

# DISTRIBUTED LAMÉ MODE RESONATORS FOR TEMPERATURE-STABLE HIGH FREQUENCY MEMS OSCILLATORS

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## ABSTRACT

This paper presents a high frequency silicon resonator design possessing a quadratic temperature coefficient of frequency (TCF) profile with high-temperature turnover point. The design takes the advantages of a square Lamé mode and extends it to a distributed arrangement, to allow efficient transduction while scaling up the frequency. A Distributed Lamé mode resonator (DLR) with frequency exceeding 160MHz and  $Q$  of 77k is measured with a motional impedance of  $3.5k\Omega$ , showing one of the highest  $f$ - $Q$ s for silicon resonators. A high-temperature turnover point above  $100^\circ\text{C}$  is achieved for DLR fabricated on a commercially available highly doped substrate, which will allow ovenized high frequency silicon oscillators that cannot be realized efficiently using a simple square shape Lamé mode resonator due to dimensional constraints. Using this method, it is possible to achieve temperature-stable silicon oscillators at high frequencies comparable to quartz oscillators, which are suitable for very high frequency (VHF) military (navigation, radar, GPS) and consumer applications (mobile phones).

## INTRODUCTION

Silicon MEMS timing resonators have drawn a great amount of attention during the past two decades due to their small size, low cost, and integration compatibility. However, silicon resonator timing elements still have not been able to replace their quartz counterpart in many applications. The bottleneck for silicon MEMS resonators is the lack of temperature stability at high resonance frequency. Temperature stability in a silicon timing element is usually achieved by ovenizing a highly-doped square Lamé mode resonator with a quadratic TCF profile at its turnover point with a resonance frequency typically below 10MHz [1]. Square Lamé mode resonators have been popularly used to attain high  $f$ - $Q$  products owing to their ability to produce  $Q$ s over a million with frequencies in the range of 10s of MHz [2,3]. However, incorporating such high  $Q$ s drops the bandwidth and it becomes more challenging for the oscillator to lock into the resonance frequency, if there are spurious modes present near the actual peak. Furthermore, the relatively low frequency of square Lamé mode resonators requires the use of up-converting frequency synthesizers to get to higher frequency oscillators, leading to additional phase noise and larger power consumption.

Cross-sectional Lamé mode resonators have been demonstrated previously for ovenization at high frequencies [4]. However, they have poor robustness over process and mainly substrate thickness variations, affecting their manufacturability as a commercial product. Other high frequency resonance modes are also achievable with silicon, however they suffer from large temperature instability. In this

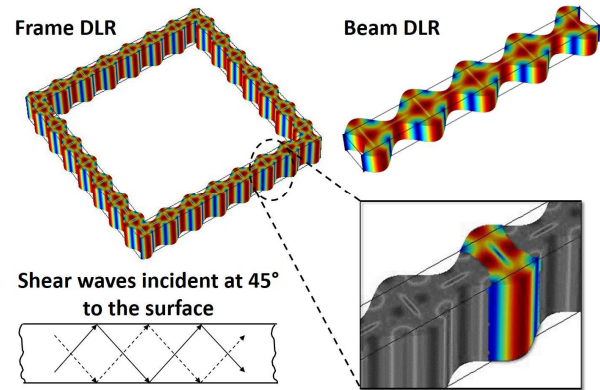


Figure 1: Animation of a propagating train of Lamé modes in the shape of frame and beam structures and (inset) highlighted deformation of one distributed element.

work, we present a resonator design that makes use of the in-plane shear nature of Lamé mode to enable high-frequency distributed resonance modes with a TCF turnover point at high temperatures. The proposed distributed Lamé mode allows us to achieve high  $f$ - $Q$  products at high frequencies ( $>100\text{MHz}$ ) with high  $Q$  factors ranging from 50k to 100k.

## DESIGN

The resonator realized is a Lamé mode bulk acoustic wave resonator constructed in a way that the mode is *distributed* across a beam or a frame structure (figure 1). In typical silicon bulk acoustic wave resonators, primary (P-wave) or secondary (S-wave) elastic waves travelling through the resonator body reach the silicon-air interface and reflect back to form a combination of P and S waves, a phenomenon also called mode conversion. However, in the special case wherein an S-wave is incident to the boundary at  $45^\circ$ , there will be no wave conversion and only S-waves are reflected [5], resulting in what is called a Lamé mode. The pure S-wave nature of Lamé mode indicates that a series of Lamé modes can result from a propagating train of S-waves at  $45^\circ$ . In other words, it is possible to actuate a series of Lamé modes distributed in a rectangular beam or a frame with a uniform width (Figure 2). The distributed configuration retains the thermal and mechanical properties of a square Lamé mode while providing more design freedom in performance scaling.

### Frequency turnover point sensitivity to doping

An important feature of the square Lamé mode is the behavior of its frequency to temperature. A square Lamé mode on a highly N-doped substrate has been known to show a turnover point at high temperatures ( $>100^\circ\text{C}$ ) for doping  $>$

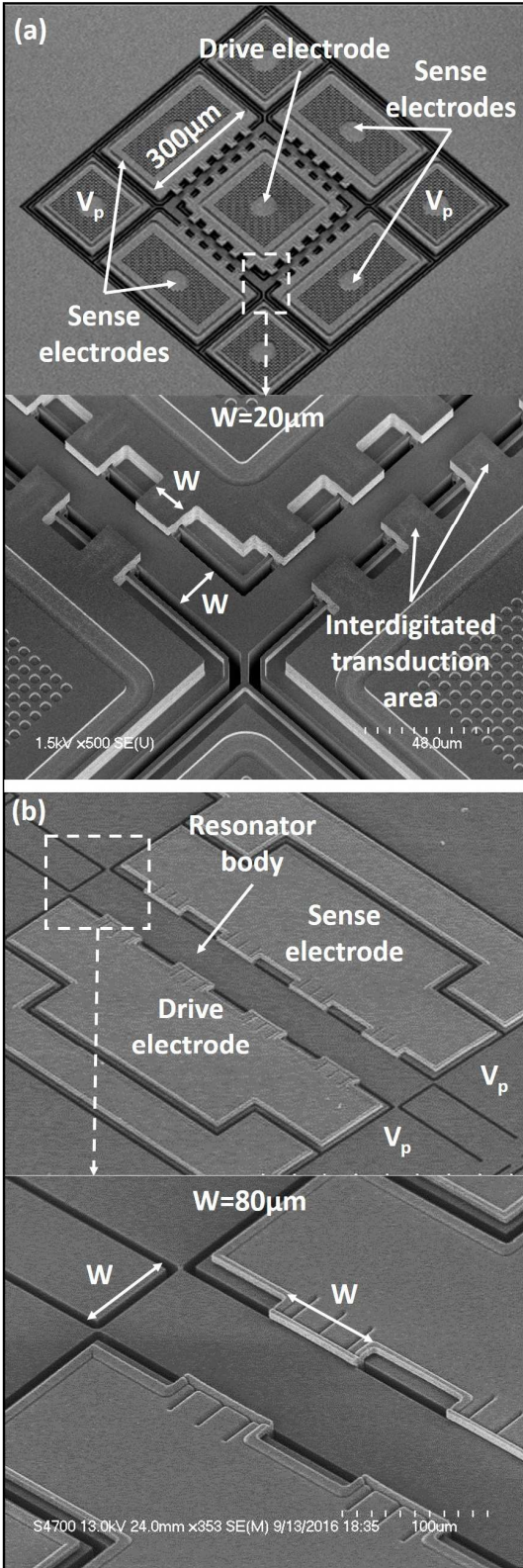


Figure 2: SEM images of DLR in frame (a) and beam (b) configurations at 167MHz and 41MHz with the zoomed in view showing the interdigitated distribution of  $W=20\mu\text{m}$  and  $80\mu\text{m}$  square Lamé modes across the lengths of the two DLR respectively.

$4 \times 10^{19} \text{cm}^{-3}$  [6]. At the turnover point, the slope of TCF curve is zero, hence a high temperature turnover point is important for highly stable oscillator design because it allows ovenization of the resonator to hold its temperature at its turnover point in order to improve frequency stability. This important TCF property is maintained for the distributed Lamé mode due to the same nature of wave propagation and energy distribution as a square Lamé mode resonator. The turnover point however, is extremely sensitive to doping variation. Figure 3 shows the simulated turnover points for a  $\langle 100 \rangle$  Lamé mode according to parameters given in [6].

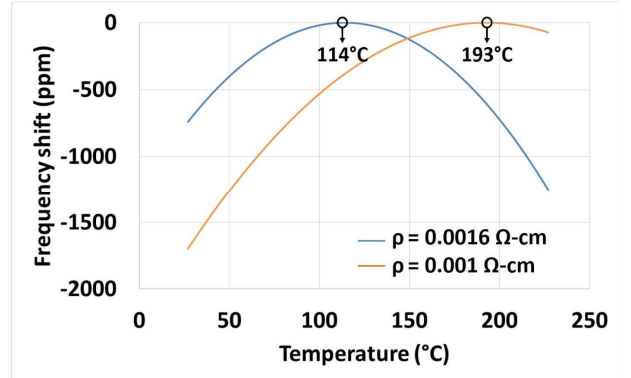


Figure 3: Simulated turnover points for  $\langle 100 \rangle$  Lamé mode for a commercially available highly doped N-type wafer with resistivity range of 0.001-0.0016 ohm-cm (corresponding to doping concentration of  $7\text{-}5 \times 10^{19} \text{cm}^{-3}$ ), showing large variation on a wafer.

#### Frequency scaling of Distributed Lamé mode resonators

While having the similar TCF behavior, DLR shows a clear advantage over the square Lamé mode in terms of frequency scalability. The frequency  $f$  of a square Lamé mode resonator aligned to the  $\langle 100 \rangle$  direction is given by [7]:

$$f = \frac{1}{\sqrt{2} \cdot W} \cdot \sqrt{\frac{c_{11} - c_{12}}{2\rho}} \quad (1)$$

Here,  $W$  is the width of the resonator,  $c_{11}$  and  $c_{12}$  are primary elastic constants, and  $\rho$  is the density of silicon. We can easily see that in order to get high operational frequency, the width of the square needs to be extremely small, which significantly sacrifices the transduction efficiency for oscillator applications. The motional impedance  $R_m$  of a square Lamé mode is given by [8]:

$$R_m \propto \frac{g^4}{QV_p^2 tW} \propto \frac{f}{QV_p^2} \quad (2)$$

where  $Q$  is the quality factor,  $t$  is the device thickness,  $g$  is the transduction gap size, and  $V_p$  is the polarization voltage. The small width  $W$  of the resonator results in a loss of transduction area, making the motional impedance increasing proportionally with the frequency. In addition, for properly designed Lamé mode resonator, the  $Q$  is close to the Akheizer limit of silicon, which decreases at high frequencies, further increasing  $R_m$  with a close to quadratic trend. Consequently, the rapid increase in motional impedance prevents one from up-scaling the operational frequency, which highly limits its

application.

This limit can be overcome with a distributed Lamé mode design. In DLR, the frequency is governed by the width of the beam and the length can be extended to a much longer multiple of the width. This effectively increases the transduction area by allowing a “digitated” electrode arrangement, thereby improving  $R_m$  and minimizing the insertion loss as compared to a square Lamé mode. For a DLR with  $N$  electrode-pair-digits, the motional impedance is given by:

$$R_m \propto \frac{g^4}{QV_p^2 \frac{N}{2} tW} \propto \frac{2}{N} \frac{f}{QV_p^2} \quad (3)$$

The number of electrode-pair-digits is divided by a factor of 2, considering the DLR has one electrode pair for every other distributed element, compared to two electrode pairs for one traditional square Lamé mode resonator.

Comparing Eqn. (2) and (3) we can see, the increase in frequency and drop in  $Q$  can be compensated by extending the resonator length and increasing the number of electrode-digits. This shows that the decoupling of width and length offers design freedom to scale the frequency without compromising transduction efficiency. For example, the frame DLR device used in this work has 24 electrode-pair-digits, offering motional impedance well below  $10k\Omega$  with a  $190nm$  gap size, which would otherwise be more than  $100k\Omega$  for a square Lamé mode resonator with the same frequency.

### Robustness to thickness variation

The DLR not only enables high-frequency Lamé modes with low motional impedance, but also has good robustness over device thickness variations. One of the major challenges in fabrication of devices on SOI wafers is the thickness variations on the device layer across the wafer. At very high doping levels close to the solid solubility of the dopant species in silicon, the thickness variation across the wafer can range from  $\pm 1\mu m$  to  $\pm 5\mu m$ .

Previous work [4] has shown incorporating the Lamé mode in a cross-sectional orientation, enables a high frequency device to be used as a Lamé mode with increased transduction area. However, due to the thickness dependency of the device, the fabrication of such a device becomes more difficult at higher frequencies since the effect of thickness variation in the device layer of an SOI wafer becomes more dominant for smaller thickness. In contrast, since the DLR is formed by in-plane S-wave reflections, the thickness dependency is eliminated and better control of the mode shape is enabled even in the presence of large thickness variations. The simulated thickness variation of  $\pm 1\mu m$  for a DLR shows as small as  $\pm 600ppb$  variation in frequency.

## EXPERIMENTAL RESULTS

Two implementations of DLR are fabricated, a beam structure with a width of  $80\mu m$  and a frame structure of width  $20\mu m$  used to demonstrate the frequency scalability using the HARPSS process which enables sub-micron-gap capacitive transduction [9-11]. These two resonators share the same

resonance frequency as would square resonators with the same side width ( $41MHz$  and  $167MHz$  respectively). The length dimensions of these two devices are much larger than their square counterparts, providing large transduction areas. Table 1 gives the properties of both the resonators.

Table 1: DLR dimensions are shown. Note that the difference in the motional impedance comes largely due to the difference in actuation gap sizes.

Dimension	Beam DLR	Frame DLR
Frequency (MHz)	41	167
Width ( $\mu m$ )	80	20
Length ( $\mu m$ )	720	300x4
Thickness ( $\mu m$ )	60 $\pm$ 1	40 $\pm$ 0.5
Gap size (nm)	300	190
Motional impedance ( $k\Omega$ )	80	3.5
Doping level ( $cm^{-3}$ )	$5-7 \times 10^{19}$	$1-2 \times 10^{18}$

The device is first characterized in a vacuum chamber at sub-mTorr pressure using a network analyzer. The frame DLR measures a motional impedance of  $3.5k\Omega$  at a  $V_p$  of

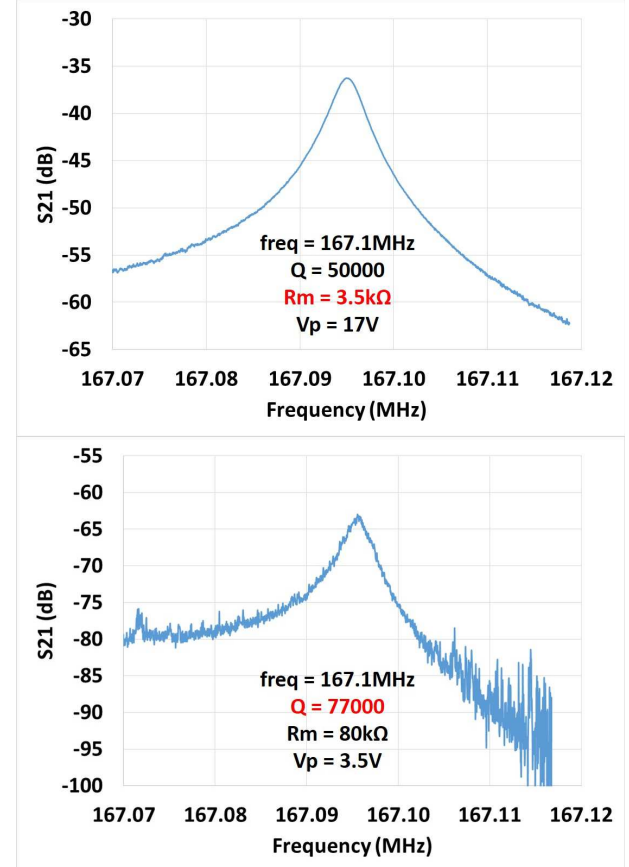


Figure 4: The low motional impedance of  $3.5k\Omega$  (insertion loss of  $-36dB$ ) at  $V_p=17V$  on the frame DLR reduces the gain requirement on the amplifiers of the oscillator circuit, making it easier to design oscillators at high frequencies (top) and  $Q$  of  $77k$  is measured for  $V_p=3.5V$  showing the  $f-Q$  product to be  $1.3 \times 10^{13}$  (bottom).



17V, which corresponds to a low insertion loss of -36dB (figure 4). When being used to build oscillators, this low motional impedance and insertion loss will reduce the gain requirement on the amplifier circuits, making it more practical to achieve high oscillation frequencies. A  $Q$  of 77k is measured on the same device for a  $V_p$  of 3.5V. The insertion loss is -64dB ( $R_m$  of 80k $\Omega$ ) which agrees with the  $V_p$  scaling, taking the  $f$ - $Q$  to be as high as  $1.3 \times 10^{13}$ , which is among the highest  $f$ - $Q$  products reported for silicon resonators. The temperature-frequency relation of the DLR fabricated on the highly-doped wafer is characterized in a temperature chamber. A quadratic TCF profile is observed on various devices as expected, with the maximum turnover point measured to be at 170°C and a minimum turnover point of 95°C (figure 5), which agrees well with the simulated value for the doping range considering the doping variation across the wafer. The TCF around the turnover point is close to zero, therefore ovenizing the device to maintain its temperature at the turnover point will allow one to build an extremely frequency stable oscillator that are suitable for high-frequency timing applications over a wide range of environment temperature.

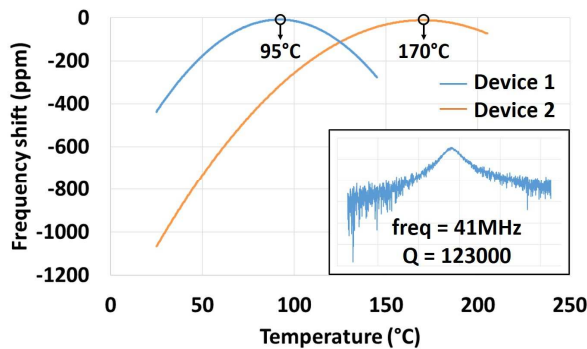


Figure 5: Measured turnover points at 95°C and 170°C for two beam DLR devices on the same wafer having a doping concentration variation of  $5$  to  $7 \times 10^{19} \text{ cm}^{-3}$  and (inset)  $Q$  of 123k is measured at 41MHz. The TCF at the two turnover points is zero.

## CONCLUSION

A method to design high frequency resonators using a distributed Lamé mode was implemented, while retaining the advantages of a square resonator including its intrinsic TCF turnover point and stability to thickness variations. A TCF turnover point was measured to be at 170°C for a doping concentration of  $5$  to  $7 \times 10^{19} \text{ cm}^{-3}$  on the fabricated DLR with a resonance frequency of 41MHz. An alternative DLR was also fabricated and characterized, measuring an  $f$ - $Q$  product of  $1.3 \times 10^{13}$  at 167MHz, with a motional impedance of 3.5k $\Omega$ , enabling much lower motional impedance as compared to square resonators, thereby allowing the up-scaling of resonance frequency. Results in this work show the capability and flexibility of DLR designs, indicating a great potential in VHF and UHF consumer and military applications.

## ACKNOWLEDGEMENT

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