THERMOELASTIC QUALITY-FACTOR ENHANCED DISK RESONATOR GYROSCOPE

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ABSTRACT

This paper proposes and demonstrates a novel method to greatly enhance the thermoelastic quality-factor (Q_{TED}) of disk resonator gyroscope (DRG) which is hanging lumped mass on the frame structure. This method could greatly reduce the resonant frequency f_0 whereas rarely affect the relaxation rate f_{Relax}. Simulations and experiments have been implemented to verify this technique. The fabricated enhanced DRG shows a two-fold improvement in quality-factor (Q) and five-fold improvement in decay time constant (τ) compared with the conventional pure frame DRG.

INTRODUCTION

Making high performance micro gyroscopes that is convenient for batch fabrication is of great interest. Disk resonator gyroscope (DRG) has received attention in recent years due to its symmetrical nested rings structure with single central anchor, which provide high thermal stability, low anchor loss and large modal mass [1-4]. One crucial metric for realizing high performance of the resonating gyroscope is the quality-factor (Q). Because high Q will result in high signal to noise ratio and high stability.

The main damping mechanism of the DRG is the thermoelastic dissipation (TED) [1]. Therefore improving Q_{TED} of the DRG would be of great importance. The strain field of the vibrating resonator will cause a temperature gradient, which will result in an irreversible heat flow. The dassipation caused by this conversion form mechanical energy to heat is called TED. The most widely used theoretical model of TED is the Zener's standard model [5]

$$Q_{TED} = \underbrace{\frac{C_V}{E\alpha^2 T_0}}_{\text{Material term}} \underbrace{\frac{1+(2\pi f_0 / f_{\mathrm{Relax}})^2}{2\pi f_0 / f_{\mathrm{Relax}}}}_{\text{Geometric term}}, \tag{1}$$

where C_V is the heat capacity of a solid, E is the Young's modulus, α is the coefficient of thermal expansion, T_0 is the nominal average temperature, f_0 and f_{Relax} are resonant frequency and relaxation rate, respectively. It can be seen that TED can be separated into two terms: the material dependent term and the geometry dependent term.

Improving Q_{TED} by optimizing the geometry design has always been attractive. Many researches have been taken to study the influence of the geometry on TED of micro resonators [6-11]. Some designing techniques to design high Q_{TED} resonators have been provided [9-11]. In this paper, we propose and demonstrate a novel geometry designing method to greatly enhance Q_{TED} of the DRG.

DESIGN AND FABRICATION

Q_{TED} Enhancing Design

Based on the Zener's model, the TED reaches the maximum if f_0 is close to f_{Relax}. f_{Relax} is inversely proportional to the relaxation time τ_{Relax}, which is the time to relax back to equilibrium when there is a temperature gradient,

$$f_{\mathrm{Relax}} = \frac{1}{\tau_{\mathrm{Relax}}} = \frac{\pi^2 \chi}{b^2}, \tag{2}$$

where χ is thermal diffusivity of the solid, b is the width of the strained beam. A lot of silicon resonators such as DRG work in the *isothermal* condition, in which $2\pi f_0/f_{\mathrm{Relax}} \ll 1$, to obtain high Q_{TED}. In order to get a higher Q_{TED}, lower f_0 and smaller b, thus higher f_{Relax}, are desired.

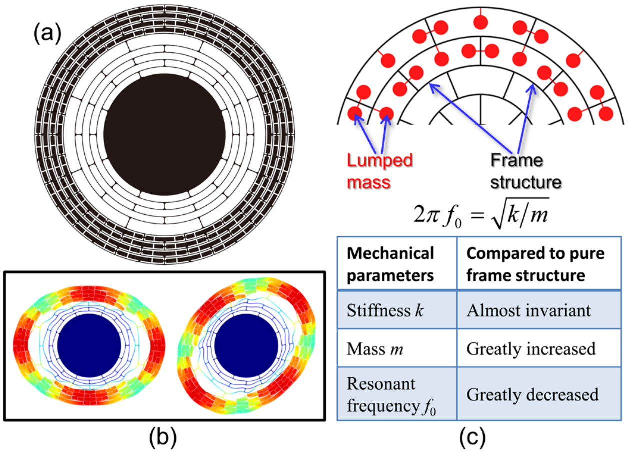

Figure 1: (a) Geometry and (b) n=2 wine-glass mode shapes of the enhanced disk resonator. (c) Schematic of hanging lumped mass on frame structure.

Table 1: Basic structural parameters of the enhanced DRG.

Parameter	**Value** (μm)
Resonator Diameter	8,000
Resonator Height	148
Width of Rings and Spokes	19
Anchor Diameter	3,760
Gap Width	15

The enhancing technique is realized by hanging lumped mass on the frame structure. The geometry of enhanced DRG with hanging lumped mass is shown in Figure 1(a). The frame structure which is fixed on a single central anchor is made of 9 nested rings. The outer five rings are interconnected with 16 spokes, whereas the inner rings are interconnected with 8 spokes. Lumped mass pieces are hanged on the outer four layers of rings and spokes. The basic structural parmeters of the enhanced DRG are shown in Table 1. The n=2 wine-glass modes are used as the driving and sensing modes, as shown in Figure 1(b).

The method to hang lumped mass on frame structure could reduce the resonant frequency f_0, as shown in Figure 1(c). Because compared to the pure frame structure, the effective stiffness k of the enhanced structure is almost invariant, whereas the effective mass m is greatly increased. Besides, there is no deformation, thus no temperature gradient, in the lumped mass. The thermoelastic dissipation mainly occurs in the flexural frame structure. Therefore the relaxation rate f_{Relax} of the enhance DRG mainly depends on the wide of the frame.

Compared to the conventional pure frame DRG with identical mechancial parameters, the enhanced DRG can provide similar f_{Relax} and greatly reduced f_0. Theoretically speaking, enhanced DRG will have much higher Q_{TED}.

TED Simulation

A TED simulation based on COMSOL is implemented to compare the enhanced DRG with a conventional DRG made by 9 nested rings. The material used in the simulation is (111) single-crystal silicon. Young's modulus, Poisson's ratio, and shear modulus are transversely and vertically isotropic for (111) silicon [12]. The structural parameters of the simulated DRGs are shown in Table 1. The temperature deviation distribution of the enhanced DRG is obtained by simulation, as shown in Figure 2. It can be seen that there is almost no temperature deviation in the lumped mass.

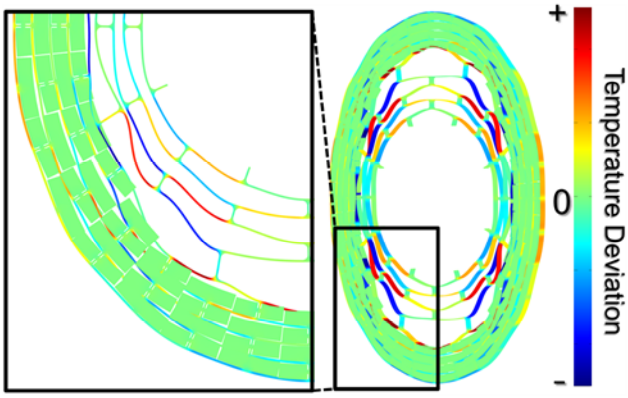

Figure 2: Temperature deviation in the enhanced DRG under n=2 wine-glass deformation.

We also calculated the effective mass m_{eff} and angular gain A_g based on the extracted shape functions of the n=2 modes [13,14]. The simulated results are summarized in Table 2. It can be seen that compared with the conventional pure frame DRG, the enhanced DRG can provide much higher Q_{TED}.

Table 2: Simulation comparison between the enhanced DRG and the conventional pure frame DRG.

Parameter	Enhanced DRG	Pure Frame DRG
f_0	5,895 Hz	13,682 Hz
Q_{TED}	251,570	101,220
m_{eff}	2.71 mg	0.47 mg
A_g	0.399	0.394

Fabrication

The DRGs were fabricated based on an aligned Au-Au thermo-compression bonding and deep reactive ion etching process. A P-type 150 μm (111) single-crystal silicon wafer is used as the structure wafer. Au bonding pads are patterned, and then some 2 μm pillars are etched on the structure wafer for bonding. Another P-type 350 μm (111) wafer is used as the substrate. A 2 μm SiO_2 layer is grown on the substrate wafer by thermal oxidization. Then the mating Au bonding pads are patterned on the oxide layer. The structure wafer and the substrate wafer are bonded by using aligned Au-Au thermo-compression bonding process. The structure wafer is patterned with Al leading pads. Lastly, the resonator structure and the electrodes are released by a deep reactive ion etching process.

The enhanced DRG prototype is fabricated, as shown in Figure 3(a). A conventional pure frame DRG is also fabricated for comparison, as shown in Figure 3(b). Each disk resonator is surrounded by 16 external capacitive electrodes. The wide slots in the disk resonators are inserted with couples of differential internal capacitive electrodes used for driving or sensing; and the narrow slots are inserted with single internal capacitive electrodes used for electrostatic tunning or parametric exciting. The capacitive gap is 15 μm.

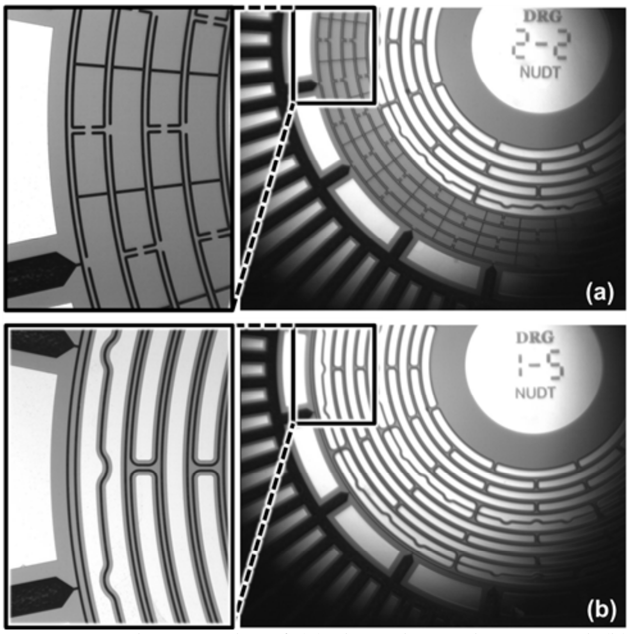

Figure 3: Photographs of (a) the enhanced DRG and (b) the pure frame DRG. The basic structural parameters of them are the same.

METHODS

Usually two methods could be used to evaluate Q of the resonator: frequency response test, and the ring-down test. When Q>100 k, the frequency sweeping is time-consuming whereas the peak of the frequency response curve is very sharp. Thus a small f_0 drift due to temperature variation could cause a very large error. Q of the enhanced DRG could be very high. Therefore the fabricated DRGs are characterized by ring-down measurement.

The experimental setup of the ring-down test is shown in Figure 4. The DRG is located in a vacuum chamber. The AC driving signal produced by the oscillator subsystem (OSC) of the frequency response analyzer (NF FRA 5087) is modulated into a 500 kHz carrier wave and then used to actuate the DRG. The capacitance variation of pickoff electrodes is converted into an AC output signal by a charge amplifier. The output signal is amplified and then high-pass filtered to reduce the low frequency noise. Then the output signal is synchronously demodulated by a multiplier and a low-pass filter. The driving signal and the output signal are putted into the Ch1 and Ch2 ports of FRA 5087, respectively. Double-pole double-throw switch S2 is used to change the testing axis. During the test, we firstly find the resonant frequency of the driving mode by frequency response test. Then actuate the DRG at resonance. Lastly, stop the actuation by turn off the switch S1 and record the decaying signal by using a NI-DAQ card in Labview.

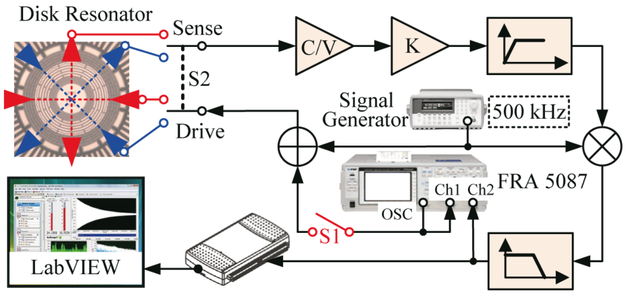

Figure 4: Block diagram of the experimental setup for ring -down measurement.

The recorded decaying signal is then filtered and the envelope is obtained using MATLAB. The decaying time constant τ is calculated by fitting the envelope with inverse exponential function $A_0\exp(-t/\tau)$. Q can be estimated based on $Q=\tau\pi f_0$.

ERESULTS AND DISCUSSION

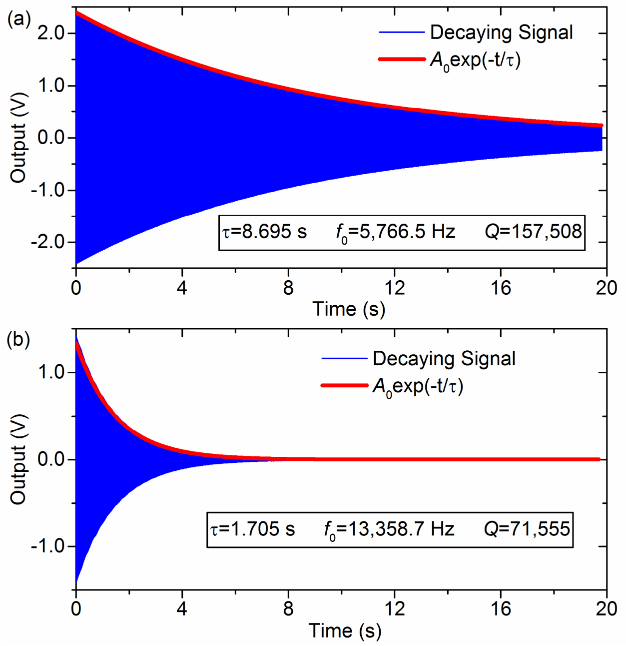

Figure 5: (a) Ring-down signal of the enhanced DRG at 0.004 Pa vaccum. (b) Ring-down signal of the pure frame DRG at 0.004 Pa vaccum.

The ring-down tests results of the DRGs at 0.004 Pa are shown in Figure 6. The longest decaying time constant τ of the enhanced DRG is tested to be 8.695 s, and Q is calculated to be 157,508. τ of the pure frame DRG is 1.705 s, and Q is calculated to be 71,555.

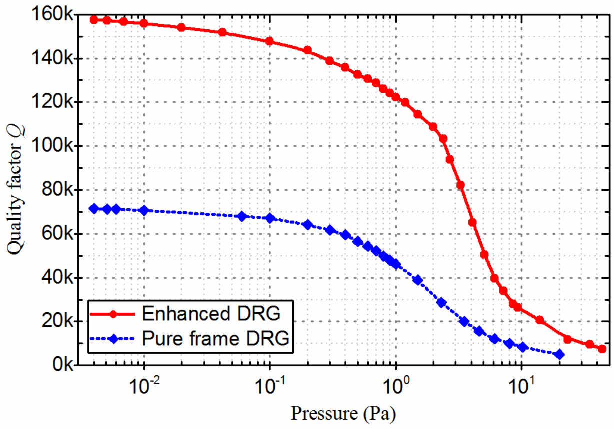

Figure 6: Tested Q at different pressure conditions.

The ring-down test is implemented at different pressure levels. The results are shown in Figure 5. It can be seen that both Q of the enhanced DRG and that of the conventional pure frame DRG increase as pressure decreases. And the improvements of the Qs are very slight when the pressure is below 0.01 Pa. The air damping is the only pressure-dependent damping component and is proportional to the pressure. Q_{air} of the enhanced DRG under 0.004 Pa vaccum and that of the pure frame DRG are calculated to be about 120 million and 292 million, respectively. There are some damping components Q_{other} caused by other unknown mechanisms rather than TED and air damping. Q_{other} of the enhanced DRG and that of the pure frame DRG are estimated to be 418,777 and 252,262, respectively. Q_{other} may be mainly contributed by anchor loss or surface loss.

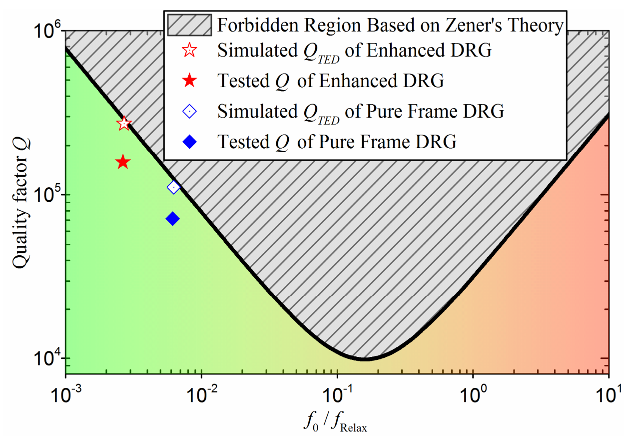

Figure 7: Q_{TED} as a function of resonant frequency f_0 / relaxation rate f_{Relax}. The simulated Q_{TED} and tested Q of the DRGs are also marked.

It can be seen from (1) that Q_{TED} is a function of the f_0/f_{Relax}, as shown in Figure 7. Since the smallest beam width of the enhanced DRG and that of the pure frame DRG are the same, thus f_{Relax} of them would be the same and can be calcuted base on (2). COMSOL simulated Q_{TED}s of the DRGs are marked in Figure 7. It can be seen that for resonators with bending deformation, if the resonant frequency and strained beam width is known, the Q_{TED}

estimated based on Zener's theory is sufficiently accurate. The tested Qs of the DRGs are also included in Figure 7. The Q improvement of the enhanced DRG is mainly contributed by the increasement of the Q_{TED}.

The enhanced DRG shows great advantages in Q and effective mass m_{eff}. The key parameters such as mechanical sensitivity S_{mech} and mechanical resolution ARW_{mech} would be also enhanced. S_{mech} and ARW_{mech} at resonance and in perfact mode-matching condition can be estimated base on [15,16]. During the calculation, the driving amplitude is assumed to be a constant 2.5 μm. The detailed comparison on key characters of the enhanced DRG with these of the pure frame DRG is summarized in Table 3.

Table 3: Character comparison between the enhanced DRG and the pure frame DRG.

Character	Enhanced DRG	Pure Frame DRG
f_0 (Tested)	5,766.5 Hz	13,358.9 Hz
Q	157,508	71,555
τ	8.695 s	1.705 s
S_{mech}	17.35 μm/(rad/s)	3.36 μm/(rad/s)
ARW_{mech}	0.0009°/√h	0.0021°/√h

CONCLUSION

This study presents the Q_{TED} enhancement of the centrally anchored DRG by hanging lumped mass on the frame structure. Compared with the conventional pure frame DRG, the enhanced DRG can halve f_0, double Q, enhance τ by 5 times, amplify m_{eff} by 5.7 times, enhance S_{mech} by 5 times, and reduce ARW_{mech} by 2.3 times. This Q_{TED} enhancing method of hanging lumped mass on frame structure can be widely used to other micro / nano resonators which are made by high thermal conductivity materials.

Our future work will focus on identifing the unknown damping components by studying the temperature dependence of Q. And Q optimization by annealing and improving the anchor structure will also be implemented. Besides, a new fabrication process based on Si-Si bonding will be developed.

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