Mathematical Operations

Arithmetic and Logical

Distance

- For pixel *p*, *q*, and *z* with coordinates (*x*,*y*), (*s*,*t*) and (*u*,*v*),
- *D* is a distance function or metric if

- $D(p,q) \geq 0$ ($D(p,q) = 0$ if and only if $p = q$)

- $D(p,q) = D(q,p)$

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

- $D(p,z) \leq D(p,q) + D(q,z)$

D_4-distance (city-block distance)

$$D_4(p,q) = |x - s| + |y - t|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with $D_4(p) = 1$ is 4-neighbors of p.

D_8-distance (chessboard distance)

$$D_8(p,q) = \max(|x-s|,|y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with $D_8(p)$ = 1 is 8-neighbors of p.

Mathematical Tools Used in Digital Image Processing

- Linear operation

- H is said to be a linear operator if, for any two images f and g and any two scalars a and b,

$$H(af + bg) = aH(f) + bH(g)$$

Arithmetic operations

- Addition

  s(x,y) = f(x,y)+g(x,y)

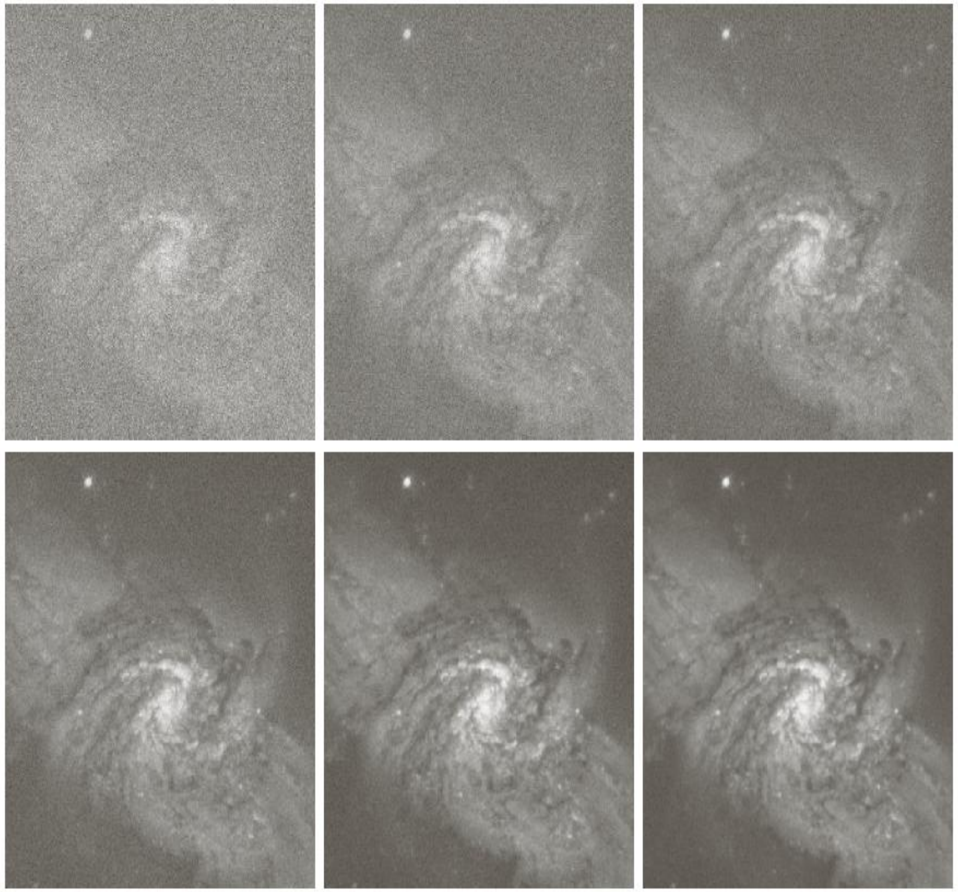
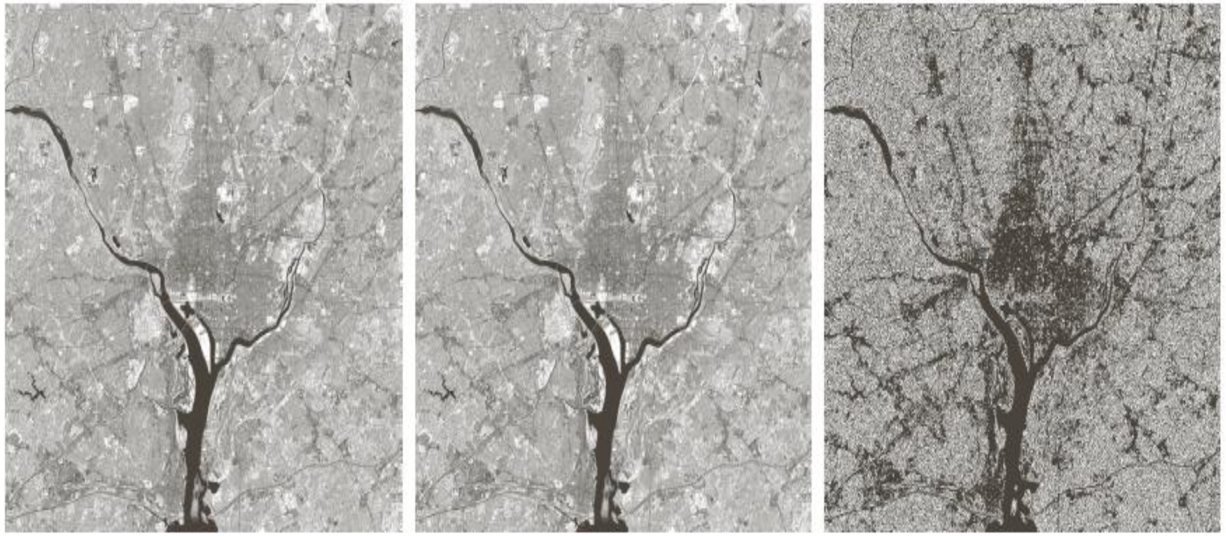


FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

Subtraction

- Subtraction to enhance differences
- $s(x,y) = f(x,y)-g(x,y)$



a b c

FIGURE 2.27 (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.

Digital subtraction angiography

a	b
c	d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

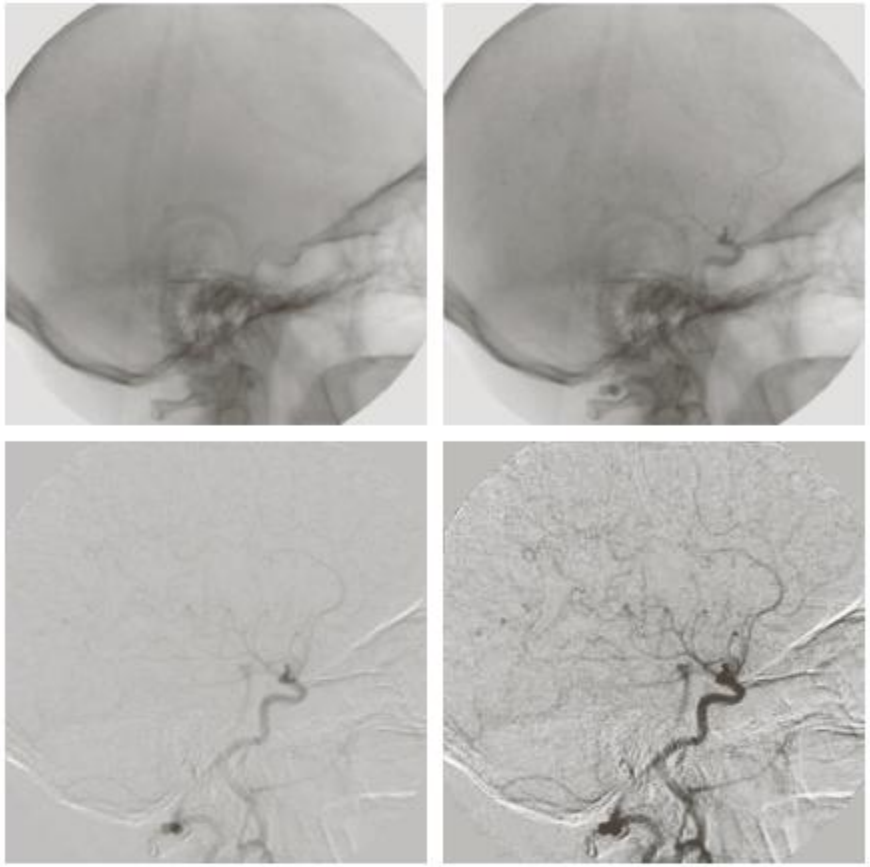


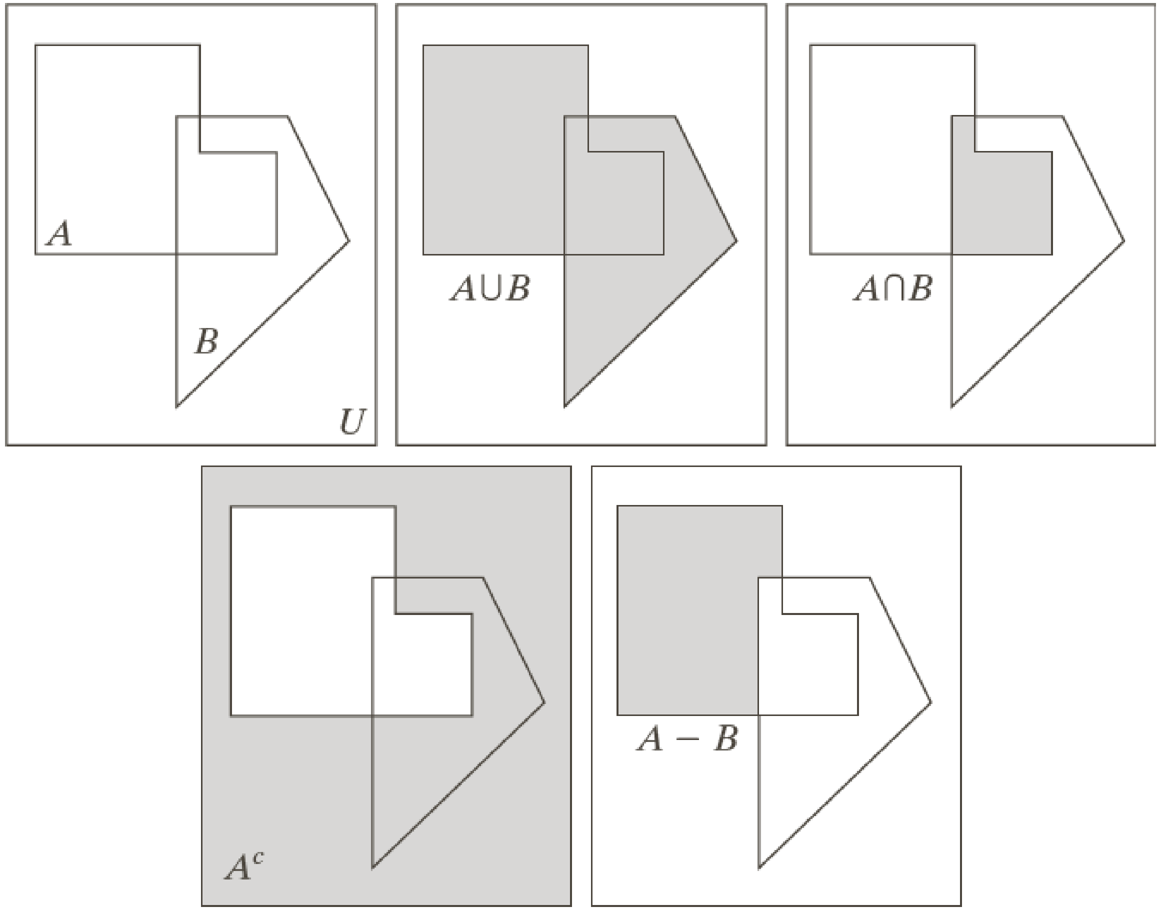
Image multiplication

- $s(x,y) = f(x,y).g(x,y)$

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Set operations

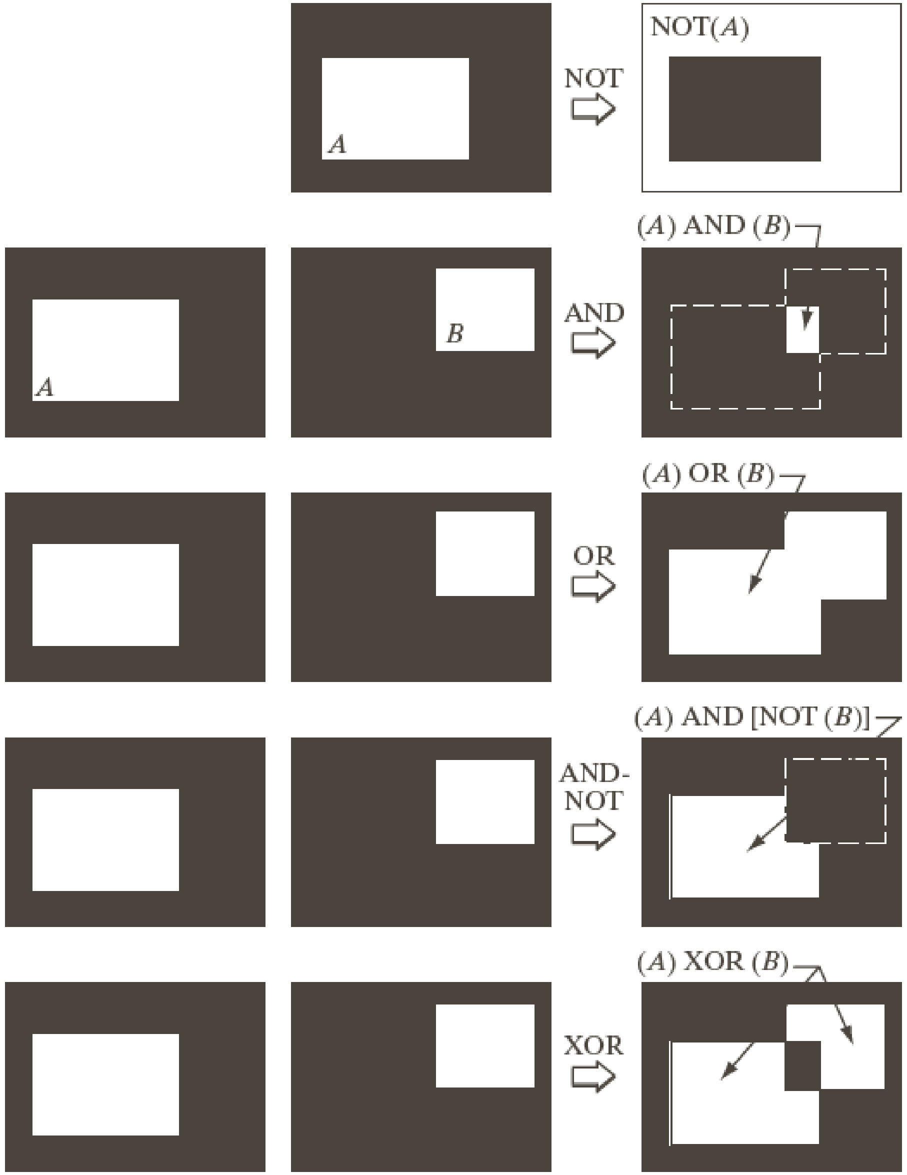


a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Logical operations

FIGURE 2.33
Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.



Single-pixel operations

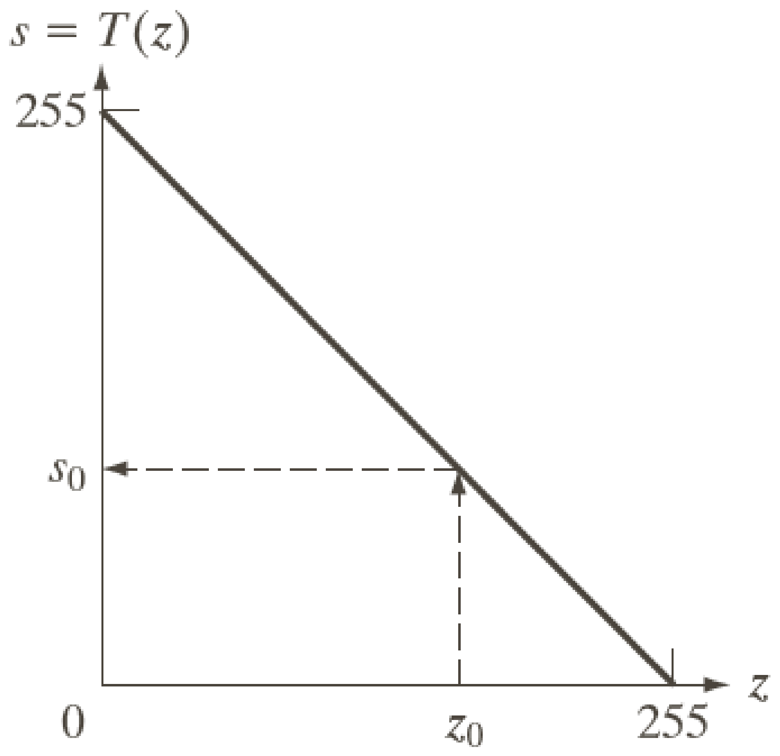


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

Neighborhood operations

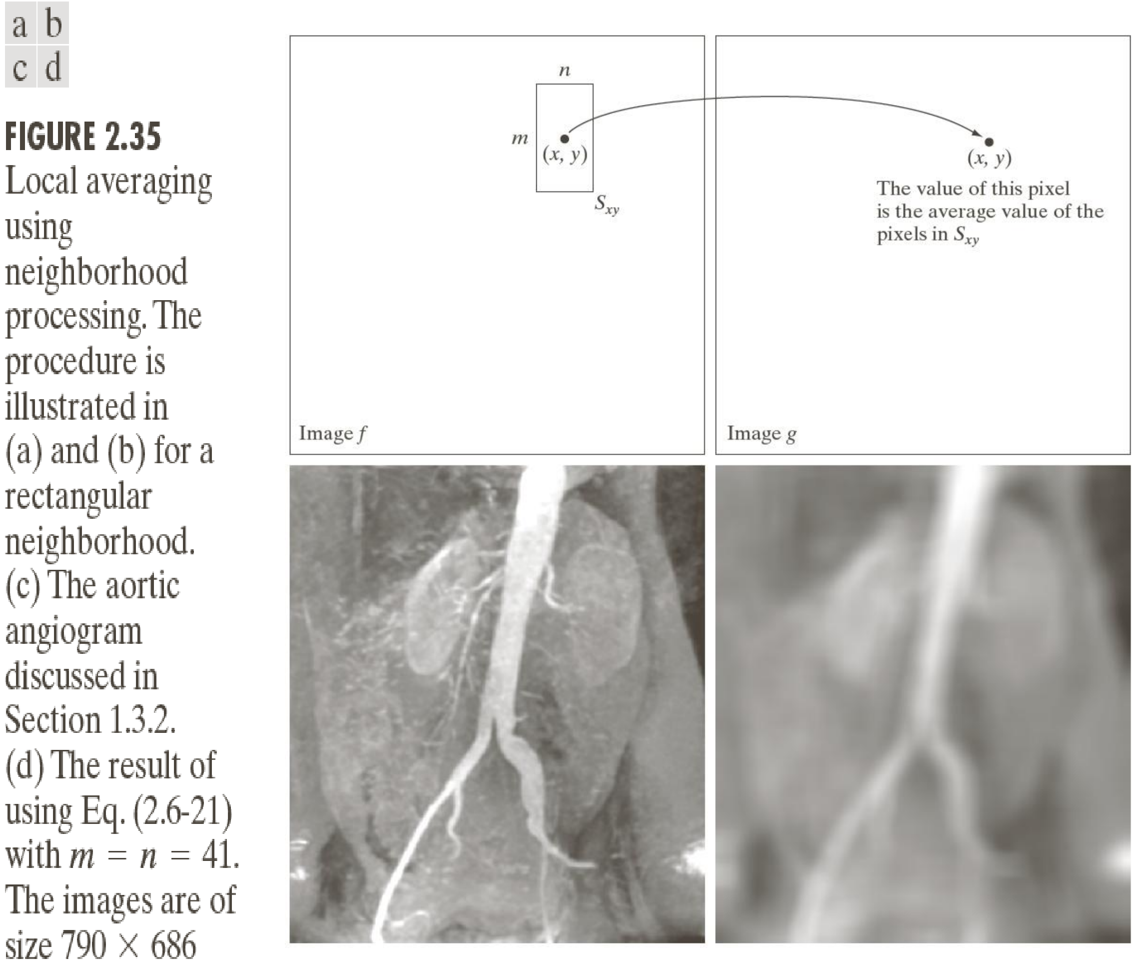


FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

Food for thought!

1. What is meant by a linear operation in image processing? State the condition that an operator must satisfy to be linear.
2. Write the mathematical expression for image addition and mention one practical application of this operation.
3. How does image subtraction help in enhancing differences between images? Give one real-world example where subtraction is used.
4. Differentiate between D4-distance and D8-distance. How does the definition of neighbors change in each case?
5. What is the difference between single-pixel operations and neighborhood operations? Give one example of each.

Programming assignments

1. Apply Pixel-wise Arithmetic & Logical Operations.

Concepts Used:

- Image addition, subtraction, multiplication
- Logical operations (AND / OR / XOR)

Tasks

1. Read two grayscale images of the same size.
2. Perform image addition, subtraction, and multiplication.
3. Apply one logical operation on binary images.
4. Display original and output images and note observations.

2. Single-pixel and Neighborhood Operations

Concepts Used:

- Neighborhood vs single-pixel operations

Tasks

1. Find out Negative of the input gray scale image.
2. Apply a 3×3 neighborhood filter for calculating average.
3. Compare Neighborhood Operations with a single-pixel operation.

AI supported self-learning on **Math & Logical Operations** (Prompts compatible with ChatGPT)

Active Learners (Learning by Doing)

1. Give me a small numerical image example and ask me to perform image addition, subtraction, and multiplication. Let me attempt first, then explain the solution.
2. Create a simple binary image and ask me to apply AND, OR, and XOR logical operations. Let me predict the output before explanation.

Reflective Learners (Learning by Thinking)

1. Explain arithmetic and logical operations in digital images step by step, and summarize their key differences at the end.
2. Explain what makes an operation linear in image processing and why linearity is important.

Sensing Learners (Concrete & Practical)

1. Explain image addition, subtraction, and multiplication using real pixel values and small grayscale matrices.
2. Explain logical operations (AND, OR, XOR) using simple binary images with pixel-level examples.

Intuitive Learners (Concepts & Patterns)

1. Explain the concept of linear operations in image processing and how arithmetic operations fit into this idea.
2. Explain how mathematical and logical operations change the interpretation of image information.

Visual Learners (Diagrams & Structure)

1. Explain arithmetic and logical image operations using tables, diagrams, or grid-based illustrations.
2. Visually compare single-pixel operations and neighborhood operations using diagrams or structured layouts.

Verbal Learners (Words & Explanation)

1. Explain arithmetic and logical image operations in simple language using everyday analogies.
2. Explain the difference between single-pixel and neighborhood operations as if teaching it to a beginner.

Sequential Learners (Step-by-Step Logic)

1. Explain how image addition, subtraction, and multiplication are performed step by step at the pixel level.
2. List the steps involved in applying a neighborhood operation and explain how it differs from a single-pixel operation.

Global Learners (Big Picture First)

1. First explain the overall role of mathematical and logical operations in digital image processing, then explain individual operations.
2. Explain why distance measures (D4 and D8) and neighborhood definitions are important before explaining how they are computed.