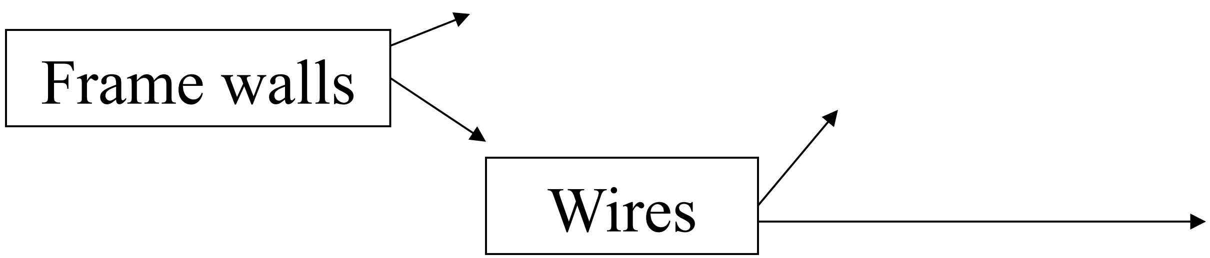
Computer Science 112
Data Structures

Lecture 23:
Shortest Path
Quicksort

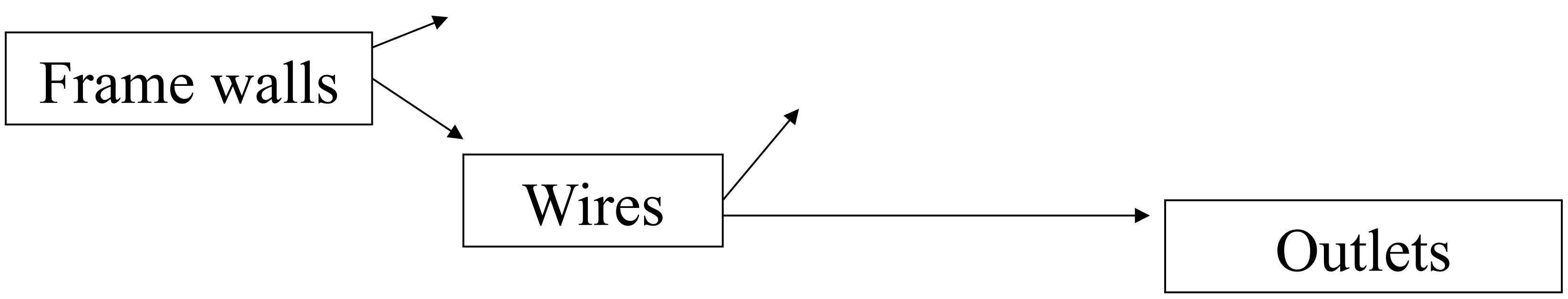
Review: Topsort Example

Frame walls

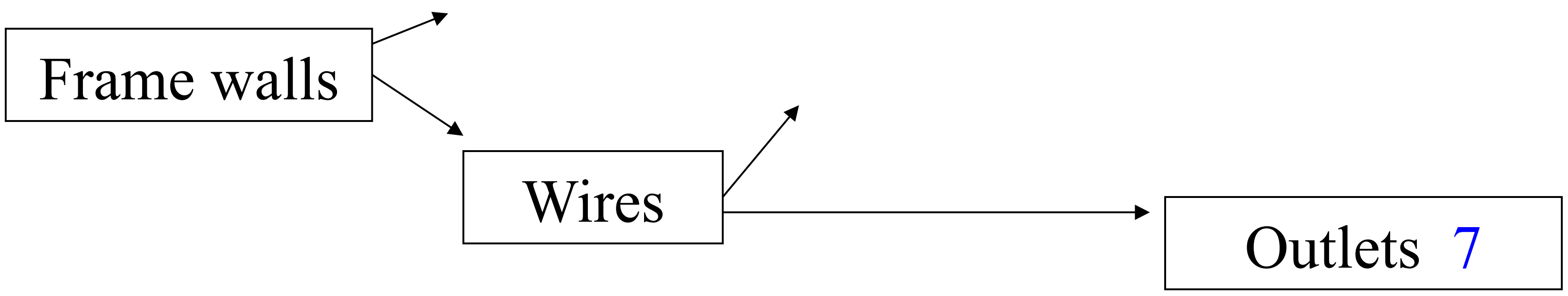
Topsort Example



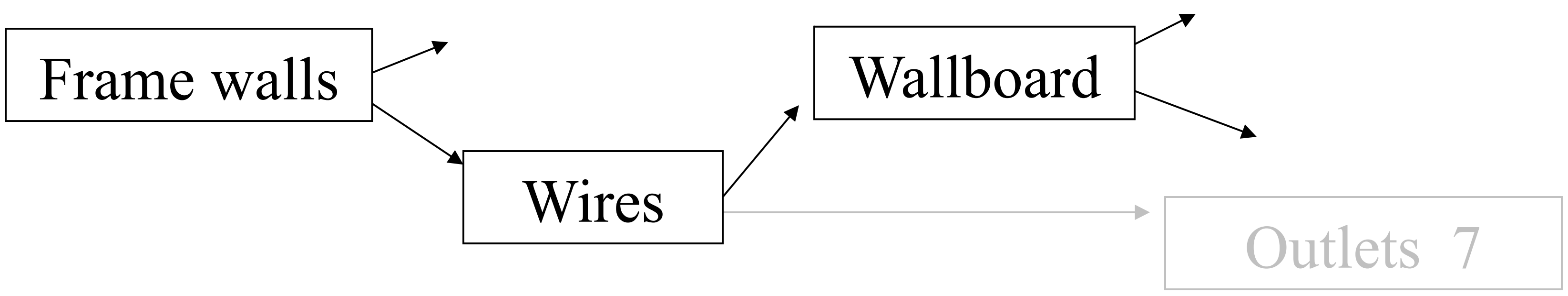
Topsort Example



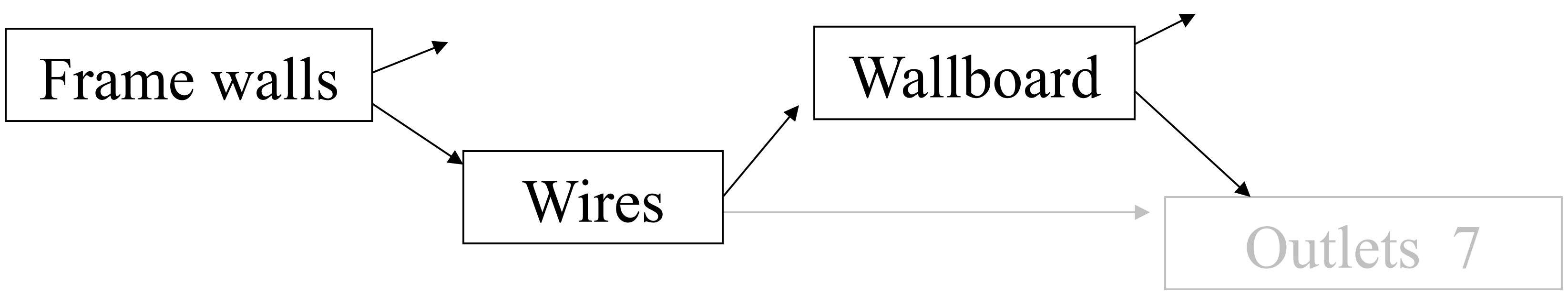
Topsort Example



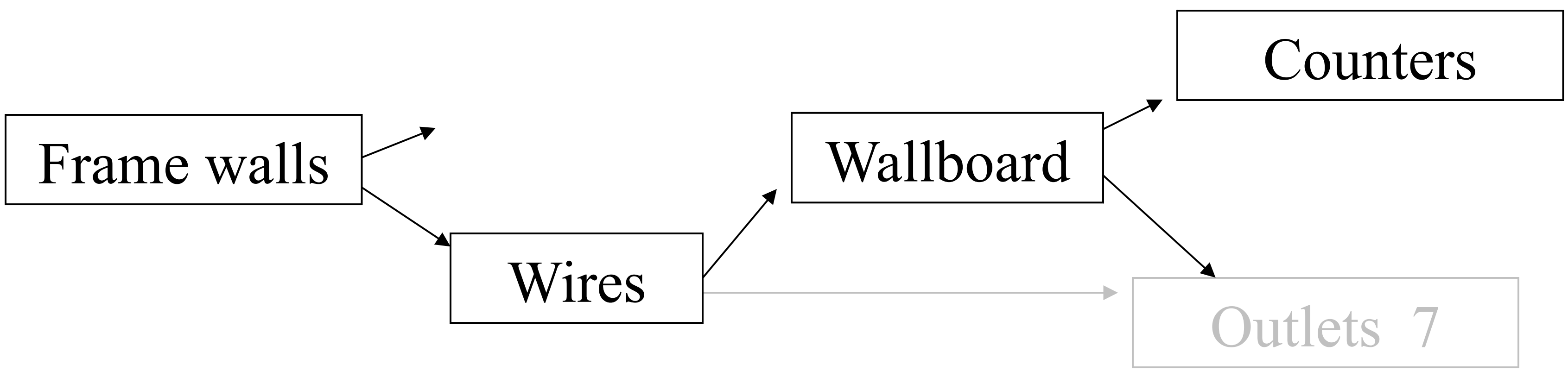
Topsort Example



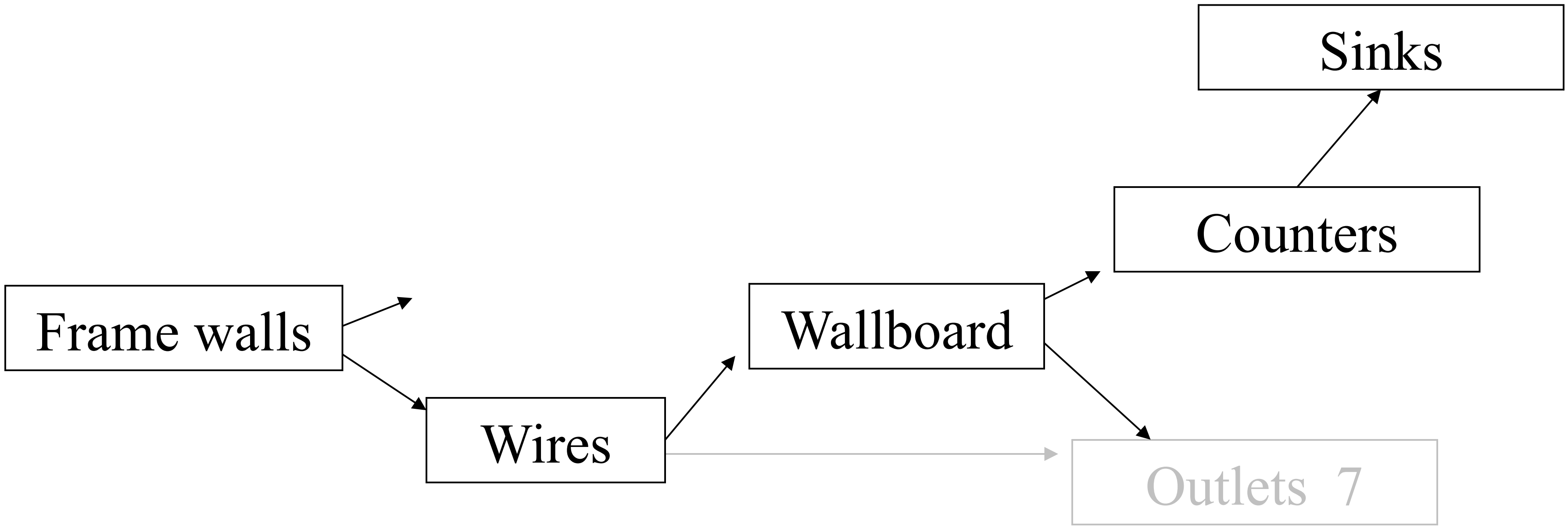
Topsort Example



Topsort Example



Topsort Example



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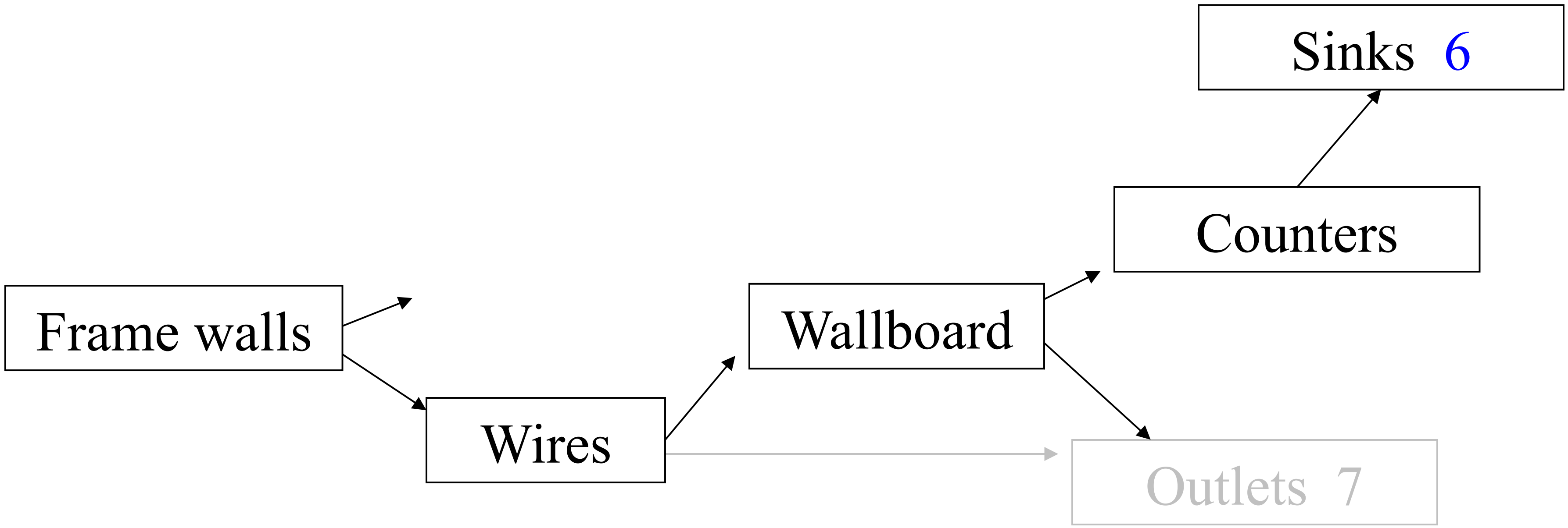

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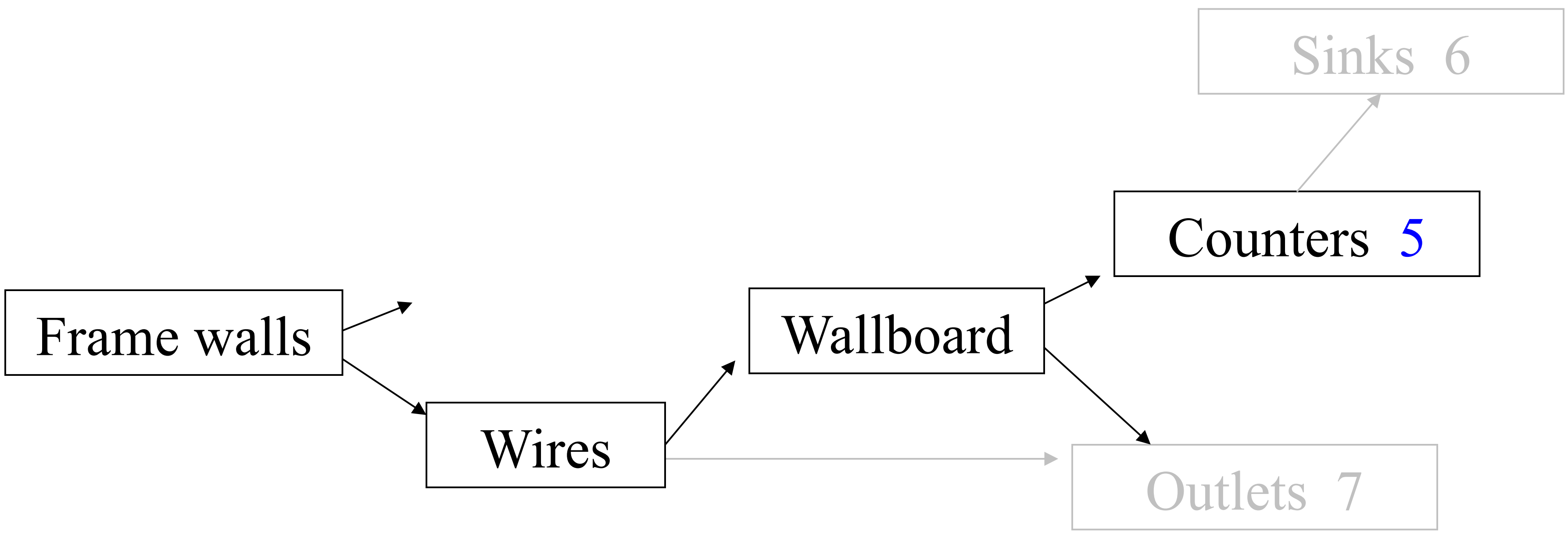
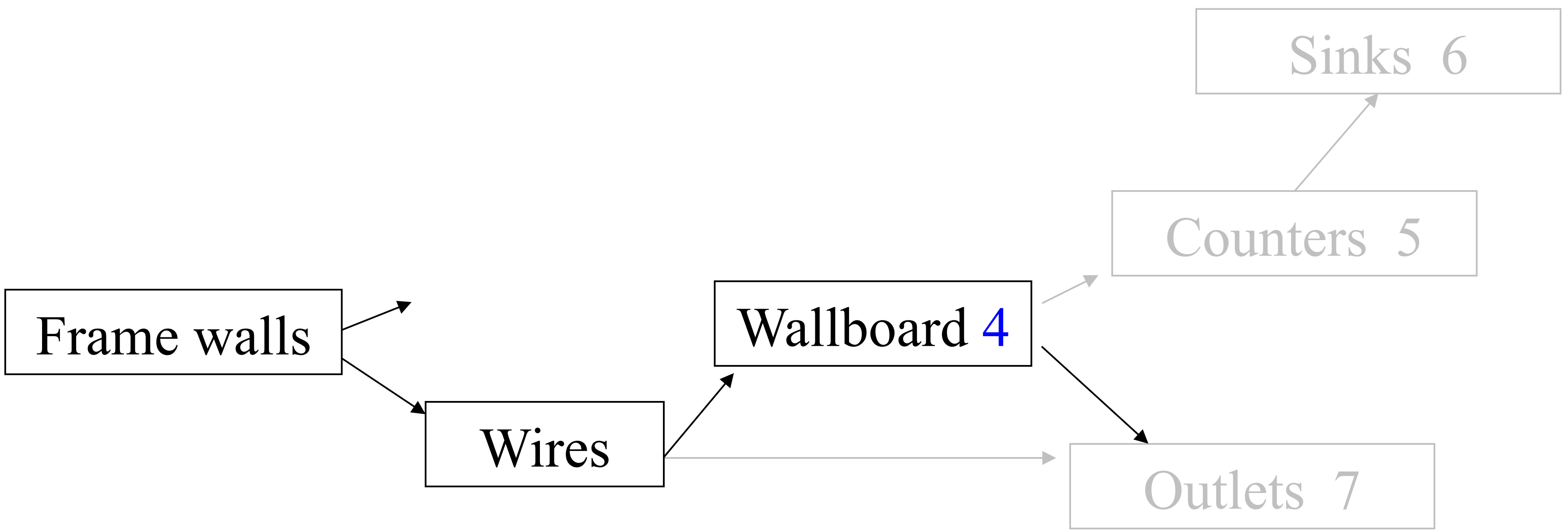
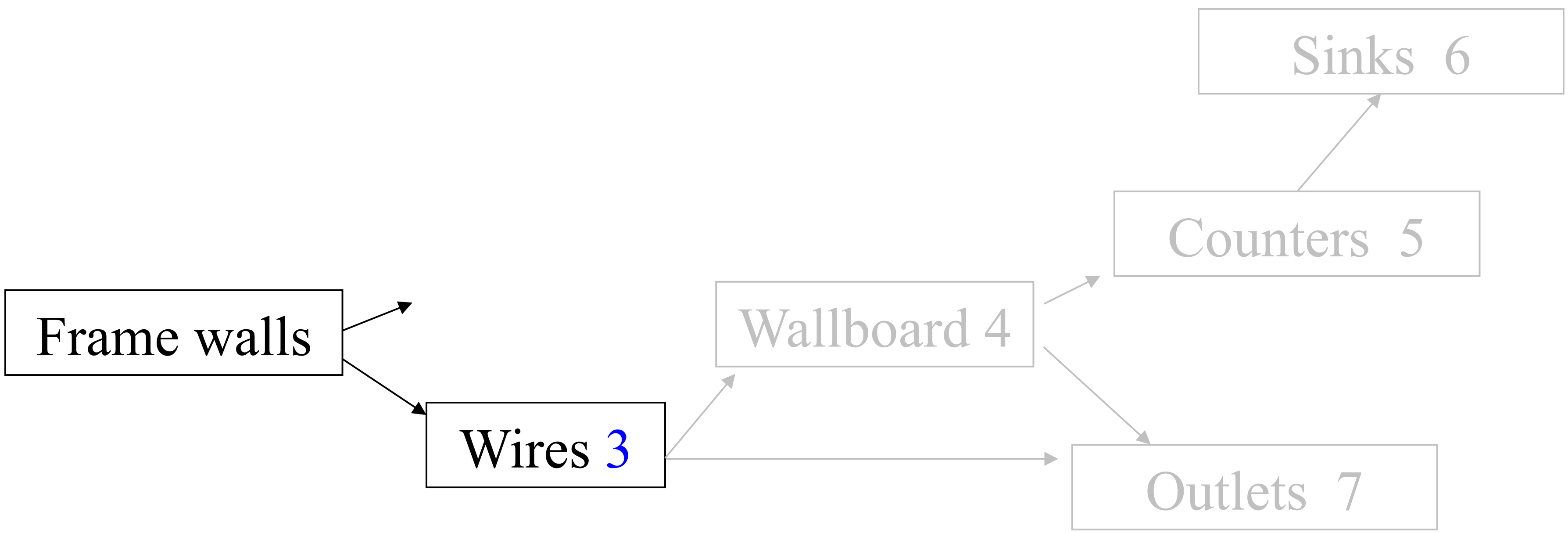


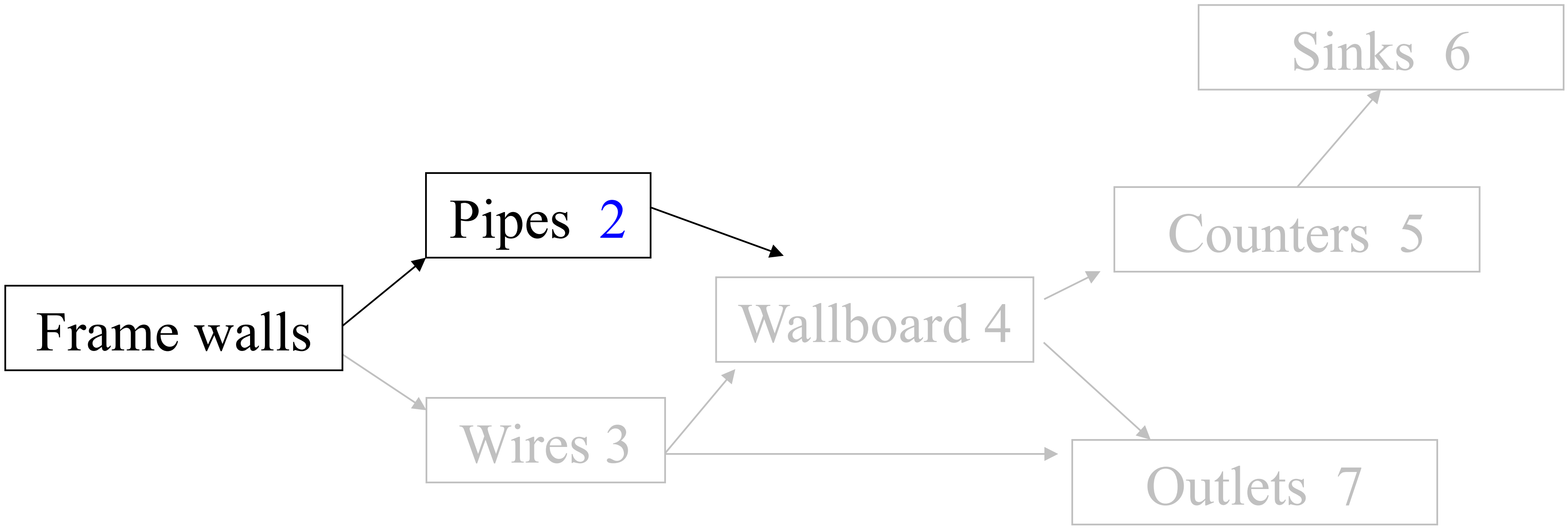
Topsort Example



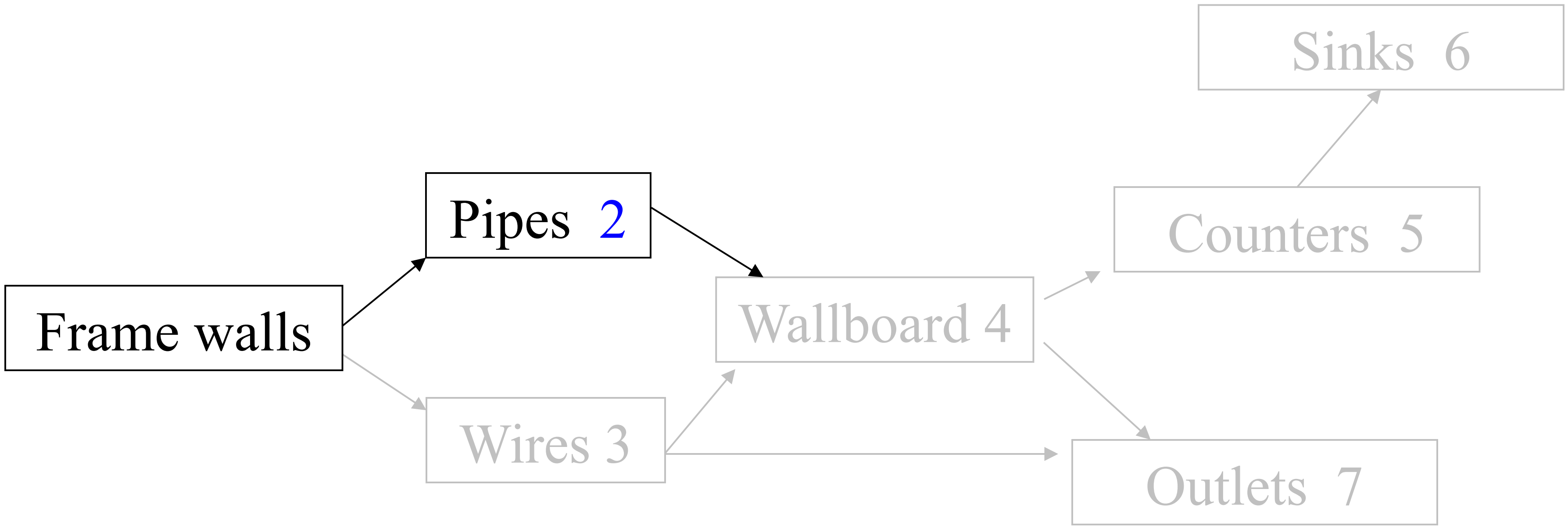
Topsort Example



Topsort Example



Topsort Example



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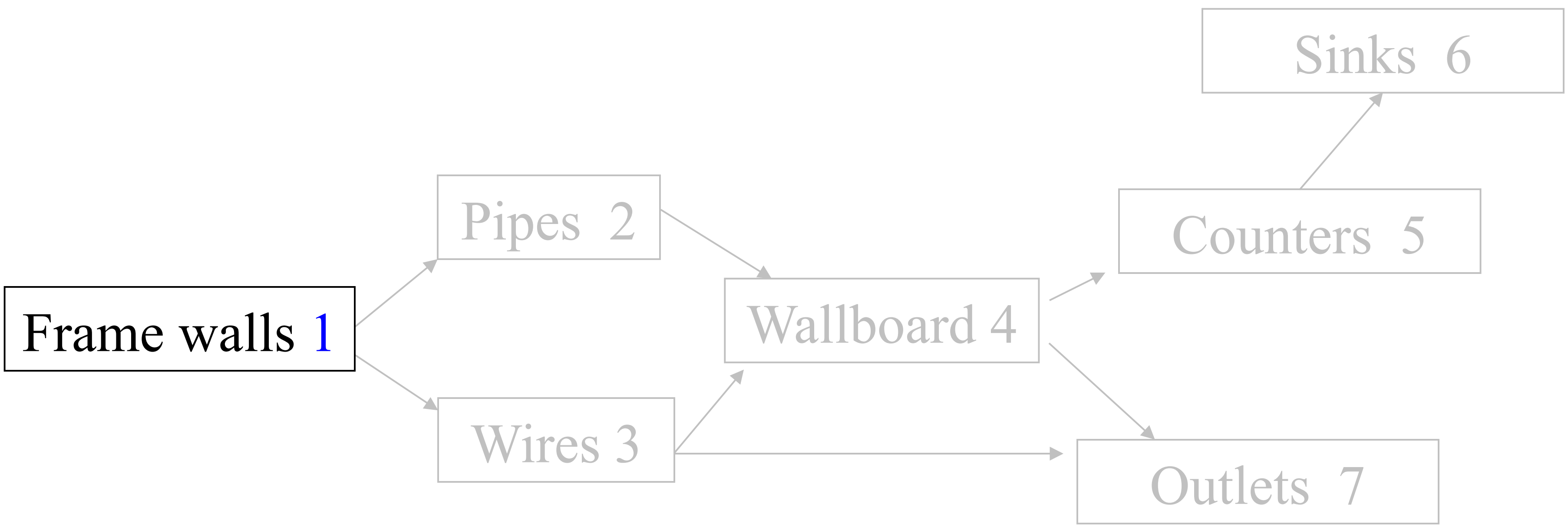
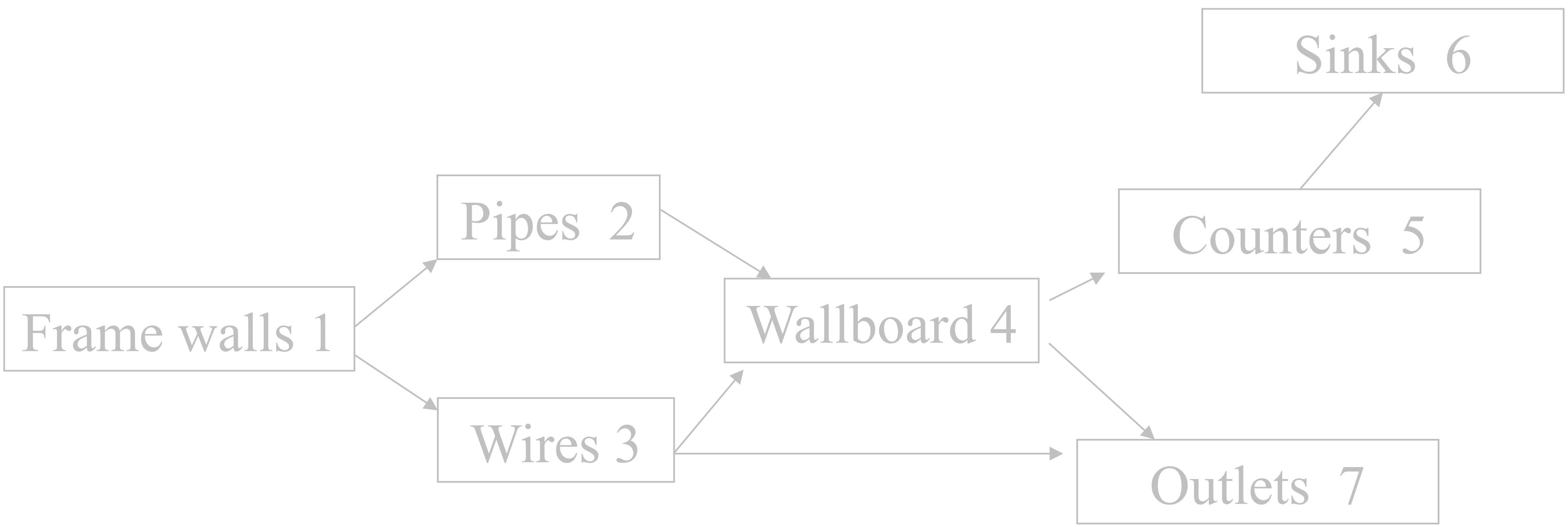
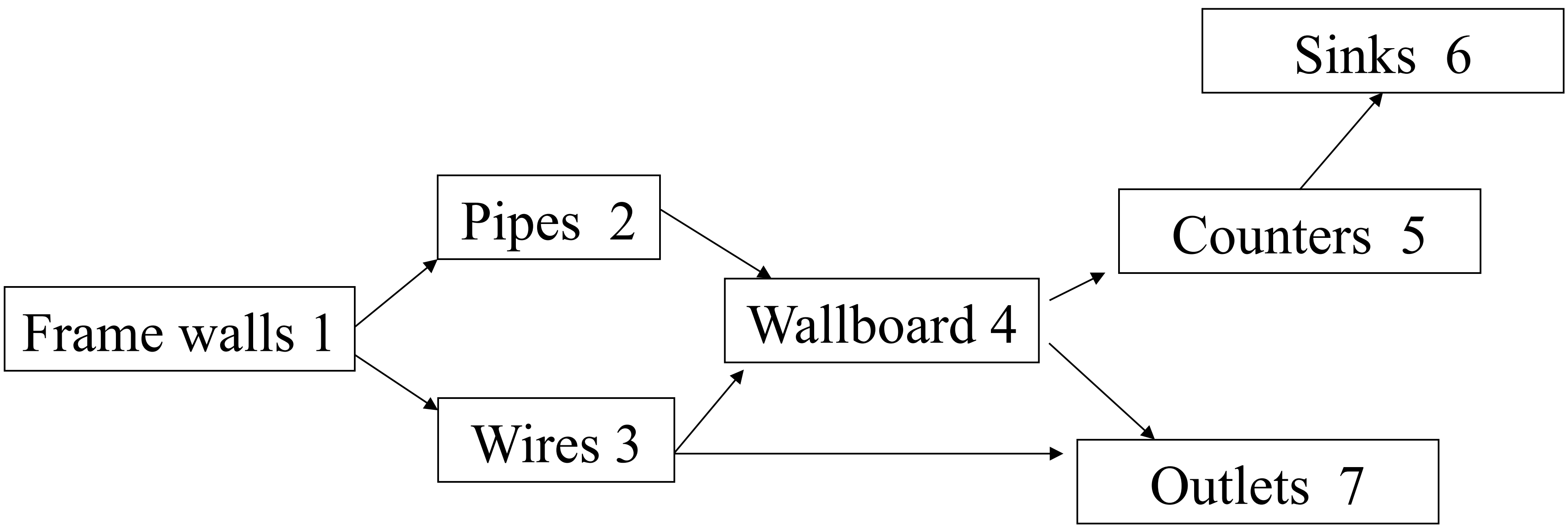


Topsort Example



Topsort Example



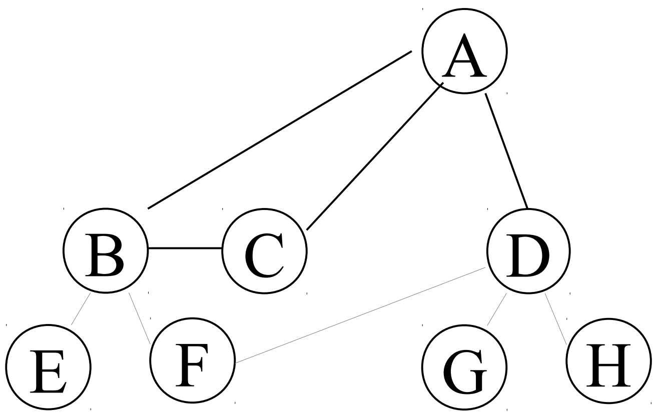
Topsort Cost

- **DFS + numbering**

  = O(n+e) + O(n) = O(n+e)

Review: Breadth First Search

- **Like breadth first search on tree**



- **Breadth First: A B C D E F G H**
- **Depth First: A B E F D G H C**

Breadth First Algorithm

- **bfsG(v):**

  visit and mark v

  enqueue v

  while not queue.empty( )

      dequeue into w

      for each neighbor n of w:

          if n not visited:

              visit and mark n

              enqueue n

Breadth First Cost

- **like depth-first:**
  - **visit every vertex**
  - **cross every edge**

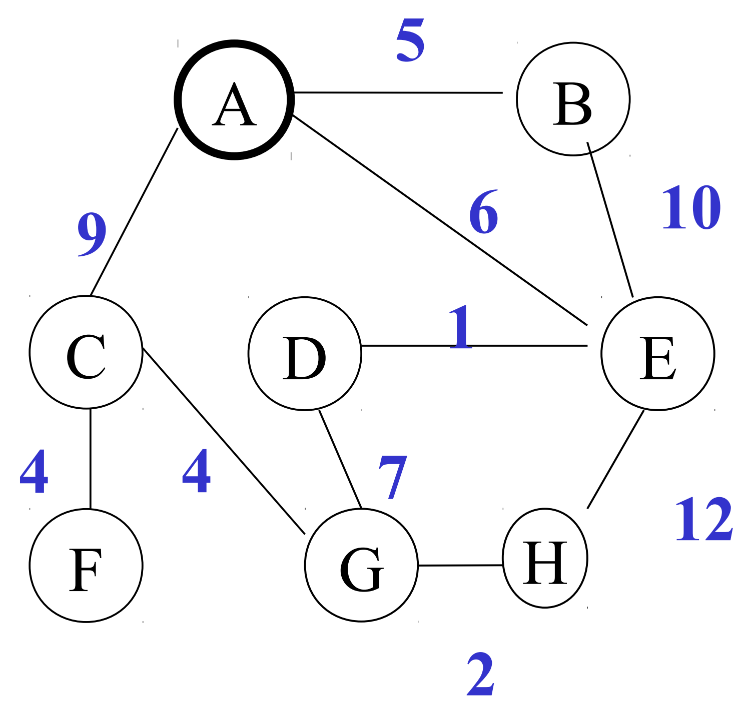
  O(n+e)

Review: Shortest Path

- **weighted graph**
  - **weights are all > 0**
- **“length” of a path = sum of weights of arcs on path**
- ~~**given start vertex, end vertex, find shortest path from start to end**~~
- **given start vertex, find shortest paths from start to each other vertex**

Shortest Paths

- How long is the path
  - AEH
  - AEDGH
- What is the shortest path from A to H?

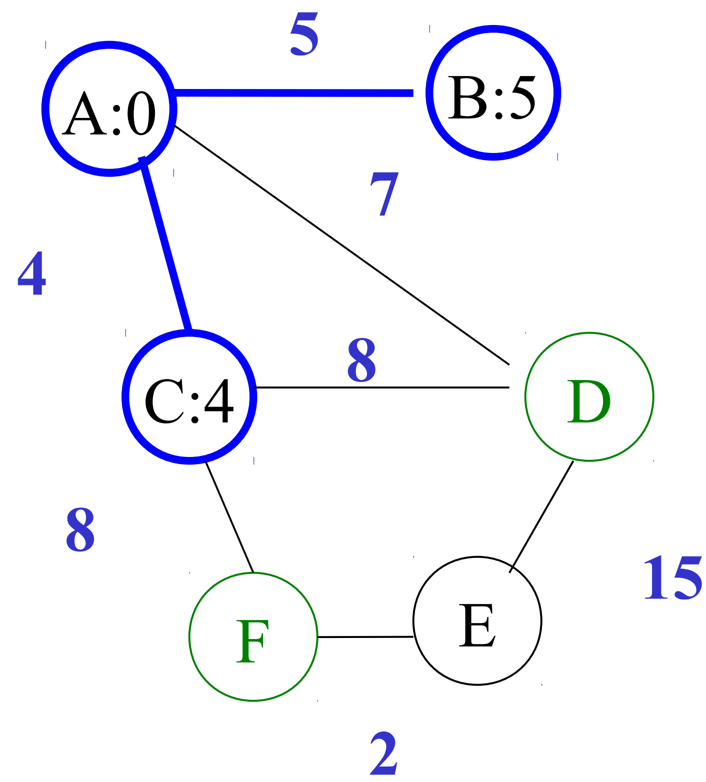


Dijkstra's Algorithm

- **Tree: nodes for which you know the shortest path from start, and the arcs on those paths**
- **Fringe: nodes that are not in the tree yet but have a neighbor in the tree**

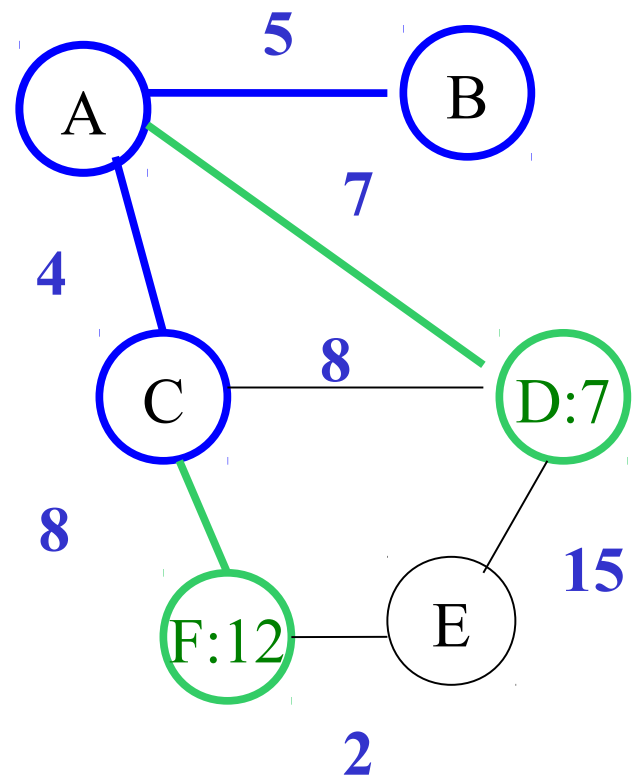
Dijkstra's Algorithm

- Vertices in the **tree** have
  - a link: last step on the shortest path to vertex from start
  - a distance: the length of that whole path



Dijkstra's Algorithm

- Vertices in the fringe have
  - a link: an arc from the tree if > 1 of these, use the arc that gives the shortest path back to start
  - a distance: the length of the path using link



Algorithm:

Put start vertex in the tree, update neighbors

While there are any vertices in fringe

Let v be vertex in fringe with

smallest distance-from-start.

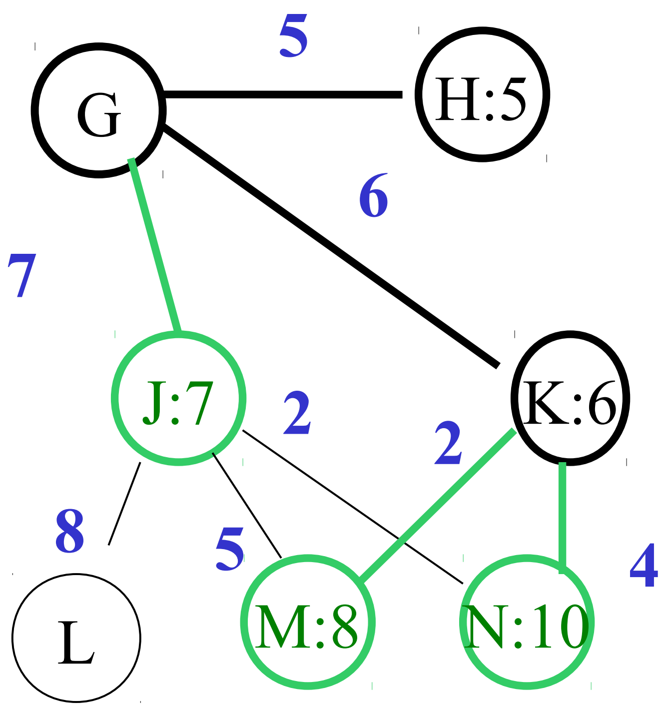
Put v in the tree.

Update neighbors

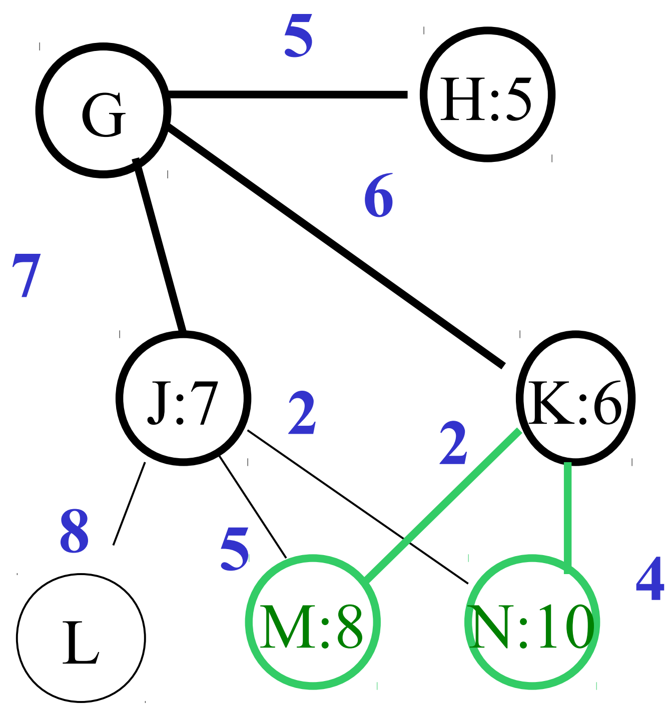
Update neighbors

- **Neighbor of v that is not in tree and not in fringe gets added to fringe with link v**
- **Neighbor of v that is in the fringe gets checked: would changing link to be v result in a smaller distance? If so, change link and distance**
- **Neighbor of v that is in tree is not changed**

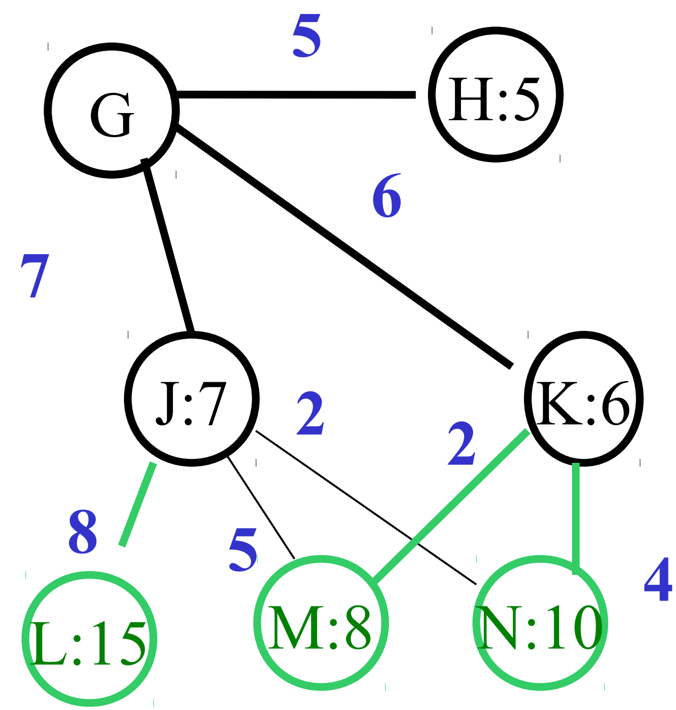
J → tree, Update neighbors



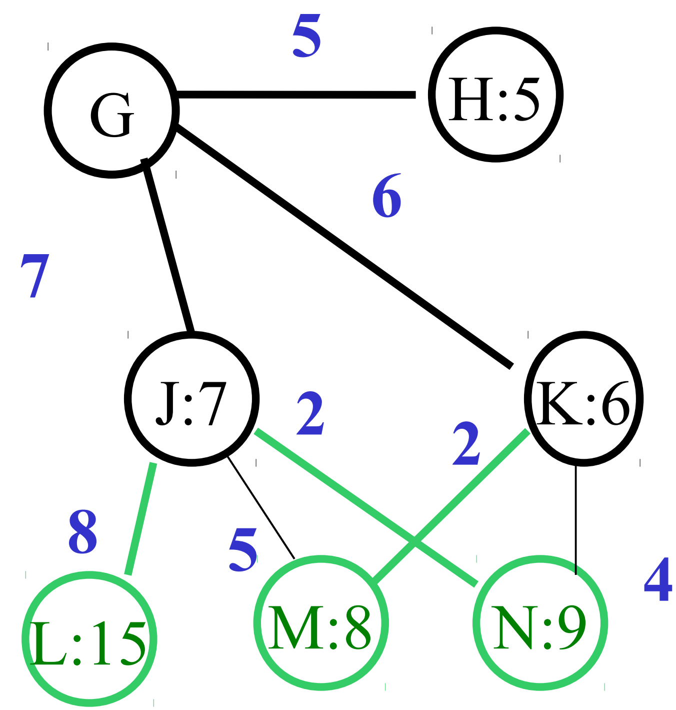
J → tree



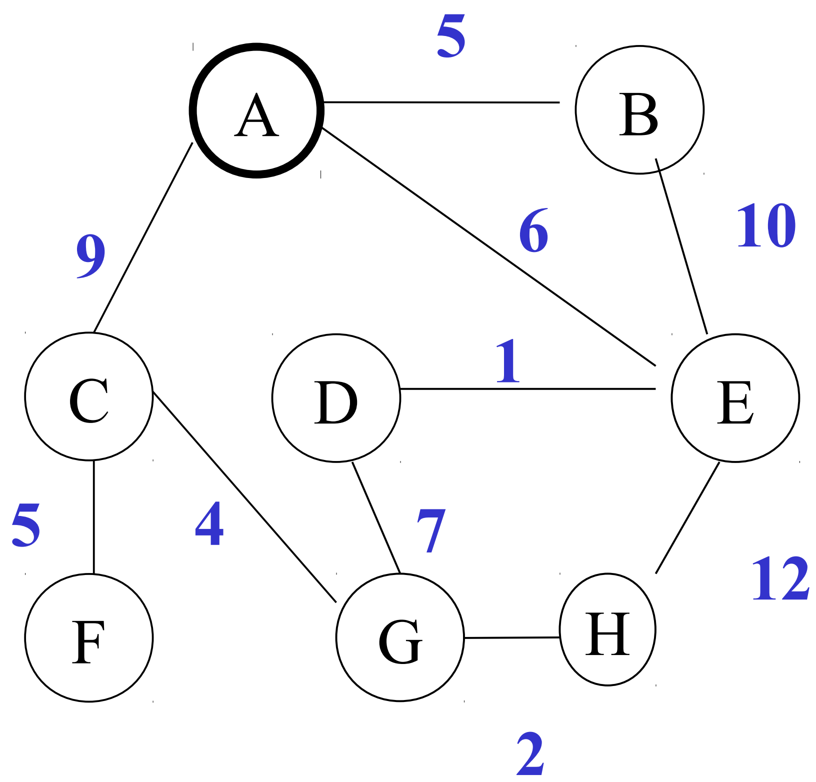
Update neighbors: neither tree nor fringe → fringe



Update neighbors: Fringe: check links



Example



Example

Tree	to B	to C	to D	to E	to F	to G	to H
A	<u>5:A</u>	9:A	∞	6:A	∞	∞	∞

Example

Tree	to B	to C	to D	to E	to F	to G	to H
A	<u>5:A</u>	9:A	∞	6:A	∞	∞	∞
B: 5A		9:A	∞	<u>6:A</u>	∞	∞	∞

Example

Tree	to B	to C	to D	to E	to F	to G	to H
A	5:A	9:A	∞	6:A	∞	∞	∞
B: 5A		9:A	∞	6:A	∞	∞	∞
E: 6A	`	9:A	7:E		∞	∞	18:E

Example

Tree	to B	to C	to D	to E	to F	to G	to H
A	<u>5:A</u>	9:A	∞	6:A	∞	∞	∞
B: 5A		9:A	∞	<u>6:A</u>	∞	∞	∞
E: 6A		9:A	<u>7:E</u>		∞	∞	18:E
D:7E		<u>9:A</u>			∞	14:D	18:E

Example

Tree	to B	to C	to D	to E	to F	to G	to H
A	<u>5:A</u>	9:A	∞	6:A	∞	∞	∞
B: 5A		9:A	∞	<u>6:A</u>	∞	∞	∞
E: 6A		9:A	<u>7:E</u>		∞	∞	18:E
D:7E		<u>9:A</u>			∞	14:D	18:E
C:9A					14:C	<u>13:C</u>	18:E

Example

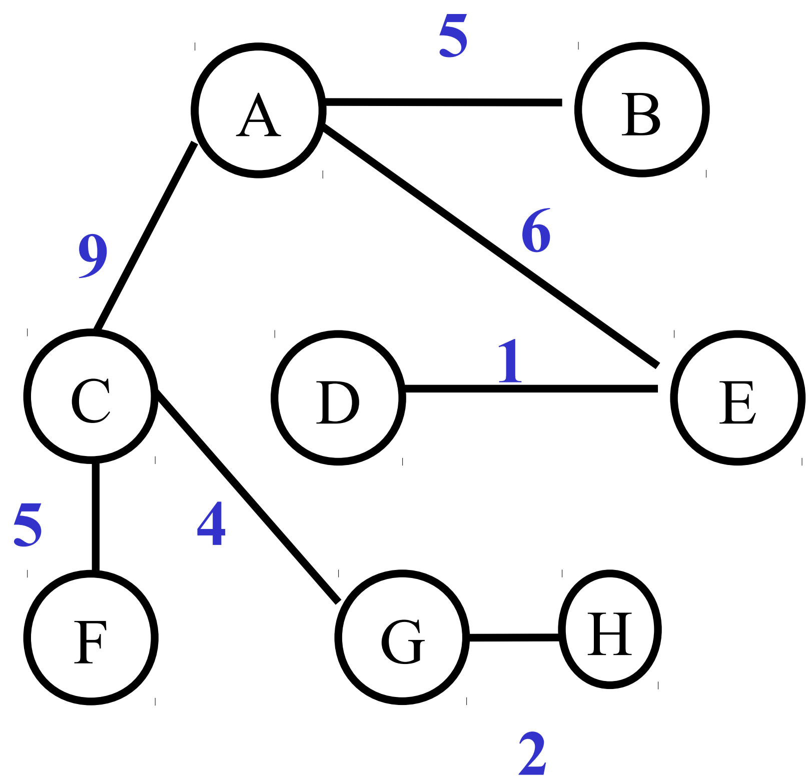
Tree	to B	to C	to D	to E	to F	to G	to H
A	<u>5:A</u>	9:A	∞	6:A	∞	∞	∞
B: 5A		9:A	∞	<u>6:A</u>	∞	∞	∞
E: 6A		9:A	<u>7:E</u>		∞	∞	18:E
D:7E		<u>9:A</u>			∞	14:D	18:E
C:9A					14:C	<u>13:C</u>	18:E
G:13C					<u>14:C</u>		15:G

Example

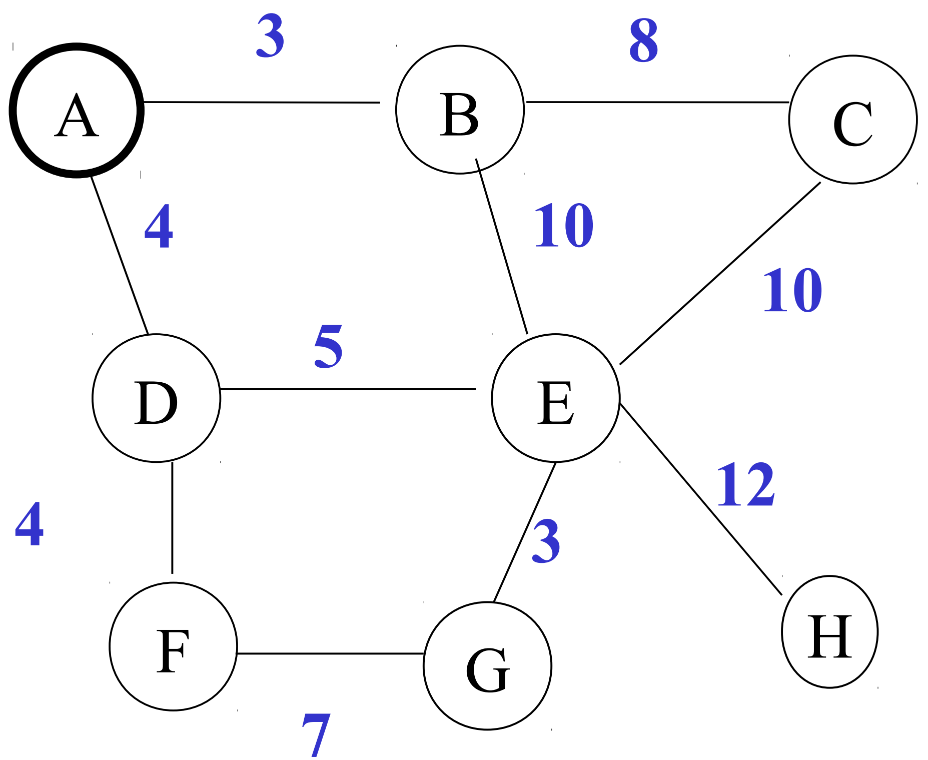
Tree	to B	to C	to D	to E	to F	to G	to H
A	<u>5:A</u>	9:A	∞	6:A	∞	∞	∞
B: 5A		9:A	∞	<u>6:A</u>	∞	∞	∞
E: 6A		9:A	<u>7:E</u>		∞	∞	18:E
D:7E		<u>9:A</u>			∞	14:D	18:E
C:9A					14:C	<u>13:C</u>	18:E
G:13C					<u>14:C</u>		15:G
F: 14C							<u>15:G</u>

Example

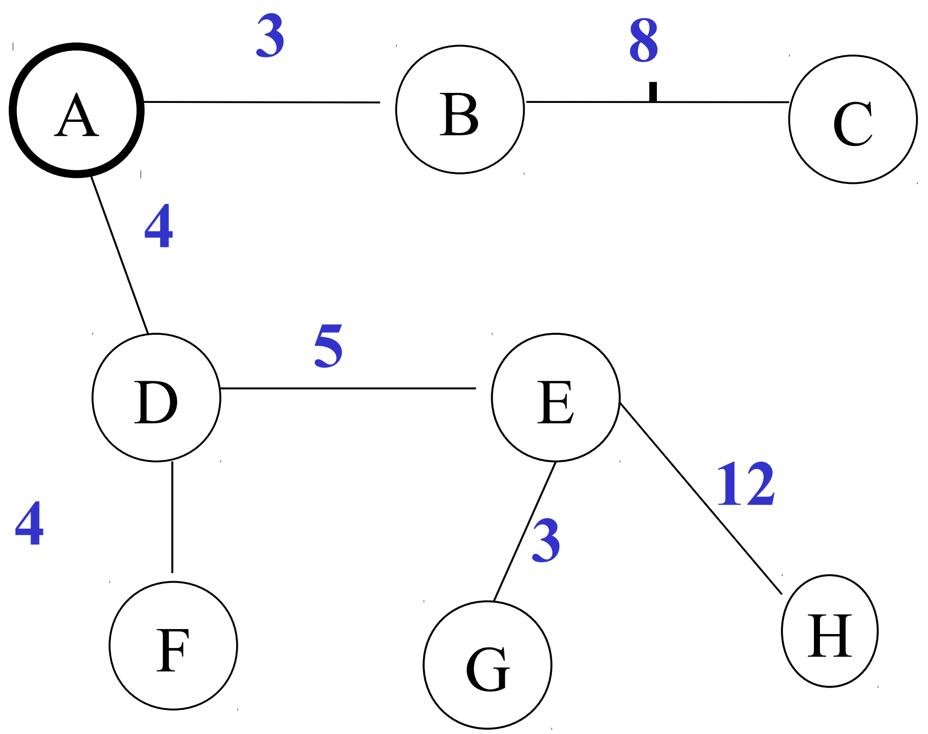
Tree	to B	to C	to D	to E	to F	to G	to H
A	<u>5:A</u>	9:A	∞	6:A	∞	∞	∞
B: 5A		9:A	∞	<u>6:A</u>	∞	∞	∞
E: 6A		9:A	<u>7:E</u>		∞	∞	18:E
D:7E		<u>9:A</u>			∞	14:D	18:E
C:9A					14:C	<u>13:C</u>	18:E
G:13C					<u>14:C</u>		15:G
F: 14C							<u>15:G</u>
H:15G							


5
A
B
6
9
C
D
1
E
5
4
F
G
H
2

Another Example



Result



New: Cost of Dijkstra's Algorithm

What are the operations to consider?

- **Picking the min-distance vertices from the fringe**
- **Adding vertices to the Tree**
- **Updating neighbors**
  - **Adding vertices to the Fringe**
  - **Updating links when needed**

Data Structures

- **For graph:**
  - adjacency matrix
  - adjacency list
- **For fringe:**
  - unordered linked list
  - ordered linked list
  - min heap

plus tree/fringe/neither marked on node

Matrix as Adjacency list, Fringe as Unordered linked list

- **Picking the min-distance vertices from the fringe**
  - **Worst case fringe is all vertices not in tree**

    $(n-1) + (n-2) + \ldots + 1 = O(n^2)$
- **Adding vertices to the Tree**

  $O(n + e)$
- **Updating neighbors**
  - **Adding vertices to the Fringe: $O(n)$**
  - **Checking and Updating links: $O(n+e)$**
- **Total: $O(n^2) + O(n) + O(n) + O(n+e) = O(n^2)$**

Matrix as Adjacency list, Fringe as min-heap

- Picking the min-distance vertices from the fringe
  - Worst case fringe is all vertices not in tree

    $\log(n-1) + \log(n-2) + \ldots + \log(1) = O(n \log n)$
- Adding vertices to the Tree

  $O(n + e)$
- Updating neighbors
  - Adding vertices to the Fringe: $O(n \log n)$
  - Checking and Updating links: $O(n + e \log n)$
- Total:

  $O(n \log n) + O(n \log n) + O(n) + O(n \log n) + O(n + e \log n)$
  $= O((n+e) \log n)$

New: Quicksort1

- **Quicksort:**
  - **Partition**
    - **Split data into two groups, all in one group < any in other group**
  - **sort groups separately**
    - **use quicksort recursively**
  - **append**
    - **if partition & sort are in-place there is nothing to do here**

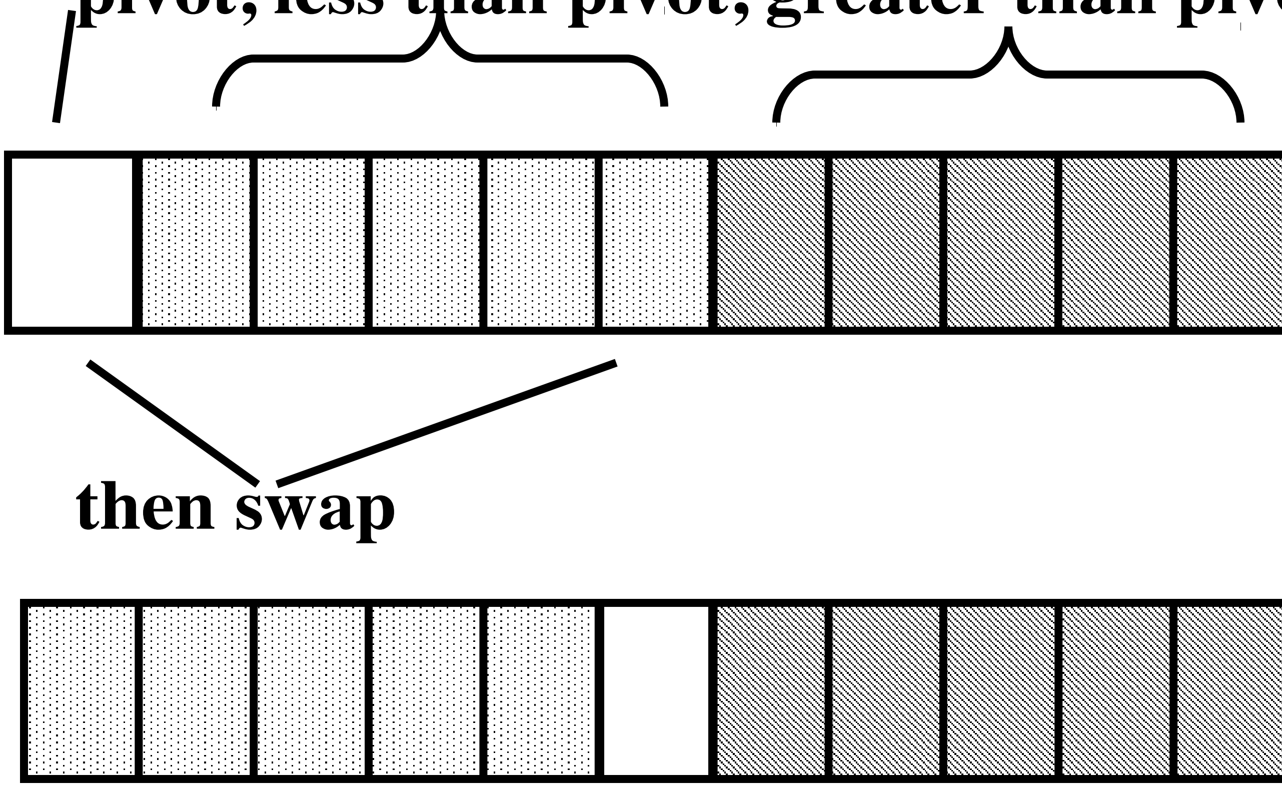
Partition

- Choose a “pivot” value from data
  - ideal would be median => equal size lists
  - but takes too long to find median
  - simplest: pivot = first
    - but in order -> worst case
  - safer: median of 1st, last, middle

Partition

- **Trick: first partition like this**

pivot, less than pivot, greater than pivot



Partition

- Use 2 pointers: left and right
  - move left from low+1 up until A[left] > pivot
  - move right from high down until A[right]<pivot
  - Swap numbers in A[left] and A[right]
  - Repeat until left>=right

Partition

1 12 5 11 3 7 6 8 15 9 10 13 2 4 14

8 12 5 11 3 7 6 1 15 9 10 13 2 4 14

8 4 5 11 3 7 6 1 15 9 10 13 2 12 14

8 4 5 2 3 7 6 1 15 9 10 13 11 12 14

1 4 5 2 3 7 6 8 15 9 10 13 11 12 14

Quicksort

- **How sort regions left & right of pivot?**
  - **Quicksort! (unless < 3 numbers in region)**
    - **Actually, insertion sort faster for small regions**
      - **size<10 or so**

Quick Sort

Unsorted:	2	5	1	8	3	9	4
pivot=4							
Partition:	2	1	3	4	5	9	8
Sort Groups:	1	2	3	4	5	8	9
Result:	1	2	3	4	5	8	9

Quick Sort

1 12 5 11 3 7 6 8 15 9 10 13 2 4 14

1 5 3 7 6 2 4   **8**   12 11 15 9 10 13 14

1 2 3   **4**   5 7 6   **8**   9 11 10   **12**   15 13 14

1 **2** 3 **4** 5 **6** 7 **8** 9 **10** 11 **12** 13 **14** 15

Complexity

- Partition takes O(n) time where n is the number of numbers to partition
- Best case: assume partition always into equal halves
  - Suppose 15 numbers in array
  - partition 0 - 14   15 compares
  - partition 0-6, 8-14   7+7=14 compares

    …   … always O(n) compares
- Each level divides partition size by 2, stop at size 1
  - log n levels
- Total: O(n log n)

Complexity

- **Worst case: always divide into 0 and all-but pivot**
  - **15 -> 14 -> 13 -> … 1: O(n) levels, total O(n^2)**
- **Average case: O(n logn) like best**

Code

- See http://www.cs.ubc.ca/~harrison/Java/QSortAlgorithm.java.html