Sharpening Filters

Sharpening (change detection)

1. First derivative
   a) must be zero in areas of constant intensity
   b) must be nonzero at the onset of an intensity step or ramp
   c) must be nonzero along ramps
2. Second derivative
   a) must be zero in constant areas
   b) must be nonzero at the onset and end of an intensity step or ramp
   c) must be zero along ramps of constant slope

Sharpening (change detection)

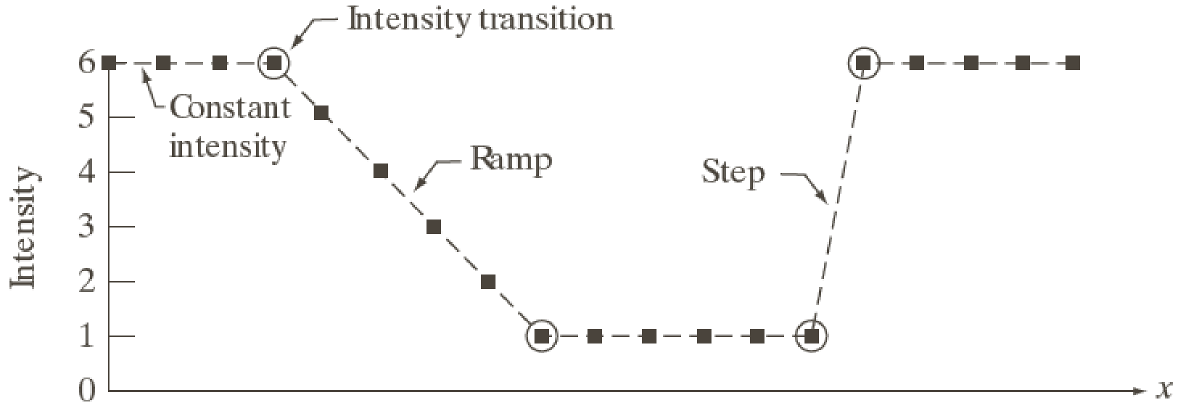
1. First derivative

$$\frac{\partial f}{\partial x} = f(x + 1) - f(x)$$

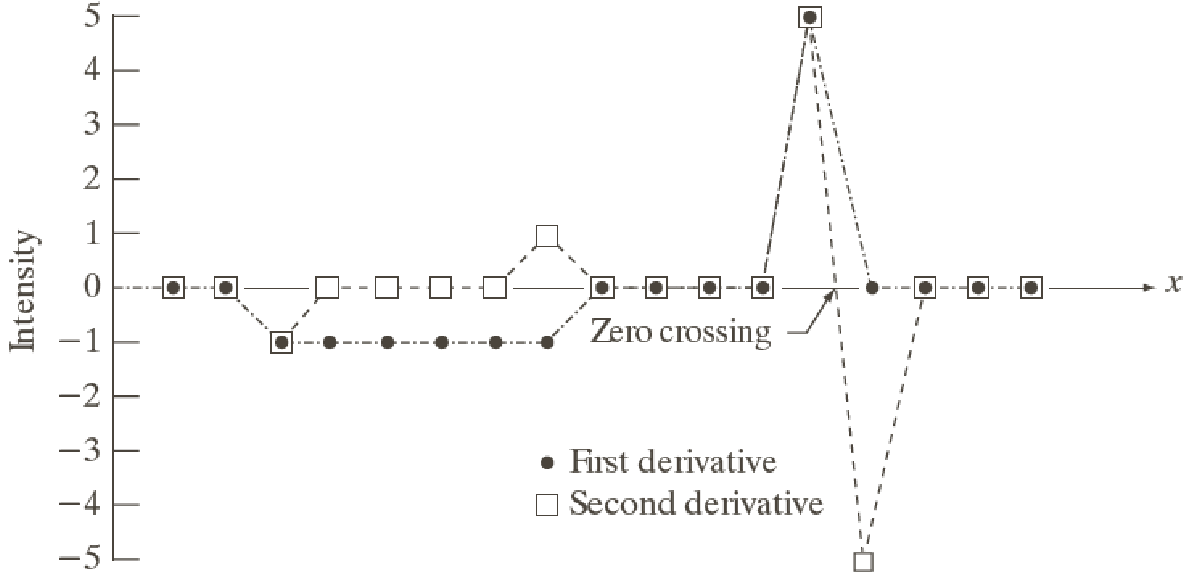
2. Second derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x + 1) + f(x - 1) - 2f(x)$$

Performance of derivatives



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Gradient (first derivative)

$$\nabla f \equiv \operatorname{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$M(x, y) = \operatorname{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

$$M(x, y) \approx |g_x| + |g_y|$$

$$\alpha(x, y) = \tan^{-1}\left[\frac{g_y}{g_x}\right]$$

Gradient operators

$$g_x = \frac{\partial f(x, y)}{\partial x} = f(x + 1, y) - f(x, y)$$

$$g_y = \frac{\partial f(x, y)}{\partial y} = f(x, y + 1) - f(x, y)$$

Gradient operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

a	
b	c
d	e
f	g

FIGURE 10.14 A 3 × 3 region of an image (the z's are intensity values) and various masks used to compute the gradient at the point labeled z_5.

Gradient operators

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

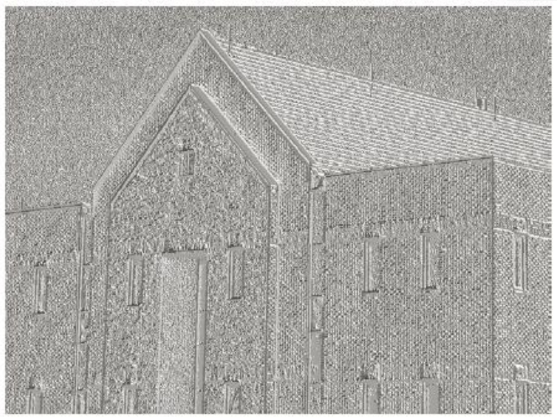
−2	−1	0
−1	0	1
0	1	2

Sobel

a b
c d

FIGURE 10.15 Prewitt and Sobel masks for detecting diagonal edges.

Gradient operators

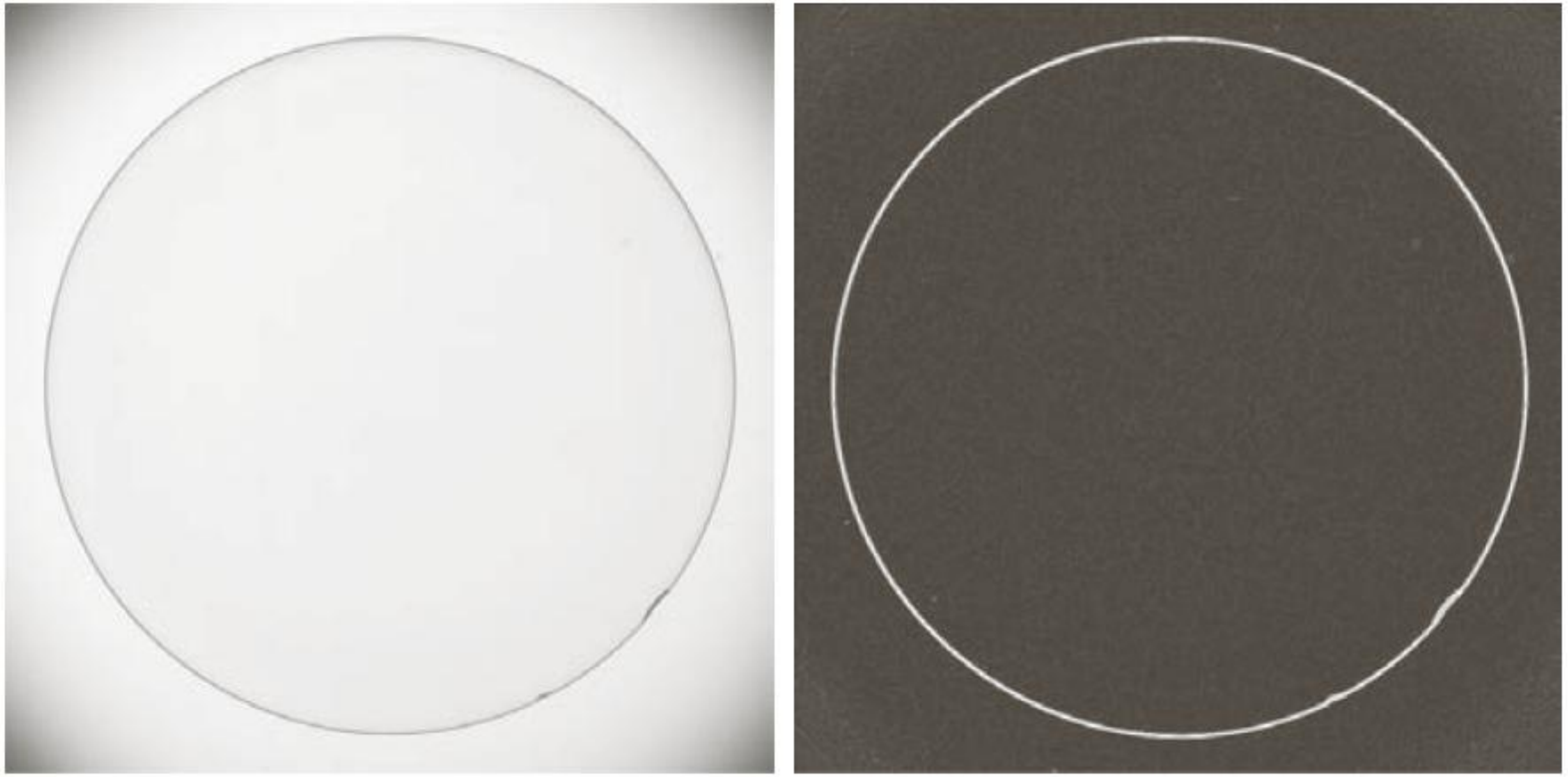


a b
c d

FIGURE 10.16 (a) Original image of size 834×1114 pixels, with intensity values scaled to the range [0, 1]. (b) $|g_x|$, the component of the gradient in the x-direction, obtained using the Sobel mask in Fig. 10.14(f) to filter the image. (c) $|g_y|$, obtained using the mask in Fig. 10.14(g). (d) The gradient image, $|g_x| + |g_y|$.

FIGURE 10.17 Gradient angle image computed using Eq. (10.2-11). Areas of constant intensity in this image indicate that the direction of the gradient vector is the same at all the pixel locations in those regions.

Gradient operators



a b

FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Pete Sites, Perceptics Corporation.)

Sobel Gradient operator in OpenCV

```
Sobel(src, dst, ddepth, dx, dy)

Sobel(input, horizontal_derivative, CV_32F, 1, 0);

Sobel(input, vertical_derivative, CV_32F, 0, 1);
```

Food for thought!

1. What is the purpose of sharpening filters in image processing?
2. Why are derivatives used in image sharpening?
3. What happens to the first derivative in regions of constant intensity?
4. What is the gradient in image processing?
5. What does the Sobel operator compute?

Programming assignment

- Implement a median filter to remove impulse noise from an image and compare its performance with a linear smoothing filter.
- **Concepts Used**
  - Image sharpening
  - First derivative
  - Gradient magnitude
  - Sobel operator
  - Edge detection
- **Tasks**
  - Read a grayscale image.
  - Apply the Sobel operator to compute:
    - Horizontal gradient (Gx)
    - Vertical gradient (Gy)
  - Compute the gradient magnitude using $|\ G\ |= \sqrt{Gx^2 + Gy^2}$
  - Display the original image, Gx, Gy, and gradient magnitude image.
  - Compare the edge-detected result with the original image and briefly comment on how edges are enhanced.

AI supported self-learning (Prompts compatible with ChatGPT)

Active Learners (Learning by Doing)

1. Provide a small grayscale intensity profile and ask me to manually compute its first derivative and identify edge locations.
2. Guide me in writing a Python/OpenCV program to compute horizontal and vertical gradients using the Sobel operator and combine them to obtain gradient magnitude.

Reflective Learners (Learning by Thinking)

1. Explain why derivatives are used for image sharpening and summarize how first and second derivatives detect intensity changes.
2. Why is the first derivative zero in constant regions but non-zero at edges? Provide a reasoning-based explanation.

Sensing Learners (Concrete & Practical)

1. Use actual pixel values to demonstrate how the Sobel operator detects edges in a small grayscale matrix.
2. Provide a real-world application where gradient-based edge detection is critical.

Intuitive Learners (Concepts & Patterns)

1. Explain the mathematical relationship between derivatives and change detection in images.
2. Compare first and second derivative behavior on intensity steps and ramps and explain the underlying pattern.

Visual Learners (Diagrams & Structure)

1. Show an original image, its horizontal gradient, vertical gradient, and gradient magnitude, and visually explain the differences.
2. Illustrate how a Sobel kernel responds differently to vertical and horizontal edges.

Verbal Learners (Words & Explanation)

1. Explain image sharpening using an analogy such as detecting sudden changes in terrain elevation.
2. Describe the concept of gradient in simple teaching language.

Sequential Learners (Step-by-Step Logic)

1. Break down the algorithm for computing gradient magnitude step by step, including convolution and magnitude calculation.
2. Explain step by step how Sobel operators compute horizontal and vertical derivatives.

Global Learners (Big Picture First)

1. Explain the overall role of sharpening filters in the image enhancement pipeline before discussing derivatives.
2. Provide a big-picture comparison between smoothing filters and sharpening filters.