MATH 354 HW #19 Section 4.2 problems 2,3,4,5 ①

(2) $2 = [2] + 0$, so $f_1 = 0$

$\frac{7}{24} = [\frac{7}{24}] + \frac{7}{24}$, so $f_2 = \frac{7}{24}$

$\frac{1}{3} = [\frac{1}{3}] + \frac{1}{3}$, so $f_3 = \frac{1}{3}$

Since f_3 is the largest we focus on the 3rd constraint

$x_1 + x_3 + \frac{3}{4}x_5 + \frac{11}{12}x_6 + \frac{13}{12}x_7 = \frac{1}{3}$, x_3 is basic

$1 = [1] + 0$, so $g_1 = 0$, $\frac{3}{4} = [\frac{3}{4}] + \frac{3}{4}$, so $g_5 = \frac{3}{4}$

$\frac{11}{12} = [\frac{11}{12}] + \frac{11}{12}$, so $g_6 = \frac{11}{12}$, $\frac{13}{12} = [\frac{13}{12}] + \frac{1}{12}$, so $g_7 = \frac{1}{12}$

Then the 1st cutting plane constraint is

$$-\frac{3}{4}x_5 - \frac{11}{12}x_6 - \frac{1}{12}x_7 + u_1 = -\frac{1}{3}$$

(3) Problem Maximize $z = x + y$

Subject to: $2x + 3y + u = 12$ $x \geq 0, y \geq 0$

$2x + y + v = 6$ x, y integers

Tableau #1

		1	1	0	0	
	C_B	x	y	u	v	x_{B_0}
	0	u	2	3	1	0	12
←	0	v	2	1	0	1	6
			-1	-1	0	0	0

$B_1 = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$, $B_1^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1/2 \end{bmatrix}$

Tableau #2

	C_{B_1}	x	y	u	v	x_{B_1}
		1	1	0	0	
←	0 u	0	2	1	-1	6
	1 x	1	1/2	0	1/2	3
		0	-1/2	0	1/2	3

$B_2 = \begin{bmatrix} 2 & 0 \\ 1/2 & 1 \end{bmatrix}$, $B_2^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/4 & 1 \end{bmatrix}$

Tableau #3

C_{B_2}	x	y	u	v	x_{B_2}
	1	1	0	0	
1 y	0	1	1/2	-1/2	3
1 x	1	0	-1/4	3/4	3/2
	0	0	1/4	1/4	9/2

Final simplex tableau

Since $3 = [3] + 0$, $f_1 = 0$, $3/2 = [3/2] + \frac{1}{2}$, $\boxed{f_2 = 1/2}$,

$\therefore$ choose second constraint to modify

$-1/4 = [-\frac{1}{4}] + 3/4$, so, $g_3 = 3/4$, $\therefore 3/4 = [3/4] + 3/4$, so $g_4 = 3/4$

1st Cutting plane constraint: $\boxed{-3/4 u - 3/4 v + u_1 = -\frac{1}{2}}$

This gives the tableau #1

Tableau #1

		1	1	0	0	0	
		x	y	u	v	u_1	
1	y	0	1	1/2	−1/2	0	3
1	x	1	0	−1/4	3/4	0	3/2
← 0	u_1	0	0	−3/4	−3/4	1	−1/2
		0	0	1/4	1/4	0	9/2

Since a basic variable is negative the dual simplex method will be used

$$B_1 = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -1/4 \\ 0 & 0 & -3/4 \end{bmatrix} \qquad B_1^{-1} = \begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & -4/3 \end{bmatrix}$$

Tableau #2

		1	1	0	0	0	
		x	y	u	v	u_1	
1	y	0	1	0	−1	2/3	$8/3 = [\frac{8}{3}] + \frac{2}{3}$
1	x	1	0	0	1	−1/3	$5/3 = [5/3] + \frac{2}{3}$
0	u	0	0	1	1	−4/3	$2/3 = [2/3] + 2/3$
		0	0	0	0	1/3	13/3

Optimal solution is $[5/3\ 8/3]$ Not integral so construct a second cutting plane constraint

Choose the u now (u is a basic variable).

For v: $1 = [1] + 0$, so $g_4 = 0$, for u_1: $-4/3 = [-4/3] + \frac{2}{3}$, so $g_5 = 2/3$

2nd Cutting plane constraint: $\boxed{-2/3 u_1 + u_2 = -2/3}$

The tableau #3 is

Tableau #3

		1	1	0	0	0	0	
		x	y	u	v	u_1	u_2	
1	y	0	1	0	−1	2/3	0	8/3
1	x	1	0	0	1	−1/3	0	5/3
0	u	0	0	1	1	−4/3	0	2/3
← 0	u_2	0	0	0	0	−2/3	1	−2/3
		0	0	0	0	1/3	0	13/3

$$B_2 = \begin{bmatrix} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & -4/3 \\ 0 & 0 & 0 & -2/3 \end{bmatrix} \qquad B_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -3/2 \end{bmatrix}$$

We get

		1	1	0	0	0	0	
C_B		x	y	u	v	u_1	u_2	X_B
1	y	0	1	0	-1	0	1	2
1	x	1	0	0	1	0	$-1/2$	2
0	u	0	0	1	1	0	-2	2
0	u_1	0	0	0	0	1	$-3/2$	1
		0	0	0	0	0	$1/2$	4

We get the optimal solution $\overline{X}_0^T = [2\ 2]$, $Z = 4$

The set of feasible solutions will be bounded by $2x+3y=12$, $2x+y=6$ and $x \geq 0$, $y \geq 0$

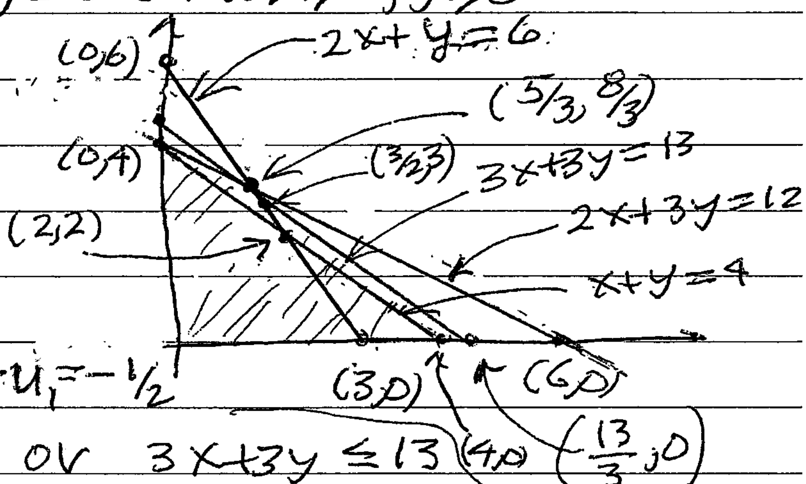
From

$u = 12 - 2x - 3y$

$v = 6 - 2x - y$

~~we get;~~

1st cutting plane constraint:



$-3/4(12-2x-3y) - 3/4(6-2x-y) + u_1 = -1/2$

so $3x + 3y + u_1 = 13$ or $\underline{\underline{3x+3y \leq 13}}$

2nd cutting plane constraint:

$-2/3(13-3x-3y) + u_2 = -2/3$ so $2x + 2y + u_2 = 8$

or $\underline{\underline{x + y \leq 4}}$

(4) Canonical Form

Maximize: $z = x + 4y$

Subject to: $x + 6y + u = 36$

$3x + 8y + v = 60$

$x \geq 0, y \geq 0, u \geq 0, v \geq 0$

x or y integers

Tableau #1

	x	y	u	v	
$\leftarrow u$	1	6	1	0	36
v	3	8	0	1	60
	−1	−4	0	0	0

Choose y as entering variable and from θ ratios: $\frac{36}{6}$ for u and $\frac{60}{3}$ for v. Choose u as departing variable.

Tableau #2

	x	y	u	v	
y	1/6	1	1/6	0	6
$\leftarrow v$	5/3	0	−4/3	1	12
	−1/3	0	2/3	0	24

Choose x as the entering variable and from the θ-ratios: $\frac{6}{1/6} = 36$ for y; $\frac{12}{5/3} = \frac{36}{5}$. Choose v as the departing variable.

Tableau #3

	x	y	u	v	
y	0	1	3/10	−1/10	24/5
x	1	0	−4/5	3/5	36/5
	0	0	2/5	1/5	132/5

The optimal solution is $x = \frac{36}{5}$, $y = \frac{24}{5}$, $z = \frac{132}{5}$

The optimal values are not integers so begin the second stage by constructing a cutting plane constraint. Since $\frac{24}{5} = \left[\frac{24}{5}\right] + \frac{4}{5}$, $f_1 = \frac{4}{5}$ and $\frac{36}{5} = \left[\frac{36}{5}\right] + \frac{1}{5}$, $f_2 = \frac{1}{5}$; choose $f_1 = 4/5$ and the 1st constraint for modifying. From $\frac{3}{10} = \left[\frac{3}{10}\right] + \frac{3}{10}$, $g_3 = 3/10$ and from $-\frac{1}{10} = \left[-\frac{1}{10}\right] + 9/10$, $g_4 = 9/10$.

1st Cutting Plane Constraint $\boxed{-\frac{3}{10}u - \frac{9}{10}v + \alpha = -\frac{4}{5}}$

With this constraint added our new tableau is as follows:

Tableau #1

	x	y	u	v	α	
y	0	1	$3/10$	$-1/10$	0	$24/5$
x	1	0	$-4/5$	$3/5$	0	$36/5$
$\leftarrow \alpha$	0	0	$-3/10$	$-9/10$	1	$-4/5$
	0	0	$2/5$	$1/5$	0	$\frac{132}{5}$

Since the basic variable α is <0 we need to use the dual simplex method

Choose α as the departing variable and from $\frac{2/5}{-3/10} = -4/3$ for u and $\frac{1/5}{9/10} = -2/9$ for v, we choose v as the arriving variable since $-2/9 > -4/3$.

We get

Tableau #2

	x	y	u	v	α	
y	0	1	$1/3$	0	$-1/9$	$44/9 = [\frac{44}{9}] + 8/9$ (f_1)
x	1	0	-1	0	$2/3$	$20/3 = [\frac{20}{3}] + 2/3$ (f_2)
v	0	0	$1/3$	1	$-10/9$	$8/9 = [\frac{8}{9}] + 8/9$ (f_3)
	0	0	$1/3$		$2/9$	$236/9$

Optimal solution is still not integral so construct another cutting plane constraint. Choose the 1st constraint to modify since $f_1 = 8/9$ is the larger of $8/9$ and $2/3$. (Could have chosen the 3rd constraint) From $\frac{1}{3} = [\frac{1}{3}] + \frac{1}{3}$, $g_1 = \frac{1}{3}$, from $-1/9 = [-\frac{1}{9}] + \frac{8}{9}$, $g_5 = 8/9$ We get

2nd Cutting Plane Constraint

$$\boxed{-\tfrac{1}{3}u - \tfrac{8}{9}\alpha + \beta = -\tfrac{8}{9}}$$

With this our Tableau is #3

	x	y	u	v	α	β	
y	0	1	$1/3$	0	$-1/9$	0	$44/9$
x	1	0	-1	0	$2/3$	0	$20/3$
v	0	0	$1/3$	1	$-10/9$	0	$8/9$
$\leftarrow \beta$	0	0	$-1/3$	0	$-8/9$	1	$-8/9$
	0	0	$1/3$	0	$2/9$	0	$236/9$

Use dual simplex, β is the departing variable

Note:

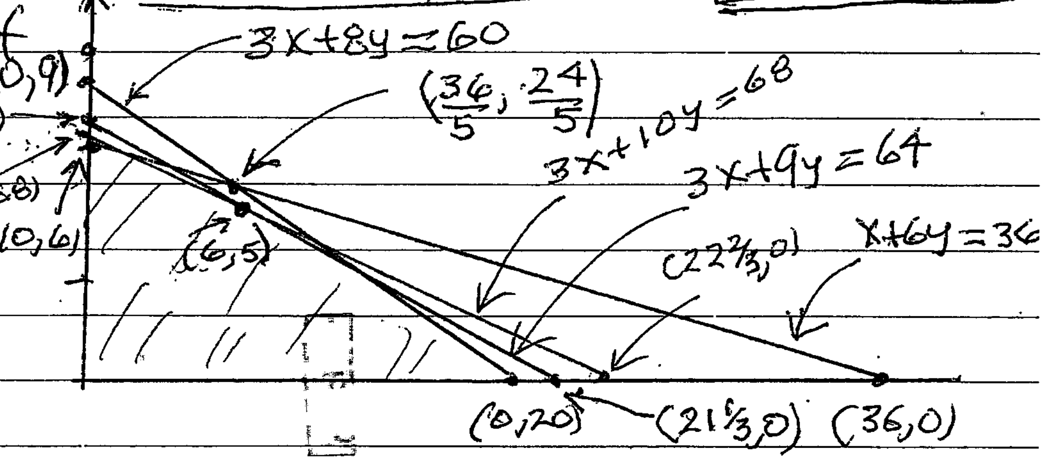
and from $\frac{1}{3}/-\frac{1}{3} = -1$ for u and $\frac{2}{9}/-\frac{8}{9} = -\frac{1}{4}$ for β
choose the β (since $-\frac{1}{4} > -1$) for the arriving variable. We get the final tableau

Tableau #4

	x	y	u	v	α	β	
y	0	1	3/8	0	0	-1/8	5
x	1	0	-3/4	0	0	3/4	6
v	0	0	3/4	1	0	-5/4	2
α	0	0	3/8	0	1	-9/8	1
	0	0	1/4	0	0	1/4	26

We see that the optimal solution is: $x=6, y=5$
$z=26$

The original constraints are:
$\boxed{x+6y \le 36}$ and $\boxed{3x+8y \le 60}$
Using $u = 36-x-6y$ $v = 60-3x-8y$
in the two cutting plane constraints we get: $-\frac{3}{10}(36-x-6y) - \frac{9}{10}(60-3x-8y) + \alpha = -\frac{4}{5}$
which gives $3x+9y+\alpha = 64$ or $\boxed{3x+9y \le 64}$
and with $\alpha = 64-3x-9y$
$-\frac{1}{3}u - \frac{8}{9}(64-3x-9y) + \beta = -\frac{8}{9}$
which gives $3x+10y+\beta = 68$ or $\boxed{3x+10y \le 68}$
The graphs of these four constraints and $x \ge 0, y \ge 0$ is to the right



(5) Canonical form: Maximize: $z = 4x + y$

Subject to: $3x + 2y + u = 5$

$2x + 6y + v = 7$

$3x + 7y + w = 6$

$x \geq 0, y \geq 0$

x and y integers

Tableau #1

	4	1	0	0	0	
	x	y	u	v	w	
← 0 u	(3)	2	1	0	0	5
0 v	2	6	0	1	0	7
0 w	3	7	0	0	1	6
	−4	−1	0	0	0	0

$B = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

$B^{-1} = \begin{pmatrix} 1/3 & 0 & 0 \\ -2/3 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

Tableau #2

		4	1	0	0	0	
		x	y	u	v	w	
4	x	1	2/3	1/3	0	0	5/3
0	v	0	14/3	−2/3	1	0	1/3
0	w	0	5	−1	0	1	1
		0	5/3	4/3	0	0	$\frac{20}{3}$

Optimal tableau

optimal solution is $[5/3, 0]^T$

The optimal solution is not integral, so we will construct a 1st Cutting Plane constraint as follows: $5/3 = [5/3] + 2/3$ so $f_1 = 2/3$;

$1/3 = [\frac{1}{3}] + 1/3$, so $f_2 = 2/3$. Choose the 1st constraint

$\frac{2}{3} = [2/3] + 2/3$, $\frac{1}{3} = [1/3] + \frac{1}{3}$ So $g_2 = 2/3$, $g_3 = \frac{1}{3}$ and

1st Cutting Plane Constraint: $\boxed{-2/3\, y - \frac{1}{3} u + \alpha = -2/3}$

Our new tableau is ↓

Tableau #1

	4	1	0	0	0	0	
	x	y	u	v	w	α	
x_1	1	2/3	1/3	0	0	0	5/3
v	0	14/3	−2/3	1	0	0	11/3
w	0	5	−1	0	1	0	1
← α	0	−2/3	−1/3	0	0	1	−2/3
	0	5/3	4/3	0	0	0	$\frac{20}{3}$

Use dual simplex Method

$\text{Max}\left[\frac{5/3}{-2/3}, \frac{4/3}{-1/3}\right] = -2\frac{1}{2}$

So choose

Tableau #2

		4	1	0	0	0	0	
		x	y	u	v	w	α	
4	x	1	0	0	0	0	1	1
0	v	0	0	-3	1	0	7	-1
0	w	0	0	$-7/2$	0	1	$15/2$	-4
1	y	0	1	$1/2$	0	0	$-3/2$	1
		0	0	$1/2$	0	0	$5/2$	5

where we have used:

$$B_1 = \begin{bmatrix} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & 14/3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & -2/3 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 15/2 \\ 0 & 0 & 0 & -3/2 \end{bmatrix}$$

The solutions are integers but not feasible so, use the dual simplex method again

$$B_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -7/2 & 0 \\ 0 & 0 & 1/2 & 1 \end{bmatrix}, \quad B_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6/7 & 0 \\ 0 & 0 & -2/7 & 0 \\ 0 & 0 & 1/7 & 1 \end{bmatrix}$$

We get

Tableau #3

	4	1	0	0	0	0	
	x	y	u	v	w	α	
x	1	0	0	0	0	1	1
v	0	0	0	1	$-\frac{6}{7}$	$4/7$	$17/7 = [\frac{17}{7}] + 3/7 \leftarrow f_2$
u	0	0	1	0	$-2/7$	$-15/7$	$8/7 = [\frac{8}{7}] + 1/7 \leftarrow f_3$
y	0	1	0	0	$\frac{1}{7}$	$-3/7$	$3/7 = [3/7] + 3/7 \leftarrow f_4$
	0	0	0	0	$1/7$	$\frac{25}{7}$	$\frac{31}{7}$

Since the optimal solution $[1\ 3/7]^T$ is not integral

So, Construct a 2nd Cutting Plane Constraint

since $\max\{f_2, f_3, f_4\} = 3/7$, choose the v row constraint for modifying. $-\frac{6}{7} = [-\frac{6}{7}] + \frac{1}{7}$, so $g_5 = \frac{1}{7}$, and $\frac{4}{7} = [\frac{4}{7}] + \frac{4}{7}$, so $g_6 = 4/7$. With this we get 2nd Cutting Plane Constraint: $\boxed{-\frac{1}{7}w - 4/7\alpha + \beta = -\frac{3}{7}}$

Our next tableau is:

Tableau #4

	x	y	u	v	w ↓	α	β	
x	1	0	0	0	0	1	0	1
v	0	0	0	1	$-6/7$	$4/7$	0	$17/7$
u	0	0	1	0	$-2/7$	$-\frac{15}{7}$	0	$8/7$
y	0	1	0	0	$1/7$	$-3/7$	0	$3/7$
← β	0	0	0	0	$-1/7$	$-4/7$	1	$-3/7$
	0	0	0	0	$\frac{1}{7}$	$\frac{25}{7}$	0	$31/7$

Dual Simplex again

Then with

$$B_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -6/7 \\ 0 & 0 & 1 & 0 & -2/7 \\ 0 & 0 & 0 & 1 & 1/7 \\ 0 & 0 & 0 & 0 & -1/7 \end{bmatrix}, \quad B_3^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -7 \end{bmatrix}$$

we get

Tableau #5

		4	1	0	0	0	0	0	
		x	y	u	v	w	α	β	
4	x	1	0	0	0	0	1	0	1 ←
1	v	0	0	0	1	0	$-\frac{20}{7}$	6	5
0	u	0	0	1	0	0	$-23/7$	2	2
0	y	0	1	0	0	0	-1	1	0 ←
0	w	0	0	0	0	1	4	-7	3
		0	0	0	0	0	$8/7$	0	4

We see that we have found an integer optimal solution $\underline{\underline{x=1,\ y=0}}$ and $\underline{\underline{z=4}}$.