# Sharpening Filters

# Laplacian (second derivative)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

# Laplacian operators

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | −4 | 1 |
| 0 | 1 | 0 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | −8 | 1 |
| 1 | 1 | 1 |

| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | 4 | −1 |
| 0 | −1 | 0 |

| | | |
|---|---|---|
| −1 | −1 | −1 |
| −1 | 8 | −1 |
| −1 | −1 | −1 |

a b
c d

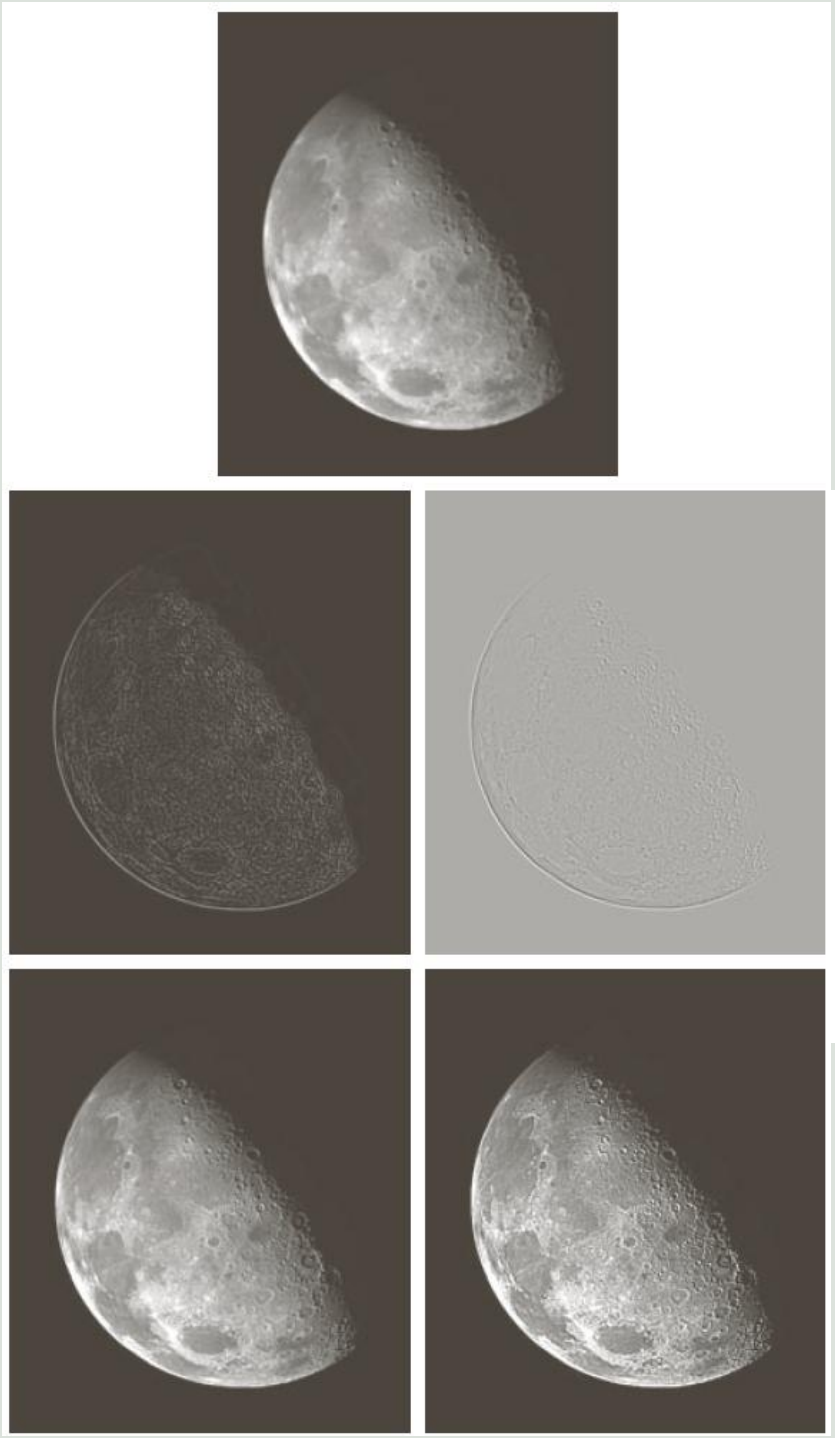
**FIGURE 3.37**
(a) Filter mask used to implement Eq. (3.6-6).
(b) Mask used to implement an extension of this equation that includes the diagonal terms.
(c) and (d) Two other implementations of the Laplacian found frequently in practice.

# Laplacian

$$g(x, y) = f(x, y) + c\left[\nabla^2 f(x, y)\right]$$

$$f_m = f - \min(f)$$

$$f_s = K\left[f_m / \max(f_m)\right]$$



a
b c
d e

**FIGURE 3.38**
(a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

# Laplacian in OpenCV

cv2.Laplacian (src, dst, ddepth, ksize)

Cv2. Laplacian(input, laplacian, CV_32F, 1);

# Food for thought!

1. What is the Laplacian operator in image processing?
2. How does the second derivative behave in constant intensity regions?
3. Why is the Laplacian useful for image sharpening?
4. How does the Laplacian respond at edges?
5. Highlight the difference between Gradient and Laplacian.

# Programming assignment

- Implement the Laplacian (second-derivative) operator to detect edges and enhance image sharpness in a grayscale image.
- **Concepts Used**
  - Second derivative
  - Laplacian operator
  - Edge detection
  - Spatial filtering
  - Image sharpening
- **Tasks**
  - Read a grayscale image.
  - Implement the Laplacian operator using a standard 3×3 Laplacian mask (4-neighbor or 8-neighbor version).
  - Apply the Laplacian filter to obtain the edge image.
  - Enhance the image by combining the Laplacian result with the original image (image sharpening).
  - Display the original image, Laplacian output, and sharpened image.
  - Briefly comment on how the Laplacian enhances edges.

# AI supported self-learning (Prompts compatible with ChatGPT)

Active Learners (Learning by Doing)

1. Provide a small grayscale matrix and ask me to manually apply a 3×3 Laplacian mask (4-neighbor version). Let me compute the result first, then explain the correct solution.
2. Guide me step by step in writing a Python/OpenCV program to apply the Laplacian operator and sharpen an image by combining it with the original.

Reflective Learners (Learning by Thinking)

1. Explain why the Laplacian is considered a second-derivative operator and summarize how it detects rapid intensity changes.
2. Why is the Laplacian zero in constant regions but large near edges? Provide a reasoning-based explanation.

Sensing Learners (Concrete & Practical)

1. Use actual pixel values to demonstrate how the Laplacian responds at edges in a small grayscale matrix.
2. Provide a practical example where Laplacian-based sharpening improves image interpretation.

Intuitive Learners (Concepts & Patterns)

1. Explain the mathematical behavior of the second derivative on intensity steps and ramps.
2. Compare the response patterns of gradient (first derivative) and Laplacian (second derivative) and explain the underlying logic.

Visual Learners (Diagrams & Structure)

1. Show an original image, its Laplacian output, and the sharpened result, and visually explain how edges are enhanced.
2. Illustrate the difference between 4-neighbor and 8-neighbor Laplacian masks.

Verbal Learners (Words & Explanation)

1. Explain the Laplacian operator using an analogy such as detecting sudden curvature changes in a road.
2. Describe the difference between gradient-based sharpening and Laplacian sharpening in simple teaching language.

Sequential Learners (Step-by-Step Logic)

1. Break down the Laplacian sharpening algorithm step by step, from mask application to combining with the original image.
2. Explain step by step how to compute the Laplacian using convolution.

Global Learners (Big Picture First)

1. Explain the overall role of second-derivative operators in image sharpening before discussing the Laplacian mask.
2. Provide a big-picture comparison between first-derivative (gradient) and second-derivative (Laplacian) sharpening techniques.