Discrete Optimization and Decision Making

Last-mile Delivery Problem

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https://github.com/paaaaat/DODM-project

Contents

Notation

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- $G = (V, A)$ a complete directed connected graph
- a set of node $V = \{0, 1, 2, ..., \bar{c}\}$ ($C = V \setminus \{0\}$)
- a set of archs $A = \{(i, j) : i, j \in V, i \neq j\}$
- a package weight w_c, $\forall c \in C$
- a service time s_c, $\forall c \in C$
- a set $K = \left\{1, 2, ..., \bar{k}\right\}$ of homogeneous vehicles each with capacity W
- a travel time $t_{i,j}$ associated with each arc $(i, j) \in A$
- a maximum returning time to depot $t_{\max}$

Decision Variables

Decision Variables

- $x_{i,j,k}$: 1 if vehicle k traverses $(i,j) \in A$, 0 otherwise
- $y_{i,k}$: starting time of the service for vehicle k at customer i
- $d_{j,k}$: route duration if j is the last node visited before the depot

Objective Function

Objective Function

$$\min \quad \sum_{k \in K} \sum_{(i,j) \in A} t_{i,j} x_{i,j,k}$$

Constraints

Constraints

- $\sum_{j \in V} \sum_{k \in K} x_{j,i,k} = 1 \quad \forall i \in C, \text{with } j \neq i$
- $\sum_{j \in C} x_{0,j,k} = 1 \quad \forall k \in K$
- $\sum_{j \in V} x_{j,i,k} = \sum_{j \in V} x_{i,j,k} \quad \forall i \in C, \forall k \in K, \text{with } j \neq i$
- $\sum_{j \in C} x_{0,j,k} = \sum_{j \in C} x_{j,0,k} \quad \forall k \in K$
- $\sum_{i \in C} w_i \left(\sum_{j \in V} x_{j,i,k} \right) \leq W \quad \forall k \in K, \text{with } j \neq i$
- $y_{j,k} \geq y_{i,k} + t_{i,j} + s_i - M(1 - x_{i,j,k}) \quad \forall i \in V, \forall j \in C, \forall k \in K \text{ with } i \neq j$
- $y_{0,k} = 0 \quad \forall k \in K$
- $d_{j,k} \geq y_{j,k} + s_j + t_{j,0} - M(1 - x_{j,0,k}) \quad \forall j \in C, \forall k \in K$
- $d_{j,k} \leq t_{\max} x_{j,0,k} \quad \forall j \in C, \forall k \in K$
- $x_{i,j,k} \in \{0, 1\} \quad \forall (i,j) \in A, \forall k \in K$
- $y_{i,k} \geq 0 \quad \forall i \in C, \forall k \in K$
- $d_{j,k} \geq 0 \quad \forall j \in V, \forall k \in K$

Time Windows and Incompatible Triplets (Module 2)

Notation

- a set $R = \{(i,j,l) | i,j,l \in C, i \neq j, j \neq l\}$ of incompatible customers
- time windows $[a_c, b_c], \forall c \in C$

Decision Variables

- $z_{i,k}$: 1 if customer i is served by vehicle k

Constraints

- $z_{i,k} = \sum_{j \in V} x_{j,i,k} \quad \forall i \in C, \forall k \in K, \text{with } j \neq i$
- $a_i z_{i,k} \leq y_{i,k} \leq b_i z_{i,k} + M\big(1 - x_{i,j,k}\big) \quad \forall i \in C, \forall j \in V, \forall k \in K$
- $z_{i,k} + z_{j,k} + z_{l,k} \leq 2 \quad \forall (i,j,l) \in R, \forall k \in K$

Distributing Workload across Drivers (Module 4)

Decision Variables

- T_k: the total route duration of vehicle k
- T_{avg}: average worload $\frac{1}{|K|}\sum_{k\in K} T_k$ over all drivers k
- $\text{dev}_k = |T_k - T_{\text{avg}}|, \text{ for each } k \in K$

Objective Function

$$\min \qquad \frac{1}{|K|}\sum_{k\in K} \text{dev}_k$$

This was introduced as a second objective function in a lexicographic manner, allowing for a 5% degradation of the first objective function's optimal value:

$$Z_A \leq (1 + 0,05) Z_A^*$$

Constraints

- $T_k = \sum_{j \in C} d_{j,k} x_{j,0,k} \quad \forall k \in K$
- $T_{\text{avg}} = \frac{1}{|K|} \sum_{k \in K} T_k$
- $\text{dev}_k \geq T_k - T_{\text{avg}} \quad \forall k \in K$
- $\text{dev}_k \geq T_{\text{avg}} - T_k \quad \forall k \in K$