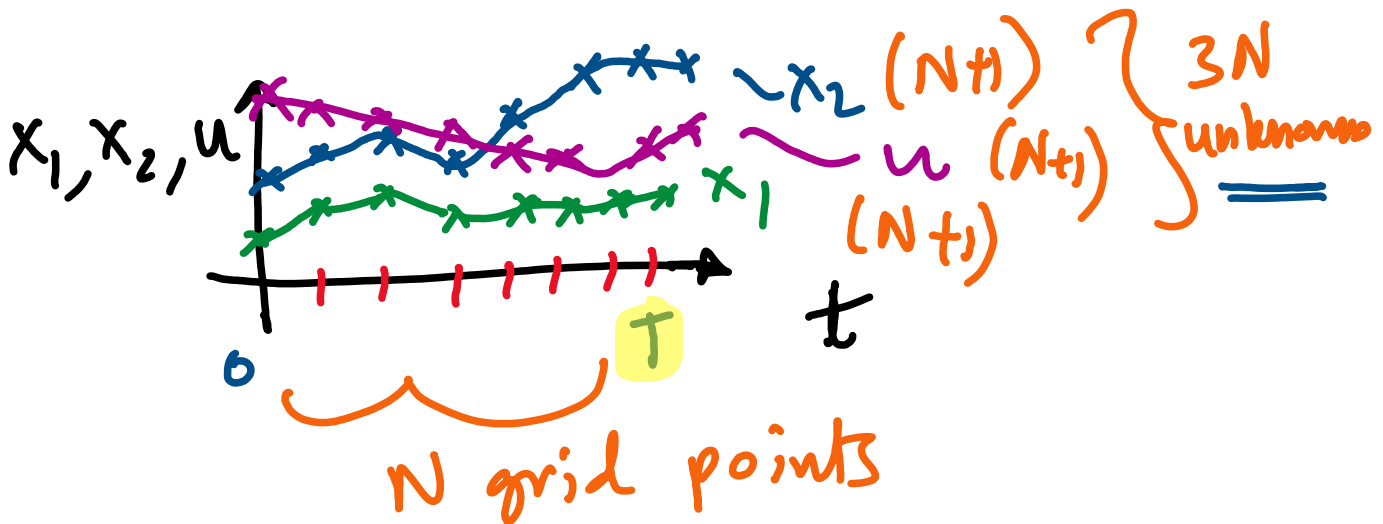Two methods:

① Collocation method

② Shooting method.   } Differ in how they handle the dynamics

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① Collocation method

Discretize the equations of motion

$$\dot{x}_1 = x_2 \quad ; \quad \dot{x}_2 = u$$

x_1, x_2, u

$x_2 (N+1)$
$u (N+1)$
$x_1 (N+1)$ } 3N unknowns

0

T

t

N grid points

(a) optimization variables

T $= 1$

$u(i)$ $\quad 0 \le i \le N = (N+1)$

$x_1(i)$ $\quad 0 \le i \le N = N+1$

$x_2(i)$ $\quad 0 \le i \le N = \underline{N+1}$

$\underline{3N+4}$

(b) cost $\quad$ minimize T

(c) constraints

$$\dot{x}_1 = x_2 \implies \frac{x_1(t+\Delta t) - x(t)}{\Delta t} = x_2(t)$$

$$x_1(i+1) - x_1(i) = x_2(i) \, \Delta t$$

$$x_1(i+1) - x_1(i) - x_2(i) \, \Delta t = 0$$

$$0 \le i \le N-1$$

N constraints

$$\dot{x}_2 = u$$

$$x_2(i+1) - x_2(i) - \Delta t \, u(i) = 0$$

$$0 \leq i \leq N-1$$

$$\underbrace{\qquad}_{N}$$

$$\left.\begin{array}{l} x_1(0) = 0 \\[1em] x_1(N) = D \\[1em] x_2(0) = 0 \\[1em] x_2(N) = 0 \end{array}\right\} \quad 4$$

Number of constraints $2N+4$

Optimization variable $3N+4$

Constraints $2N+4$

Free variables $(3N+4) - (2N+4) = N$

There are infinitely many solutions but the optimization will bias it a single solution