Denavit-Hartenberg (DH) Convention Handout

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The DH convention is a popular convention to represent the kinematics of robot manipulators.

$$
\begin{aligned}
\mathbf{H}_i^{i-1} &= \mathbf{H}_z(\theta_i)\mathbf{H}_z(d_i)\mathbf{H}_x(a_i)\mathbf{H}_x(\alpha_i) \\
&= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{1}
$$

where $s\theta_i = \sin\theta_i$, $c\theta_i = \cos\theta_i$, $s\alpha_i = \sin\alpha_i$, $c\theta_i = \cos\alpha_i$. These parameters are known as link length a_i, link twist α_i, link offset d_i, and joint angle θ_i. Normally, it take 3 positions and 3 orientations, a total of 6 numbers to describe a link, however, the DH uses only 4 numbers.

Algorithm for using DH for forward kinematics There are three steps.

1. **Assign coordinate frames:**
   (a) Assign z_i along the axis of actuation for each link, where $i = 0, 1, 2, ...(n-1)$.
   (b) Assign the base frame $o_0 - x_0 - y_0 - z_0$. The z_0 has already been assigned. Assign x_0 arbitrarily. Assign y_0 based on x_0 and z_0 using right hand rule.
   (c) Now assign coordinate frames $o_i - x_i - y_i - z_i$ for $i = 1, 2, ..., n-1$. z_i is already attached in first step. Next we assign x_i using these rules.
      i. **z_{i-1} and z_i are not coplanar:** In this case, there is a unique shorted distance segment that is perpendicular to z_{i-1} and z_i. Choose this as x_i axis. The origin o_i is where x_i intersects z_i. The y_i is found from right hand rules.
      ii. **z_{i-1} and z_i parallel:** In this case, there infinitely many perpendiculars. Choose any of these perpendiculars for x_i. Furthermore, where x_i intersects z_i we draw the origin x_i. Finally, y_i is found from the right hand rule. To make equations simpler, choose x_i such that is passes through o_{i-1}. This will make $d_i = 0$. Also, since z_{i-1} is parallel to z_i, $\alpha_i = 0$.

iii. z_{i-1} **and** z_i **intersect:** In this case, x_i is chosen to be normal to the plane formed by z_{i-1} and z_i. There will be two possible directions for x_i, one of them is chosen arbitrarily and o_i is obtained by the intersection of $z - i$ and x_i. Finally y_i is obtained from right hand rule. Also, since z_{i-1} intersects z_i, $a_i = 0$.

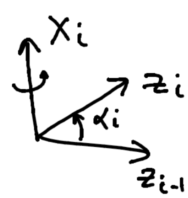
(d) Finally we need to attach an end effector frame, $o_n - x_n - y_n - z_n$. Attach z_n to be the same direction as z_{n-1}. Now depending on the relation between z_n and z_{n-1}, attach frame x_n. Finally, attach y_n using the right hand rule.

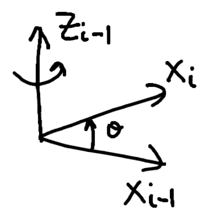
2. **Generate a table for DH parameter:** Now generate the DH table as follows.

Link	a_i	α_i	d_i	θ_i
1				
2				
.				
.				
.				
n				

Here is a cheat sheet to help populate the table

$a_i \longrightarrow$ distance betⁿ z_{i-1} & z_i along x_i
$\alpha_i \longrightarrow$ angle betⁿ z_{i-1} & z_i along x_i
$d_i \longrightarrow$ distance betⁿ x_{i-1} & x_i along z_{i-1}
$\theta_i \longrightarrow$ angle betⁿ x_{i-1} & x_i along z_{i-1}





3. **Apply DH transformation to evaluate forward kinematics:** Finally, use the DH formulate to link two adjacent frames

$$\mathbf{H}_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The position and orientation of the end-effector is found using the formula

$$\mathbf{H}_n^0 = \mathbf{H}_1^0 \mathbf{H}_2^1 \mathbf{H}_3^2 ... \mathbf{H}_n^{n-1} = \begin{bmatrix} \mathbf{R}_n^0 & \mathbf{d}_n^0 \\ \mathbf{0} & 1 \end{bmatrix}$$

The position of the end-effector is $\mathbf{d}_n^0$ and the orientation is $\mathbf{R}_n^0$. From $\mathbf{R}_n^0$, we can recover the Euler angles for the end-effector frame.