MATH 354 HW #14 Section 3.3 #2, 4, 6

②

c_B	4 x_1	$\frac{5}{3}$ x_2	$\frac{4}{3}$ x_3	3 x_4	−1 x_5	0 x_6	$-\frac{2}{3}$ x_7	x_B
−1 x_5	0	$\frac{4}{3}$	$\frac{2}{3}$	0	1	0	$-\frac{1}{3}$	4
0 x_6	0	$\frac{1}{3}$	$\frac{2}{3}$	1	0	1	$-\frac{1}{3}$	10
4 x_1	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	0	0	$\frac{1}{6}$	4
	0	−5/3	−4/3	−1	0	0	5/3	12

Entries in objective row going from constraint column under x_1 to the column x_7.

$$z_1 - c_1 = \bar{c}_B^T \bar{t}_1 - c_1 = [-1\ 0\ 4]\begin{bmatrix}0\\0\\1\end{bmatrix} - 4 = 4 - 4 = \underline{\underline{0}}$$

$$z_2 - c_2 = \bar{c}_B^T \bar{t}_2 - c_2 = [-1\ 0\ 4]\begin{bmatrix}4/3\\1/3\\1/3\end{bmatrix} - \frac{5}{3} = 0 - 5/3 = \underline{\underline{-5/3}}$$

$$z_3 - c_3 = \bar{c}_B^T \bar{t}_3 - c_3 = [-1\ 0\ 4]\begin{bmatrix}2/3\\2/3\\1/6\end{bmatrix} - \frac{4}{3} = 0 - \frac{4}{3} = \underline{\underline{-4/3}}$$

$$z_4 - c_4 = \bar{c}_B^T \bar{t}_4 - c_4 = [-1\ 0\ 4]\begin{bmatrix}0\\1\\1/2\end{bmatrix} - 3 = 2 - 3 = \underline{\underline{-1}}$$

$$z_5 - c_5 = \bar{c}_B^T \bar{t}_5 - c_5 = [-1\ 0\ 4]\begin{bmatrix}1\\0\\0\end{bmatrix} - (-1) = -1 + 1 = \underline{\underline{0}}$$

$$z_6 - c_6 = \bar{c}_B^T \bar{t}_6 - c_6 = [-1\ 0\ 4]\begin{bmatrix}0\\1\\0\end{bmatrix} - 0 = 0 - 0 = \underline{\underline{0}}$$

$$z_7 - c_7 = \bar{c}_B^T \bar{t}_7 - c_7 = [-1\ 0\ 4]\begin{bmatrix}-1/3\\-1/3\\1/6\end{bmatrix} - (-2/3) = 1 + 2/3 = \underline{\underline{5/3}}$$

The objective function value for the tableau's basic variable solution is

$$z = \bar{c}_B^T \bar{x}_B = [-1\ 0\ 4]\begin{bmatrix}4\\10\\4\end{bmatrix} = -4 + 16 = 12$$

Note that $\bar{c}_B$ is determined by the basic variables x_5, x_6, x_1. In the order x_5, x_6, x_1, the associated columns form the identity

$$\begin{array}{ccc} \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \\ x_5 \quad x_6 \quad x_1 \end{array}$$

The numbers above x_5, x_6, x_1 are, respectively, $[-1\ 0\ 4] = \bar{c}_B^T$. So, $\bar{c}_B = \begin{bmatrix}-1\\0\\4\end{bmatrix} \begin{array}{l}\leftarrow x_5\\ \leftarrow x_6\\ \leftarrow x_1\end{array}$

Note the correspondence

(4)

Final Tableau of Phase 1 Modified for Beginning Phase 2

4. Maximize $z = 3x_1 + x_2 + 3x_3$.

		3	1	3	0	0	0	0	
$\mathbf{c}_B$		x_1	x_2	x_3	x_4	x_5	x_6	y_1	$\mathbf{x}_B$
$\cancel{0}\ 3$	x_1	1	1	0	2	0	$\frac{2}{3}$	0	2
$\cancel{0}\ 3$	x_3	0	-1	1	-1	0	-1	0	$\frac{5}{2}$
$\cancel{0}\ 0$	x_5	0	2	0	3	1	1	0	$\frac{2}{3}$
$\cancel{1}\ 0$	y_1	0	$-\frac{3}{2}$	0	$-\frac{1}{2}$	0	-2	1	0
		0	$\cancel{\frac{3}{2}}\ -1$	0	$\cancel{\frac{1}{2}}\ 3$	0	$\cancel{2}\ -1$	0	$\cancel{0}\ \frac{27}{2}$

Initial Tableau for Phase 2

(A) <u>Step 1</u> Remove all columns from the final tableau of Phase 1 above which are for nonbasic artificial variables. (There are none)

<u>Step 2</u> Replace the row above the variable symbols with the coefficients from the original objective function. Each artificial variable gets a zero. (Done)

<u>Step 3</u> In the column under $\overline{C}_B$, enter the appropriate numbers from the new row given in step 2 (Done)

<u>Step 4</u> Calculate the entries in the new objective row with the formula $z_j - c_j = \overline{C}_B^T E_j - c_j$ (Done, see below)

<u>Step 4</u> <u>Calculations:</u> $z_1 - c_1 = [3\ 3\ 0\ 0]\begin{bmatrix}1\\0\\0\\0\end{bmatrix} - 3 = \underline{0}$

$$z_2 - c_2 = [3\ 3\ 0\ 0]\begin{bmatrix}1\\-1\\2\\-3/2\end{bmatrix} - 1 = 3 - 3 - 1 = \underline{-1}$$

$$z_3 - c_3 = [3\ 3\ 0\ 0]\begin{bmatrix}0\\1\\0\\0\end{bmatrix} - 3 = \underline{0}, \quad z_4 - c_4 = \underline{3}, \quad z_5 - c_5 = \underline{0}$$

$$z_6 - c_6 = -1, \quad z_7 - c_7 = \underline{0}, \quad z = \overline{C}_B^T X_B = [3\ 3\ 0\ 0]\begin{bmatrix}2\\5/2\\2/3\\0\end{bmatrix} = \frac{27}{2}$$

<u>Decisions on</u> <u>Entering and</u> <u>Departing Variables</u>

<u>Choose x_2</u> as the entering variable. Since we have an artificial variable as a basic variable (See (B) of the handout " More Calculations --") we have to be careful to follow the rule for choosing the departing variable. Specifically, in the <u>row</u> for the basic

(and artificial) variable y_1, if the entry in the pivotal column (the column under x_2) is negative (which it is, it is $-3/2$) we must choose y_1 as the departing variable.

With these choices we get the Initial Tableau for Phase 2 given above.

(B) Construction of Tableau #2 Pivot on $-\frac{2}{3}$

Tableau #2

		3	1	3	0	0	0	0	
$\bar{C}_B$		x_1	x_2	x_3	x_4	x_5	x_6	y_1	$\bar{x}_B$
3	x_1	1	0	0	5/3	0	−2/3	2/3	2
3	x_3	0	0	1	−2/3	0	1/3	−2/3	5/2
0	x_5	0	0	0	7/3	1	−5/3	4/3	2/3
1	x_2	0	1	0	1/3	0	4/3	$-\frac{2}{3}$	0
		0	0	0	$\frac{10}{3}$	0	$\frac{1}{3}$	−2/3	$\frac{27}{2}$

Calculate the objective row:

$$z_1 - c_1 = \bar{c}_B^T \bar{t}_1 - c_1 = [3\ 3\ 0\ 1]\begin{bmatrix}1\\0\\0\\0\end{bmatrix} - 3 = 3 - 3 = 0$$

$$z_2 - c_2 = \bar{c}_B^T \bar{t}_2 - c_2 = [3\ 3\ 0\ 1]\begin{bmatrix}0\\0\\0\\1\end{bmatrix} - 1 = 0$$

$$z_3 - c_3 = 0$$

$$z_4 - c_4 = \bar{c}_B^T \bar{t}_4 - c_4 = [3\ 3\ 0\ 1]\begin{bmatrix}5/3\\-2/3\\7/3\\1/3\end{bmatrix} - 0 = \frac{10}{3}$$

$$z_5 - c_5 = \frac{1}{3}, \quad z_6 - c_6 = -\frac{2}{3}$$

$$z = \bar{c}_B^T \bar{x}_B = [3\ 3\ 0\ 1]\begin{bmatrix}2\\5/2\\2/3\\0\end{bmatrix} = \frac{27}{2}$$

$$\bar{x}_0 = \begin{bmatrix}2\\0\\5/2\\0\\2/3\end{bmatrix}$$

Since the artificial variable y_1 is no longer basic, drop that column. The resulting tableau is maximal (No negative entries in the objective row). So, $\bar{x}_0 = \cancel{\begin{bmatrix}2\\5/2\\2/3\\0\end{bmatrix}}$ is the optimal solution with $z = \frac{27}{2}$.

(6) In canonical form we have

Maximize: $z = 2x_1 + x_2 + 3x_3$

Subject to

$$\begin{aligned} 2x_1 - x_2 + 3x_3 + x_4 \qquad\qquad &= 6 \\ x_1 + 3x_2 + 5x_3 \qquad + x_5 \quad &= 10 \\ 2x_1 \qquad + x_3 \qquad\qquad + x_6 &= 7 \end{aligned}$$

with $x_j \geq 0,\ j = 1, 2, \ldots, 6$

(A) (Initial) Tableau #1

		2	1	3 ↓	0	0	0	
$\bar{C}_{B_0}$		x_1	x_2	x_3	x_4	x_5	x_6	$\bar{X}_{B_0}$
0	x_4 ←	2	−1	3	1	0	0	6
0	x_5	1	3	5	0	1	0	10
0	x_6	2	0	1	0	0	1	7
		−2	−1	−3	0	0	0	0

i) Calculation of objective row

$$\begin{cases} z_1 - c_1 = \bar{C}_{B_0}^T t_1 - c_1 = [0\ 0\ 0]\begin{bmatrix}2\\1\\2\end{bmatrix} - c_1 = -2 \\ z_2 - c_2 = -c_2 = -1 \\ z_3 - c_3 = -c_3 = -3 \\ z_4 - c_4 = -c_4 = -0 = 0 = z_5 - c_5 = z_6 - c_6 \end{cases}$$

ii) Calculation of value of objective function for initial basic solution $= \bar{C}_B^T X_B = [0\,0\,0]\begin{bmatrix}6\\10\\7\end{bmatrix} = 0$

iii) Choose x_3 as entering variable

θ-ratios: $\frac{6}{3} = 2$ for x_4; $\frac{10}{5} = 2$ for x_5, $\frac{7}{1} = 7$ for x_6

A tie, choose x_4 for departing variable.

(B) For tableau #2, the basic variables are x_3, x_5, x_6

$$\bar{C}_{B_1} = \begin{bmatrix}3\\0\\0\end{bmatrix}, \quad B_1 = \begin{array}{c} \begin{bmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\ x_3\ \ x_5\ \ x_6 \end{array}$$

Calculate B_1^{-1}

by row reduction $[B_1 \,\vdots\, I] = \left[\begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \longrightarrow$$

(next page)

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 1 & 0 & -5/3 & 1 & 0 \\ 0 & 0 & 1 & -1/3 & 0 & 1 \end{array}\right]. \text{ So } B_1^{-1} = \begin{bmatrix} 1/3 & 0 & 0 \\ -5/3 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix}$$

Construct tableau #2 $\quad \bar{c}_{B_1} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$

$${}_1\bar{t}_1 = B_1^{-1}\,{}_0\bar{t}_1 = \begin{bmatrix} 1/3 & 0 & 0 \\ -5/3 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ -10/3+1 \\ -2/3+2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ -7/3 \\ 4/3 \end{bmatrix}$$

$${}_1\bar{t}_2 = B_1^{-1}\,{}_0\bar{t}_2 = \begin{bmatrix} 1/3 & 0 & 0 \\ -5/3 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 5/3+3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 14/3 \\ 1/3 \end{bmatrix}$$

$${}_1\bar{t}_3 = B_1^{-1}\,{}_0\bar{t}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ {}_1\bar{t}_4 = B_1^{-1}\,{}_0\bar{t}_4 = \begin{bmatrix} 1/3 \\ -5/3 \\ -1/3 \end{bmatrix};\ {}_1\bar{t}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix};\ {}_1\bar{t}_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\bar{x}_{B_1} = B_1^{-1}\,\bar{x}_{B_0} = \begin{bmatrix} 1/3 & 0 & 0 \\ -5/3 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 10 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ -10+10 \\ -2+7 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

Tableau #2

		2	1	3	0	0	0	
$\bar{c}_{B_1}$		x_1	x_2	x_3	x_4	x_5	x_6	$\bar{x}_{B_1}$
3	x_3	2/3	−1/3	1	1/3	0	0	2
← 0	x_5	−7/3	14/3	0	−5/3	1	0	0
0	x_6	4/3	1/3	0	−1/3	0	1	5
		0	−2	0	1	0	0	6

i) Calculation of objective row

$$z_1 - c_1 = \bar{c}_{B_1}\,{}_1\bar{t}_1 - c_1 = [3\ 0\ 0]\begin{bmatrix} 2/3 \\ -7/3 \\ 4/3 \end{bmatrix} - 2 = 0$$

$$z_2 - c_2 = \bar{c}_{B_1}\,{}_1\bar{t}_2 - c_2 = [3\ 0\ 0]\begin{bmatrix} -1/3 \\ 14/3 \\ 1/3 \end{bmatrix} - 1 = -2$$

$$z_3 - c_3 = 3 - 3 = 0$$

$$z_4 - c_4 = 1,\ z_5 - c_5 = 0,\ z_6 - c_6 = 0$$

ii) $z = \bar{c}_{B_1}^T \bar{x}_{B_1} = [3\ 0\ 0]\begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} = \underline{\underline{6}}$

iii) Choose x_2 to be the entering variable

Θ-ratios: $\frac{0}{14/3} = 0$ for x_5: $\frac{5}{1/3} = 15$ for x_6

Choose x_5 as the departing variable.

(C) For tableau #3, the basic variables are X_3, X_2, X_6.

$$\bar{C}_{B_2} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{array}{c} \begin{bmatrix} 1 & -1/3 & 0 \\ 0 & 14/3 & 0 \\ 0 & 1/3 & 1 \end{bmatrix} \\ \begin{matrix} X_3 & X_2 & X_6 \end{matrix} \end{array}$$

Calculate B_2^{-1} by row reduction

$$[B_2 \vdots I] = \left[\begin{array}{ccc|ccc} 1 & -1/3 & 0 & 1 & 0 & 0 \\ 0 & 14/3 & 0 & 0 & 1 & 0 \\ 0 & 1/3 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -1/3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 3/14 & 0 \\ 0 & 0 & 1 & 0 & -1/14 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1/14 & 0 \\ 0 & 1 & 0 & 0 & 3/14 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{14} & 1 \end{array}\right], \text{ So, } B_2^{-1} = \begin{bmatrix} 1 & 1/14 & 0 \\ 0 & 3/14 & 0 \\ 0 & -1/14 & 1 \end{bmatrix}$$

<u>Construct tableau</u> #3

$${}_2\bar{t}_1 = B_2^{-1}\,{}_1\bar{t}_1 = \begin{bmatrix} 1 & 1/14 & 0 \\ 0 & 3/14 & 0 \\ 0 & -1/14 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ -7/3 \\ 4/3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \\ 3/2 \end{bmatrix}$$

$${}_2\bar{t}_2 = B_2^{-1}\,{}_1\bar{t}_2 = \begin{bmatrix} 1 & 1/14 & 0 \\ 0 & 3/14 & 0 \\ 0 & -1/14 & 1 \end{bmatrix} \begin{bmatrix} -1/3 \\ 14/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$${}_2\bar{t}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; {}_2\bar{t}_4 = \begin{bmatrix} 3/14 \\ -5/14 \\ 19/42 \end{bmatrix}, \; {}_2\bar{t}_5 = \begin{bmatrix} 1/14 \\ 3/14 \\ -1/14 \end{bmatrix}, \; {}_2\bar{t}_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\bar{X}_{B_2} = B^{-1}\bar{X}_{B_1} = \begin{bmatrix} 1 & 1/14 & 0 \\ 0 & 3/14 & 0 \\ 0 & -1/14 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

<u>Tableau #3</u>

		↓ 2	1	3	0	0	0	
$\bar{C}_{B_2}$		X_1	X_2	X_3	X_4	X_5	X_6	$\bar{X}_{B_2}$
3	X_3	1/2	0	1	3/14	1/14	0	2
1	X_5	−1/2	1	0	−5/14	3/14	0	0
← 0	X_6	3/2	0	0	19/42	−1/14	1	5
		−1	0	0	2/7	3/7	0	6

i) Calculation of objective row

$$\begin{cases} z_1 - c_1 = [3\ 1\ 0]\begin{bmatrix} 1/2 \\ -1/2 \\ 3/2 \end{bmatrix} - 2 = 1 - 2 = 1 \\ z_2 - c_2 = [3\ 1\ 0]\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 1 = 0, \end{cases}$$

$$z_3 - c_3 = [3\ 1\ 0]\begin{bmatrix}1\\0\\0\end{bmatrix} - 3 = 0, \quad z_4 - c_4 = \frac{2}{7}$$

$$z_5 - c_5 = \frac{3}{7}, \quad z_6 - c_6 = 0, \quad z = \bar{c}_{B_2}^T \bar{x}_{B_2} = [3\ 1\ 0]\begin{bmatrix}2\\0\\5\end{bmatrix} = 6$$

(D) For tableau #4, the basic variables are x_3, x_2, x_1 since x_1 is the entering variable and from the Θ-ratios: $\frac{2}{1/2} = 4$ for x_5; $\frac{5}{3/2} = \frac{10}{3}$ for x_6, the departing variable is x_6

$$\bar{c}_{B_3} = \begin{bmatrix}3\\1\\2\end{bmatrix} \qquad B_3 = \begin{array}{c} \begin{bmatrix}1 & 0 & 1/2\\0 & 1 & -1/2\\0 & 0 & 3/2\end{bmatrix} \\ \begin{array}{ccc} x_3 & x_2 & x_1 \end{array} \end{array}$$

Calculate B_3^{-1} by row reduction

$$[B_3 \,\vdots\, I] = \left[\begin{array}{ccc|ccc}1 & 0 & 1/2 & 1 & 0 & 0\\0 & 1 & -1/2 & 0 & 1 & 0\\0 & 0 & 3/2 & 0 & 0 & 1\end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc}1 & 0 & 1/2 & 1 & 0 & 0\\0 & 1 & -1/2 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 2/3\end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 0 & -1/3\\0 & 1 & 0 & 0 & 1 & 1/3\\0 & 0 & 1 & 0 & 0 & 2/3\end{array}\right] \quad \text{So } B_3^{-1} = \begin{bmatrix}1 & 0 & -1/3\\0 & 1 & 1/3\\0 & 0 & 2/3\end{bmatrix}$$

Then $${}_3\bar{t}_1 = B_3^{-1}\,{}_2\bar{t}_1 = \begin{bmatrix}1 & 0 & -1/3\\0 & 1 & 1/3\\0 & 0 & 2/3\end{bmatrix}\begin{bmatrix}1/2\\-1/2\\3/2\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$${}_3\bar{t}_2 = B_3^{-1}\,{}_2\bar{t}_2 = \begin{bmatrix}1 & 0 & -1/3\\0 & 1 & 1/3\\0 & 0 & 2/3\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad {}_3\bar{t}_3 = \begin{bmatrix}1\\0\\0\end{bmatrix},$$

$${}_3\bar{t}_4 = \begin{bmatrix}4/63\\-13/63\\19/63\end{bmatrix}, \quad {}_3\bar{t}_5 = \begin{bmatrix}2/21\\4/21\\-1/21\end{bmatrix}, \quad {}_3\bar{t}_6 = \begin{bmatrix}-1/3\\1/3\\2/3\end{bmatrix}$$

$$\bar{x}_{B_3} = B_3^{-1}\bar{x}_{B_2} = \begin{bmatrix}1 & 0 & -1/3\\0 & 1 & 1/3\\0 & 0 & 2/3\end{bmatrix}\begin{bmatrix}2\\0\\5\end{bmatrix} = \begin{bmatrix}1/3\\5/3\\10/3\end{bmatrix}$$

We get for tableau #4

Tableau #4

		2	1	3	0	0	0	
$\bar{C}_{B_3}$		X_1	X_2	X_3	X_4	X_5	X_6	$\bar{X}_{B_3}$
3	X_3	0	0	1	$4/63$	$2/21$	$-1/3$	$1/3$
1	X_2	0	1	0	$-13/63$	$4/21$	$1/3$	$5/3$
2	X_1	1	0	0	$19/63$	$-1/21$	$2/3$	$10/3$
		0	0	0	$37/63$	$\frac{8}{21}$	$\frac{2}{3}$	$\frac{28}{3}$

where the entries in the objective row are:

$$z_1 - c_1 = \bar{C}_{B_3}^T \bar{t}_{3,1} - c_1 = [3\ 1\ 2]\begin{bmatrix}0\\0\\1\end{bmatrix} - 2 = 2-2=0$$

$$z_2 - c_2 = [3\ 1\ 2]\begin{bmatrix}0\\1\\0\end{bmatrix} - 1 = 1-1=0,\quad z_3 - c_3 = 0$$

$$z_4 - c_4 = [3\ 1\ 2]\begin{bmatrix}4/63\\-13/63\\19/63\end{bmatrix} - 0 = \frac{37}{63},\quad z_5 - c_5 = \frac{8}{21}$$

$$z_6 - c_6 = [3\ 1\ 2]\begin{bmatrix}-1/3\\1/3\\2/3\end{bmatrix} - 0 = 2/3,\quad z = \bar{C}_{B_3}^T \bar{X}_{B_3} = [3\ 1\ 2]\begin{bmatrix}1/3\\5/3\\10/3\end{bmatrix} = \frac{28}{3}$$

Since there are no negative entries in the objective row, we have an optimal solution

$$\bar{X}_0 = \begin{bmatrix}\frac{10}{3}\\5/3\\1/3\end{bmatrix} = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \quad \text{with } z = \frac{28}{3}$$