Histogram Matching (Specification)

$$s = T(r) = \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j \quad \text{--- (1)}$$

$$s = T(r) = (L-1) \int_0^r p_r(w)\, dw \quad \text{--- (2)}$$

$$G(z) = (L-1) \int_0^z p_z(t)\, dt \quad \text{--- (3)}$$

$$G(z) = s = T(r) \Rightarrow z = G^{-1}(s) = G^{-1}[T(r)]$$

Given

$$p_r(r) = \begin{cases} 2r/(L-1)^2 & ; \quad 0 \le r \le (L-1) \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$s = T(r) = (L-1)\int_0^r \frac{2w}{(L-1)^2}\, dw = r^2/(L-1) \quad \text{— (1)}$$

Desired

$$p_z(z) = \begin{cases} 3z^2/(L-1)^3 & ; \quad 0 \le z \le (L-1) \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$s = G(z) = (L-1)\int_0^z \frac{3t^2}{(L-1)^3}\, dt = z^3/(L-1)^2 \quad \text{— (2)}$$

$$Z = \left[(L-1)^2 s\right]^{1/3} = \left[(L-1) r^2\right]^{1/3}$$

Discrete domain

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

Given

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \; ; \quad k = 0, 1, 2, \ldots, L-1$$

Desired

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

$$q = 0, 1, 2, \ldots, L-1$$

$$z_q = G^{-1}(s_k) = G^{-1}[T(r_k)]$$

64×64 I/p image

Intensity range (0 − 7)

r_k	n_k	$P_r(r_k)$
0	790	0.19
1	1023	0.25
2	850	0.21
3	656	0.16
4	329	0.08
5	245	0.06
6	122	0.03
7	81	0.02

$$S_k = \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j$$

$k = 0, 1, 2, \ldots, 7$

$$P_r(r_k) = \frac{n_k}{MN}$$

$S_0 = 1.33 \rightarrow 1$

$S_1 = 3.08 \rightarrow 3$

$S_2 = 4.55 \rightarrow 5$

$S_3 = 5.67 \rightarrow 6$

$S_4 = 6.23 \rightarrow 6$

$S_5 = 6.65 \rightarrow 7$

$S_6 = 6.86 \rightarrow 7$

$S_7 = 7 \rightarrow 7$

$$G(z_q) = (L-1)\sum_{i=0}^{q} P_z(z_i)$$

z_q	Specified $P_z(z_k)$	Calculated $P_z(z_k)$
0	0.00	0.00
1	0.00	0.00
2	0.00	0.00
3	0.15	0.19
4	0.20	0.25
5	0.30	0.21
6	0.20	0.24
7	0.15	0.11

$G(z_0) = 0.00 \rightarrow 0$

$G(z_1) = 0.00 \rightarrow 0$

$G(z_2) = 0.00 \rightarrow 0$

$G(z_3) = 1.05 \rightarrow 1$

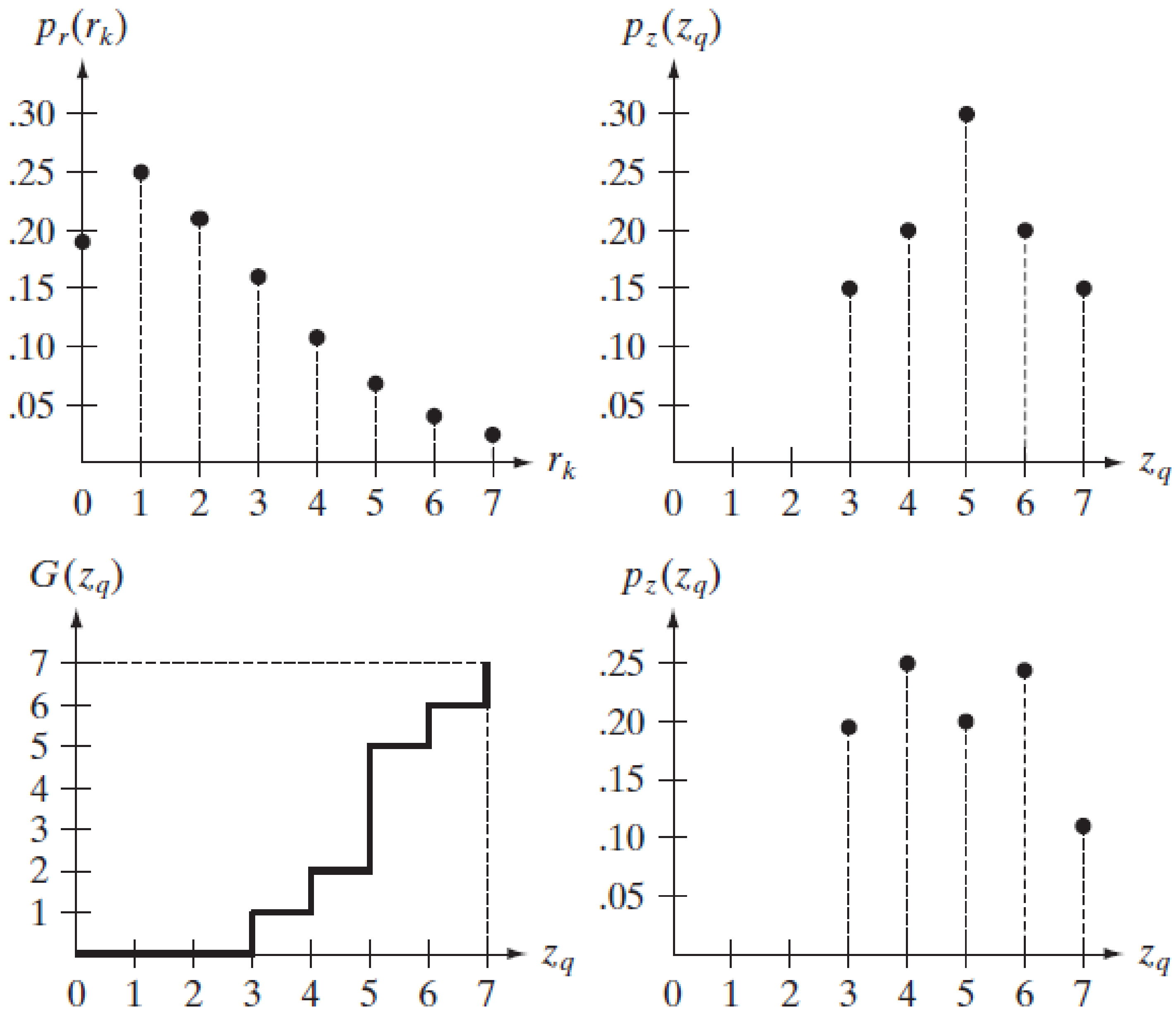
$G(z_4) = 2.45 \rightarrow 2$

$G(z_5) = 4.55 \rightarrow 5$

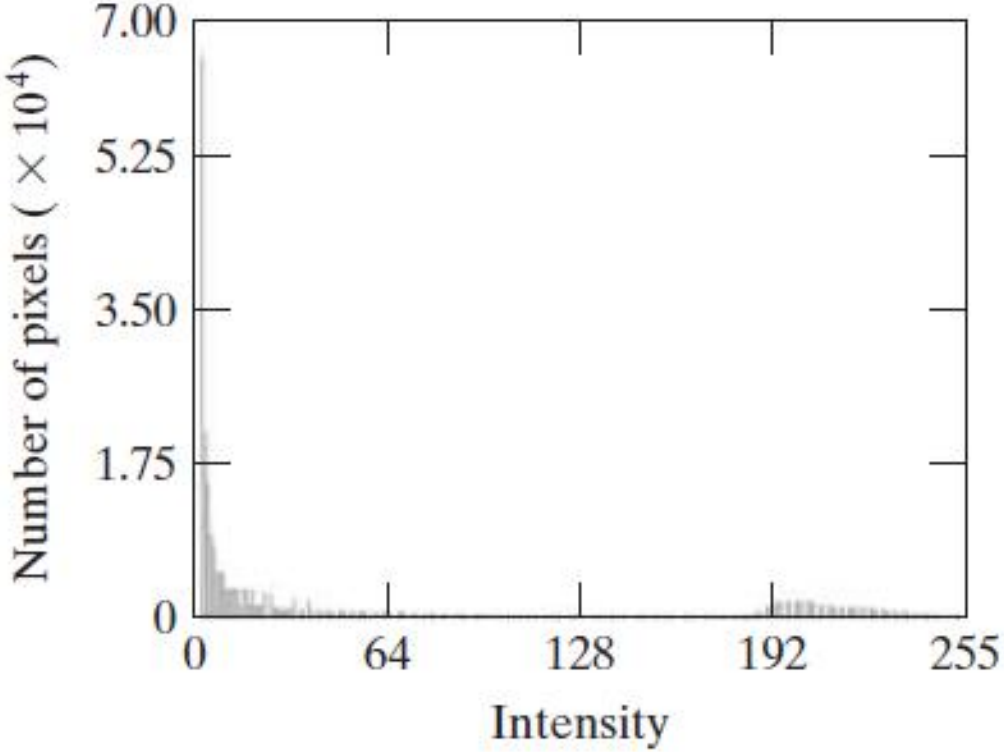
$G(z_6) = 5.95 \rightarrow 6$

$G(z_7) = 7 \rightarrow 7$

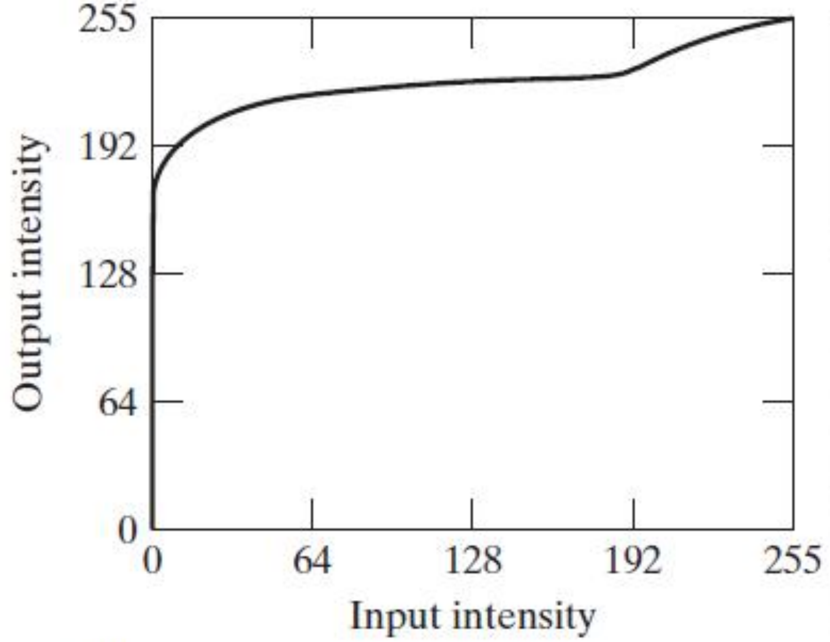
z_q	$G(z_q) = s$	r	$p_z(z_k)$
0	0	–	0.00
1	0	–	0.00
2	0	–	0.00
3	1	0	0.19
4	2	1	0.25
5	5	2	0.21
6	6	3 & 4	0.24
7	7	5, 6 & 7	0.11

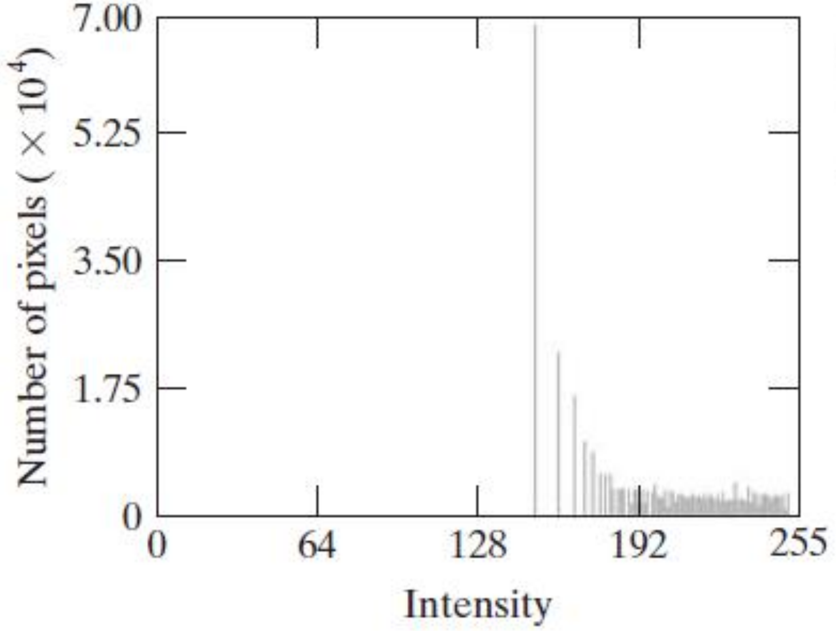


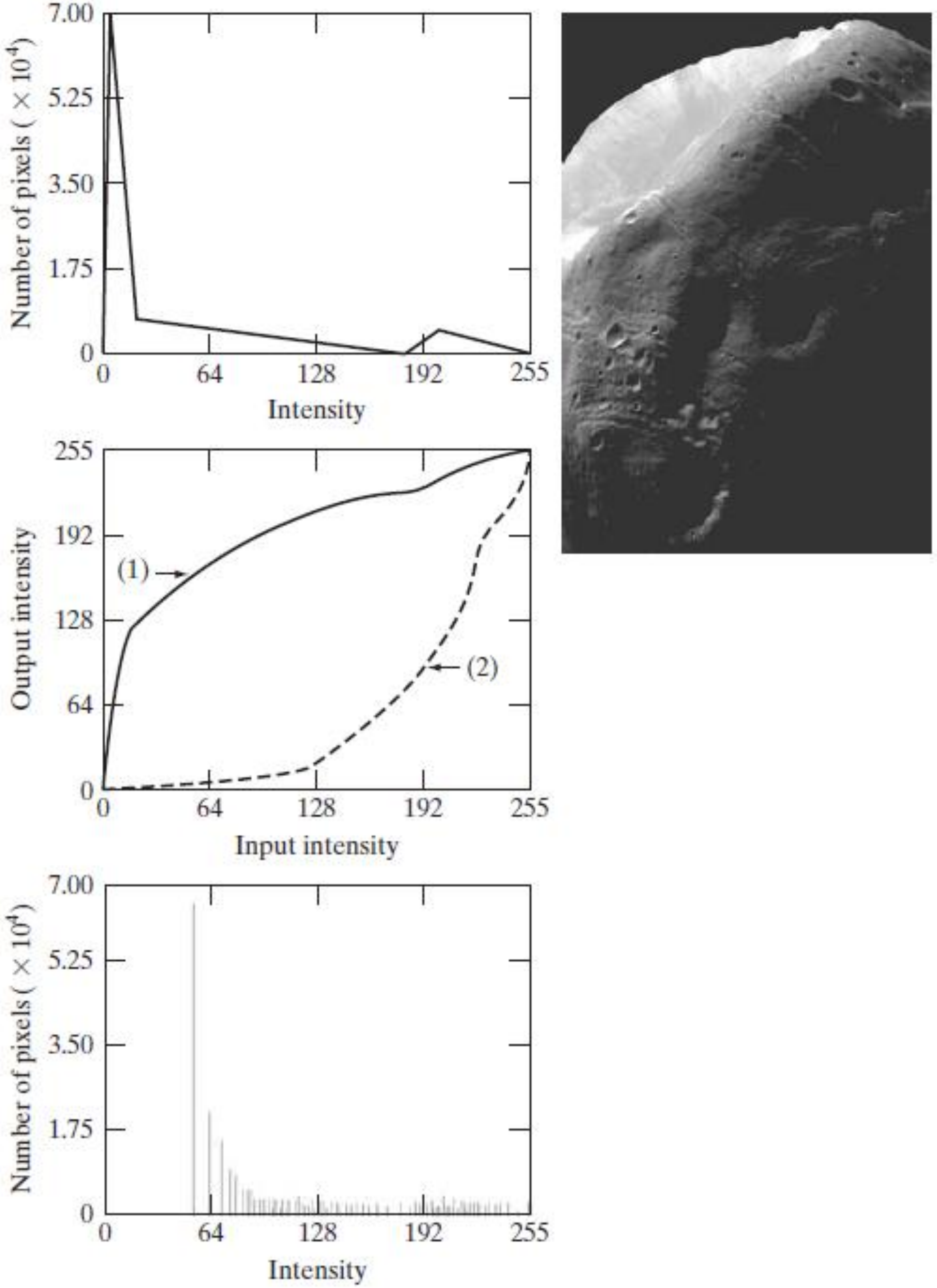
$p_r(r_k)$
.30
.25
.20
.15
.10
.05
r_k
0 1 2 3 4 5 6 7
$p_z(z_q)$
.30
.25
.20
.15
.10
.05
z_q
0 1 2 3 4 5 6 7
$G(z_q)$
7 6 5 4 3 2 1
z_q
0 1 2 3 4 5 6 7
$p_z(z_q)$
.25
.20
.15
.10
.05
z_q
0 1 2 3 4 5 6 7



7.00
5.25
3.50
1.75
0
Number of pixels (× 10^4)
0
64
128
192
255
Intensity


255
192
128
64
0
0
64
128
192
255
Output intensity
Input intensity


7.00
5.25
3.50
1.75
0
0
64
128
192
255
Number of pixels (× 10^4)
Intensity


Number of pixels (× 10^4)
7.00
5.25
3.50
1.75
0
0
64
128
192
255
Intensity
Output intensity
255
192
128
64
0
(1)
(2)
Input intensity
Number of pixels (× 10^4)
7.00
5.25
3.50
1.75
0
64
128
192
255
Intensity

Food for thought!

1. What is histogram matching?
2. How is histogram matching different from histogram equalization?
3. Why is histogram matching used in image processing?
4. What is the main requirement for performing histogram matching?
5. Is histogram matching a single-pixel or neighborhood operation? Explain briefly.

Programming assignment

- Implement histogram matching to transform the intensity distribution of an input image so that it resembles a given reference image.
- **Concepts Used**
  - Image histogram
  - Normalized histogram
  - Cumulative distribution function (CDF)
  - Intensity transformation
  - Histogram matching (specification)
- **Tasks**
  - Read an input grayscale image and a reference image.
  - Compute the histograms and normalized histograms of both images.
  - Calculate the CDF for the input and reference histograms.
  - Create a mapping function to match the input histogram with the reference histogram.
  - Generate the histogram-matched image.
  - Display the original, reference, and processed images along with their histograms.
  - Briefly comment on how closely the output histogram matches the reference and how the image appearance changes.

AI supported self-learning (Prompts compatible with ChatGPT)

Active Learners (Learning by Doing)

1. Give me a small grayscale input image matrix and a reference histogram. Ask me to manually compute the mapping function for histogram matching. Let me try first, then explain the solution.
2. Create a numerical example where I calculate the CDF for an input image and a reference image and then match their intensity values step by step.

Reflective Learners (Learning by Thinking)

1. Explain histogram matching step by step starting from histogram computation to generating the matched image, and summarize why each step is necessary.
2. Explain clearly how histogram matching differs from histogram equalization and when each should be used.

Sensing Learners (Concrete & Practical)

1. Explain histogram matching using actual pixel values from a small grayscale image and a reference image.
2. Show a practical example where histogram matching makes one image resemble another in appearance.

Intuitive Learners (Concepts & Patterns)

1. Explain histogram matching as an intensity transformation based on cumulative probability distributions.
2. Explain conceptually why matching histograms changes the visual style or tone of an image.

Visual Learners (Diagrams & Structure)

1. Show the histograms of an input image and a reference image, then visually demonstrate how histogram matching aligns them.
2. Use graphs to explain the relationship between normalized histogram, CDF, and the mapping function.

Verbal Learners (Words & Explanation)

1. Explain histogram matching in simple language using an analogy such as adjusting one photo to look like another.
2. Explain the difference between histogram equalization and histogram matching as if teaching a beginner.

Sequential Learners (Step-by-Step Logic)

1. List the steps required to perform histogram matching, from reading the images to generating the output.
2. Explain step by step how the mapping function is derived using the CDFs of the input and reference images.

Global Learners (Big Picture First)

1. First explain the overall purpose of histogram matching in image enhancement, then explain how it works.
2. Explain where histogram matching fits within the complete digital image processing pipeline.