A Spectroscopic Binary Star is the one where we observe a periodic shift in spectral lines. This happens if the period of a binary system is not prohibitively long and if the orbital motion has a component along the line of sight. More detailed analysis about this star system can be found in the given reference [1].

Let us consider two SB stars of mass m_1 and m_2 with semi major axis a_1 and a_2 and time period T. From Kepler's third law, we have -

$$T^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3 \tag{1}$$

where

$$a = a_1 + a_2 \tag{2}$$

and G is the universal gravitation constant. Therefore,

$$T^2 = \frac{4\pi^2}{G(m_1 + m_2)} (a_1 + a_2)^3 \tag{3}$$

Rearranging,

$$(a_1 + a_2)^3 = T^2 \ \frac{G\ (m_1 + m_2)}{4\pi^2} \tag{4}$$

Now, if we assume an orbit with very small eccentricity ($e << 1$) , then the orbital speeds of the stars are essentially constant and given by $v_1 = 2\pi a_1/T$ and $v_2 = 2\pi a_2/T$. We use the expression of orbital velocity to get an expression for a_1, a_2 and use it in equation (4)

$$a_1 + a_2 = \frac{T}{2\pi}(v_1 + v_2)$$
$$\frac{T^3}{8\pi^3}(v_1 + v_2)^3 = T^2 \ \frac{G\ (m_1 + m_2)}{4\pi^2}$$

Thus upon solving, we get

$$m_1 + m_2 = \frac{T}{2\pi G}(v_1 + v_2)^3 \tag{5}$$

The radial velocity of stars is calculated as -

$$v_r = z \cdot c \tag{6}$$

where z is the redshift value of the star and c is the speed of light. Writing v_1, v_2 in terms of radial velocity-

$$v_1 = v_{1radial}/\sin i$$
$$v_2 = v_{2radial}/\sin i$$

where i is the angle between the plane of an orbit and the plane of the sky. Thus,

$$m_1 + m_2 = \frac{T}{2\pi G} \frac{(v_{1r} + v_{2r})^3}{\sin^3 i} \tag{7}$$

In the COM frame,

$$m_1 a_1 = m_2 a_2 \tag{8}$$

and using expression of a_1, a_2 we get,

$$\frac{m_1}{m_2} = \frac{v_{2r}}{v_{1r}} \tag{9}$$

Using equation 8 to get an expression of v_{2r} and putting it in equation 6, we get

$$m_1 + m_2 = \frac{T}{2\pi G} \frac{v_{1r}^3}{\sin^3 i} (1 + \frac{m_1}{m2})^3 \tag{10}$$

Thus the **Mass Function** is given by -

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{T}{2\pi G} \frac{(v_{1r})^3}{\sin^3 i} \tag{11}$$

[1] Carroll and Ostlie, An Introduction to Modern Astrophysics, Chapter 7 Binary Star Systems

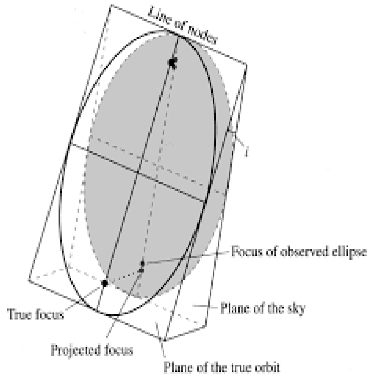

Abbildung 1: An elliptical orbit projected onto the plane of the sky produces an observable elliptical orbit (Credit: Carroll and Ostlie)

The length of semi-major axis is given by -

$$a = a_1 + a_2 = \frac{T}{2\pi}(v_1 + v_2) \tag{12}$$

Thus the **Projected semi-major axis is given by**

$$a' = a \sin i = \frac{T}{2\pi}(v_{1r} + v_{2r}) \tag{13}$$