MATH 354 HW# 6 Section 2.1 # 2,4,6

(2) Maximize: $z = x_1 + 3x_2 + 5x_3$

Subject to:
$$2x_1 - 5x_2 + x_3 \le 3$$
$$x_1 + 4x_2 \le 5$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$$

Canonical formulation: (Introduce slack variables)

Maximize: $z = x_1 + 3x_2 + 5x_3$

subject to:
$$2x_1 - 5x_2 + x_3 + u = 3$$
$$x_1 + 4x_2 + v = 5$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ u \ge 0,\ v \ge 0$$

Matrix form for constraints

$$\begin{bmatrix} 2 & -5 & 1 & 1 & 0 \\ 1 & 4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ u \\ v \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Initial tableau: Note: Objective function is written
$$-x_1 - 3x_2 - 5x_3 + z = 0$$

	x_1	x_2	x_3	u	v	
u	2	-5	1	1	0	3
v	1	4	0	0	1	5
	-1	-3	-5	0	0	0

u and v are the initial basic variables

x_1, x_2, x_3 are the initial nonbasic variables

④

Tableau #1

	x_1	x_2	x_3	x_4	
x_4	$3/2$	0	$5/3$	1	6
x_2	$2/3$	1	2	0	8
	-4	0	-2	0	12

x_2 and x_4 are basic

x_1 and x_3 are nonbasic

1st Choose nonbasic entering variable to be x_1 since the entry -4 in its objective row is the most negative entry in the objective row.

2nd Check the θ-ratios in the pivotal column, the column under x_1. For x_3 the ratio is $\frac{6}{3/2} = 4$. For x_4 the ratio is $\frac{8}{2/3} = 12$. The smallest ratio is for x_3. So, choose x_3 to be the departing variable

3rd The pivot is the entry $3/2$ under x_1. Carry out the pivoting operation.

i) Divide pivot row by $3/2$ and enter in tableau #2

Tableau #2

	x_1	x_2	x_3	x_4	
x_1	1	0	$10/9$	$2/3$	4
x_2	0	1	$34/27$	$-4/9$	$\frac{16}{3}$
	0	0	$\frac{22}{9}$	$\frac{8}{3}$	28

ii) Multiply new 1st row by $-2/3$ and add to second row to make second row entry under x_1 equal to zero

iii) Multiply new 1st row by 4 and add to the objective row to make the entry under x_1 equal to zero

iv) The new basic variables are x_1 and x_2.

⑥ Initial tableau

	x_1	x_2	x_3	x_4	x_5	
x_3	$2/3$	0	1	$3/5$	0	$3/2$
x_2	$3/2$	1	0	1	0	$5/2$
x_5	5	0	0	$2/9$	1	$2/3$
	4	0	0	-5	0	$7/3$

x_2, x_3, x_5 are the initial basic variables
x_1 and x_4 are nonbasic

1st Choose x_4 for the entering variable (because of
2nd Check the θ-ratios for the x_4 column
x_3 ratio $= \frac{3/2}{3/5} = 5/2$, x_2 ratio $= \frac{5/2}{1}$, x_5 ratio $= \frac{2/3}{2/9} = 3$
We see that their is a tie for smallest ratio
(We will carry out the calculations for both choices)

Case: Choose x_2 for the departing variable
3rd Pivot on entry 1 in the pivotal column
i) Divide pivot row by 1
ii) Multiply pivot row by $-3/5$ and add to 1st row
Multiply pivot row by $-2/9$ and add to 3rd row
iii) Multiply pivot row by 5 and add to objective row

to get:

	x_1	x_2	x_3	x_4	x_5	
x_3	$-7/30$	$-3/5$	1	0	0	0
x_4	$3/2$	1	0	1	0	$5/2$
x_5	$14/3$	$-2/9$	0	0	1	$1/9$
	$23/2$	5	0	0	0	$\frac{89}{6}$

Note: Degeneracy, a basic variable with value $=0$.

x_3, x_4, x_5 basic variables.

Case: Choose x_3 for the departing variable
3rd Pivot on the $3/5$ in the x_4 column (see tableau at top of this page

	x_1	x_2	x_3	x_4	x_5	
x_4	$10/9$	0	$5/3$	1	0	$5/2$
x_2	$7/18$	1	$-5/3$	0	0	0
x_5	$\frac{385}{81}$	0	$-10/27$	0	1	$1/9$
	$86/9$	0	$25/3$	0	0	$\frac{89}{6}$

Note: Degeneracy a basic variable with value $=0$.

x_4, x_2, x_5 basic variables.