# ENHANCED FREQUENCY STABILITY IN A NON-LINEAR MEMS OSCILLATOR EMPLOYING PHASE FEEDBACK

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## ABSTRACT

Microelectromechanical system (MEMS) oscillators are conventionally designed so that the resonator is operated within the linear regime. This paper investigates an optimal low noise point for a nonlinear MEMS oscillator where phase noise to frequency noise conversion is minimized. A closed loop oscillator consisting of a digitally controlled phase shifter is implemented and measurements of frequency stability are conducted under varying phase feedback conditions. The measured results show that for a specific value of feedback phase, the oscillation frequency is independent of phase fluctuations. Enhanced frequency stability is achieved when the MEMS resonator is biased at the optimal phase feedback condition in the non-linear regime.

## INTRODUCTION

Advanced microelectromechanical system resonators with high sensitivity and stability have received much interest for applications in timing, sensing and wireless communications [1,2]. Miniaturization provides significant application benefits; however, non-linear effects are also more readily apparent in micro- and nano-mechanical resonators as a result of dimensional scaling. Oscillators incorporating MEMS resonators for frequency references or sensing applications are conventionally designed so that the gain control scheme addresses resonator operation within the linear regime to avoid the detrimental impact of resonator nonlinearities on phase noise and frequency stability. More recent research has theoretically explored optimally biasing the resonator the resonator within the non-linear regime with analytical models predicting improvements in oscillator phase noise relative to operation in the linear regime [3]. Previous theoretical work by Greywall and Yurke *et al* report that by biasing the resonator suitably in the non-linear regime [4,5], fluctuations in feedback phase would be cancelled when the slope of the frequency-to-feedback phase passes through zero. Recent follow-on experimental work on NEMS oscillators demonstrated reductions in phase noise of the oscillator and suppression of amplitude-phase conversion of thermomechanical noise when the resonator is biased in the non-linear regime using phase feedback though frequency stability measurements were not separately presented [6]. Subsequently Zou *et al* experimentally showed that gains in frequency stability are possible when the resonator is biased optimally in the non-linear regime where resonator biasing was adjusted by controlling the drive amplitude and quality factor [7].

This paper demonstrates an approach to realize a low noise MEMS oscillator by employing digitally controlled phase feedback to bias the resonator optimally in the non-linear regime. Oscillation is started up in an open-loop configuration and operation is then switched to a closed-loop mode. The oscillation frequency in a closed loop configuration varies continuously with the feedback phase. Moreover, at the optimal operating point, enhanced frequency stability is achieved with significant improvements relative to operation within the linear region.

## EXPERIMENTAL METHODS

### Resonator design and characterization

The MEMS resonator tested is a vacuum packaged single-crystal silicon double-ended tuning fork (DETF). Fig. 1 shows a picture of the experimental set-up and the

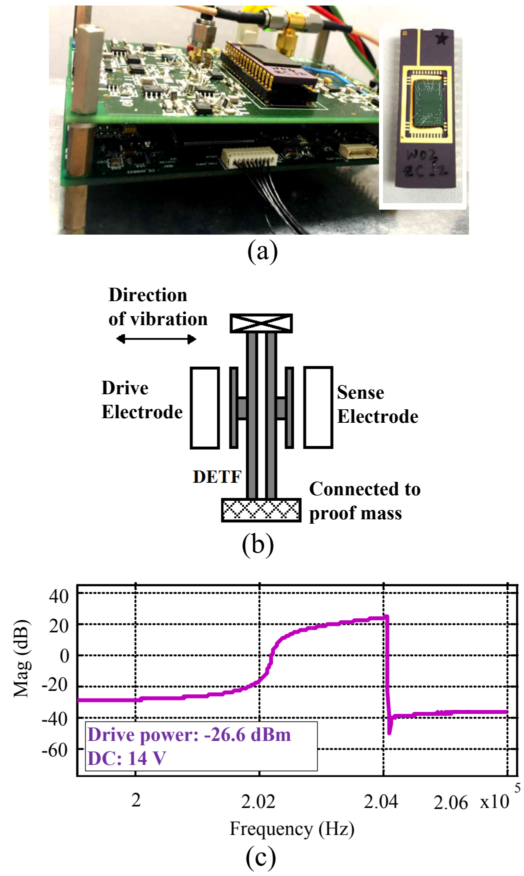


*Figure 1. Experimental set-up. (a) vacuum packaged DETF and electrical circuits, (b) structural configuration and (c) representative open-loop response of the DETF (driven by -26.6 dBm at bias voltage14 VDC).*

scheme used for driving the resonator and sensing the response. The DETF is electrostatically driven by applying using a parallel-plate electrode configuration while an alternate symmetrical set of parallel-plate electrodes is employed for sensing the motional response as shown in Fig. 1 (b). The resonator body is polarized by applying a dc voltage to the resonator with respect to both electrodes. The open-loop response of the resonator with a designed frequency about 194.3kHz and a quality factor 25,000 is measured via a network analyzer (Agilent N9915A Microwave Analyzer). A representative result is shown in Fig. 1 (c), when the resonator is driven using an ac voltage of 30 $mV_{p\text{-}p}$ and a dc polarization voltage 14 V.

**Oscillator Design**

A closed-loop oscillator circuit is designed consisting of a trans-impedance amplifier (TIA), a band pass filter (BPF), a comparator to facilitate digital phase control, a digital phase shifter (DPS) and a voltage divider as shown in the schematic diagram in Fig. 2. When the resonator is driven in the nonlinear region, the TIA and BPF read out the motional response of the DETF. The signal is then converted to a square wave by the comparator and captured by the DPS (TMS320F28335, Delfino Microcontroller). As shown in Fig. 2, a capture and timer unit within the DPS is employed to realize the phase shift within the feedback loop. A calibration of the feedback phase is performed using a test signal derived from a waveform generator. Results captured on an oscilloscope show that the phase can be shifted electronically with a resolution of 1.8° within a full span from 0° to 360°, and this value is specified as the feedback phase. Additionally, the output frequency of the DPS can be tuned from 150 to 300 kHz limited by the length of the registers (Timer unit), meeting the requirements for generating the drive signal for the resonator under test. According to Barkhausen criterion, the loop phase shift at the oscillation frequency should be

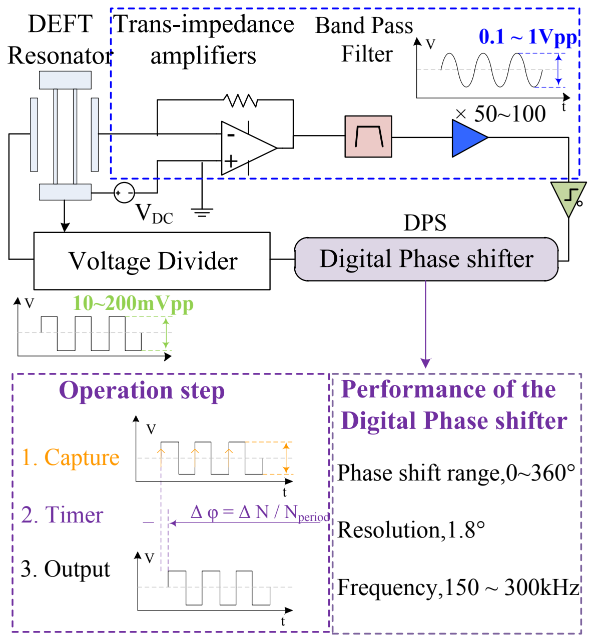


*Figure 2. Schematic diagram and operation of the closed-loop control for DETF in nonlinear regime.*

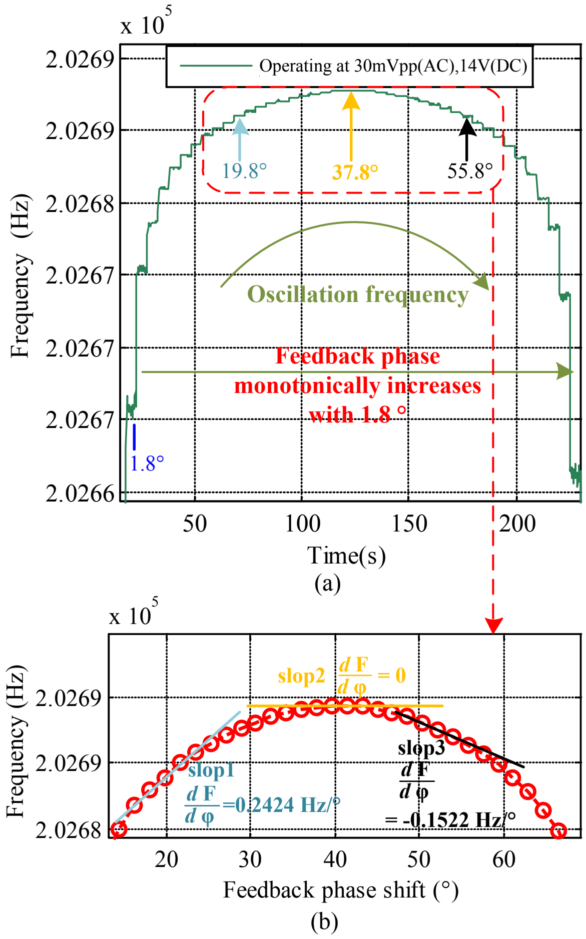


*Figure 3. Measured oscillation frequency variation with feedback phase, (a) frequency variation plotted in the time domain as the feedback phase is varied, (b) frequency vs. feedback phase plotted for a set of feedback phase values.*

0°. This is achieved by varying the feedback phase using the phase shifter. The operating point of the nonlinear MEMS resonator is also adjusted by varying the feedback phase for a specified drive voltage. The square wave derived from the DPS is then attenuated to 30 $mV_{p\text{-}p}$ using a voltage divider, and this signal is then used to drive the resonator.

An alternative approach to oscillation start-up is devised using this feedback topology. This oscillation start-up approach differs from that employed in conventional positive feedback oscillators. First, the resonator is excited using the drive signal derived from the microcontroller in an open-loop mode. Next, the DPS captures the signal from the read-out unit and adjusts the feedback phase to adjust the loop phase shift to be 0° or an integral multiple of 2π. Finally, the DPS outputs the drive signal to realize oscillation in a closed loop configuration.

## RESULTS AND DISCUSSION

**Oscillation frequency variation under varying feedback phase**

Upon applying a 14 VDC to bias the resonator, the oscillation amplitude grows to a steady-state value biasing the resonator within the nonlinear region. The oscillation frequency of the DETF can be altered in real-time by adjusting the feedback phase using the DPS in the time domain as shown in Fig. 3 (a). Fig. 3 (b) plots the oscillator output frequency vs. feedback phase for a set of feedback phase values. The feedback phase monotonically increases by 1.8°, and the oscillation frequency increases first and then decreases when the feedback phase crosses a value of 37.8°for a drive voltage of 30 $mV_{p-p}$. It can be noticed from the output signal in Fig. 3 (a) that the frequency stability is improved as the feedback phase approaches 37.8°. Specifically, for comparison purposes, at feedback phases of 19.8° and 55.8°, the slope of the oscillation frequency to feedback phase is found to be 0.24 Hz/° and -0.15 Hz/°, respectively as compared to a value of approximately zero at 37.8° as shown in Fig. 3(b).

**Frequency stability tests**

Further, it is observed experimentally in Fig. 4 that a low noise operating point exists where the measured minimum Allan deviation is 12.5 ppb under an integration time of 0.3 s for a feedback phase of 37.8°. The minimum Allan deviation at specific feedback phase values of 55.8°, 19.8° and 1.8°, is found to be 65.9 ppb, 434 ppb and 1855 ppb, respectively. Accordingly, it is experimentally demonstrated that the optimal operating point for improved frequency stability in an oscillator incorporating a non-linear MEMS resonator corresponds to the case when the slope of the oscillation frequency with respect to feedback phase is a minimum. Moreover, this value of minimum frequency stability is an improvement over previous results for comparable devices operating in the linear regime [7]. This improvement is attributed to two specific attributes of the oscillator topology presented here. First, in comparison to the linear oscillator, the signal amplitude is elevated. Second, the optimal low noise operating point corresponds to the turning point on the frequency-phase graph as illustrated in figure 3(b) for a given drive amplitude indicating that the impact of phase-to-frequency noise conversion is dominant in such a feedback configuration.

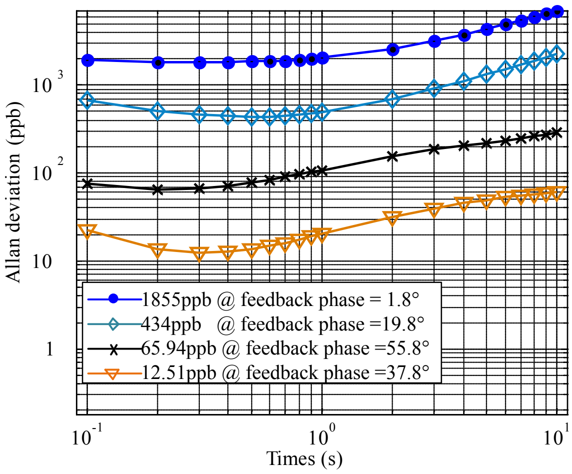


*Figure 4. Frequency stability tests at various feedback phase settings. The low noise operating point is observed at a feedback phase of 37.8° when the minimum Allan deviation is 12.5ppb.*

# CONCLUSION

This work demonstrates enhanced frequency stability for a nonlinear MEMS oscillator employing phase feedback by minimizing phase noise to frequency noise conversion. Resonator operation can be switched from open-loop mode to closed-loop with the feedback phase adjusted to maintain an overall loop phase shift of 0° or an integral multiple of 2π. A closed-loop oscillator composed of low-noise readout circuits and a digital phase shifter is presented, where the oscillation frequency can be altered accordingly in real-time by controlling the feedback phase. Moreover, a low noise operating point exists in the nonlinear regime, where the Allan deviation reaches 12.51 ppb for a feedback phase of 37.8° corresponding to the turning point in the graph of oscillation frequency versus feedback phase.

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