

Simulation of Ideal Gas

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Assumptions Using KMT

- Small particles in constant random linear motion
- No interactive forces
- Average kinetic energy depends on temperature
- Maxwell-Boltzmann distribution for total velocities (but each vector component follows normal curve)

Additional Assumptions

- Ignore particle-particle collisions
- Argon gas

What I Want to Verify

Ideal Gas Law: $PV=nRT$

Compressibility Factor: $Z=PV/nRT$ (equals 1 for ideal)

Boyle's Law: $P_1V_1=P_2V_2$

Gay-Lussac's Law: $P_1/T_1=P_2/T_2$

```

= Sim[nParticles_, xParticles_, m_, Length_] := {
  Clear[xPos, trajData];

  trajData = {};
  vTotalArray = {};
  F = 0;
  dpTotal = 0;
  nParticlesUsed = nParticles;
  xParticlesUsed = xParticles[[1 ;; nParticlesUsed]];

  For[i = 1, i ≤ nSteps, i++,
    v = {vTherm, vTherm, vTherm};
    vTotal = Sqrt[v[[1]]^2 + v[[2]]^2 + v[[3]]^2];
    AppendTo[vTotalArray, vTotal];

    For[n = 1, n ≤ nParticlesUsed, n++,
      xPos = xParticlesUsed[[n]][[i]];
      xPos += v dt;

      If[xPos[[1]] < 0, xPos[[1]] = -xPos[[1]]; dp = Abs[2 m v[[1]]]; dpTotal += dp; v[[1]] = -v[[1]];];
      If[xPos[[1]] > Length, xPos[[1]] = 2 * Length - xPos[[1]]; dp = Abs[2 m v[[1]]]; dpTotal += dp; v[[1]] = -v[[1]];];
      If[xPos[[2]] < 0, xPos[[2]] = -xPos[[2]]; dp = Abs[2 m v[[2]]]; dpTotal += dp; v[[2]] = -v[[2]];];
      If[xPos[[2]] > Length, xPos[[2]] = 2 * Length - xPos[[2]]; dp = Abs[2 m v[[2]]]; dpTotal += dp; v[[2]] = -v[[2]];];
      If[xPos[[3]] < 0, xPos[[3]] = -xPos[[3]]; dp = Abs[2 m v[[3]]]; dpTotal += dp; v[[3]] = -v[[3]];];
      If[xPos[[3]] > Length, xPos[[3]] = 2 * Length - xPos[[3]]; dp = Abs[2 m v[[3]]]; dpTotal += dp; v[[3]] = -v[[3]];];

      AppendTo[xParticlesUsed[[n]], xPos];
    ];

  ];

  F += dpTotal / time;
}

```

Simulation of Particles in a Box

```

In[3646]:= RT1 = 8.314*273; (*units J/mol = (kg m2/s2)/mol*)
M = 39.9/1000; (*molar mass: units kg*)
vTherm := Random[NormalDistribution[0, Sqrt[(RT1)/(M)]]] (*units m/s*)
m = 6.6335209*(10^(-26)); (*atomic mass of Argon in kg*)

length = 100;
nParticles = 400;
time = 100; (*units sec*)
dt = .05;
nSteps = Round[time / dt];

xParticles = {};
For[i = 1, i ≤ nParticles, i++,
  AppendTo[xParticles, {{RandomReal[{0, length}], RandomReal[{0, length}], RandomReal[{0, length}]}}];
]

Sim[nParticles, xParticles, m, length];

area = 6*(length^2);
V1 = length^3;
P1 = F/area;
n = nParticlesUsed/(6.02*(10^23));
Z = (P1V1)/(nRT1);

Print["total force: ", F, " N"]
Print["total pressure: ", P1, " Pa"]
Print["compressibility: ", Z]

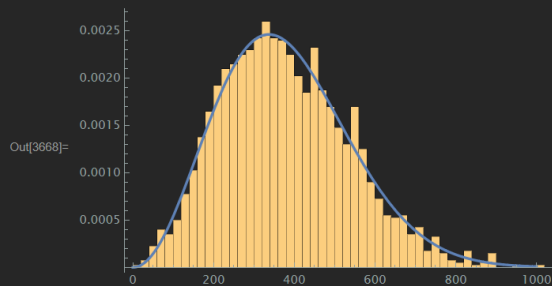
σ = ((RT1)/M)^0.5;
MaxwellCurve = Plot[PDF[MaxwellDistribution[σ], x], {x, 0, 1000}, PlotRange → {{0, 1000}, {0, .0035}}, PlotStyle → Thick];
vTotalDist = Histogram[vTotalArray, bins = 50, "PDF"];
Show[vTotalDist, MaxwellCurve]

total force: 8.98662×10-20 N

total pressure: 1.49777×10-24 Pa

compressibility: 0.993137

```



Verifying Ideal Gas Law

Conditions:

Argon molar mass, atomic mass

T=273K

Time=100sec

dt=.05sec

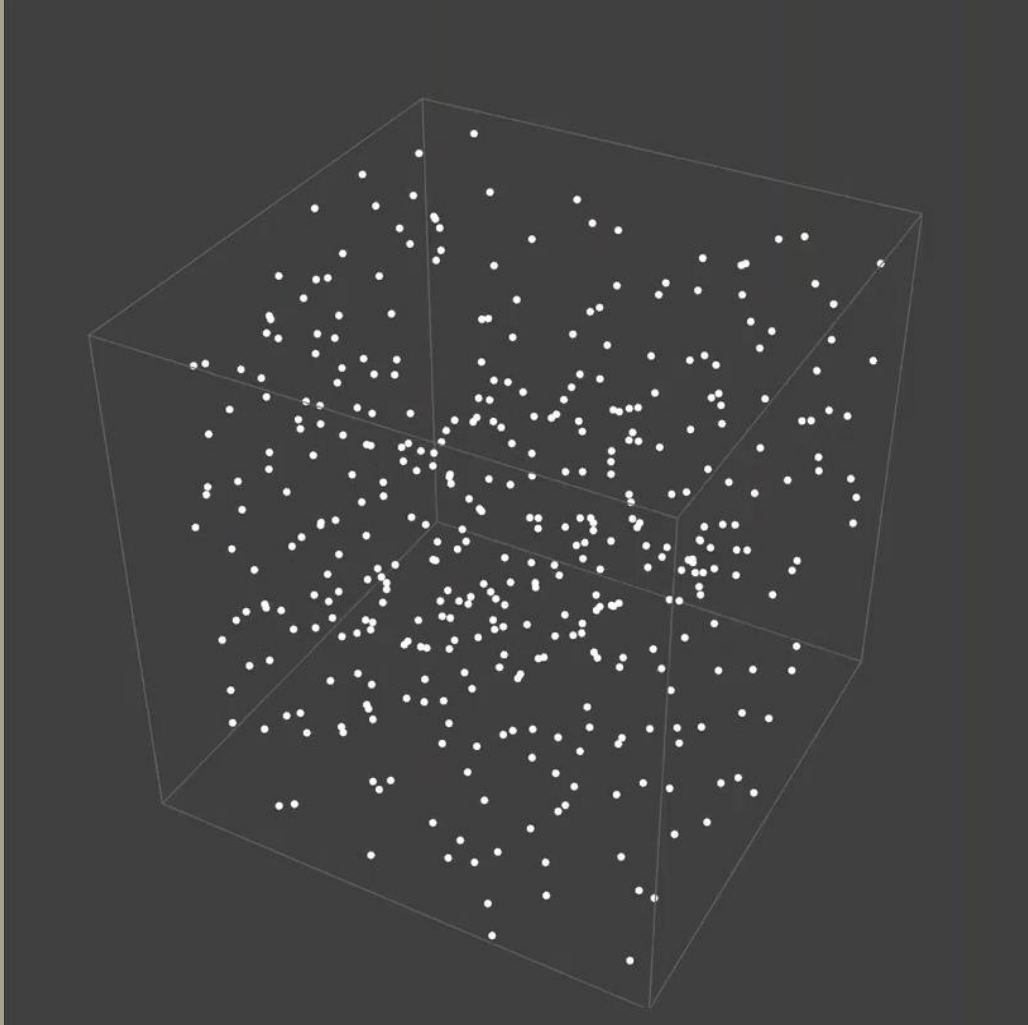
Box Length=100m

of Particles=400

Result:

Z=0.99137

Very close to 1!



Conditions:
 $T=273\text{K}$
Box Length=1000m
of Particles=400

```
RT1 = 8.314*273; (*units J/mol = (kg m2/s2)/mol*)
M = 39.9/1000; (*molar mass: units kg*)
vTherm := Random[NormalDistribution[0, Sqrt[(RT1) / (M)]]] (*units m/s*)
m = 6.6335209* (10^(-26)); (*atomic mass of Argon in kg*)

nParticles = 400;
length = 1000;
time = 100; (*units sec*)
dt = .15;
nSteps = Round[time / dt];

Sim[nParticles, xParticles, m, length];

animationArray = {};
For[i = 1, i <= nSteps + 1, i++,
  positionAllParticles = {};
  For[n = 1, n <= nParticlesUsed, n++,
    position = xParticlesUsed[[n]][i];
    AppendTo[positionAllParticles, position];
  ];
  AppendTo[animationArray, positionAllParticles]
]
v = Map[Point, animationArray];
animation = Animate[Graphics3D[{White, v[[i]]}, PlotRange -> {{0, length}, {0, length}, {0, length}}, {i, 1, Length[v], 1}, AnimationRate -> 100, RefreshRate -> 100,
```

```

In[3678]:= DiffVolume[originalLength_, multiplier_, nRuns_] := {
  pressuresRatioVolume = {};
  lengthMultiplier = multiplier^(1/3);
  length = lengthMultiplier * originalLength;

  For[k = 1, k ≤ nRuns, k++,
    RT1 = 8.314 * 273; (*units J/mol = (kg m2/s2)/mol*)
    M = 39.9 / 1000; (*molar mass: units kg*)
    vTherm := Random[NormalDistribution[0, Sqrt[(RT1) / (M)]]]; (*units m/s*)
    m = 6.6335209 * (10^(-26)); (*atomic mass of Argon in kg*)

    nParticles = 400;
    time = 100; (*units sec*)
    dt = .05;
    nSteps = Round[time / dt];

    xParticles = {};
    For[i = 1, i ≤ nParticles, i++,
      AppendTo[xParticles, {RandomReal[{0, length}], RandomReal[{0, length}], RandomReal[{0, length}]}]];
    ];

    Sim[nParticles, xParticles, m, length];

    area = 6 * (length^2);
    V2 = length^3;
    P2 = F / area;
    n = nParticlesUsed / (6.02 * (10^23));
    Z = (P2 V2) / (n RT1);

    pressuresRatioVolume = AppendTo[pressuresRatioVolume, P1 / P2];

     $\sigma = ((RT1) / M)^{0.5}$ ;
    MaxwellCurve = Plot[PDF[MaxwellDistribution[ $\sigma$ ], x], {x, 0, 1000}, PlotRange → {{0, 1000}, {0, .0035}}];
    vTotalDist = Histogram[vTotalArray, bins = 50, "PDF"];
  ];
}

```

Verifying Boyle's Law: Simulation at Different Volume

```
In[3679]:= DiffVolume[100, 8, 1];
```

```
Print["total force: ", F]  
Print["total pressure: ", P2]  
Print["compressibility: ", Z]  
Print["P1/P2 = ", P1 / P2]  
Print["V2/V1 = ", V2 / V1]
```

```
Show[MaxwellCurve, vTotalDist]
```

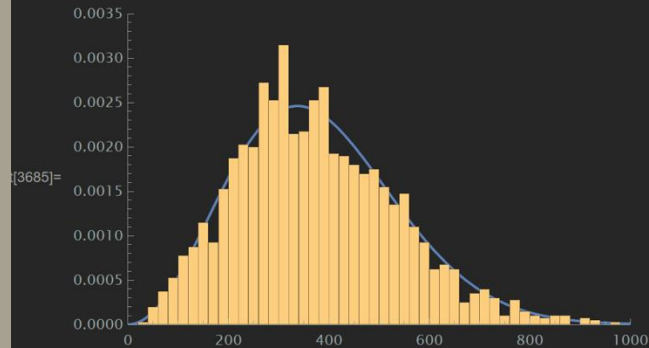
```
total force:  $4.34085 \times 10^{-20}$ 
```

```
total pressure:  $1.80869 \times 10^{-25}$ 
```

```
compressibility: 0.959438
```

```
P1/P2 = 8.28099
```

```
V2/V1 = 8
```



Condition: $V2/V1=8$

Result with 1 Run:
 $P1/P2=8.28099$

Result with 10 Runs:
Avg $P1/P2=7.93793$
Variation from Ideal =
 0.77584%

Below is the result if the simulation is run 10 times and the average pressure ratio is taken.

```
In[3686]:= multiplierVolume = 8;  
nRuns = 10;
```

```
In[3688]:= DiffVolume[100, multiplierVolume, nRuns];
```

```
In[*]:= Print[pressuresRatioVolume]  
{7.89328, 7.26403, 9.67065, 7.92187, 7.69867, 8.26179, 7.91543, 7.13317, 8.76449, 6.85595}
```

```
In[*]:= averagePressureRatioVolume = Mean[pressuresRatioVolume]  
Print["variation from ideal: ", Abs[(averagePressureRatioVolume - multiplierVolume) / multiplierVolume] * 100], "%"]
```

```
Out[*]:= 7.93793
```

```
variation from ideal: 0.77584%
```

```

DiffTemp[originalTemp_, multiplier_, nRuns_] := {
  pressuresRatioTemp = {};
  RT2 = multiplier * 8.314 * originalTemp; (*units J/mol = (kg m2/s2)/mol*)

  For[k = 1, k ≤ nRuns, k++,
    M = 39.9 / 1000; (*molar mass: units kg*)
    vTherm := Random[NormalDistribution[0, Sqrt[(RT2) / (M)]]]; (*units m/s*)
    m = 6.6335209 * (10^(-26)); (*atomic mass of Argon in kg*)

    length = 1000;
    nParticles = 400;
    time = 100; (*units sec*)
    dt = .05;
    nSteps = Round[time / dt];

    xParticles = {};
    For[i = 1, i ≤ nParticles, i++,
      AppendTo[xParticles, {{RandomReal[{0, length}], RandomReal[{0, length}], RandomReal[{0, length}]}}]];
    ];

    Sim[nParticles, xParticles, m, length];

    area = 6 * (length^2);
    V1 = length^3;
    P2 = F / area;
    n = nParticlesUsed / (6.02 * (10^23));
    Z = (P2 V1) / (n RT2);

    pressuresRatioTemp = AppendTo[pressuresRatioTemp, P1 / P2];

    σ = ((RT2) / M) ^0.5;
    MaxwellCurve = Plot[PDF[MaxwellDistribution[σ], x], {x, 0, 1800}, PlotRange → {{0, 1800}, {0, .0035}}];
    vTotalDist = Histogram[vTotalArray, bins = 30, "PDF"];
  ];
}

```

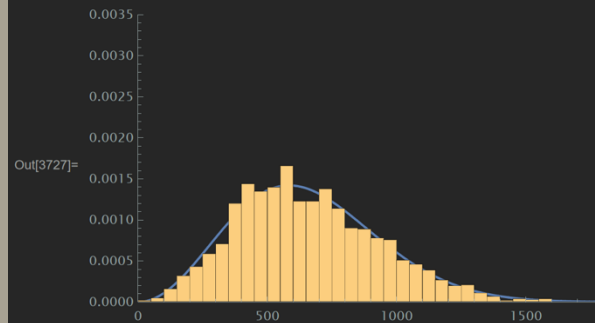
Verifying Gay-Lussac's Law: Simulation at Different Temperature


```
In[3721]:= DiffTemp[273, 3, 1];
```

```
Print["total force: ", F]  
Print["total pressure: ", P2]  
Print["compressibility: ", Z]  
Print["P1/P2 = ", P1 / P2]  
Print["T1/T2 = ", RT1 / RT2]
```

```
Show[MaxwellCurve, vTotalDist]
```

```
total force:  $2.69011 \times 10^{-19}$   
total pressure:  $4.48351 \times 10^{-24}$   
compressibility: 0.990971  
P1/P2 = 0.331541  
T1/T2 = 0.333333
```



```
In[3728]:= multiplierTemp = 3;
```

```
nRuns = 10;
```

```
In[3730]:= DiffTemp[273, multiplierTemp, nRuns];
```

```
In[ ]:= Print[pressuresRatioTemp]
```

```
{0.357164, 0.310868, 0.317409, 0.358936, 0.298064, 0.330713, 0.343533, 0.335827, 0.303814, 0.350567}
```

```
In[ ]:= averagePressureRatioTemp = Mean[pressuresRatioTemp]
```

```
Print["variation from ideal: ",
```

```
Abs[(averagePressureRatioTemp - (1 / multiplierTemp)) / (1 / multiplierTemp) * 100],  
"%"]
```

```
Out[ ]:= 0.33069
```

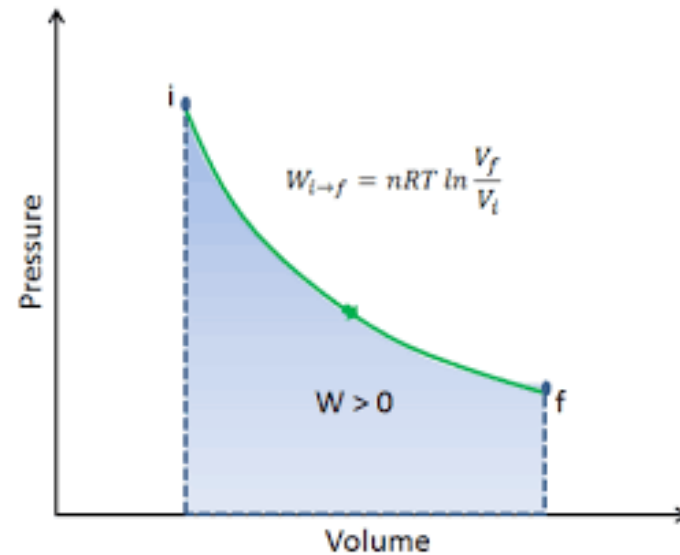
```
variation from ideal: 0.793144%
```

Condition: $T1/T2=1/3$

Result with 1 Run:
 $P1/P2=0.331541$

Result with 10 Runs:
Avg $P1/P2=0.33069$
Variation from Ideal =
 0.7933144%

Isothermal Compression



$$W_{i \rightarrow f} = \int_{V_i}^{V_f} p \, dV = \int_{V_i}^{V_f} \frac{nRT}{V} \, dV = nRT \int_{V_i}^{V_f} \frac{1}{V} \, dV = nRT \ln \frac{V_f}{V_i}$$

```

In[ ]:= IsothermalCE[nParticles_, h1_, h2_, hstep_, m_, Length_] := {
  PVarray = {};
  Zarray = {};

  For[h = h1, h ≤ h2, h += hstep,
    Clear[xPos, trajData];
    xParticles = {};

    trajData = {};
    vTotalArray = {};
    F = 0;
    dpTotal = 0;

    For[o = 1, o ≤ nParticles, o++,
      AppendTo[xParticles, {(RandomReal[{0, Length}], RandomReal[{0, Length}], RandomReal[{0, h}])}]];
    ];

    For[i = 1, i ≤ nSteps, i++,
      v = {vTherm, vTherm, vTherm};
      vTotal = Sqrt[v[[1]]^2 + v[[2]]^2 + v[[3]]^2];
      AppendTo[vTotalArray, vTotal];

      For[n = 1, n ≤ nParticles, n++,
        xPos = xParticles[[n]][[1]];
        xPos += v dt;

        If[xPos[[1]] < 0, xPos[[1]] = -xPos[[1]]; dp = Abs[2 m v[[1]]]; dpTotal += dp; v[[1]] = -v[[1]];];
        If[xPos[[1]] > Length, xPos[[1]] = 2 * Length - xPos[[1]]; dp = Abs[2 m v[[1]]]; dpTotal += dp; v[[1]] = -v[[1]];];
        If[xPos[[2]] < 0, xPos[[2]] = -xPos[[2]]; dp = Abs[2 m v[[2]]]; dpTotal += dp; v[[2]] = -v[[2]];];
        If[xPos[[2]] > Length, xPos[[2]] = 2 * Length - xPos[[2]]; dp = Abs[2 m v[[2]]]; dpTotal += dp; v[[2]] = -v[[2]];];
        If[xPos[[3]] < 0, xPos[[3]] = -xPos[[3]]; dp = Abs[2 m v[[3]]]; dpTotal += dp; v[[3]] = -v[[3]];];
        If[xPos[[3]] > h, xPos[[3]] = 2 * h - xPos[[3]]; dp = Abs[2 m v[[3]]]; dpTotal += dp; v[[3]] = -v[[3]];];

        AppendTo[xParticles[[n]], xPos];
      ];

    ];

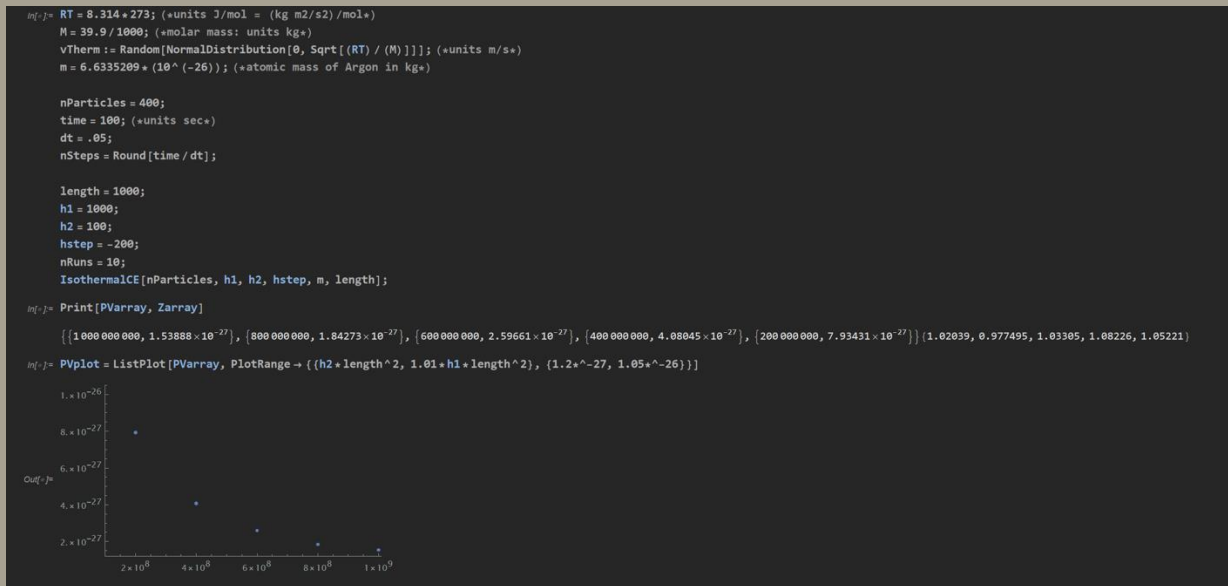
    F += dpTotal / time;

    area = 4 * Length * h + 2 * Length^2;
    V = h * Length^2;
    P = F / area;
    n = nParticles / (6.02 * (10^23));
    Z = (P V) / (n R T);

    AppendTo[PVarray, {V, P}];
    AppendTo[Zarray, Z];
  ]
}

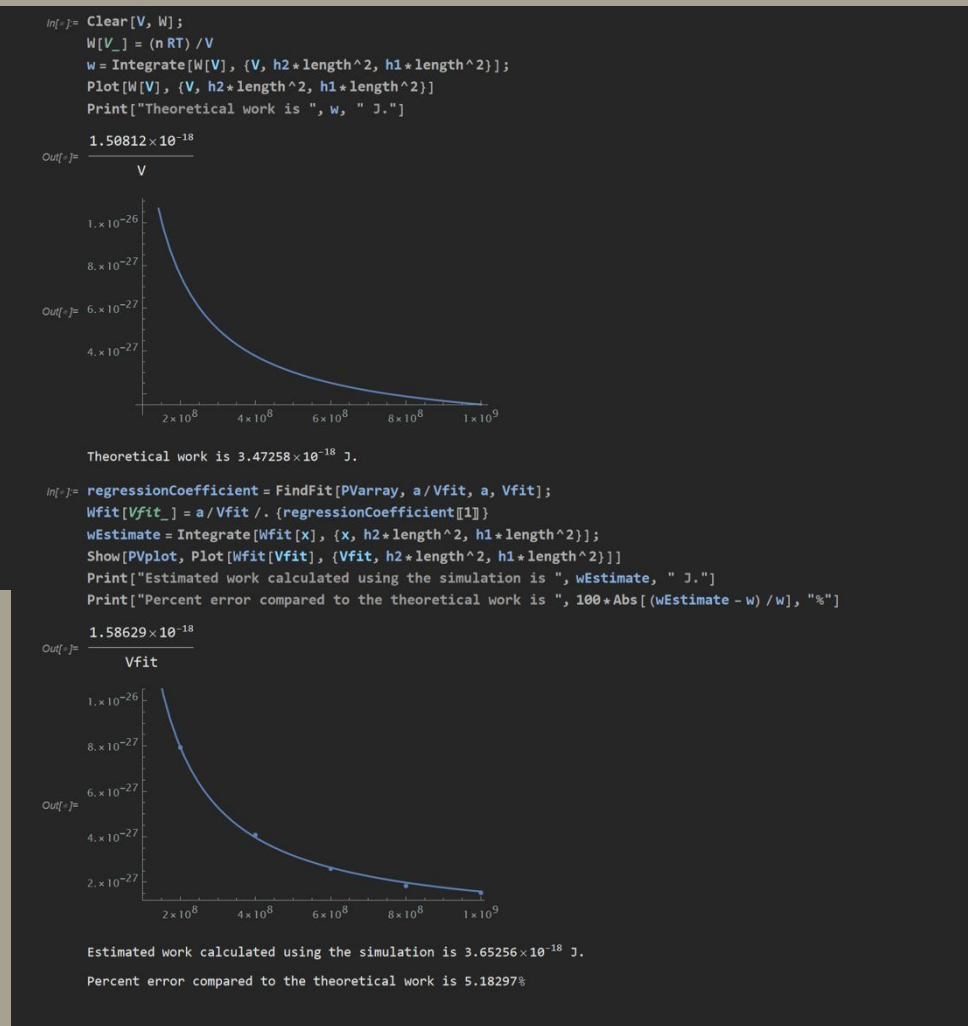
```

Simulation for Isothermal Compression



Estimated work using
simulation: about 3.653E-18J

Error compared to
theoretical integral = about
5.183%



Elastic Collisions of Two Particles

total momentum before = total momentum after

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

total kinetic energy before = total kinetic energy after

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

```
In[2714]:= Clear[v1x, v2x, v1y, v2y, v1z, v2z];
sol = Solve[m v1[[1]] + m v2[[1]] == m v1x + m v2x && 0.5 m v1[[1]]^2 + 0.5 m v2[[1]]^2 == 0.5 m v1x^2 + 0.5 m v2x^2 && m v1[[2]] + m v2[[2]] == m v1y + m v2y && 0.5 m v1[[2]]^2 + 0.5 m v2[[2]]^2 == 0.5 m v1y^2 + 0.5 m v2y^2 &&
m v1[[3]] + m v2[[3]] == m v1z + m v2z && 0.5 m v1[[3]]^2 + 0.5 m v2[[3]]^2 == 0.5 m v1z^2 + 0.5 m v2z^2 && v1x != v1[[1]], {v1x, v2x, v1y, v2y, v1z, v2z}]

finalSol = {1, 1, 1, 1, 1, 1};
For[i = 1, i <= Length[sol], i++,
  If[sol[[i]][[1]][[2]] != v1[[1]], finalSol[[1]] = sol[[i]][[1]];];
  If[sol[[i]][[2]][[2]] != v2[[1]], finalSol[[2]] = sol[[i]][[2]];];
  If[sol[[i]][[3]][[2]] != v1[[2]], finalSol[[3]] = sol[[i]][[3]];];
  If[sol[[i]][[4]][[2]] != v2[[2]], finalSol[[4]] = sol[[i]][[4]];];
  If[sol[[i]][[5]][[2]] != v1[[3]], finalSol[[5]] = sol[[i]][[5]];];
  If[sol[[i]][[6]][[2]] != v2[[3]], finalSol[[6]] = sol[[i]][[6]];];
]
finalSolArray = finalSol /. Rule -> List
v1x = finalSolArray[[1]][[2]]
v2x = finalSolArray[[2]][[2]]
v1y = finalSolArray[[3]][[2]]
v2y = finalSolArray[[4]][[2]]
v1z = finalSolArray[[5]][[2]]
v2z = finalSolArray[[6]][[2]]
```

Out[2719]= -31.2912

Out[2720]= -204.708

Out[2721]= 206.899

Out[2722]= 8.18265

Out[2723]= -235.141

Out[2724]= 148.155

```
In[2725]:= v1[[1]]
v2[[1]]
v1[[2]]
v2[[2]]
v1[[3]]
v2[[3]]
```

Out[2725]= -204.708

Out[2726]= -31.2912

Out[2727]= 8.18265

Out[2728]= 206.899

Out[2729]= 148.155

Out[2730]= -235.141

```

//clear- Sim[particles, n, length] := {
  Clear[abs, v, particles, vParticles];

  F = 0;
  dpTotal = 0;
  collisionData = {};
  isrunning = {};

  (*-----*)

  nParticlesLoop = {};
  For[n = 1, n <= nParticles, n++,
    AppendTo[nParticlesLoop, {RandomReal[{0, length}], RandomReal[{0, length}], RandomReal[{0, length}]}];
  ];
  nParticles = {nParticlesLoop};

  vTotalArray = {};
  nParticlesLoop = {};
  For[n = 1, n <= nParticles, n++,
    vTotal = Sqrt[Add[3]^2 + Add[3]^2 + Add[3]^2];
    AppendTo[nParticlesLoop, {vTherm, vTherm, vTherm}];
    AppendTo[vTotalArray, vTotal];
  ];
  nParticles = {nParticlesLoop};

  (*-----*)

  For[i = 1, i <= nSteps, i++,
    allParticlesAtOneTime = {};
    allVelocitiesAtOneTime = {};

    (*-----*)

    positionsAtTime = nParticles[i];
    distanceArray = {};

    For[k = 1, k <= length[positionsAtTime], k++,
      firstParticlePos = positionsAtTime[k];
      distanceWithEachParticle = {};

      For[l = 1, l <= length[positionsAtTime], l++,
        If[l < k,
          secondParticlePos = positionsAtTime[l];
          xdist = secondParticlePos[1] - firstParticlePos[1];
          ydist = secondParticlePos[2] - firstParticlePos[2];
          zdist = secondParticlePos[3] - firstParticlePos[3];
          distance = Sqrt[xdist^2 + ydist^2 + zdist^2];
          AppendTo[distanceWithEachParticle, {{k, l}, distance}];
        ];
      ];
      AppendTo[distanceArray, distanceWithEachParticle];
    ];

    collidingParticles = {};
    For[k = 1, k <= length[distanceArray], k++,
      distanceWithParticleCenter = distanceArray[k];
      For[l = 1, l <= length[distanceWithParticleCenter], l++,
        If[distanceWithParticleCenter[l][2] < 2 r,
          AppendTo[collidingParticles, distanceWithParticleCenter[l][1]];
        ];
      ];
      collidingParticlesIDarray = {};
      For[k = 1, k <= length[collidingParticles], k++,
        For[l = 1, l <= length[collidingParticles[k]], l++,
          AppendTo[collidingParticlesIDarray, collidingParticles[k][1]];
          AppendTo[collidingParticlesIDarray, collidingParticles[k][1]];
        ];
      ];
      collidingParticles = DeleteDuplicates[collidingParticlesIDarray];
      AppendTo[collisionData, collidingParticles];

      If[length[collidingParticles] == 2,
        AppendTo[isrunning, 1];
        particle1 = collidingParticles[1];
        particle2 = collidingParticles[2];
        v1 = nParticles[1][particle1];
        v2 = nParticles[1][particle2];

        nParticles[1][particle1] = v2;
        nParticles[1][particle2] = v1;
      ];

      (*-----*)

      For[n = 1, n <= nParticles, n++,
        nPos = nParticles[1][n];
        v = nParticles[1][n];
        nPos += v dt;

        If[nPos[1] < 0, nPos[1] = -nPos[1]; dp = Abs[2 n v[1]]; dpTotal += dp; v[1] = -v[1];];
        If[nPos[1] > length, nPos[1] = 2*length - nPos[1]; dp = Abs[2 n v[1]]; dpTotal += dp; v[1] = -v[1];];
        If[nPos[2] < 0, nPos[2] = -nPos[2]; dp = Abs[2 n v[2]]; dpTotal += dp; v[2] = -v[2];];
        If[nPos[2] > length, nPos[2] = 2*length - nPos[2]; dp = Abs[2 n v[2]]; dpTotal += dp; v[2] = -v[2];];
        If[nPos[3] < 0, nPos[3] = -nPos[3]; dp = Abs[2 n v[3]]; dpTotal += dp; v[3] = -v[3];];
        If[nPos[3] > length, nPos[3] = 2*length - nPos[3]; dp = Abs[2 n v[3]]; dpTotal += dp; v[3] = -v[3];];

        AppendTo[allParticlesAtOneTime, nPos];
        AppendTo[allVelocitiesAtOneTime, v];
        vTotal = Sqrt[v[1]^2 + v[2]^2 + v[3]^2];
        AppendTo[vTotalArray, vTotal];
      ];

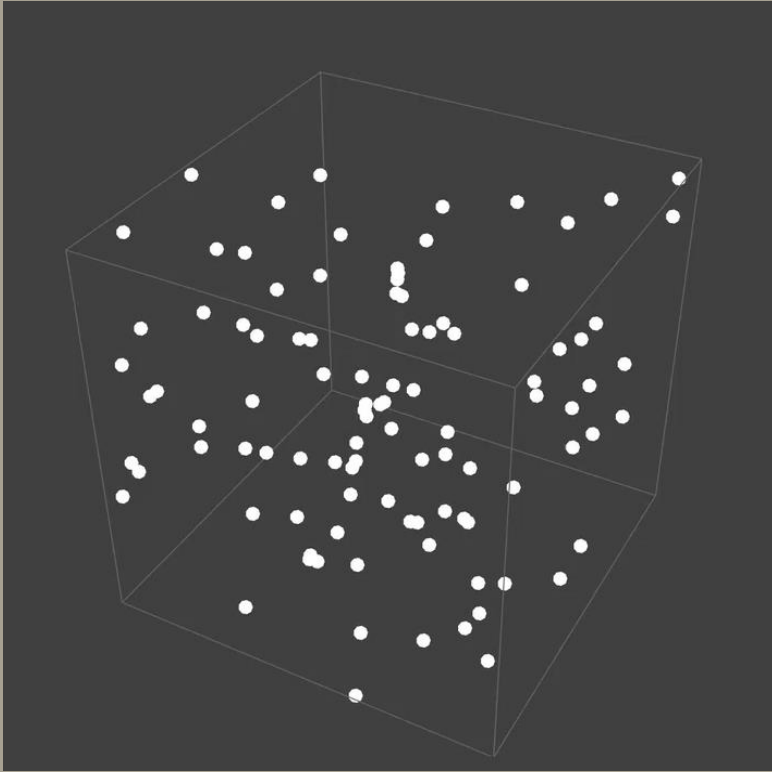
      (*-----*)

      AppendTo[nParticles, allParticlesAtOneTime];
      AppendTo[nParticles, allVelocitiesAtOneTime];
    ];

    F += dpTotal/time;
  }

```

Simulation Including Elastic Collisions of Two Particles at a Time



```
{(), (), (), (), (), (), (10, 18), (), (), (), (), (), (), (), (22, 31), (3, 51), (), (), (6, 100), (), (), (),
(), (), (), (), (17, 62), (), (), (), (), (), (), (), (), (), (20, 53), (), (), (), (), (), (), (), (),
(), (), (24, 95), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (),
(), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (), (67, 85), (), (5, 84), (), (), (), ()}
```

Conditions:

$T=273\text{K}$

Box Length= $1\text{E}-8\text{m}$

of Particles=100

time= $2\text{E}-11\text{ sec}$, $dt=2\text{E}-13\text{ sec}$



```
In[3765]:= RT1 = 8.314 * 273; (*units J/mol = (kg m2/s2) / mol*)
M = 39.9 / 1000; (*molar mass: units kg*)
vTherm := Random[NormalDistribution[0, Sqrt[(RT1) / (M)]]] (*units m/s*)
m = 6.6335209 * (10^(-26)); (*atomic mass of Argon in kg*)
r = 9.8 * 10^(-11); (*atomic radius of Argon in m*)

length = 1 * 10^(-8);
nParticles = 100;
time = 2 * 10^(-11); (*units sec*)
dt = 2 * 10^(-13);
nSteps = Round[time / dt];

Sim[nParticles, m, length];
```