

# **Computer Science 112**

## **Data Structures**

### **Lecture 08:**

### **Stacks**

### **Queues**

# Review: ArrayLists

- **Partially full arrays**
  - Make array “big enough”
  - keep a variable `numInUse`: number of elements actually in use
  - if `numInUse == array.size`, we are out of room
  - **allocate a longer array and copy shorter => longer**
- **operations on ArrayLists**
  - constructor, add, add at index, get, set, remove
  - see `DriveAL.java` on Sakai

# Amortized big-O

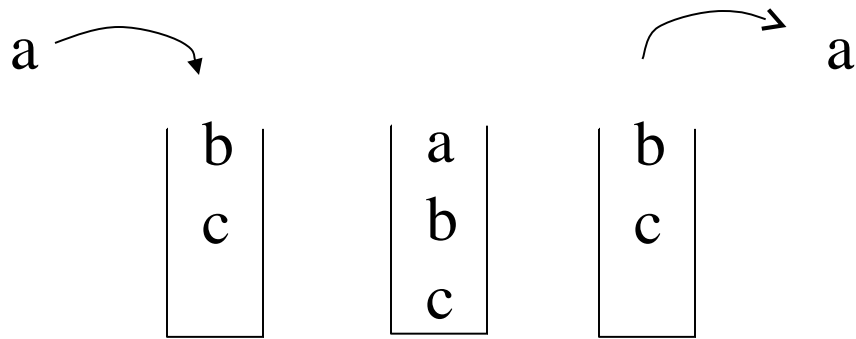
- **Increasing numInUse when space is available is very cheap**
- **Increasing capacity costs more as  $n$  gets larger**
  - **but we do it increasingly rarely**
- **Average total work to increase size step by step to  $n$  is  $O(n)$**

# Review: Stacks

- **Motivation: Last In First Out**
- **Metaphor: stack of trays in cafeteria**
- **Operations**
- **Example use: match parens**
- **Implementations:**
  - **ArrayList**
  - **Array**
  - **Linked List**
- **Big-Os**

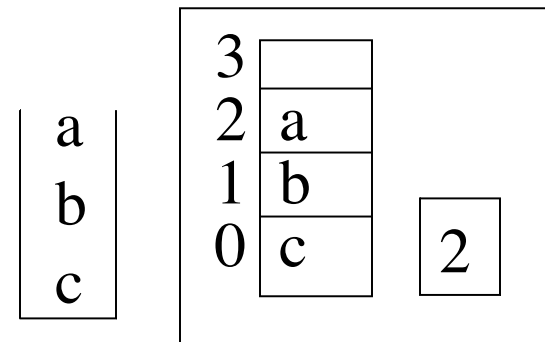
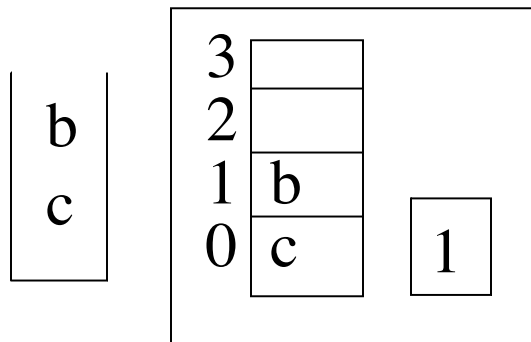
# Stacks

- **Last in first out:**



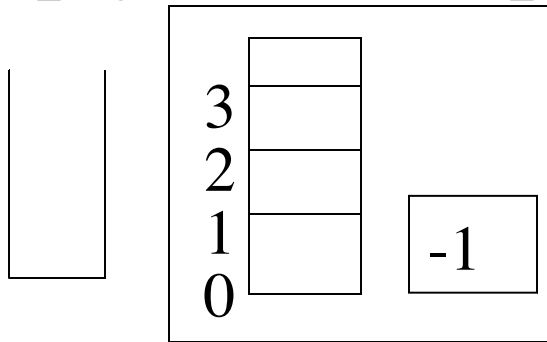
# New: Implementing Stacks

- **Stacks can be implemented using**
  - **Arrays / ArrayLists**
  - **Linked lists**
- **Arrays:**
  - **Array holds data, also need int “top of stack”**

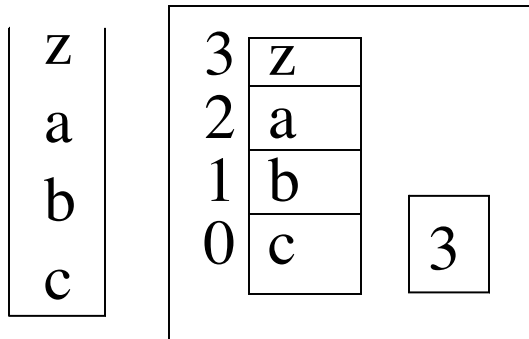


# Implementing Stacks

- **Empty stack:  $\text{top} == -1$**



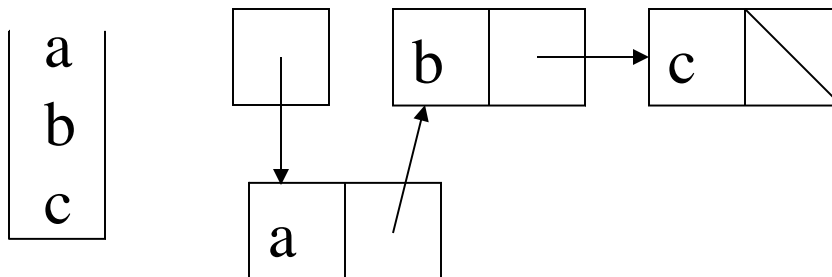
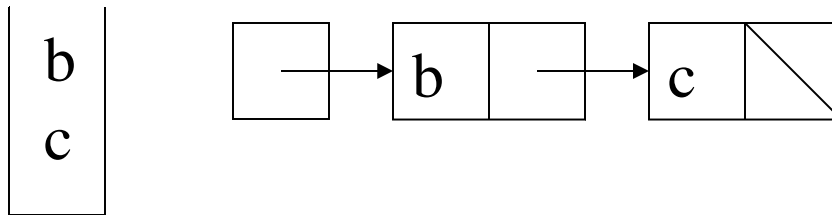
- **Full stack:  $\text{top} == \text{array size} - 1$**



See ALStack.java

# Stacks as linked lists

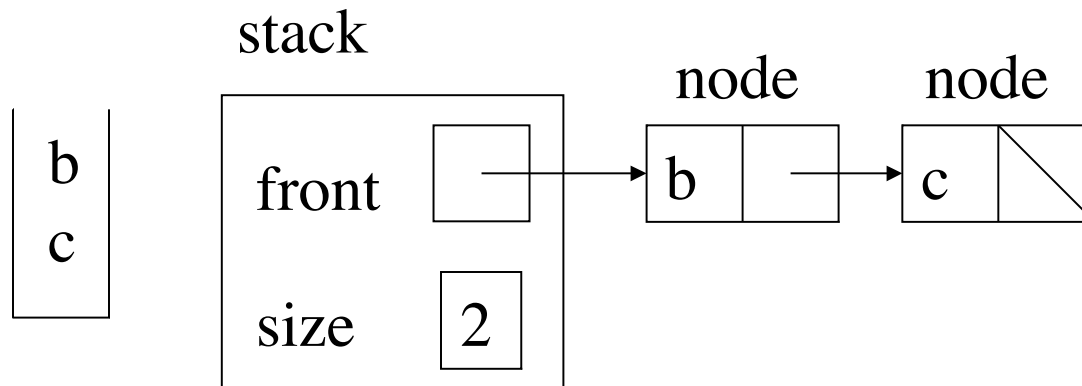
- **Front of list is top of stack**





# Stacks as linked lists

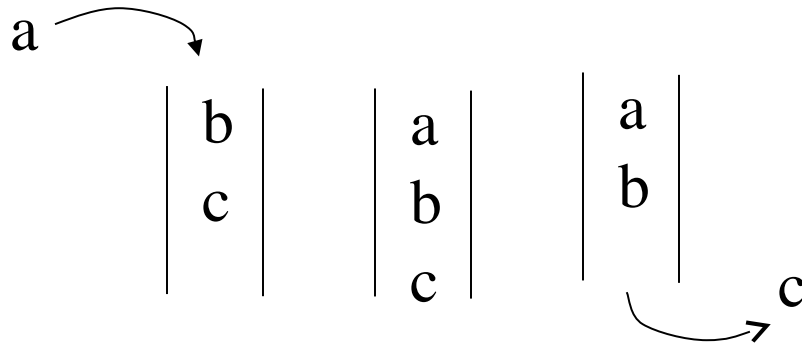
- **Only operation that is not fast is size**
- **So we also have a field to keep track of size**



See Stack.java

# Queues

- **First in first out: Queue**



# Operations

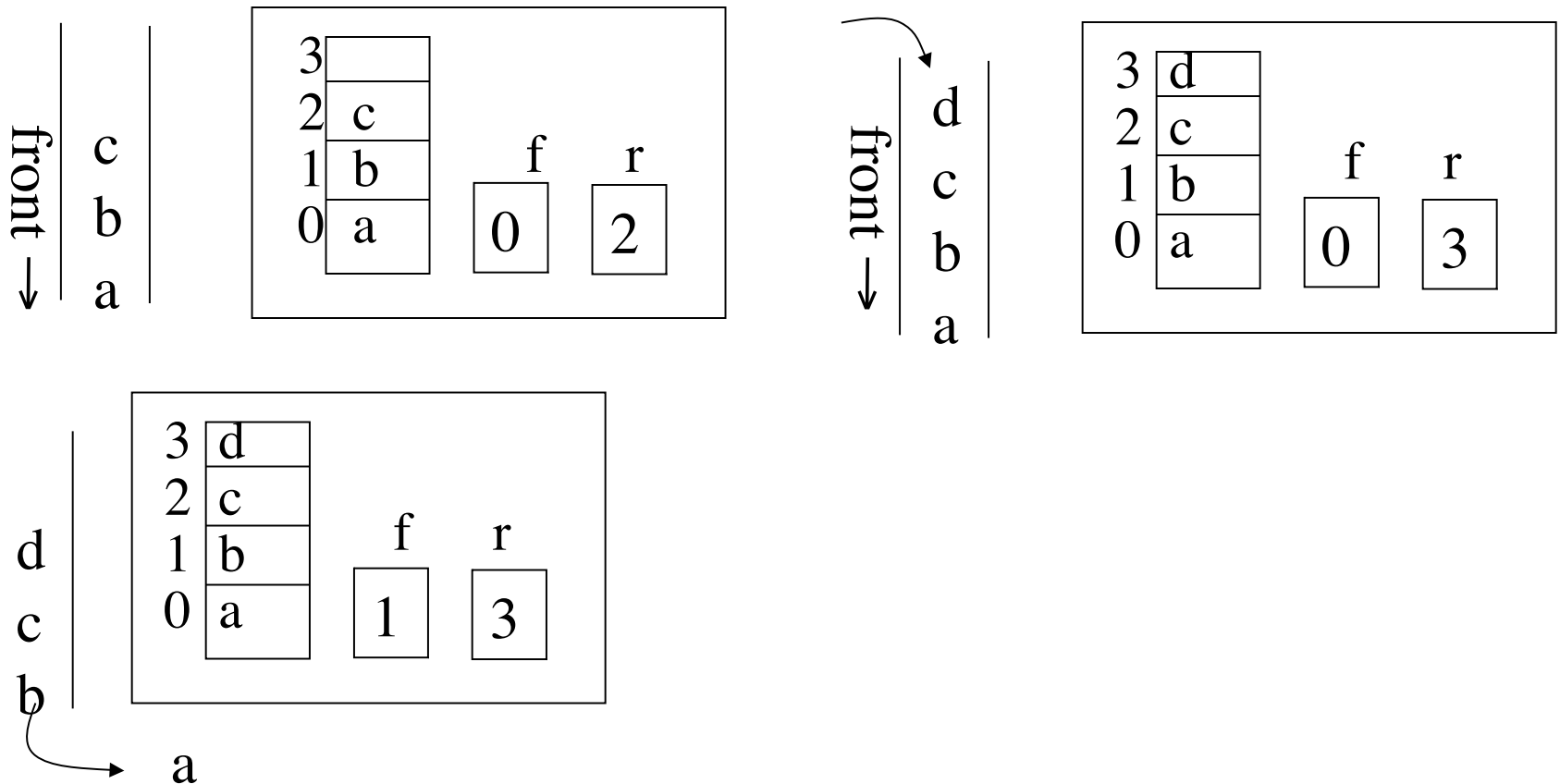
- **Queue<T>**
  - public void **enqueue**(T data)
  - public T **dequeue**( )
  - public boolean **isEmpty**( )
  - public int **size**( )
  - public void **clear**( )

# Implementing Queues

- **Queues need to be accessed at both ends, so implementations are a bit messier**
  - **Arrays: need two ints to keep track of both front and back**
  - **linked lists: use circular lists or have two pointers**

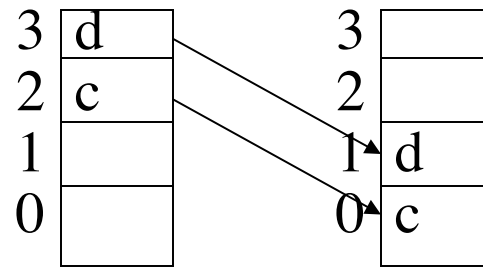
# Queues as arrays

- **Keep track of both front & rear**



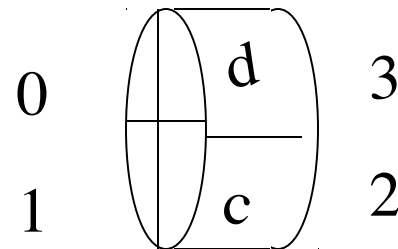
# Queues as arrays

- **Problem: how to reuse space emptied by dequeue?**
  - **Could move data down:  $O(n)$**



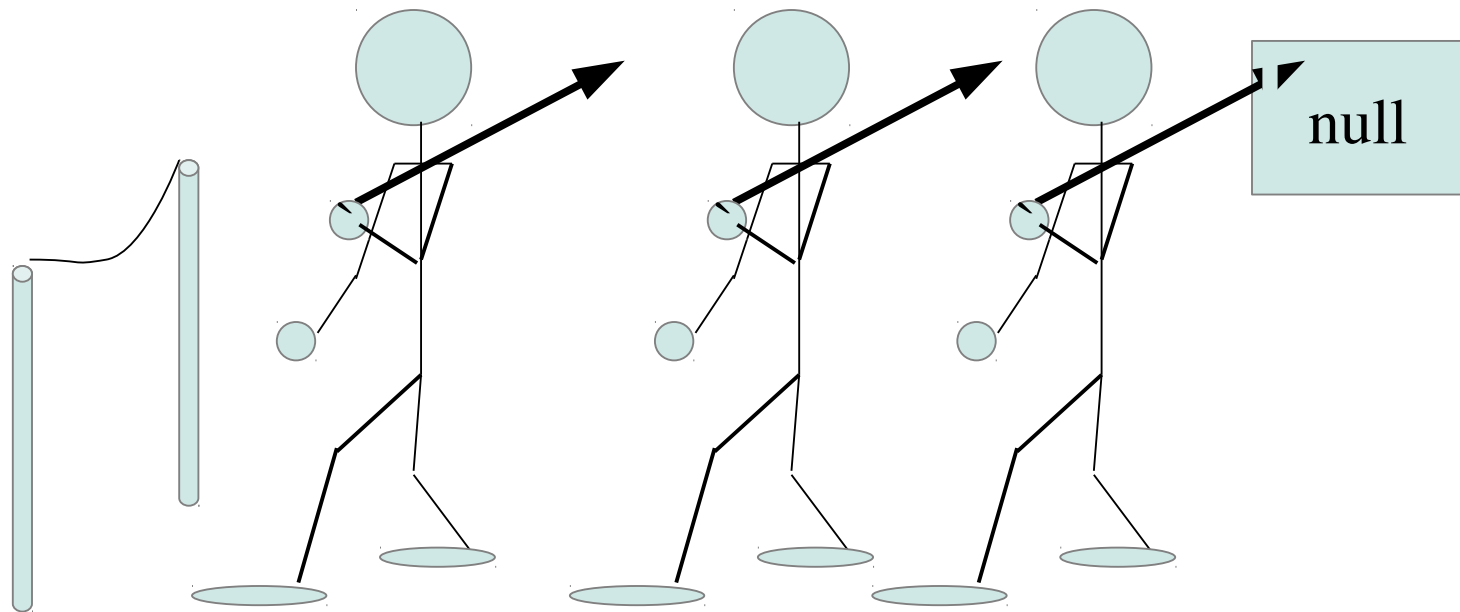
- **Treat array as circular**

**$\text{front} = (\text{front} + 1) \% \text{size}$**



# Queues as linked lists

- **Problem: Need to access both ends**
- **Solution: Linked list with head/tail pointer or circular linked list**
- **Which end of the list should be the front of the queue?**
  - **enqueue is  $O(1)$  time whether at head or tail**
  - **dequeue is  $O(1)$  at head but  $O(n)$  at tail (Why?)**
  - **so more efficient when front is head**





# Sequential Search

- **Is target value in this linked list / array?**

**=> Is target the first element?      No**

**Is target the second element?      No**

**...**

**Is target the 35<sup>th</sup> element?      Yes!**

# Cost

- **Operation to count:**
  - test if target equals element
- **Best case?**
- **Worst case?**
- **Average case ... ????**

# Input Cases

- **We group the inputs into *cases*.**
- **A case is either**
  - **A specific input**  
target is 5, array is {4, 6}
  - **Or a group of inputs, all with the same cost**  
target is not in the array

# Average Cost

- when we say “average cost” we mean a probability-weighted average.
- If all input cases are **equally likely**, average cost is

$$\frac{\sum_i C(i)}{N}$$

- $C(i)$  is the cost of input case  $i$
- $N$  is number of different input cases

# Average Cost

## Equal Probabilities

- Suppose 3 possible input cases, with equal probabilities:

Input Case $i$	Cost $C(i)$
Target in element 0	1
Target in element 1	2
Target in element 2	3

$$\begin{aligned}\text{Average Cost} &= \text{Total Cost} / N \\ &= (1+2+3) / 3 = 2\end{aligned}$$

# Average Cost

## Different Probabilities

- If different input cases have different probabilities, average cost is

$$\sum_i (C(i) * P(i))$$

- $C(i)$  is the cost of input case  $i$
- $P(i)$  is the probability of input case  $i$

# Average Cost

## Different Probabilities

- Suppose 3 possible input cases:

i	Input Case i	Cost C(i)	Probability P(i)
1	target not in array	2	1/2
2	target in element 0	1	1/4
3	target in element 1	2	1/4

$$\begin{aligned}\text{Average Cost} &= C(1)P(1) + C(2)P(2) + C(3)P(3) \\ &= 2 * 1/2 + 1 * 1/4 + 2 * 1/4 = 1.75\end{aligned}$$

# Average Cost of Sequential Search

- Target is in the array, equal probability at each position, length  $n$

$$\text{Average cost} = (1 + 2 + \dots + n) / n$$

$$= (n*(n+1)/2) / n$$

$$= (n+1) / 2$$