H4: nonlinear equations

4.1 introduction	
nonlinear phenomena	= effects that aren't directly proportional to their causes > nonlinear equations can be represented by: $\mathbf{f}(\mathbf{x}) = \mathbf{y}$. > subtract y to find: $\mathbf{f}(\mathbf{x}) = \mathbf{0}$. > root finding problem
4.2 number of solutions	
solutions of a nonlinear problem	= correspond to the points where a curved hyperplane f(x) intersect > no general statements about number of solutions
multiple roots	a smooth function f has multiple roots if: $f(x^*) = f'(x^*) = f''(x^*) = \cdots = f^{(m-1)}(x^*) = 0$ then x* is a root of multiplicity m
4.3 sensitivity	
sensitivity of a root	= if x* is a root, how much does x* for small changes to the parameters of f
condition number	= parameter for sensitivity in one dimension = $\frac{1}{\|f'(x^*)\|}$ > if f'(x) is small near x*, the error of the root is big At a multiple root x*, f'(x*) = 0 > condition number is infinite
condition number in multiple dim.	In multiple dim. this is the Jacobian **J** > $\|\mathbf{J}_f^{-1}(x)\|$.
4.4 convergence rates and stopping criteria	
convergence rate	= the effectiveness with which a certain algorithm reaches its solution def: Let $\mathbf{e}_k = \mathbf{x}_k - x^*$ be the error at iteration k, where $\mathbf{x}_k$ is the approximate solution at iteration k and x^* the (usually unknown) true solution. An iterative method is said to converge with rate r if $\lim_{k\to\infty} \frac{\|\mathbf{e}_{k+1}\|}{\|\mathbf{e}_k\|^r} = C$ for some finite constant $C > 0$.
cost of solving a system	= depends on number of iterations + amount of iterations needed
types of convergence	• $r = 1$ and $C < 1$: *linear* convergence • $r > 1$: *superlinear* convergence • $r = 2$: *quadratic* convergence • $r = 3$: *cubic* convergence
stopping criterion	look at the relative change in the solutions: $\|\mathbf{x}_{k+1} - \mathbf{x}_k\| / \|\mathbf{x}_k\| < \varepsilon,$ with ε the *error tolerance*

4.5 solving nonlinear equations in one dimension	
4.5.1 bisection method	