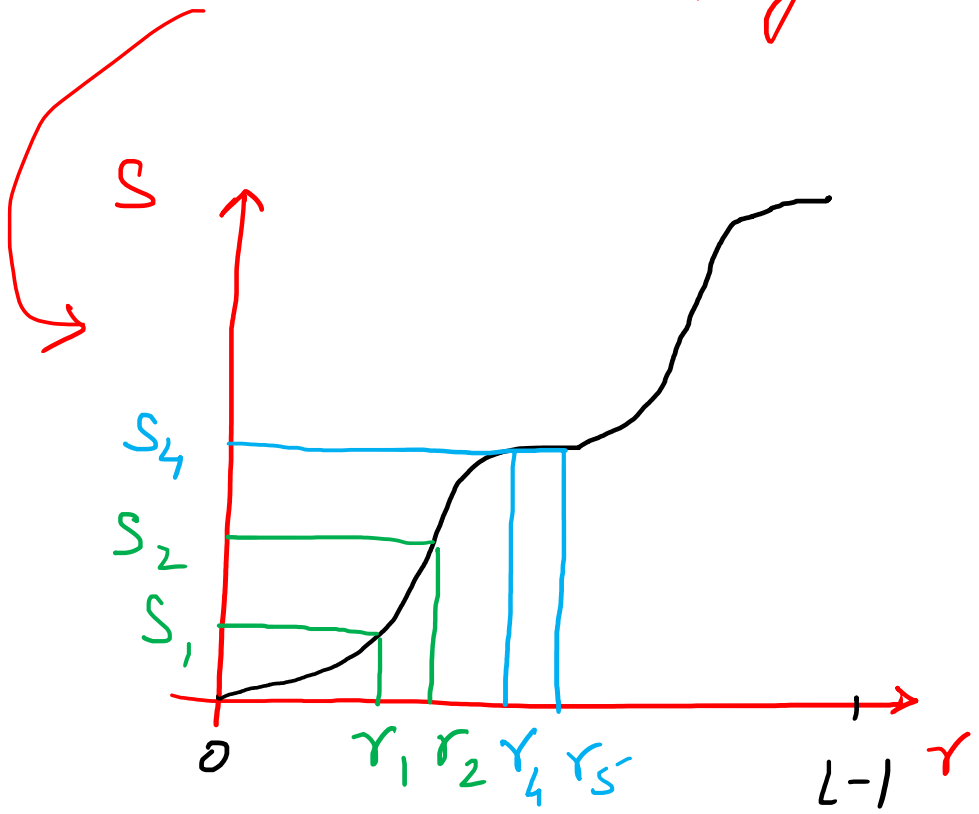
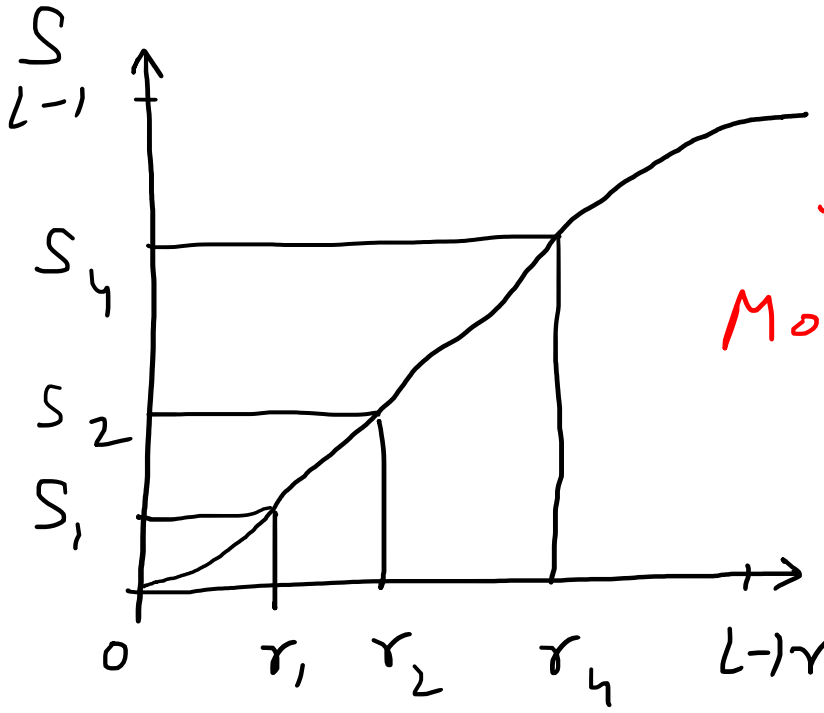
Histogram Equalization

$$S = T(r) \qquad 0 \le r \le L-1$$

a) Monotonically increasing fun^n



$0 \le r \le L-1$



Strictly Monotonically increasing fun^n

b) $0 \leq T_r \leq L-1$ for $0 \leq r \leq L-1$

c) $r = T^{-1}(s)$ $\quad 0 \leq s \leq L-1$

a') Strictly Monotonically increasing fun^{n}.

Continuous Domain

PDF $\rightarrow$ $p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$ (1)

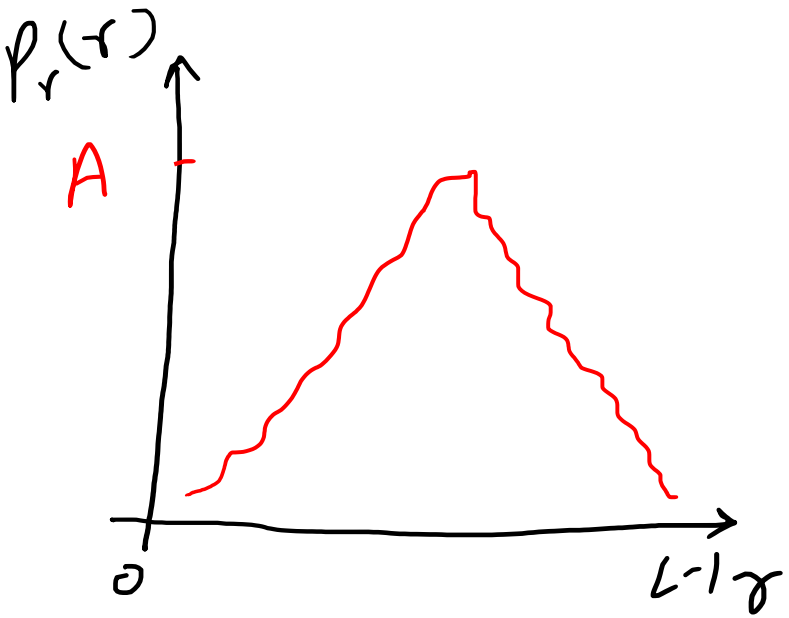
CDF $\leftarrow$ $s = T(r) = (L-1) \int_0^r p_r(w)\, dw$ (2)

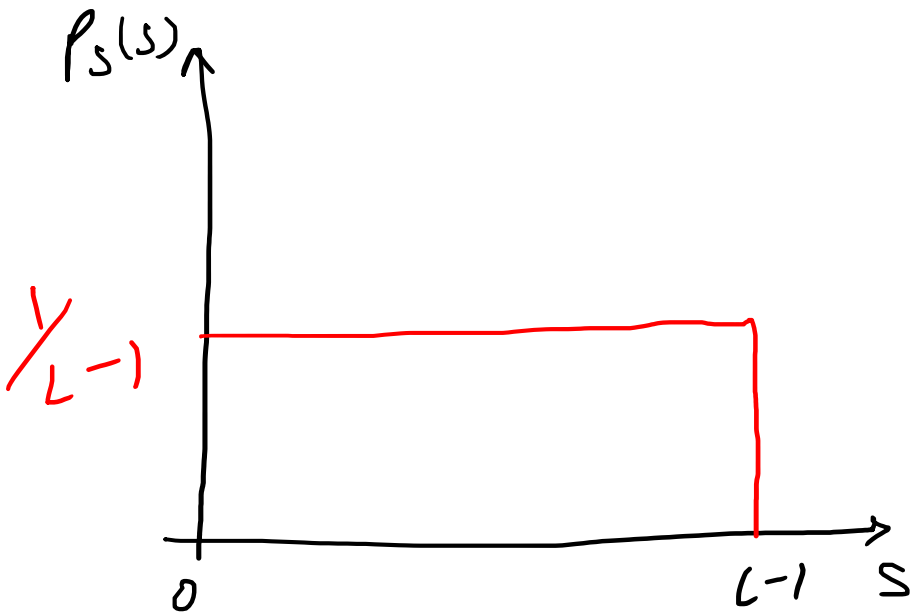
$$\frac{ds}{dr} = \frac{d}{dr}\left[(L-1) \int_0^r p_r(w)\, dw \right] = (L-1) \frac{d}{dr}\left[\int_0^r p_r(w)\, dw \right]$$

$$= (L-1)\, p_r(r)$$

$$\frac{dr}{ds} = \frac{1}{(L-1)\, p_r(r)}$$

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = \left| \frac{1}{L-1} \right|$$





a) $$P_r(r) = \begin{cases} 2r/(L-1)^2 & ; \quad 0 \le r \le L-1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$S = T(r) = (L-1) \int_0^r P_r(w)\, dw$$

$$= (L-1) \int_0^r \frac{2w}{(L-1)^2}\, dw$$

$$= \frac{1}{(L-1)} \int_0^r 2w\, dw = \frac{r^2}{(L-1)}$$

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

$$s = \frac{r^2}{L-1} \Rightarrow \frac{ds}{dr} = \frac{d}{dr}\left[\frac{r^2}{L-1}\right] = \frac{2r}{(L-1)}$$

$$\Rightarrow \frac{dr}{ds} = \frac{(L-1)}{2r}$$

$$p_s(s) = \frac{2r}{(L-1)^2} \times \frac{(L-1)}{2r} \Rightarrow \boxed{p_s(s) = \frac{1}{(L-1)}}$$

Discrete Domain (Digital Images)

$$p_r(r_k) = \frac{n_k}{MN} \quad ; \quad k = 0, 1, 2, \ldots L-1$$

$$s_k = T(r_k) = (L-1) \sum_{j=0}^{k} p_r(r_j)$$

$$s_k = \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j$$

Food for thought!

1. What is histogram equalization?
2. Why is histogram equalization used in image processing?
3. What role does the normalized histogram play in histogram equalization?
4. How does histogram equalization affect frequently occurring intensity values?
5. Is histogram equalization a single-pixel or neighborhood operation? Explain briefly.

Programming assignment

- Implement histogram equalization to enhance the contrast of a grayscale image and analyze how pixel intensity distribution changes after transformation.
- **Concepts Used**
  - Image histogram
  - Normalized histogram
  - Cumulative distribution function (CDF)
  - Intensity transformation
  - Histogram equalization
- **Tasks**
  - Read a low-contrast grayscale image.
  - Compute and display the original histogram.
  - Calculate the normalized histogram and cumulative distribution function (CDF).
  - Use the CDF to implement histogram equalization and generate the enhanced image.
  - Display the equalized image and its histogram.
  - Compare the original and processed images and briefly comment on contrast improvement.

AI supported self-learning (Prompts compatible with ChatGPT)

Active Learners (Learning by Doing)

1. Give me a small grayscale image matrix and ask me to manually compute its histogram, normalized histogram, and cumulative distribution function. Let me attempt first, then explain the solution.
2. Create a numerical example where I perform histogram equalization step by step using the CDF and explain how pixel intensities change.

Reflective Learners (Learning by Thinking)

1. Explain histogram equalization step by step starting from histogram computation to intensity transformation, and summarize why each step is necessary.
2. Explain why histogram equalization improves contrast and how it redistributes frequently occurring intensity values.

Sensing Learners (Concrete & Practical)

1. Explain histogram equalization using actual pixel values from a small grayscale image.
2. Show a practical example of a low-contrast image and explain how histogram equalization enhances visibility.

Intuitive Learners (Concepts & Patterns)

1. Explain histogram equalization as an intensity transformation based on probability distributions.
2. Explain how the cumulative distribution function reshapes the gray-level distribution of an image.

Visual Learners (Diagrams & Structure)

1. Show the histogram of an image before and after equalization and explain the visual differences.
2. Use graphs to explain the relationship between normalized histogram, CDF, and intensity mapping.

Verbal Learners (Words & Explanation)

1. Explain histogram equalization in simple language using everyday analogies such as spreading values evenly.
2. Explain the role of normalized histogram and CDF as if teaching it to someone new to image processing.

Sequential Learners (Step-by-Step Logic)

1. List the steps involved in histogram equalization from reading the image to generating the enhanced output.
2. Explain step by step how the transformation function is derived from the normalized histogram.

Global Learners (Big Picture First)

1. First explain the overall purpose of histogram equalization in image enhancement, then explain how it works.
2. Explain where histogram equalization fits within the complete digital image processing pipeline.