: none"> <li>&gt; if the photon has sufficient energy, the atom releases an electron</li> <li>&gt; therefore the threshold for emitting electrons depends on frequency of light</li> </ul> <p>it also explains why there is no time delay, since the atoms absorb all the energy from one photon, instead of it dissipating throughout the cathode</p> <p>This is called the Einstein relation:</p> $\frac{1}{2}mv_{\max}^2 = h\nu - W$ <p>where W is the workfunction</p> <ul style="list-style-type: none"> <li>= minimum energy required for an electron to leave an atom</li> <li>&gt; determines the threshold frequency <math>\nu_t</math>:</li> </ul> $h\nu_t = W.$
Millikans experiments	<p>= measurement of stopping potential in relation to frequency</p> <p>&gt; equation:</p> $V_0 = \frac{h}{e}\nu - \frac{W}{e}.$ 
Wave-particle duality	<p>Light is proven to be both a wave and a particle:</p> <ul style="list-style-type: none"> <li>- wave: diffraction and interference</li> <li>- particle: photo-electric effect, black body</li> </ul>
<b>1.3 the Compton effect</b>	
Compton effect: experiment	<p>Irradiate a graphite target with a monochromatic X-ray beam with wavelength <math>\lambda_0</math></p> <ul style="list-style-type: none"> <li>&gt; measure the intensity of the scattered radiation as a function of wavelength</li> </ul> <p>Results:</p> <p>The scattered wavelength had two components: <math>\lambda_0</math>, the same wavelength</p> <p style="text-align: right;"><math>\lambda_1</math>, a new wavelength with <math>\lambda_1 &gt; \lambda_0</math></p>  <p>Figure 1.7 Schematic diagram of Compton's apparatus.</p> 

## Compton effect: explanation

take a free electron at rest

> after collision, the electron recoils

> relativistic: for the electron moving at speed  $v$ , the energy is:

$$E = \frac{mc^2}{(1 - v^2/c^2)^{1/2}}. \quad (1.31)$$

The kinetic energy  $T$  of the particle is defined as the difference between its total energy  $E$  and its rest mass energy  $mc^2$ , so that

$$T = E - mc^2. \quad (1.32)$$

The corresponding momentum of the particle is

$$\mathbf{p} = \frac{m\mathbf{v}}{(1 - v^2/c^2)^{1/2}} \quad (1.33)$$

and from (1.31) and (1.33) we see that the energy and momentum are related by

$$E^2 = m^2c^4 + p^2c^2. \quad (1.34)$$

> the velocity of the photon is  $c$

> its energy:  $E = h\nu = hc/\lambda$  is finite

> mass of a photon is zero, because of 1.31

> the momentum:

$$p = E/c = h/\lambda.$$

> Let us now apply these formulae to the situation depicted in Fig. 1.9, where a photon of energy  $E_0 = hc/\lambda_0$  and momentum  $\mathbf{p}_0$  (with  $p_0 = E_0/c = h/\lambda_0$ ) collides with an electron of rest mass  $m$  initially at rest. After the collision, the photon has an energy  $E_1 = hc/\lambda_1$  and a momentum  $\mathbf{p}_1$  (with  $p_1 = E_1/c = h/\lambda_1$ ) in a direction making an angle  $\theta$  with the direction of incidence, while the electron recoils with a momentum  $\mathbf{p}_2$  making an angle  $\phi$  with the incident direction. Conservation of momentum yields  $\mathbf{p}_0 = \mathbf{p}_1 + \mathbf{p}_2$  or, in other words

$$p_0 = p_1 \cos \theta + p_2 \cos \phi \quad (1.36a)$$

$$0 = p_1 \sin \theta - p_2 \sin \phi \quad (1.36b)$$

from which we find that

$$p_2^2 = p_0^2 + p_1^2 - 2p_0p_1 \cos \theta. \quad (1.37)$$

Conservation of energy yields the relation

$$E_0 + mc^2 = E_1 + (m^2c^4 + p_2^2c^2)^{1/2} \quad (1.38)$$

and therefore, if we denote by  $T_2$  the kinetic energy of the electron after the collision, we have

$$\begin{aligned} T_2 &= (m^2c^4 + p_2^2c^2)^{1/2} - mc^2 \\ &= E_0 - E_1 = c(p_0 - p_1) \end{aligned} \quad (1.39)$$

so that

$$p_2^2 = (p_0 - p_1)^2 + 2mc(p_0 - p_1). \quad (1.40)$$

Combining (1.40) with (1.37) we then find that

$$\begin{aligned} mc(p_0 - p_1) &= p_0p_1(1 - \cos \theta) \\ &= 2p_0p_1 \sin^2(\theta/2). \end{aligned} \quad (1.41)$$

Multiplying both sides of (1.41) by  $h/(mcp_0p_1)$  and using the fact that  $\lambda_0 = h/p_0$  and  $\lambda_1 = h/p_1$ , we finally obtain

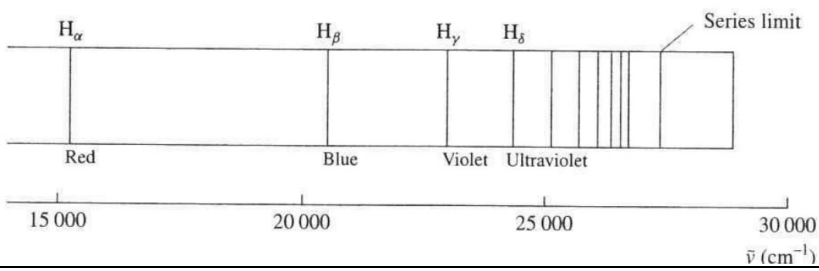
$$\Delta\lambda = \lambda_1 - \lambda_0 = 2\lambda_C \sin^2(\theta/2) \quad (1.42)$$

where the constant  $\lambda_C$  is given by

$$\lambda_C = \frac{h}{mc} \quad (1.43)$$

Now  $\lambda_C$  is the Compton wavelength

### 1.4 atomic spectra and the Bohr model of the hydrogen atom

emission lines	= light from an incandescent f-gas composed of discrete wavelengths
absorption lines	= lines missing from a continuum when light is passed through a gas
	>> emission an absorption lines overlap > each element has its own characteristic line spectrum
Balmer series	<p>= line spectrum of hydrogen</p> <p>&gt; lines come closer together, until a limit of <math>\lambda = 3646 \text{ \AA}</math>.</p> <p>formula:</p> $\lambda = C \frac{n^2}{n^2 - 4} \quad (1.44)$ <p>where <math>C</math> is a constant equal to <math>3646 \text{ \AA}</math>, and <math>n</math> is an integer taking on the values 3,4,5...</p> 
wave numbers of Balmer series	<p>wave number: <math>\tilde{\nu} = 1/\lambda = \nu/c</math>.</p> <p>&gt; for Balmer series:</p> $\tilde{\nu} = R_H \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$ <p>with <math>R_H</math> = Rydberg constant</p>
general Balmer-Rydberg formula	<p>For other series of atomic hydrogen:</p> $\tilde{\nu}_{ab} = R_H \left( \frac{1}{n_a^2} - \frac{1}{n_b^2} \right); \quad n_a = 1, 2, \dots$ $n_b = 2, 3, \dots$ <p>Now:</p> <p>As seen from (1.47), the wave number <math>\tilde{\nu}_{ab}</math> of a line in the atomic hydrogen spectrum can be expressed in the form</p> $\tilde{\nu}_{ab} = T_a - T_b \quad (1.48)$ <p>that is as the difference of two <i>spectral terms</i></p> $T_a = R_H/n_a^2, \quad T_b = R_H/n_b^2. \quad (1.49)$ <p>Consequence:</p> <p>a consequence, if the wave numbers of three spectral lines are associated with three terms as</p> $\tilde{\nu}_{ij} = T_i - T_j, \quad \tilde{\nu}_{jk} = T_j - T_k, \quad \tilde{\nu}_{ik} = T_i - T_k \quad (1.50)$ <p>we have</p> $\tilde{\nu}_{ik} = (T_i - T_j) + (T_j - T_k) = \tilde{\nu}_{ij} + \tilde{\nu}_{jk} \quad (1.51)$ <p>&gt; for elements other than hydrogen this formula still holds up, but with more complicated <math>T_a</math> and <math>T_b</math></p>

### 1.4.2 the nuclear atom

planetary model of the atom

Positive charge in nucleus which holds the mass  
negative charge around the nucleus which gives volume

- > problem: the  $e^-$  on curved trajectory accelerate
  - > radiate electromagnetic waves
  - > lose energy
  - >  $e^-$  collapse on the nucleus

But: experiments confirm that there should be a planetary model

### 1.4.3 Bohr's model of the hydrogen atom

Bohr's model

planetary model of atom but:

- $e^-$  only have a certain set of stable orbits, stationary states
- $e^-$  do not radiate em-waves

- > these stationary states have certain energy levels
- > radiation only takes place when switching between energy levels

Bohr frequency relation

For two energy states  $E_a$  and  $E_b$ , an  $e^-$  can switch only if its hit by a photon for which:

$$h\nu = E_b - E_a$$

on the other hand if the  $e^-$  lowers energy state, it sends out a photon with this energy

Bohr's model analytically

assume the magnitude of the orbital angular momentum  $L$  of the  $e^-$  in a circular orbit around the nucleus can only take one of the values:

$$L = nh/2\pi = n\hbar,$$

with  $n = 1, 2, 3, \dots$  the quantum number

Take an  $e^-$  moving around a nucleus with infinite mass

> for a speed  $v$  non-relativistic it holds:

$$\frac{Ze^2}{(4\pi\epsilon_0)r^2} = \frac{mv^2}{r},$$

now because angular momentum is quantised

$$L = mvr = n\hbar, \quad n = 1, 2, 3, \dots$$

From (1.54) and (1.55), we obtain the allowed values of  $v$  and  $r$

$$v = \frac{Ze^2}{(4\pi\epsilon_0)\hbar n}$$

and

$$r = \frac{(4\pi\epsilon_0)\hbar^2 n^2}{Zme^2}.$$

the kinetic energy:

$$T = \frac{1}{2}mv^2 = \frac{m}{2\hbar^2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^2}$$

and if we choose  $E=0$  for the  $e^-$  at infinite distance,  $r = \infty$ , the potential energy is:

$$V = -\frac{Ze^2}{(4\pi\epsilon_0)r} = -\frac{m}{\hbar^2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^2}.$$

Thus the total energy is:

$$E_n = -\frac{m}{2\hbar^2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^2}, \quad n = 1, 2, 3, \dots$$

>  $n$  can go from 1 up until  $\infty$

> the energy spectrum corresponding to the bound states of the one-electron atom contains an infinite number of discrete energy levels

Bohr's model analytically	<p>the transitions between energies can thus be written down as:</p> $\nu_{ab} = \frac{m}{4\pi\hbar^3} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \left( \frac{1}{n_a^2} - \frac{1}{n_b^2} \right)$ <p>Or in the case of hydrogen:</p> $\tilde{\nu}_{ab} = R(\infty) \left( \frac{1}{n_a^2} - \frac{1}{n_b^2} \right)$ <p>where the constant <math>R(\infty)</math> is given by</p> $R(\infty) = \frac{m}{4\pi c \hbar^3} \left( \frac{e^2}{4\pi\epsilon_0} \right)^2$ <p>with <math>\tilde{\nu}_{ab} = \nu_{ab}/c</math>.</p> <p>&gt; this is the exact same form as the Balmer-Rydberg function</p>
Ionisation energy	<p>= the energy needed to remove one <math>e^-</math> from an atom in its ground state</p> <p>We have just seen that you can describe the energy as:</p> $E_n = -I_P/n^2, \quad n = 1, 2, 3, \dots$ <p>where</p> $I_P = \frac{m}{2\hbar^2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 = hcR(\infty)Z^2 = 13.6 Z^2 \text{ eV.}$ <p>for a one-electron system</p> <p>&gt; zero energy is achieved when the <math>e^-</math> is at infinite distance  &gt; the ionisation energy is the <math>E_n</math> for <math>n=1</math>, so <math>E_n = I_P</math></p>
ionisation	<p>an atom is initially in a bound state of negative energy <math>E_n</math>  &gt; when it absorbs energy greater than the binding energy <math> E_n </math> it gets ionised  &gt; ejected electron will have positive energy</p> <p>thus: states of positive energy are unbound states of the nucleus-electron system</p>
some quantities of Bohr's theory	<p>For <math>Z=1</math></p> $a_0 = \frac{(4\pi\epsilon_0)\hbar^2}{me^2} = 5.29 \times 10^{-11} \text{ m.} \quad (1.66)$ <p>The velocity <math>v_0</math> of the electron in the first Bohr orbit of the hydrogen atom is seen from (1.56) to be given by</p> $v_0 = \frac{e^2}{(4\pi\epsilon_0)\hbar} = \alpha c \quad (1.67)$ <p>where we have introduced the dimensionless constant</p> $\alpha = \frac{e^2}{(4\pi\epsilon_0)\hbar c} \quad (1.68)$ <p>&gt; <math>\alpha</math> is the fine structure constant: <math>\alpha = 1/137</math>  &gt; thus <math>v_0 = c/137</math>  &gt; <math>e^-</math>-speeds are non-relativistic</p> <p>And energy can be rewritten as:</p> $E_n = -\frac{1}{2}mc^2 \frac{(Z\alpha)^2}{n^2}.$ <p>&gt; if <math>Z</math> is not too large, <math>E_n</math> is small with respect to <math>mc^2</math>  &gt; non-relativistic energy</p>

### 1.4.3 finite nucleus mass

Bohr's model with finite mass

let  $m$  be the electron mass and  $M$  the nucleus mass  
 > centre of mass is will be at rest or in uniform motion  
 > centre of mass is set to be the centre of the coordinate system

Now the angular momentum is:

$$L = \mu v r$$

with  $\mu$  the reduced mass:

$$\mu = \frac{mM}{m+M}$$

So Bohr's quantisation condition becomes:

$$L = \mu v r = n\hbar, \quad n = 1, 2, 3, \dots$$

Now the kinetic energy becomes:

$$T = \mu v^2 / 2.$$

and the potential energy stays the same, since it doesn't depend on mass:

$$V = -Ze^2 / (4\pi\epsilon_0)r$$

So the total energy

$$E_n = -\frac{\mu}{2\hbar^2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^2}$$

Now we find for the radius:

$$r = \frac{(4\pi\epsilon_0)\hbar^2 n^2}{Z\mu e^2} = \frac{n^2}{Z} \frac{m}{\mu} a_0 = \frac{n^2}{Z} a_\mu$$

where  $a_\mu = (m/\mu)a_0$  is the *modified Bohr radius*.

The Rydberg constant is also modified:

$$R(M) = \frac{\mu}{m} R(\infty) = \frac{1}{1 + (m/M)} R(\infty).$$

isotopic shift

= effect where different isotopes of the same element have slightly different spectra  
 > caused by the dependence on  $\mu$

### 1.4.4 limitations of the Bohr model

limitations of Bohr's model

- hypotheses that  $e^-$  doesn't radiate energy in stationary orbit and that only circular orbits are allowed don't have any proof
- model doesn't account for 2 or more  $e^-$  systems
- no method to calculate rate of transitions between stationary orbits

### 1.4.5 Bohr correspondence principle

correspondence principle

= quantum theory results must tend asymptotically to those obtained from classical physics in the limit of large quantum numbers

> example

in a Bohr system with one  $e^-$ , the frequency of the radiation emitted by a transition from  $E_n$  to  $E_{n-1}$  is given by:

$$\nu = \frac{m}{4\pi\hbar^3} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \left[ \frac{1}{(n-1)^2} - \frac{1}{n^2} \right].$$

for large  $n$ , we can simplify:

Now, when  $n \gg 1$ , one has  $(n-1)^{-2} - n^{-2} \simeq 2n^{-3}$  and therefore, for large  $n$ ,

$$\nu \simeq \frac{m}{2\pi\hbar^3} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^3}.$$

If we calculate according to classical physics:

$$\nu_{cl} = \frac{v}{2\pi r} = \frac{m}{2\pi\hbar^3} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^3}$$

> correspondence



### 1.4.6 the Franck and Hertz experiment

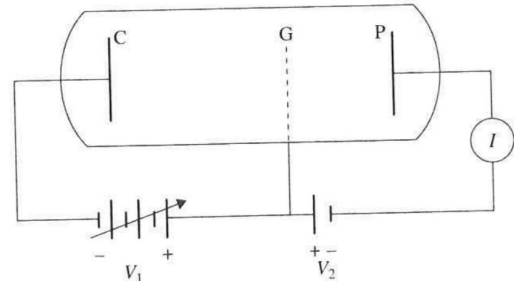
Franck&Hertz: experiment

We have a tube filled with mercury  
 > connect a cathode C that ejects  $e^-$  towards a wire grid G  
 > these  $e^-$  are accelerated using a potential  $V_1$ :

$$mv^2/2 = eV_1$$

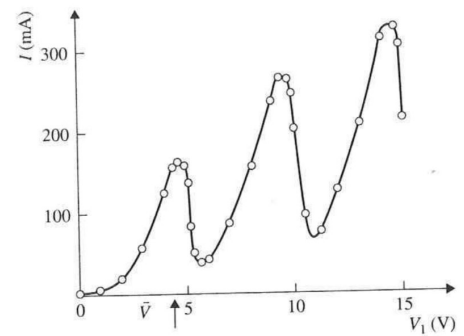
When the  $e^-$  get to G, we read out a current I  
 > behind G, we have a collection plate P with a lower potential  $V_2$

Now we gradually increase  $V_1$  and measure the current I



Franck&Hertz: results

When  $V_1$  is gradually increased, we see increases in I, followed by sharp dips



Franck&Hertz: explanation

When the  $e^-$  leave C, they have a certain energy  
 > when colliding with a mercury atom, they can transfer this energy  
 > however: the mercury atom will only absorb this energy when it matches the energy required to excite an  $e^-$  to jump an orbit

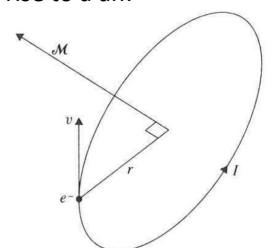
When the voltage applied to the system matches this energy, we see a sharp dip in current flow

> this is because the  $e^-$  lose their energy to the mercury atoms  
 > and therefore they cannot reach the grid G

When the voltage doesn't match this energy, we see an increase in voltage

> this is because the  $e^-$  collide elastically with the mercury atoms  
 > the  $e^-$  retain their energy

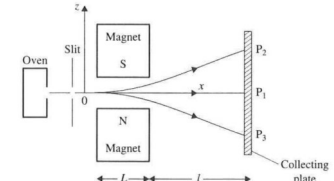
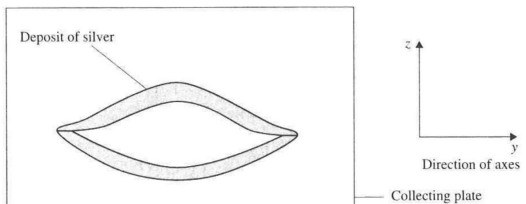
### 1.5 The Stern-Gerlach experiment: angular momentum and spin

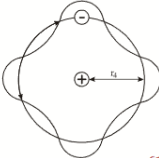
magnetic dipole moment of an $e^-$	<p>For a circulating current <math>I</math> enclosing a small plane area <math>dA</math>, gives rise to a an:</p> $\mathcal{M} = I dA$ <p>Now for a <math>e^-</math> with charge <math>e</math>, velocity <math>v</math> on an orbit with radius <math>r</math>:</p> $I = \frac{ev}{2\pi r}$ <p>The enclosed area is <math>\pi r^2</math>:</p> $\mathcal{M} = -\frac{e}{2m} \mathbf{L}$ <p>With <math>\mathbf{L} = \mathbf{r} \times \mathbf{p}</math> = orbital angular momentum</p> 
magnetic dipole and Bohr's quantisation rule	<p>Bohr's quantisation rule suggests <math>\hbar</math> is a natural unit of angular momentum &gt; we rewrite the magnetic dipole moment as:</p> $\mathcal{M} = -\mu_B (\mathbf{L}/\hbar)$ <p>where</p> $\mu_B = \frac{e\hbar}{2m}$ <p>&gt; where <math>\mu_B</math> = Bohr magneton:</p> $\mu_B = 9.27 \times 10^{-24} \text{ J T}^{-1}$

#### 1.5.1 interaction with a magnetic field

atom in a magnetic field	<p>If an atom with magnetic moment <math>\mathbf{M}</math> is placed in a magnetic field <math>\mathbf{B}</math>: The potential energy is:</p> $W = -\mathcal{M} \cdot \mathbf{B}$ <p>Where the system experiences a torque:</p> $\mathbf{\Gamma} = \mathcal{M} \times \mathbf{B}$ <p>and thus a net force:</p> $\mathbf{F} = -\nabla W$ <p>Which has special components:</p> $F_x = \mathcal{M}_y \frac{\partial B_z}{\partial x}, \quad F_y = \mathcal{M}_x \frac{\partial B_z}{\partial y}, \quad F_z = \mathcal{M}_z \frac{\partial B_z}{\partial z}$
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#### 1.5.2 Stern-Gerlach experiment

Stern-Gerlach: experiment	<p>in a vacuum, narrow atomic beam of silver atoms enters inhomogeneous magnetic field &gt; falls in collecting plate where it can be detected</p> 
Stern-Gerlach: results	<p>The magnetic moment of the atoms is completely random &gt; the force on each atom is:</p> $F_x = \mathcal{M}_z \frac{\partial B_z}{\partial x}, \quad F_y = \mathcal{M}_z \frac{\partial B_z}{\partial y}, \quad F_z = \mathcal{M}_z \frac{\partial B_z}{\partial z}$ <p>But since the magnet is xz-symmetrical we find: <math>\partial B_z / \partial y = 0</math>. Also, apart from edge effects, <math>\partial B_z / \partial x = 0</math> Only a force in the z direction</p> <p>&gt; hypothesis: since the magnetic moment in z-direction is completely random: <math display="block">-\mathcal{M} \leq \mathcal{M}_z \leq \mathcal{M}</math> we expect a deposit spread continuously over a symmetrical region</p> <p>Reality: we find two separate deposits:</p> 

Stern-Gerlach: explanation	<p><math>\mathbf{M}_z</math> can range from <math>(-M_z)_{\max}</math> to <math>(M_z)_{\max}</math> and <math>L_z</math> from <math>(-L_z)_{\max}</math> to <math>(L_z)_{\max}</math>  &gt; we denote the observed multiplicity of values of <math>M_z</math> by <math>\alpha</math></p> <p>Let us now postulate that <math>L</math> changes in steps of <math>\hbar</math>:</p> $L = l\hbar,$ <p>and thus for <math>L_z</math> is also quantised:</p> $L_z = m\hbar$ <p>with <math>l</math> zero or positive  with <math>m</math> positive and negative</p> <p>Now we find a link between <math>m</math> and <math>l</math>:  <math>m</math> must take on values: <math>-l, -l+1, \dots, l-1, l</math>,</p> <p>And the multiplicity <math>\alpha = (2l+1)</math></p> <p>&gt;&gt; result: quantisation of the z-component of angular momentum and the existence of electron spin</p>
<b>1.5.3 electron spin</b>	
electron spin	<p>assign each <math>e^-</math> with an intrinsic magnetic moment <math>\mathbf{M}_s</math>  &gt; can only take two values: <math>\pm M_s</math></p> <p>This magnetic moment is due to an intrinsic angular moment/spin, denoted by <math>\mathbf{S}</math>  &gt; we have: <math>\mathbf{M}_s = -g_s \mu_B (\mathbf{S}/\hbar)</math></p> <p>with <math>g_s</math> the spin gyromagnetic ratio</p>
electron spin and the Stern-Gerlach experiment	<p>Introduce a spin quantum number analogous to <math>l</math>  &gt; multiplicity of the spin component is given by: <math>(2s+1)</math>  &gt; for an <math>e^-</math>: <math>s = 1/2</math></p> <p>Now the magnetic moment if an atom is due to its orbital angular momentum and its spin angular momentum:  <math>\mathbf{M} = -\mu_B (\mathbf{L} + g_s \mathbf{S})/\hbar.</math></p>
<b>1.6 De Broglie's hypothesis. Wave properties of matter and the genesis of quantum mechanics</b>	
De Broglie's hypothesis	<p>= material particles also possess wave-like properties  &gt; wave-particle duality is universal</p> <p>The energy of a photon is given by:  <math>E = h\nu</math>  and the photon momentum is given by:  <math>p = h\nu/c = h/\lambda,</math></p> <p>Now for a particle, De Broglie suggested:</p> $\nu = \frac{E}{h} \quad \text{and} \quad \lambda = \frac{h}{p}.$ <p>Non-relativistically this can also be approximated as</p> $\lambda = \frac{h}{mv}.$
Explanation for De Broglie's hyp.	<p>In a Bohr system with one <math>e^-</math>, let's suppose an <math>e^-</math> has a circular orbit with radius <math>r</math>  &gt; we associate a standing wave with this stable orbit  &gt; a whole number of wavelengths must fit into one circumference <math>2\pi r</math>:</p> $n\lambda = 2\pi r \quad n = 1, 2, 3, \dots$ <p>Since <math>\lambda = h/p</math> and <math>L = rp</math>, we immediately find the condition</p> $L = nh/2\pi = n\hbar$ <p>which we already have proven</p> 

experimental confirmation of De Broglie's hypothesis

For a macroscopic particle:

$m = 10^{-3} \text{ kg}$ ,  $v = 1 \text{ m/s} \rightarrow \lambda \simeq 6.6 \times 10^{-31} \text{ m} = 6.6 \times 10^{-21} \text{ \AA}$ ,  
> undetectable

For a non-relativistic  $e^-$ :

$m = 9.1 \cdot 10^{-31} \text{ kg}$ , accelerated by a potential difference  $V_0$

> kinetic energy:  $mv^2/2 = eV_0$ , then:

$$\lambda = \frac{h}{(2meV_0)^{1/2}} = \frac{12.3}{[V_0(\text{Volts})]^{1/2}} \text{ \AA}$$

> very small

> if we want to use interference, there are no slits small enough

> we can however use a crystal whereupon  $e^-$  are reflected

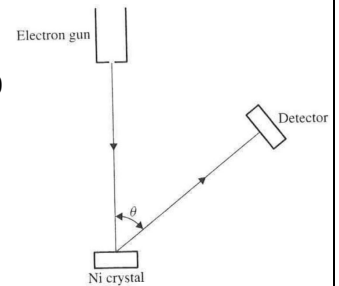
### 1.6.1 Davisson-Germer experiment

Davisson-Germer: experiment

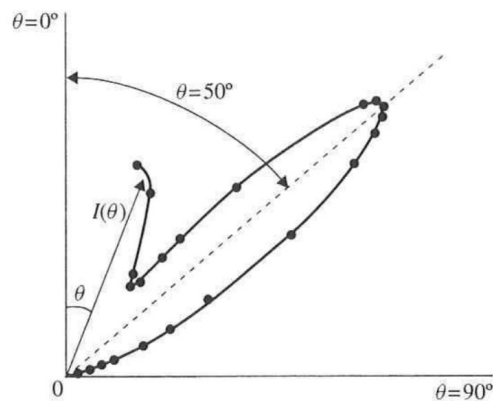
accelerate  $e^-$  with kinetic energy  $eV_0$

> strike at normal incidence the surface of a crystal

> measure  $N(\theta)$  = number of atoms scattered at an angle  $\theta$



> results: max at  $0^\circ$  > min at  $35^\circ$  > max at  $50^\circ$



Davisson-Germer: explanation

Bragg condition of constructive interference:

$$n\lambda = 2d \sin \theta_B$$

where  $d$  is the spacing of Bragg planes and  $n$  an integer

Now if we look at the figure, we can formulate for  $D$  = spacing of atoms in the crystal:

$$d = D \sin \alpha, \text{ with } \alpha = \pi/2 - \theta_B.$$

This the condition becomes:

$$n\lambda = D \sin \theta.$$

From other experiment we know for the crystal:  $D = 2.15 \text{ \AA}$ .

> thus we find:

$$\lambda = (2.15 \sin 50^\circ) \text{ \AA} = 1.65 \text{ \AA}.$$

And through De Broglie's hypothesis we find:  $1.65 \text{ \AA}$ .

>> experiment confirms hypothesis

