CMSC 498L: Introduction to Deep Learning **Released: Feb-04. Due Feb-11.**

Assignment 1

Name: Enter Name Here *UID:* Enter UID Here

Instructions:

- Submit the assignment on ELMS.
- Assignments have to be formatted in LaTeX. You can use overleaf for writing your assignments.
- Submit only the compiled PDF version of the assignment.
- Refer to policies (collaboration, late days, etc.) on the course website.

1 Probability

1. **Density function**. Let p be a Gaussian distribution with zero mean and variance of 0.1. Compute the density of p at 0.

   Sol:

2. **Conditional probability**. A student is taking a one-hour-time-limit makeup examination. Suppose the probability that the student will finish the exam in less than x hours is $x/2$, $\forall$x $\in [0, 1]$. Given that the student is still working after 0.75 hour, what is the conditional probability that the full hour will be used?

   Sol:

3. **Bayes rule**. Consider the probability distribution of you getting sick given the weather in the table below.

Sick?	Weather			
	sunny	rainy	cloudy	snow
yes	0.144	0.02	0.016	0.02
no	0.576	0.08	0.064	0.08

Compute $\mathrm{P}(\,\text{sick} = \text{yes} \mid \text{Weather} = \text{rainy}\,)$.

Sol:

2 Calculus and Linear Algebra

For each of the following questions, we expect to see all the steps for reaching the solution.

1. Compute the derivative of the function $f(z)$ with respect to z $\left(\text{i.e., } \frac{df}{dz}\right)$, where

$$f(z) = \frac{1}{1 + e^{-z}}$$

Sol:

2. Compute the derivative of the function $f(w)$ with respect to w_i, where $w, x \in \mathbb{R}^{\mathrm{D}}$ and

$$f(w) = \frac{1}{1 + e^{-w^T x}}$$

Sol:

3. Compute the derivative of the loss function $J(w)$ with respect to w, where

$$J(w) = \frac{1}{2} \sum_{i=1}^{m} \left| w^T x^{(i)} - y^{(i)} \right|$$

Sol:

4. Compute the derivative of the loss function $J(w)$ with respect to w, where

$$J(w) = \frac{1}{2}\left[\sum_{i=1}^{m}\left(w^T x^{(i)} - y^{(i)}\right)^2\right] + \lambda\|w\|_2^2$$

Sol:

5. Compute the derivative of the loss function $J(w)$ with respect to w, where

$$J(w) = \sum_{i=1}^{m} \left[y^{(i)} \log \left(\frac{1}{1 + e^{-w^T x^{(i)}}} \right) + \left(1 - y^{(i)} \right) \log \left(1 - \frac{1}{1 + e^{-w^T x^{(i)}}} \right) \right]$$

Sol:

6. Compute $\nabla_w f$, where $f(w) = \tanh\left[w^T x\right]$.

   Sol:

7. Find the solution to the system of linear equations given by Ax=b, where

$$A = \begin{pmatrix} 2 & 1 & -1 \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 8 \\ -11 \\ -3 \end{pmatrix}.$$

Sol:

8. Find the eigenvalues and associated eigenvectors of the matrix:

$$A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}$$

Sol:

3 Activation functions

For each of the following activation functions, write their equations and their derivatives. Plot the functions and derivatives, with $x \in [-5, 5]$ and $y \in [-10, 10]$ plot limits. (No need to submit the code for plots.)

1. Relu

   Sol:

2. Tanh

   Sol:

3. Softmax

   Sol:

4. Sigmoid

   Sol:

5. Leaky ReLU

   Sol:

6. ELU (plot with $\alpha = 0.3$)

   Sol:

7. Sinc

   Sol: