MATH 354 HW #11 Section 3.1

Problems 2, 3, 5, 6

(2) Consider the LPP

Minimize: $z' = 6x_1 + 6x_2 + 8x_3 + 9x_4 \longrightarrow \bar{b} = \begin{bmatrix} 6 \\ 6 \\ 8 \\ 9 \end{bmatrix}, \bar{c} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

Subject to: $x_1 + 2x_2 + x_3 + x_4 \geq 3$

$2x_1 + x_2 + 4x_3 + 9x_4 \geq 8 \longrightarrow A^T = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 4 & 9 \end{bmatrix}$

$x_j \geq 0, j = 1, 2, 3, 4$

The dual will be Maximize: $z = \bar{c}^T \bar{w}$ $\quad \bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

Subject to:

$A\bar{w} \leq \bar{b}, \bar{w} \geq 0$

Which is, specifically Maximize: $z = 3w_1 + 8w_2$

Subject to

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 4 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \longrightarrow \begin{aligned} w_1 + 2w_2 &\leq 6 \\ 2w_1 + w_2 &\leq 6 \\ w_1 + 4w_2 &\leq 8 \\ w_1 + 9w_2 &\leq 9 \end{aligned} \qquad \begin{aligned} w_1 &\geq 0 \\ w_2 &\geq 0 \end{aligned}$$

(3) Consider the LPP

Maximize: $z = 3x_1 + 2x_2 + 5x_3 + 7x_4$

Subject to

$$\begin{aligned} 3x_1 + 2x_2 + x_3 \quad\quad\quad &\leq 8 \\ 5x_1 + x_2 + 2x_3 + 4x_4 &= 7 \\ 4x_1 \quad\quad + x_3 - 2x_4 &\leq 12 \end{aligned} \qquad x_j \geq 0,\ j = 1, 2, 3, 4$$

According to the rules for constructing the dual, the second constraint, being an equality, will have the effect of having the second variable in the dual problem unrestricted. We have

$$\bar{c} = \begin{bmatrix} 3 \\ 2 \\ 5 \\ 7 \end{bmatrix}, \bar{b} = \begin{bmatrix} 8 \\ 7 \\ 12 \end{bmatrix} \quad A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 5 & 1 & 2 & 4 \\ 4 & 0 & 1 & -2 \end{bmatrix} \text{ so}$$

the form of the dual will be: $\bar{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

$\underline{\text{Minimize}}$ $z' = \bar{b}^T \bar{w} = 8w_1 + 7w_2 + 12w_3$

$\underline{\text{Subject to:}}$

$$A^T \bar{w} = \begin{bmatrix} 3 & 5 & 4 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 4 & -2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \longrightarrow \begin{aligned} 3w_1 + 5w_2 + 4w_3 &\geq 3 \\ 2w_1 + w_2 &\geq 2 \\ w_1 + 2w_2 + w_3 &\geq 5 \\ 4w_2 - 2w_3 &\geq 7 \end{aligned} \leftarrow \bar{C}$$

$w_1 \geq 0, w_3 \geq 0,$

$\underline{w_2 \text{ unrestricted}}$

Note: $x_j \geq 0, j = 1,2,3,4$ has the effect of having all constraints with "$\geq$".

Primal	Dual
(5) $\underline{\text{Maximize}}$: $Z = 3X_1 + X_2 + 4X_3$	$\underline{\text{Minimize}}$: $z' = 18w_1 + 12w_2$
$\underline{\text{Subject to}}$: $3X_1 + 3X_2 + X_3 = \leq 18$	$\underline{\text{Subject to}}$: $3w_1 + 2w_2 \geq 3$
$2X_1 + 2X_2 + 4X_3 = 12$	$3w_1 + 2w_2 = 1$
$X_1 \geq 0, X_3 \geq 0$, $\underline{X_2 \text{ unrestricted}}$	$w_1 + 4w_2 \geq 4$
	$w_1 \geq 0$,
	w_2 unrestricted

(6) $\underline{\text{Minimize}}$: $z' = 5x_1 + 2x_2 + 6x_3$

$\underline{\text{Subject to}}$: $4x_1 + 2x_2 + x_3 \geq 12$

$-3x_1 - 2x_2 - 3x_3 \geq -6$

$x_1 \geq 0, x_2 \geq 0$

$\underline{x_3 \text{ unrestricted}}$

$$\bar{b} = \begin{bmatrix} 5 \\ 2 \\ 6 \end{bmatrix}, \quad \bar{c} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & 2 & 1 \\ -3 & -2 & -3 \end{bmatrix}$$

Primal

$\underline{\text{Maximize}}$: $Z = 12w_1 - 6w_2$

$\underline{\text{Subject to}}$: $4w_1 - 3w_2 \leq 5$

$2w_1 - 2w_2 \leq 2$

$w_1 - 3w_3 = 6$

$w_1 \geq 0, w_2 \geq 0$