CMSC420, Spring 2018 **Sections: 0301**

Homework #2

Posted : Tuesday, 02-06-2018, 12:00pm

Due (early): Thursday, 02-15-2018, 12:00pm

Due (late): Saturday, 02-17-2018, 11:59pm

Student's first and last name: _____ *Grade (grader only):*_____

Student's UID: _____

P1	P2	P3

University Honor Pledge:

I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination.

Print the text of the University Honor Pledge below:

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Signature: _____

Problem 1: Theory *(3 pts each for 15 pts total)*

Fill-in the blanks below. You do **not** need to justify your answers.

(a) The maximum **absolute value** of **imbalance** allowed in a subtree of an AVL Tree is... **1**

(b) **True** or **False**: AVL trees are **guaranteed** to have **exactly** $\log_2 n$ height for n nodes. **False**

(c) Splay trees display *amortized* **Logarithmic ($O(\log_2 n)$)** complexity of insertion.

(d) **True** or **False**: There are no **perfect** splay trees with more than one node. **False**

(e) **True** or **False**: A Red-**black** tree over n nodes *can* have a height **greater** than an AVL Tree over the same set of nodes. **True**

SOME EMPTY SPACE FOR YOU IF YOU NEED IT BELOW

Problem 2: AVL & AVL-G Trees *(55 pts)*

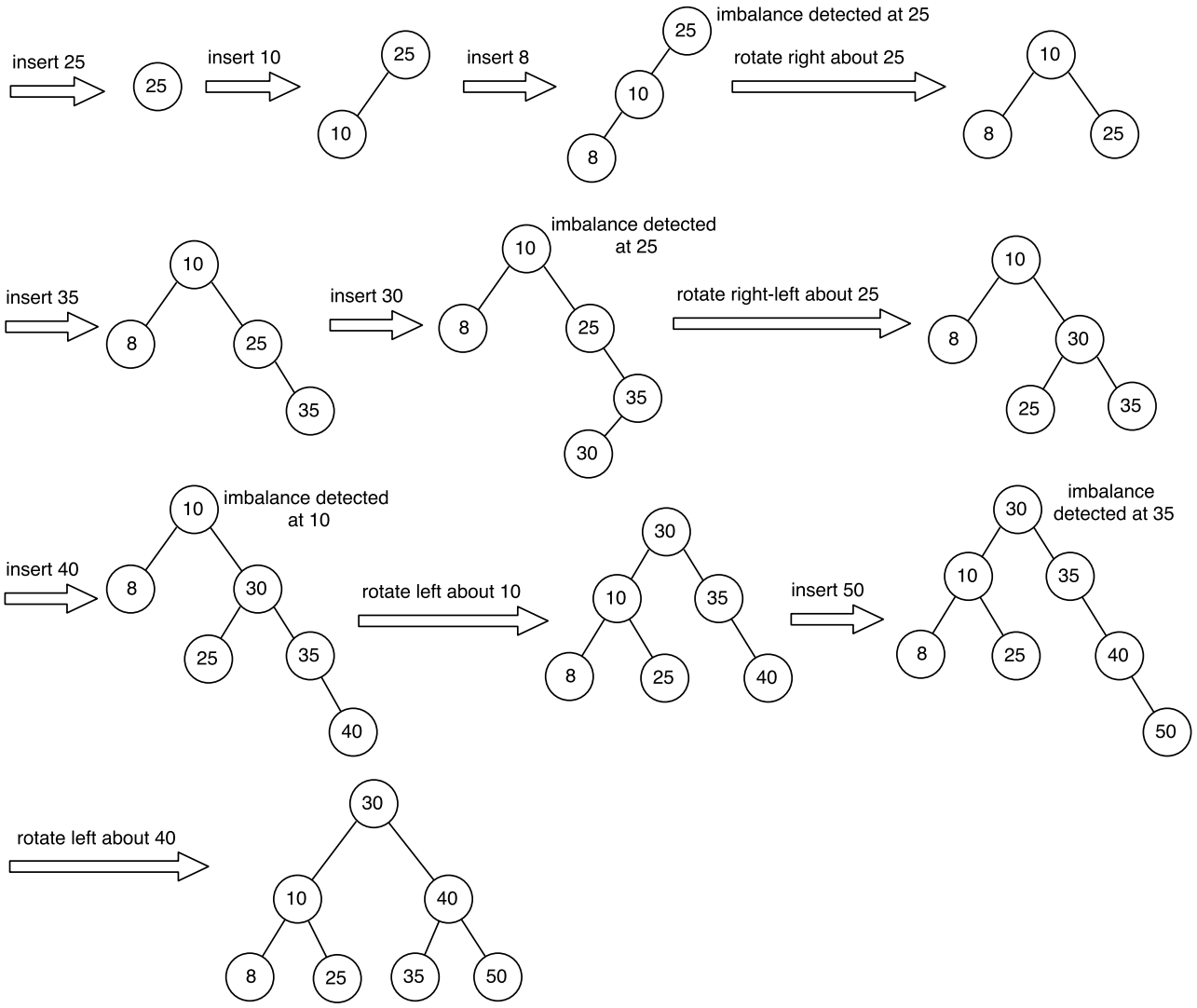
Question (a): Insertions (20 pts)

Insert the following integers in an *initially empty* AVL tree **in the order they are given**:

$$25, 10, 8, 35, 30, 40, 50$$

For every insertion, mention **whether rotations are required anywhere to maintain the AVL condition**. You can answer by telling us something like: *"Inserting key k_1 causes an imbalance detected at the node that contains k_2, which can be rectified by an "L-R" rotation that makes our tree look like this: (shape)"*. Remember that in order to restore the imbalance **globally**, we might need **more than one rotation**!

BEGIN YOUR ANSWER TO QUESTION (a) BELOW THIS LINE



CONTINUE YOUR ANSWER TO QUESTION (a) BELOW THIS LINE

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Question (b): Deletions (10 pts)

In the AVL tree shown in figure 1, show the result of deleting 10.

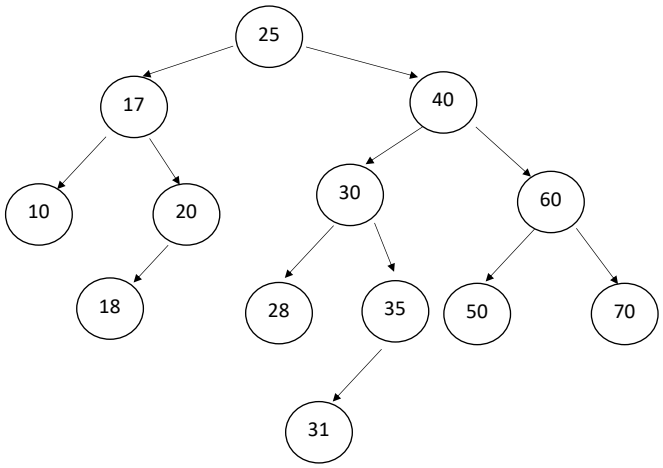
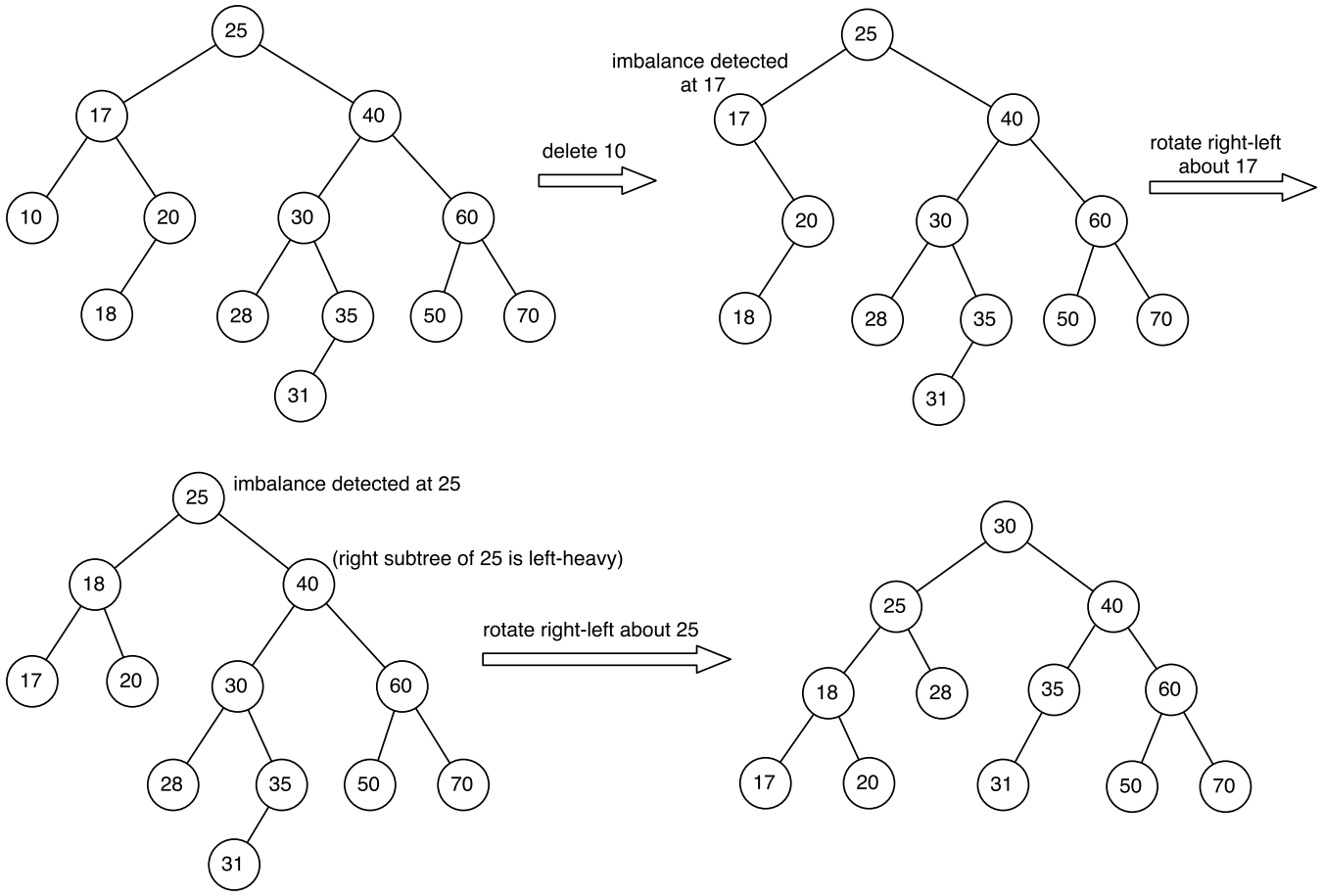


Figure 1: An AVL Tree.

Whenever a rotation has to be made, mention **what kind** and **about which subtree**. For example, you can say *"we are rotating the node that contains* 20 *to the right", or, even simpler since in AVL trees nodes only contain a single key, "we are rotating* 17 *to the left"*. After that, draw the tree that results after the rotation you showed. Repeat until done with **all** necessary rebalancings.

BEGIN YOUR ANSWER TO QUESTION (b) BELOW THIS LINE



CONTINUE YOUR ANSWER TO QUESTION (b) BELOW THIS LINE

Question (c): Trade-offs of AVL & AVL-G Trees (25 pts)

Insert the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, **in the order shown**, in a:

(i) Binary Search Tree.

(ii) AVL (AVL-1) Binary Search Tree.

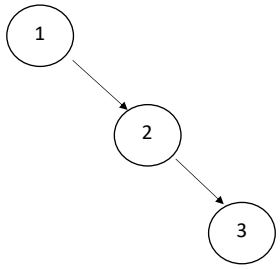
(iii) AVL - 2 Binary Search Tree.

(iv) AVL - 3 Binary Search Tree.

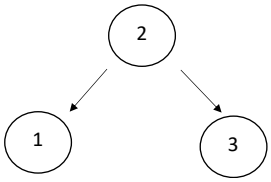
For the Binary Search Tree, we only want you to show us **the result of inserting all of those numbers in the tree.** For the AVL and AVL-G trees, we would like you to answer as follows: suppose that you have inserted a certain number of elements in your tree, **without triggering any rotations**. However, a new insertions triggers at least one rotation *somewhere*. Give us the shape of the tree **right at the point of the insertion that triggers the rotation(s)**. Then, explain **where** rotations are needed and **why**. Finally, show the resulting tree **after** the rotation. Repeat until done.

To start you out and give you an idea of how we would like you to answer, we give away a model solution to the insertion of the first 3 (three) integers in the list for the AVL-1 BST:

The first three insertions lead to an AVL tree that looks like this:



When backtracking to the root, having updated the height of the node containing (2) from 0 *to* 1*, we detect an imbalance of* -2*. We can solve it via a left rotation about the root (*1*). The resulting tree is shown below:*



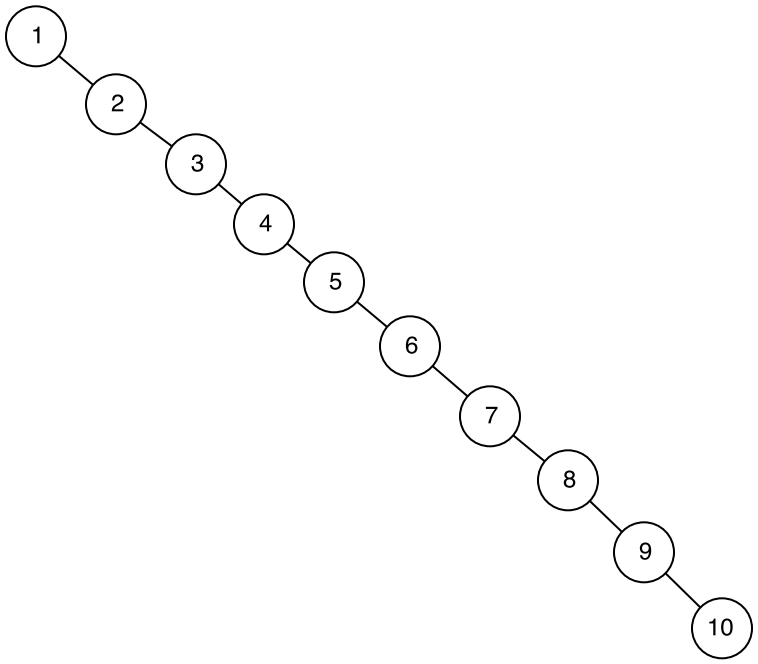
After you are done with those shapes, you will do some **cost analysis**. Assume that the cost of a **single** rotation (left or right) is c. Answer the following questions:

- For **all** of the trees, **how many nodes are visited when searching for the key 10**? You should include **both** the root **and** the node that contains 10.

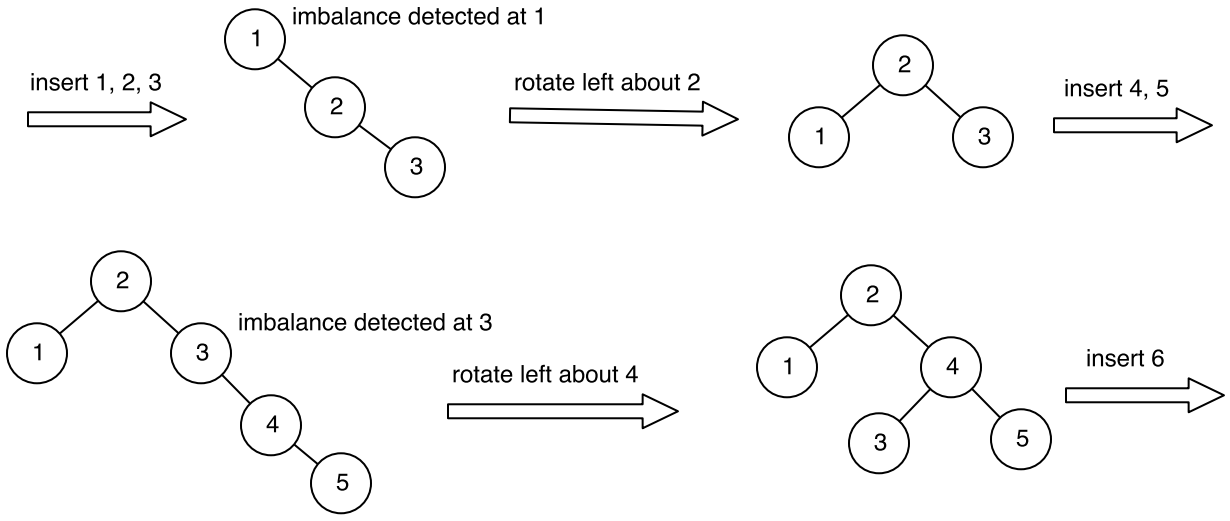
- For the AVL and AVL-G trees, what is the **total cost of their rotations** after inserting **all** the numbers in the given order?

BEGIN YOUR ANSWER TO QUESTION (c) BELOW THIS LINE

(*i*) Binary Search Tree:

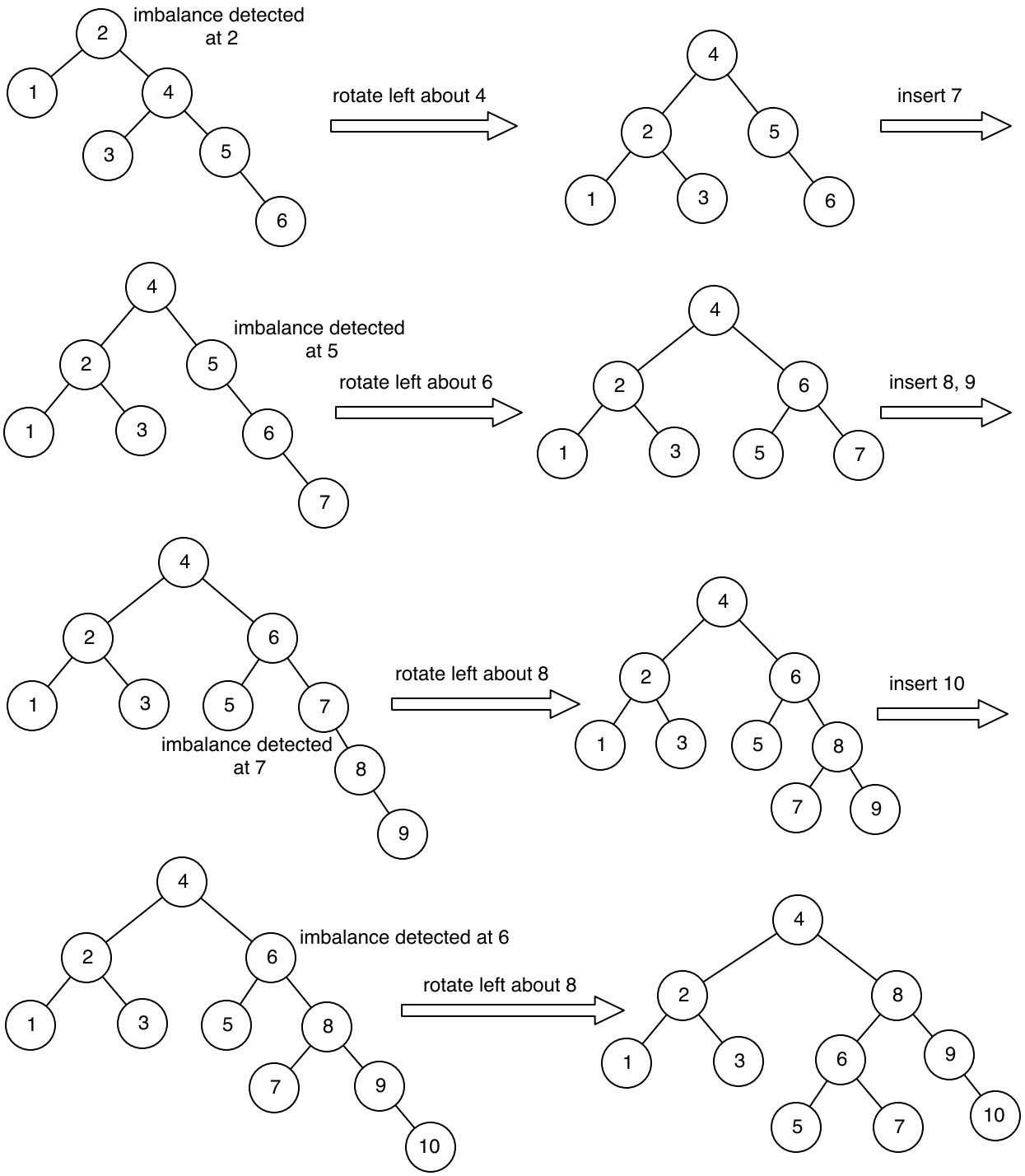


(*ii*) AVL-1 Binary Search Tree:



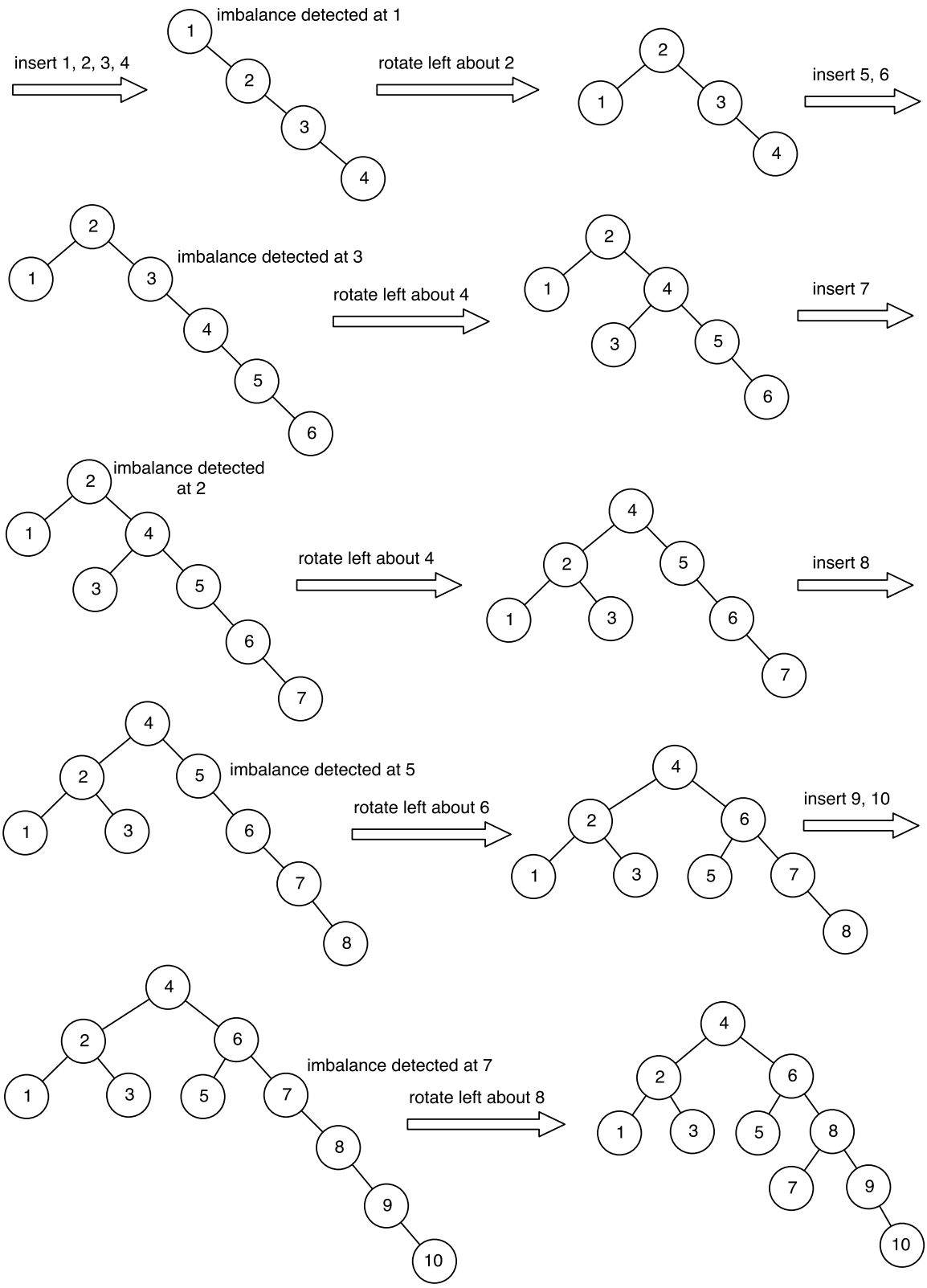
CONTINUE YOUR ANSWER TO QUESTION (c) BELOW THIS LINE

(*ii*) AVL-1 Binary Search Tree (Continued):



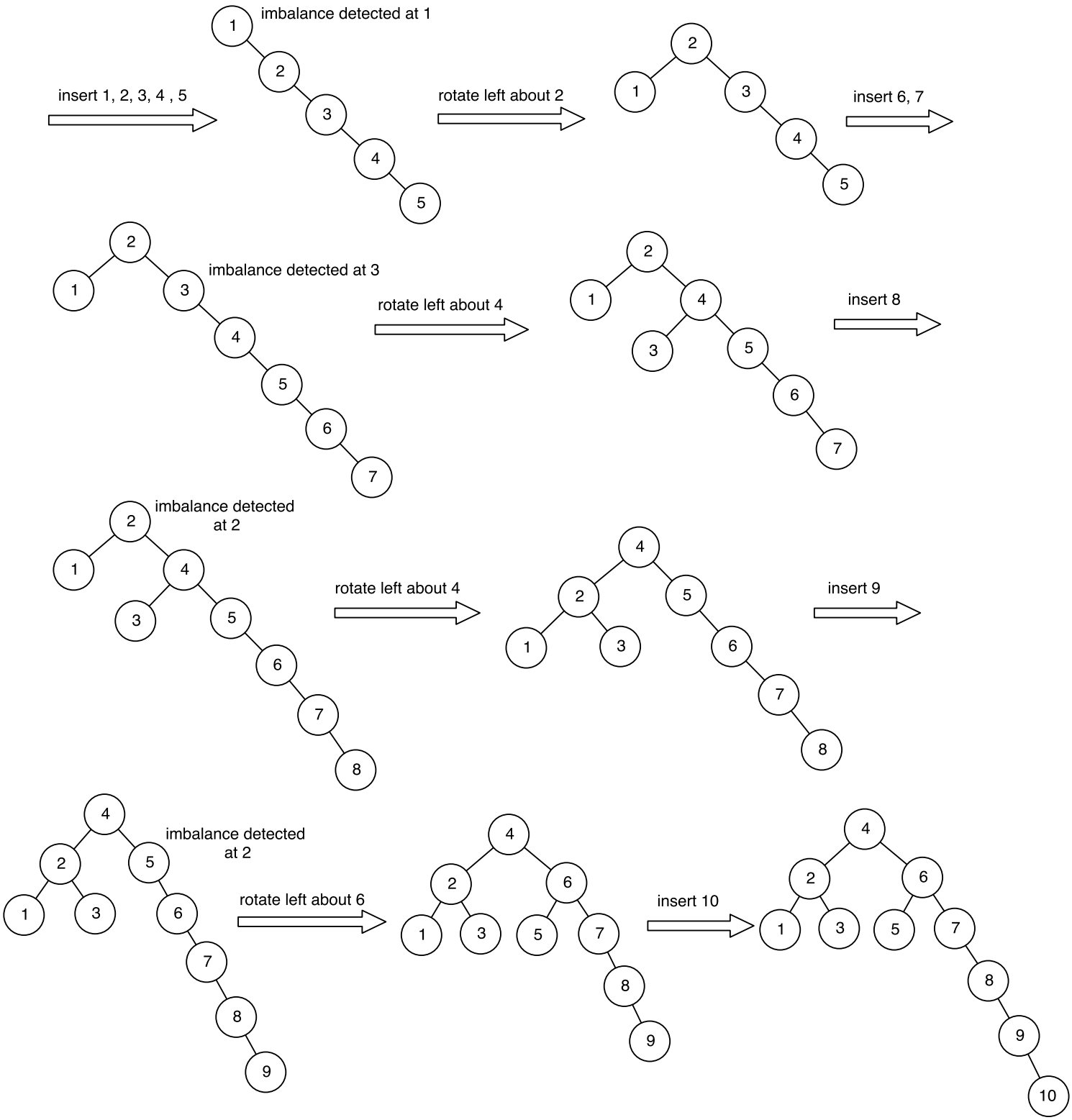
CONTINUE YOUR ANSWER TO QUESTION (c) BELOW THIS LINE

(*iii*) AVL-2 Binary Search Tree:



CONTINUE YOUR ANSWER TO QUESTION (c) BELOW THIS LINE

(*iv*) AVL-3 Binary Search Tree:



CONTINUE YOUR ANSWER TO QUESTION (c) BELOW THIS LINE

- number of nodes visited when searching 10:

  binary search tree : 10
  AVL-1 binary search tree : 4
  AVL-2 binary search tree : 5
  AVL-3 binary search tree : 6

- total cost of rotations after finishing insertions:

  AVL-1 binary search tree : $6c$
  AVL-2 binary search tree : $5c$
  AVL-3 binary search tree : $4c$

Problem 3: Splay Trees *(30 pts)*

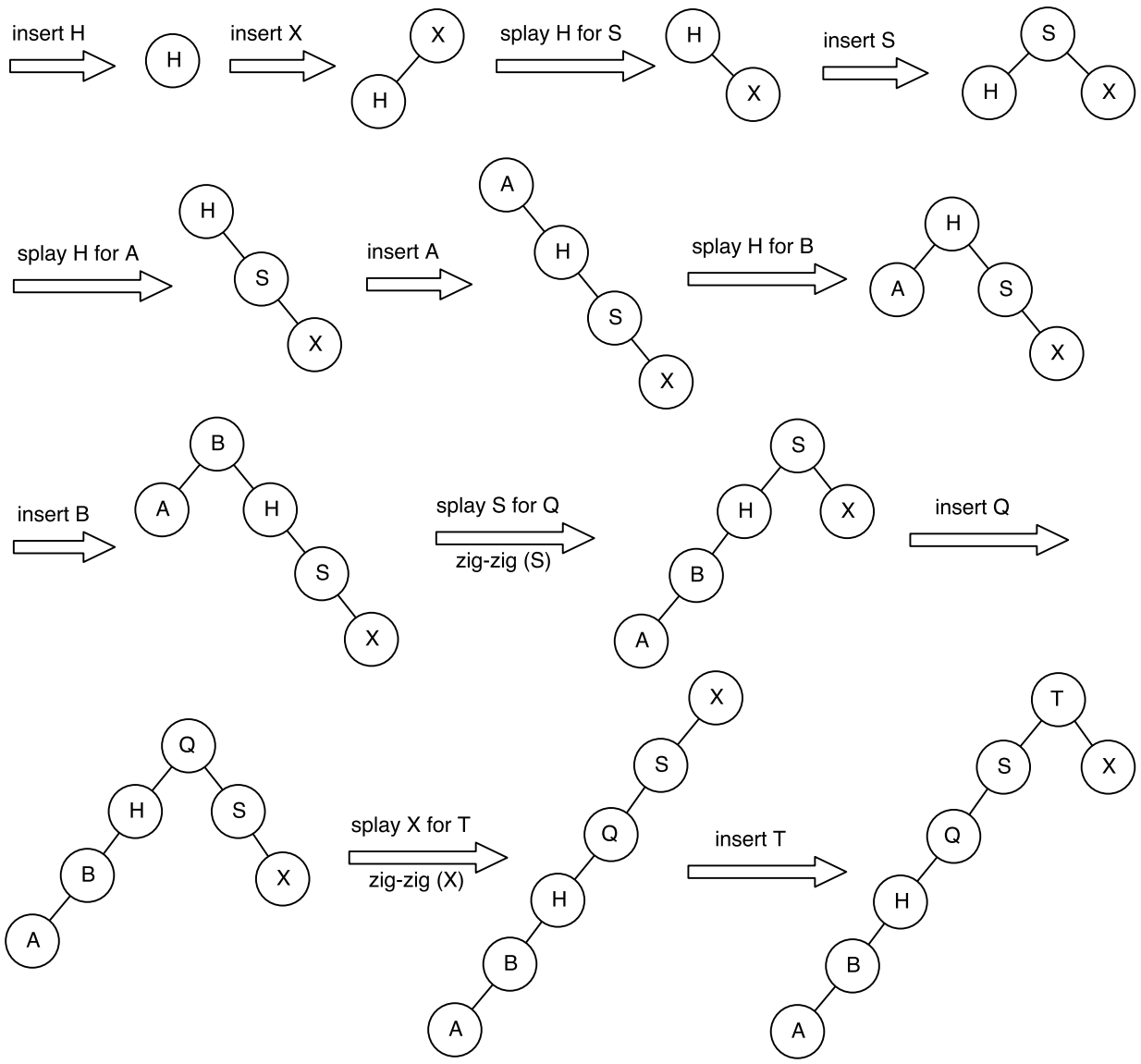
Question (a): Insertions (20 pts)

Insert the following uppercase English characters in an **initially empty** splay tree **in the order they are given**:

$$H,\ X,\ S,\ A,\ B,\ Q,\ T$$

For every insertion, mention **which key is splayed for on which node** (or node's subtree). For example, you can say *"we are splaying the node that contains the key 'C' for the key 'Q' "*. After this, show the tree that results by the splaying. Then, tell us how the new key is inserted in the tree by drawing the relevant tree.

BEGIN YOUR ANSWER TO QUESTION (a) BELOW THIS LINE



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Question (b): Deletions (10 pts)

In the splay tree of figure 2, show the result of deleting the key 'I'

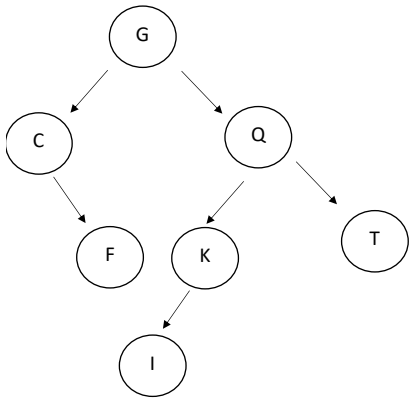
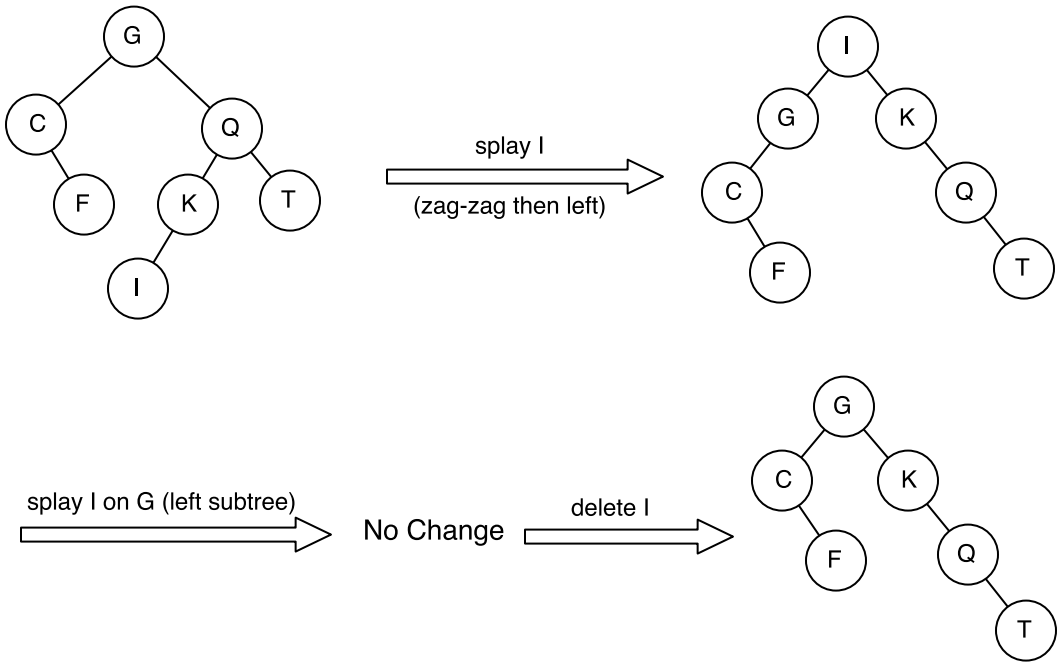


Figure 2: A splay tree.

Whenever a splaying is made, mention **which key is splayed for on which node** (or node's subtree). For example, you can say *"we are splaying the node that contains the key 'C' for the key 'Q'.* After that, draw the tree that results after splaying. Repeat until done.

BEGIN YOUR ANSWER TO QUESTION (b) BELOW THIS LINE



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