

## HW6

1. For each of the following schedules, state whether it is conflict-serializable. If yes, provide all equivalent serial schedules. If no, state why it is not conflict-serializable. ( $ri(X)$  denotes a read on object X for transaction  $T_i$ .  $wj(Y)$  denotes a write on object Y for transaction  $T_j$ .)

1)  $r1(X), r3(Y), r2(Y), w3(Y), r3(X), r2(Z), w1(X), w2(Z), r1(Z), w1(Z)$

**Solution:** We start with an empty graph with three vertices labeled  $T_1, T_2, T_3$ . We go through each operation in the schedule:

$r1(X)$ : no subsequent writes to X from other transactions, so no new edges;

$r3(Y)$ : no subsequent writes to Y from other transactions, so no new edges;

$r2(Y)$ : Y is subsequently written by  $T_3$ , so add edge  $T_2 \rightarrow T_3$ ;

$w3(Y)$ : no subsequent operations to Y, so no new edges;

$r3(X)$ : X is subsequently written by  $T_1$ , so add edge  $T_3 \rightarrow T_1$ ;

$r2(Z)$ : Z is subsequently written by  $T_1$ , so add edge  $T_2 \rightarrow T_1$ ;

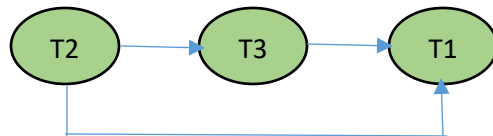
$w1(X)$ : no subsequent operations to X, so no new edges;

$w2(Z)$ : Z is subsequently read by  $T_1$ , so add edge  $T_2 \rightarrow T_1$ ;

$r1(Z)$ : no subsequent writes to Z from other transactions, so no new edges;

$w1(Z)$ : no subsequent operations, so no new edges;

We end up with the following precedence graph:



This graph has no cycles, so the original schedule must be conflict-serializable. The equivalent serial schedule is  $T_2$ - $T_3$ - $T_1$ .

2)  $r1(X), r2(Y), r3(Y), w3(Y), r2(Z), w1(X), r3(X), r1(Z), w2(Z), w1(Z)$

**Solution:** We start with an empty graph with three vertices labeled  $T_1, T_2, T_3$ . We go through each operation in the schedule:

$r1(X)$ : no subsequent writes to X from other transactions, so no new edges;

$r2(Y)$ : Y is subsequently written by  $T_3$ , so add edge  $T_2 \rightarrow T_3$ ;

$r3(Y)$ : no subsequent writes to Y from other transactions, so no new edges;

$w3(Y)$ : no subsequent operations to Y, so no new edges;

$r2(Z)$ : Z is subsequently written by  $T_1$ , so add edge  $T_2 \rightarrow T_1$ ;

$w1(X)$ : X is subsequently read by  $T_3$ , so add edge  $T_1 \rightarrow T_3$ ;

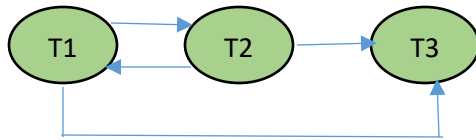
$r3(X)$ : no subsequent operations to X, so no new edges;

$r1(Z)$ : Z is subsequently written by  $T_2$ , so add edge  $T_1 \rightarrow T_2$ ;

$w2(Z)$ : Z is subsequently written by  $T_1$ , so add edge  $T_2 \rightarrow T_1$ ;

$w1(Z)$ : no subsequent operation, so no new edges;

We end up with the following precedence graph:



This graph has a cycle, so the original schedule is not conflict-serializable.

3)  $r_1(X), w_1(X), r_3(Y), r_1(Z), w_3(Y), r_2(Y), r_2(Z), r_3(X), w_1(Z), w_2(Z)$

**Solution:** We start with an empty graph with three vertices labeled T1, T2, T3. We go through each operation in the schedule:

$r_1(X)$ : no subsequent writes to X from other transactions, so no new edges;

$w_1(X)$ : X is subsequently read by T3, so add edge  $T1 \rightarrow T3$ ;

$r_3(Y)$ : no subsequent writes to Y from other transactions, so no new edges;

$r_1(Z)$ : Z is subsequently written by T2, so add edge  $T1 \rightarrow T2$ ;

$w_3(Y)$ : Y is subsequently read by T2, so add edge  $T3 \rightarrow T2$ ;

$r_2(Y)$ : no subsequent operations to Y, so no new edges;

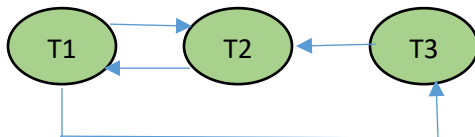
$r_2(Z)$ : Z is subsequently written by T1, so add edge  $T2 \rightarrow T1$ ;

$r_3(X)$ : no subsequent operations to X, so no new edges;

$w_1(Z)$ : Z is subsequently written by T2, so add edge  $T1 \rightarrow T2$ ;

$w_2(Z)$ : no subsequent operations, so no new edges;

We end up with the following precedence graph:



This graph has cycles, so the original schedule is not conflict-serializable.

2. Let T1, T2, T3 be the following transactions:

T1:  $r_1(D), w_1(B), w_1(D)$

T2:  $r_2(C), w_2(B)$

T3:  $r_3(D), w_3(D), w_3(B)$

For each of the following schedules, state whether it is possible under 2PL protocol? Give your reasons.

1)  $r_1(D), r_2(C), w_2(B), w_1(B), r_3(D), w_3(D), w_1(D), w_3(B)$

**Solution:** Let's try to add the lock/unlock steps to the schedule,  
 $l_1(D), r_1(D), l_2(C), r_2(C), l_2(B), u_2(C), w_2(B), u_2(B), l_1(B), w_1(B), \dots$

The next operation is  $r_3(D)$ , before that operation  $T_3$  needs to get a lock on  $D$ , however,  $D$  is currently locked by  $T_1$ .  $T_3$  cannot lock  $D$  until  $T_1$  has unlocked  $D$ . If  $T_1$  unlock  $D$  at this moment, according to 2PL, then  $T_1$  couldn't get lock on any other resources after that, then the subsequent operation  $w_1(D)$  of  $T_1$  cannot proceed. Thus, this schedule is not possible under 2PL protocol.

2)  $r_3(D), r_1(D), w_1(B), w_3(D), r_2(C), w_2(B), w_1(D), w_3(B)$

**Solution:** Let's try to add the lock/unlock steps to the schedule,  
 $l_3(D), r_3(D), \dots$

The next operation is  $r_1(D)$ , before that operation  $T_1$  needs to get a lock on  $D$ , however,  $D$  is currently locked by  $T_3$ . According to 2PL protocol,  $T_3$  won't release the lock on  $D$  until it locks  $B$ . Suppose  $T_3$  has already locked  $B$  at this moment. However,  $T_3$  won't release the lock on  $D$  until it has finished all the operations on  $D$  (after  $w_3(D)$ ). In other words, in this case it is impossible for  $r_1(D)$  to occur before  $w_3(D)$ . Thus, this schedule is not possible under 2PL protocol.

3)  $r_2(C), r_1(D), w_2(B), r_3(D), w_3(D), w_3(B), w_1(B), w_1(D)$

**Solution:** Let's try to add the lock/unlock steps to the schedule,  
 $l_2(C), r_2(C), l_1(D), r_1(D), l_2(B), u_2(C), w_2(B), u_2(B), \dots$

The next operation is  $r_3(D)$ ,  $T_3$  requires to lock  $D$  at this moment before proceeds, however,  $T_1$  won't release the lock on  $D$  before it locks  $B$ . Suppose  $T_1$  has already acquired the lock on  $B$  at this moment. However,  $T_1$  won't release the lock on  $D$  until it has finished all the operations on  $D$  (after  $w_1(D)$ ). In other words, in this case it is impossible for  $r_3(D)$  to occur before  $w_1(D)$ . Thus, this schedule is not possible under 2PL protocol.

4)  $r_1(D), r_2(C), w_2(B), r_3(D), w_1(B), w_3(D), w_3(B), w_1(D)$

**Solution:** Let's try to add the lock/unlock steps to the schedule,  
 $l_1(D), r_1(D), l_2(C), r_2(C), l_2(B), u_2(C), w_2(B), u_2(B), \dots$

The next operation is  $r_3(D)$ .  $T_3$  needs to lock  $D$  before this read, however,  $D$  is currently locked by  $T_1$ . According to 2PL,  $T_1$  won't unlock  $D$  until it has acquired all the locks on other resources. Suppose  $T_1$  has already acquired the lock on  $B$  at this moment. However,  $T_1$  won't release the lock on  $D$  until it has finished all the operations on  $D$  (after  $w_1(D)$ ). In other words, in this case it is impossible for  $r_3(D)$  to occur before  $w_1(D)$ . Thus, this schedule is not possible under 2PL protocol.

Here are some example schedules which are consistent with 2PL protocol.

1)  $r_3(D), r_2(C), w_3(D), w_2(B), r_1(D), w_3(B), w_1(B), w_1(D)$

Let's try to add the lock/unlock steps to the schedule:

$l_3(D), r_3(D), l_2(C), r_2(C), w_3(D), l_2(B), u_2(C), w_2(B), u_2(B), l_3(B), u_3(D), l_1(D), r_1(D), w_3(B), u_3(B), l_1(B), w_1(B), u_1(B), w_1(D), u_1(D).$

Pull out the lock/unlock operations of all the three transactions:

T1: l1(D), l1(B), u1(B), u1(D)

T2: l2(C), l2(B), u2(C), u2(B)

T3: l3(D), l3(B), u3(D), u3(B)

Which are consistent with the 2PL protocols.

2) r2(C), r1(D), w1(B), w2(B), w1(D), r3(D), w3(D), w3(B)

Let's try to add the lock/unlock steps to the schedule:

l2(C), r2(C), l1(D), r1(D), l1(B), w1(B), u1(B), l2(B), u2(C), w2(B), u2(B), w1(D), u1(D), l3(D), r3(D), w3(D), l3(B), u3(D), w3(B), u3(B).

Pull out the lock/unlock operations of all the three transactions:

T1: l1(D), l1(B), u1(B), u1(D)

T2: l2(C), l2(B), u2(C), u2(B)

T3: l3(D), l3(B), u3(D), u3(B)

Which are consistent with the 2PL protocols.

3. Let us begin with two bars: Cabana and Old Tavern. Cabana has local patrons A and remote patrons B, while Old Tavern has local patrons C and remote patrons D. Now a new bar New Tavern is opened in this area. Then Cabana and Old Tavern begin to lose clients. Suppose we have the followings two transactions:

T1: Cabana loses all patrons to Old Tavern. First locals, than the rest

T2: Old Tavern loses all patrons to New Tavern, first locals than the rest

Given the following schedule S:

T2: Insert local patrons of Old Tavern into New Tavern

T1: Insert local patrons of Cabana into Old Tavern

T2: Delete local patrons of Old Tavern

T1: Delete local patrons of Cabana

T2: Insert remote patrons of Old Tavern into New Tavern

T2: Delete remote patrons of Old Tavern

T1: Insert remote patrons of Cabana into Old Tavern

T1: Delete remote patrons of Cabana

What patrons will each bar have after the execution of this schedule (in terms of A, B, C, D or empty)? Is the schedule serializable (result equivalent to a serial schedule)?

**Solution:** Let's denote the local patrons and remote patrons of Cabana as C\_lp, C\_rp, respectively. Similarly, the local patrons and remote patrons of Old Tavern are OT\_lp, OT\_rp. For New Tavern, we have NT\_lp, NT\_rp for its local patrons and remote patrons. The initial values are:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
A	B	C	D	empty	empty

We will record the values for these variables after each operation in the schedule:

After T2 inserts local patrons of Old Tavern into New Tavern:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
A	B	C	D	C	empty

After T1 inserts local patrons of Cabana into Old Tavern:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
A	B	A, C	D	C	empty

After T2 deletes local patrons of Old Tavern:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
A	B	empty	D	C	empty

After T1 deletes local patrons of Cabana:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
empty	B	empty	D	C	empty

After T2 inserts remote patrons of Old Tavern into New Tavern:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
empty	B	empty	D	C	D

After T2 deletes remote patrons of Old Tavern:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
empty	B	empty	empty	C	D

After T1 inserts remote patrons of Cabana into Old Tavern:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
empty	B	empty	B	C	D

After T1 deletes remote patrons of Cabana:

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
empty	empty	empty	B	C	D

The results from the given schedule and two serial executions of T1, T2 are compared in the following table.

C_lp	C_rp	OT_lp	OT_rp	NT_lp	NT_rp
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Result	empty	empty	empty	B	C	D
T1 → T2	empty	empty	empty	empty	A,C	B,D
T2 → T1	empty	empty	A	B	C	D

The result is not equivalent to any of the serial schedule (T1 → T2, T2 → T1), so the given schedule is not serializable.