

Project Stage 4

Probability

I once heard that the odds of winning Mega Millions Jackpot in US is much like knowing that your pet will sneeze sometime during next 9 years and trying to predict when it will happen in the precision of seconds [year / month / day / hour / minute / second] I thought it would be interesting to investigate this claim in the scope of this assignment.

Rules And Winning Conditions

Mega Millions numbers are chosen from 2 sets of numbers. 5 numbers are drawn from one set of 56 numbered white balls and one Mega Ball (Golden) number is drawn from a second set of 46 numbered gold balls. The odds of winning Mega Millions are calculated by combining the odds for both sets of numbers for all prize levels. The first, third, fifth, sixth, eighth and ninth prize level odds are determined by the chances of choosing a given number of white balls correctly, plus the gold Mega Ball. The second, fourth and seventh prize level odds are determined by the chances of choosing a given number of white balls correctly and the gold Mega Ball incorrectly. The order of the items chosen is irrelevant.

Preliminary Calculations

1. How many combinations of 5 numbers can be drawn from 56 unique numbers can be calculated with “56-Choose-5” since the order does not matter:

$$C^{56}_5 = 56! / 5! * (56-5)! = 3,819,816$$

2. How many combinations of 1 number can be drawn from 46 unique numbers:

$$C^{46}_1 = 46! / 1! * (46-1)! = 46$$

Level 9 – Jackpot

Winning Condition: Match all five numbers plus the Golden Mega Ball

From the preliminary calculations we already know that the chances of choosing the right 5 numbers from 56 number set and one correct golden number from 46 number set are $1/3,819,816$ and $1/46$ consequently, therefore the probability of guessing both at once would simply be a product of their individual probabilities:

$$P(\text{Jackpot}) = 1/3,819,816 * 1/46 = 1/175,711,536$$

Now let's compare this probability to the claim of odds of pet-sneezing problem:

There are 283,824,000 seconds in 9 years, so the probability of picking the right moment would be $1/283,824,000$. This is nearly twice less likely as winning the jackpot, but still quite close.

Level 8 Win

Winning Condition: Only match all 5 numbers

This would be the probability of choosing 5 from 56 correctly times probability of not choosing 1 golden from 46 correctly:

$$P(\text{lvl 2}) = 1/3,819,816 * 45/46 = 1/3,904,700.80$$

Level 7 Win

Winning Condition: Only match 4 numbers from 56 plus one mega ball

The number of ways in which 4 numbers can be chosen correctly out of 5 numbers drawn from 56 unique numbers would be:

$$[5!/4! * (5-4)!] * [(56-5)!/((56-5)-(5-4))!(5-4)!] = 255$$

Meaning, that there are 255 different ways in which 4 numbers out of 5 numbers

drawn from a set of 56 numbers can be chosen correctly. Thus, the chances are 255 in 3,819,816 of correctly choosing 4 out of 5 numbers in the first portion of Mega Millions. The chance of correctly choosing the Mega Ball is simply 1 in 46. So the total probability would be:

$$P(\text{lvl } 7) = 255/3,819,816 * 1/46 = 1/689,064.85$$

Level 6 Win

Winning Condition: Only match 4 numbers and no mega ball

This would be equal to chances of getting 4 numbers correct out of 5 numbers drawn from 56

unique numbers times chances of not guessing one mega ball:

$$P(\text{lvl } 6) = 255/3,819,816 * 46/46 = 1/15,312.55$$

Level 5 Win

Winning Condition: match 3 numbers plus one mega ball

The number of ways in which 3 numbers can be chosen correctly out of 5 numbers drawn from 56 unique numbers would be:

$$[5!/3! * (5-3)!] * [(56-5)!/((56-5)-(5-3))!(5-3)!] = 12,750$$

Meaning, that there are 12,750 different ways in which 3 numbers out of 5 numbers drawn from a set of 56 numbers can be chosen correctly. Thus, the chances are 12,750 in 3,819,816 of correctly choosing 3 out of 5 numbers in the first portion of Mega Millions. The chance of correctly choosing the Mega Ball is simply 1 in 46. So the total probability would be:

$$P(\text{lvl } 5) = 12,750/3,819,816 * 1/46$$

Level 4 Win

Winning Condition: match 2 numbers plus one mega ball

For the same reasons as in level 5, number of ways of getting 2/5 numbers right will be:

$$[5!/2! * (5-2)!] * [(56-5)!/((56-5)-(5-2))!(5-2)!] = 208,250$$

And the probability of winning level 4 would be:

$$P(4) = 208,250/3,819,816 * 1/46 = 1/843.75$$

Level 3 Win

Winning Condition: match 3 numbers only, no mega ball

We've already calculated that the chances of getting 3 numbers correct out of 5 numbers drawn from 56 unique numbers are 12,750 in 3,819,816. The chance of correctly choosing the Mega Ball is 1 in 46. Therefore, the chances of incorrectly choosing the Mega Ball are, conversely, 45 in 46. So the total probability would be:

$$P(3) = 12,750 / 3,819,816 * 45/46 = 1/ 306.25$$

Level 2 Win

Winning Condition: match 1 number plus one mega ball

Number of ways how 1 number can be chosen correctly out of 5, when drawn from 56 numbers will be:

$$[5!/1! * (5-1)!] * [(56-5)!/((56-5)-(5-1))!(5-1)!] = 1,249,500$$

Since probability of guessing one mega ball is 1/46, the total probability will be:

$$P(2) = 1,249,500/3,819,816 * 1/46 = 1/140.63$$

Level 1 Win

Winning Condition: match just 1 mega ball

Number of ways how 0 out of 5 can be guessed correctly when drawn from 56 number set is the following:

$$[5!/1! * (5-1)!] * [(56-5)!/((56-5)-(5-1))!(5-1)!] = 2,349,060$$

This means that there are 2,349,060 different ways in which no numbers out of 5 numbers drawn from a field of 56 numbers are chosen correctly.

Thus, the chances are 2,349,060 in 3,819,816 of correctly choosing 0 out of 5 numbers in the first portion of Mega Millions, the total probability would, then be:

$$P(1) = 2,349,060/3,819,816 * 1/46 = 1/74.80$$