# Computer Science 112
# Data Structures

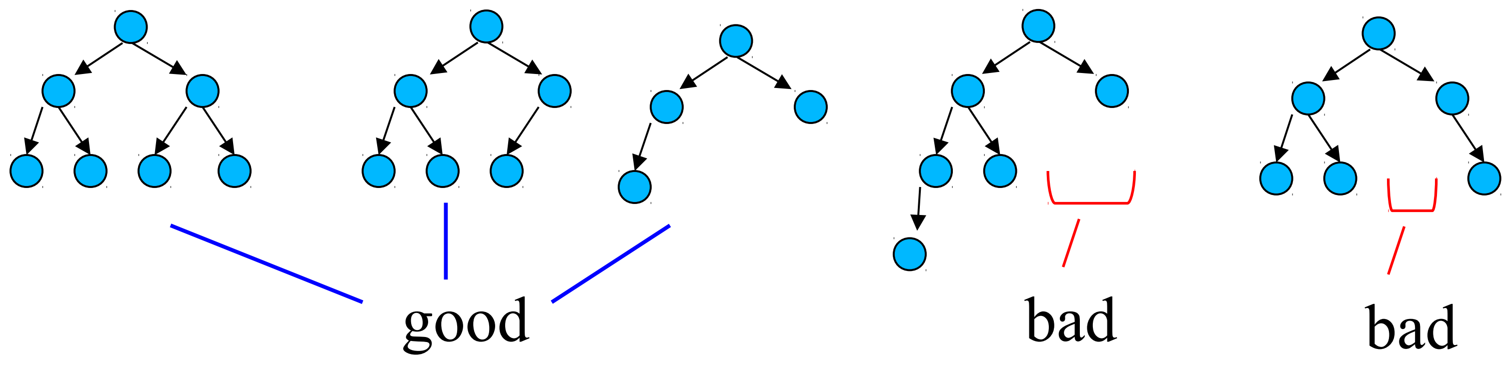
## Lecture 18:
## More Heaps
## A few practice questions for exam

## Review: Priority Queues

- **Each data item has a priority**
- **Add items to queue in any order**
- **Remove items in priority order**
  - **add A:5, B:3, C:6**
  - **remove C**
  - **add D:8**
  - **remove D, remove A**

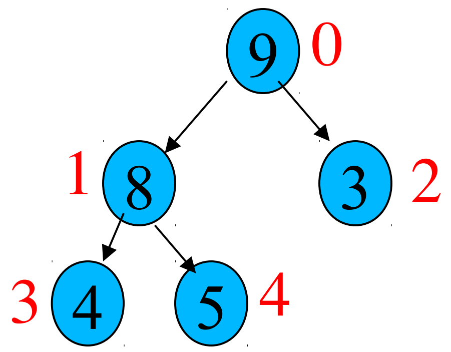
# Heap

- **A heap is a way to implement a priority queue with O(log n) complexity**
- **A heap is a complete binary tree**
  - **all levels except maybe the last are full**
  - **last level filled from left to right**



# Heap Representation

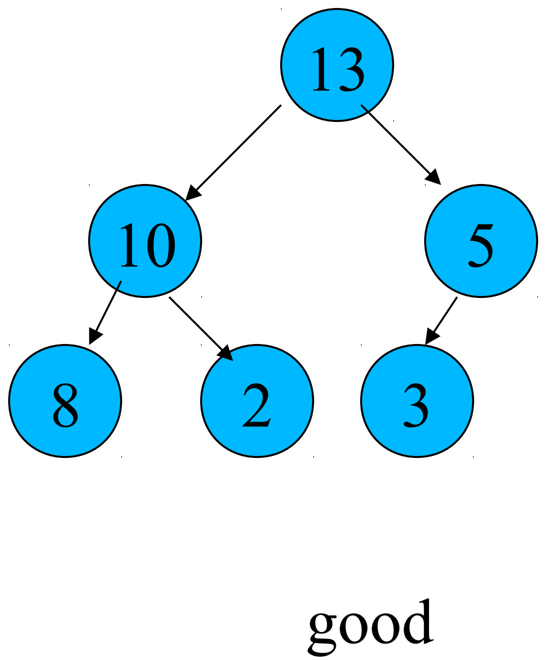
- Store heap in an array
  - For node at index j, children are at 2j+1 and 2j+2
  - Root at index **0**

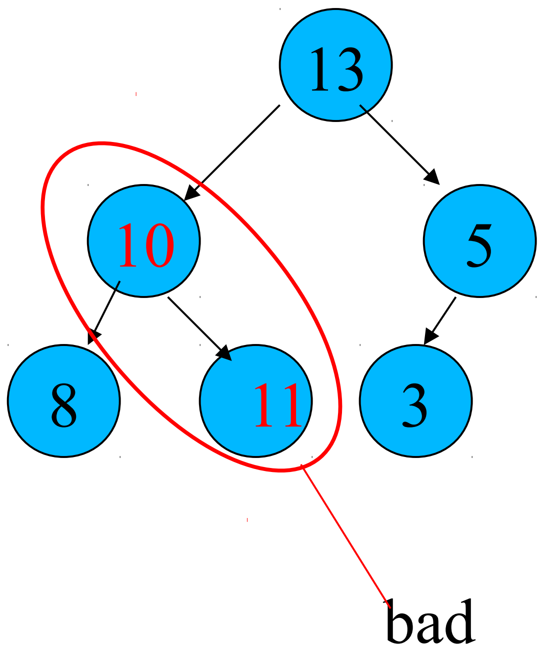


| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 9 | 8 | 3 | 4 | 5 |

## Heap

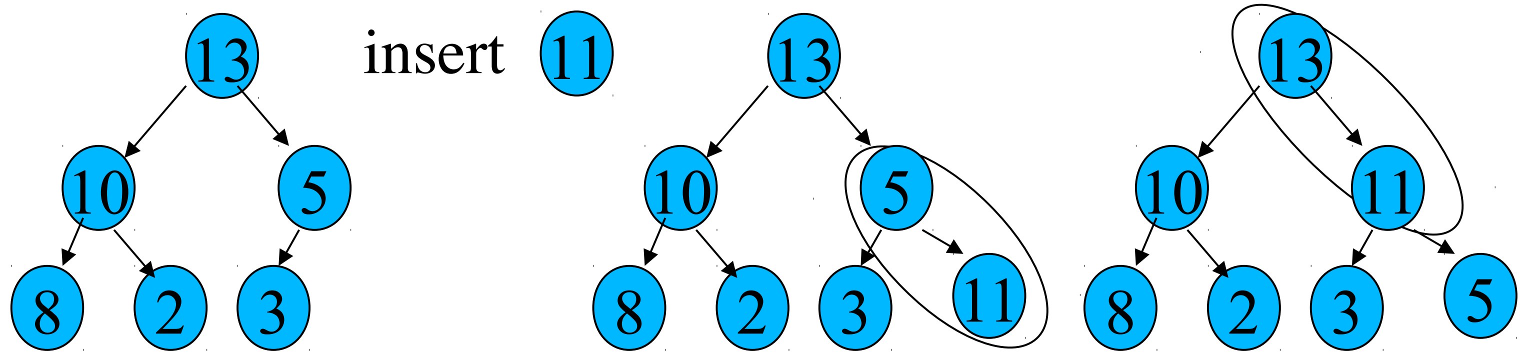
- **The number at a node is greater than the number at any descendant**

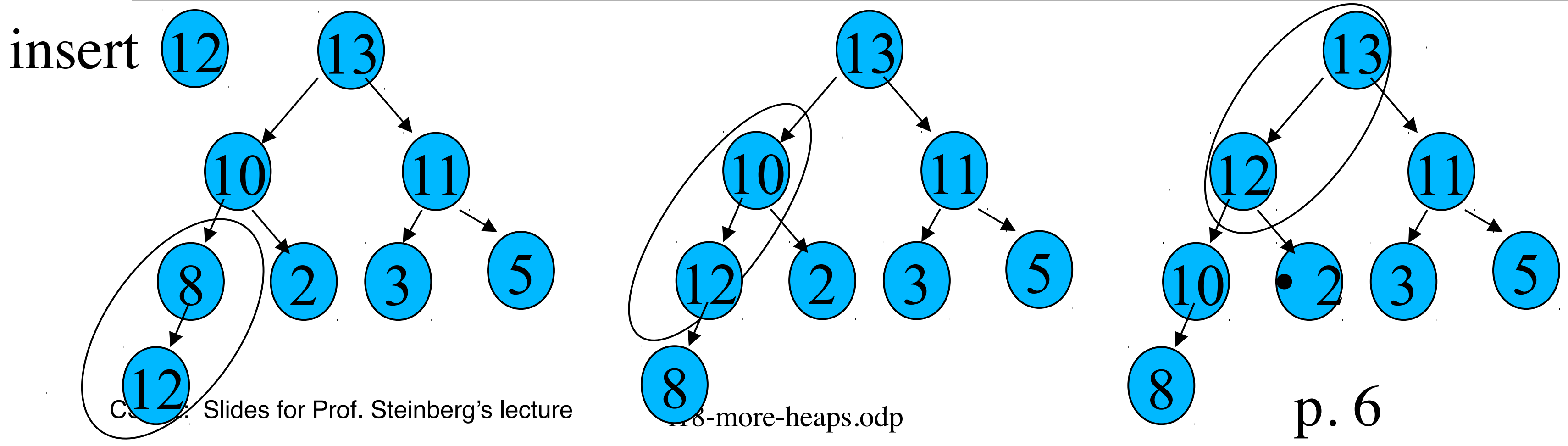




# Heap Insert

- **Add node at end of last level**
- **Move up restoring order (*filter up*)**



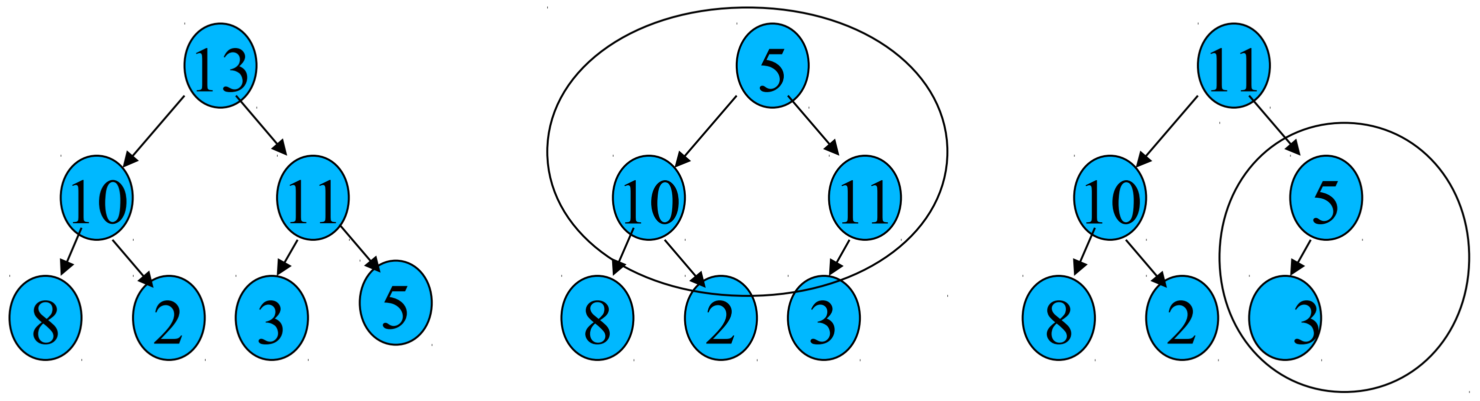


## Big O of insertion

- O(H) where H is height of while tree
  = O(log(n)) where n is number of nodes

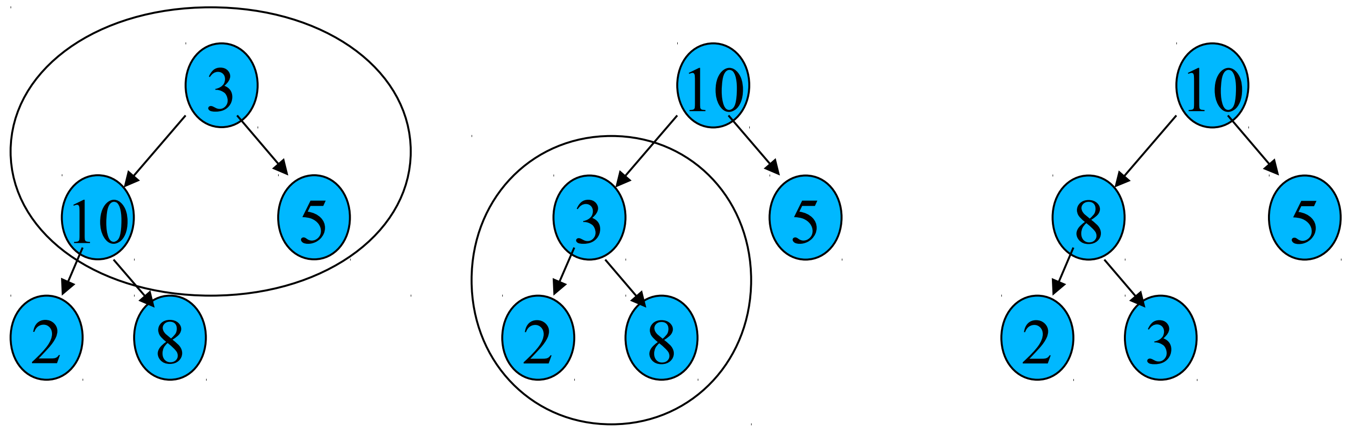
# Heap Deletion

- **Copy out data at root**
- **Delete last node on last row & put data in root**
- **Move down restoring order (*filter down*)**



## Heap Deletion

- Compare current node and two children
  - if current node largest, stop
  - if left child is largest swap current and left
  - similar if right child is largest



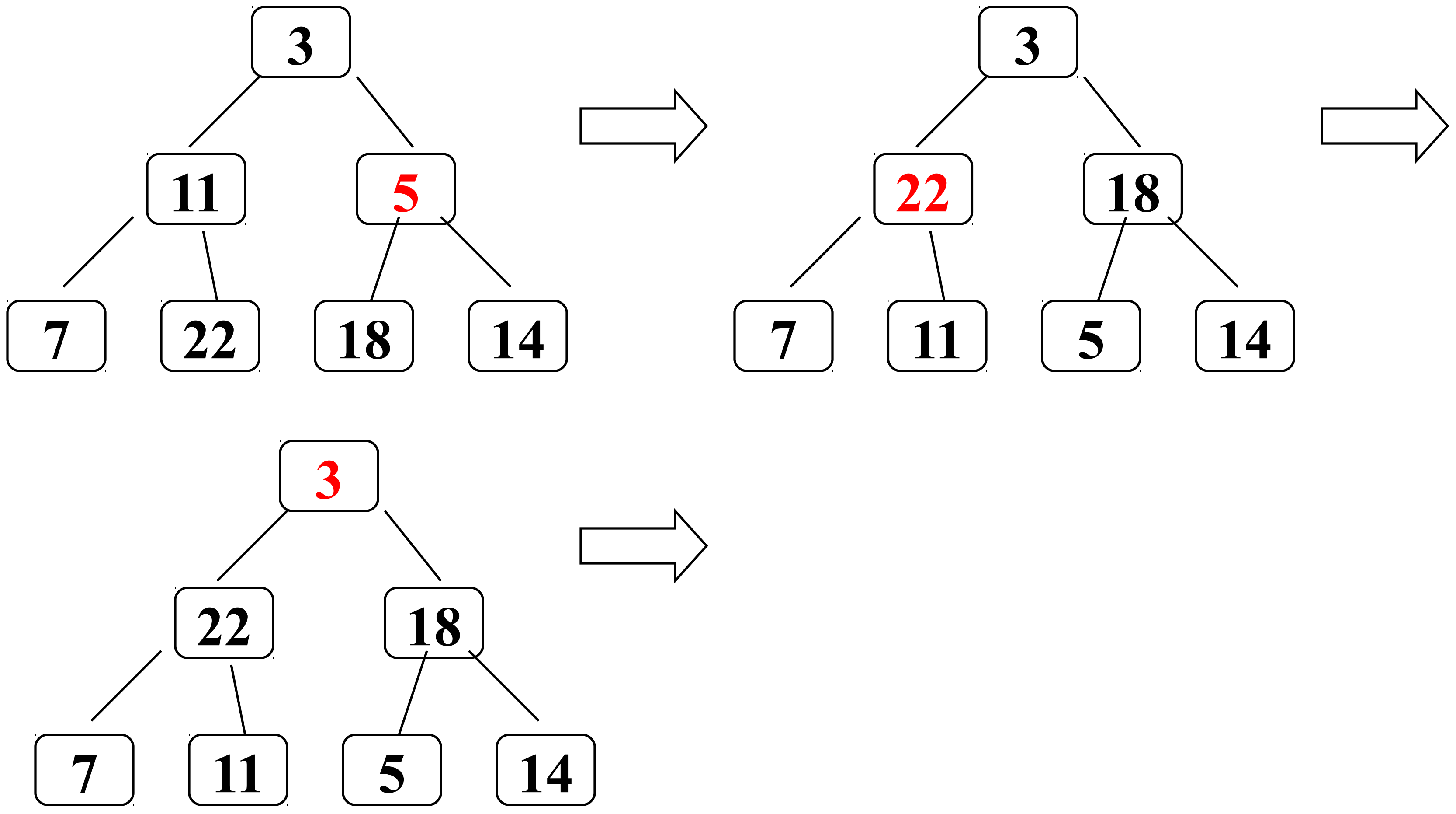
## Big O of deletion

- O(H) where H is height of whole tree
  = O(log(n)) where n is number of nodes

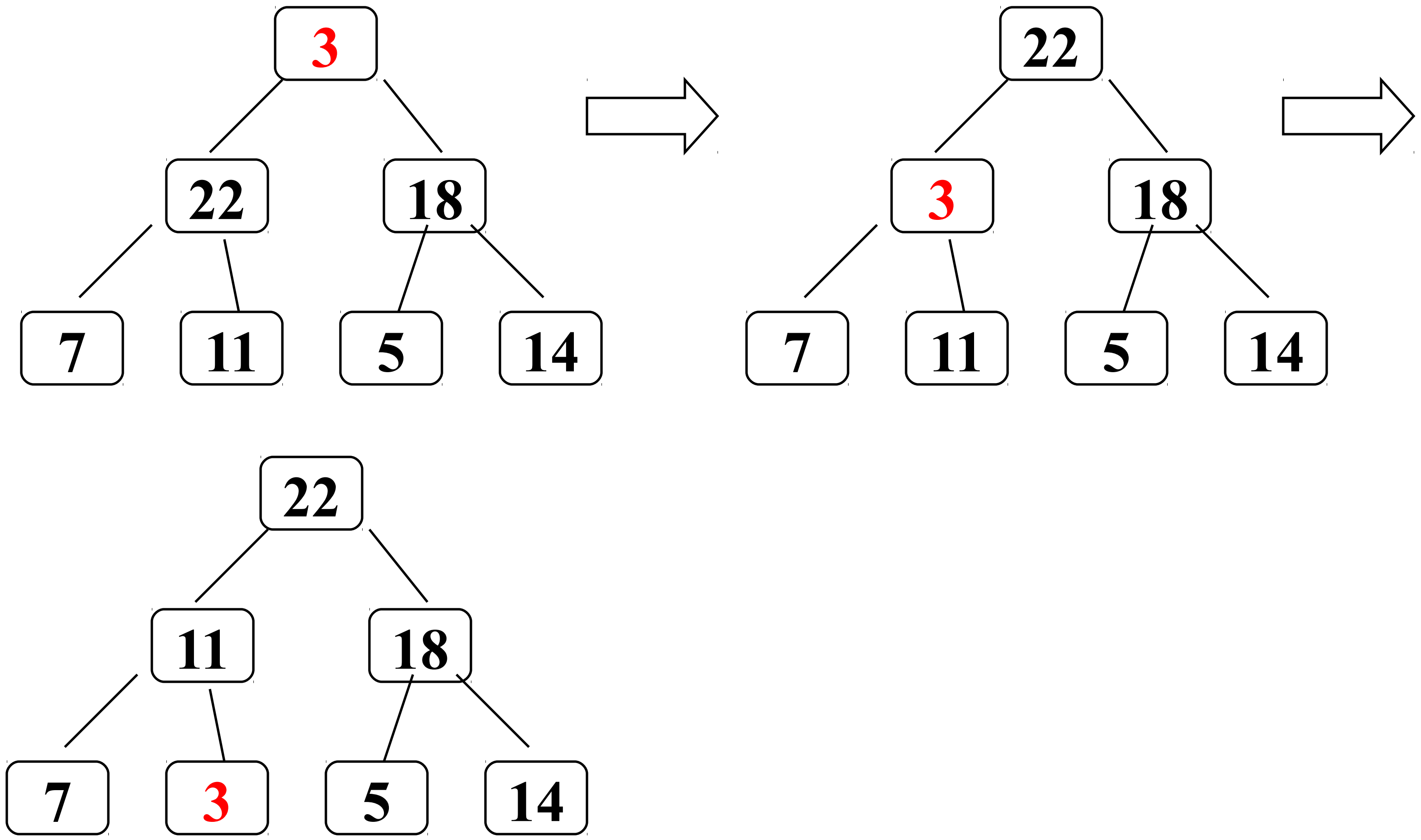
## Building a Heap from an Array

- Go from last non-leaf to index 0
  - At each node, do filter-down

# Building a Heap from an Array



# Building a Heap from an Array



## Building a Heap from an Array

- Work at a node is O(h) where h is height of subtree rooted at that node
- In a complete binary tree, majority of nodes close to bottom, so adds up to O(n)

## New:  Implementing a heap

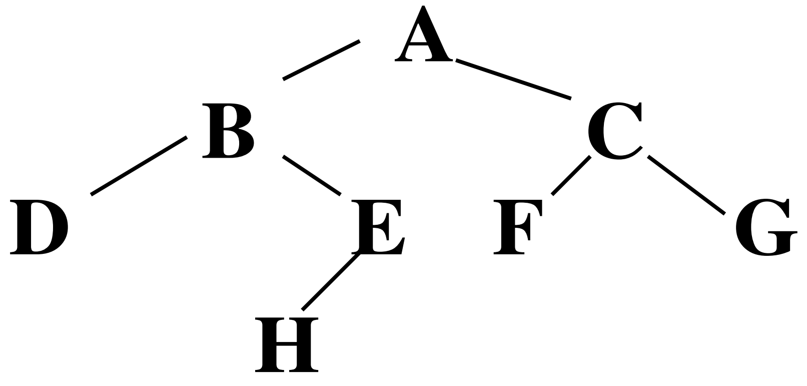
- See trees/Heap.java

# Heap Sort

- Insert all the data into the heap, one by one: n*O(log n) = O(n log n)
- Remove all the data from the heap, one by one: n*O(log n) = O(n log n)
- Total: O(n log n)

## Practice for exam

**For this tree, if we process the nodes in a pre-order traversal, in what order will the nodes be processed? Inorder?**



**Suppose you used an array to implement a hash table. Suppose each entry in the array is a hash bucket represented as a linked list, i.e. collisions are handled by chaining. Suppose the hash table is to represent a set of positive ints, and the hashing function is hash(n) = n mod 10 (ie remainder of n/10). Suppose we add the following numbers to the table:**

**100412 532 300 442 713**

**Assuming these numbers are representative of the data we will enter, have we chosen a good hash function? Why or why not?**

**Draw the hash table resulting from entering this data in the given order. Assume entries are added at the head of a list where possible.**

**100 412 532 300 442 713**

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  |  |  |

**Suppose we test each of the following numbers to see if it is in the table. (If a number is not in the table do not add it – table contents stay unchanged.) For each number, how many times do we compare it with a number in the hash table?**

**200**
**543**
**532**
**400**

**Suppose we had a BinarySearchTree made out of Nodes declared as follows:**

```
public class Node{
        public int data;
       public Node lst; // left sub tree
       public Node rst; // right sub tree
// …. etcetera}
```

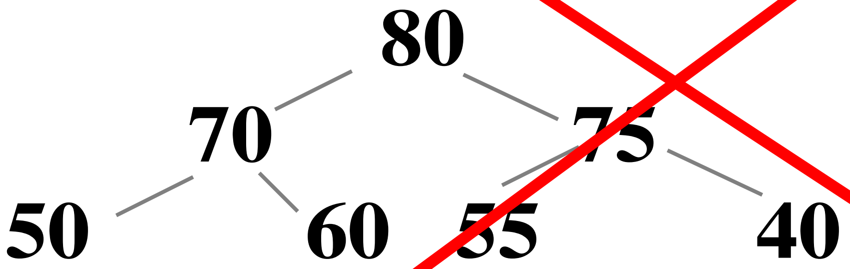
**Finish the following code to print out the data in a tree in numerical order.  It must be recursive.**

```
public static printData(Node root){


if (________________________){


System.out.println(root.data);
```

## Heaps not on exam 2 !!

~~For the MaxHeap below~~

~~What would be the first three numbers removed?~~

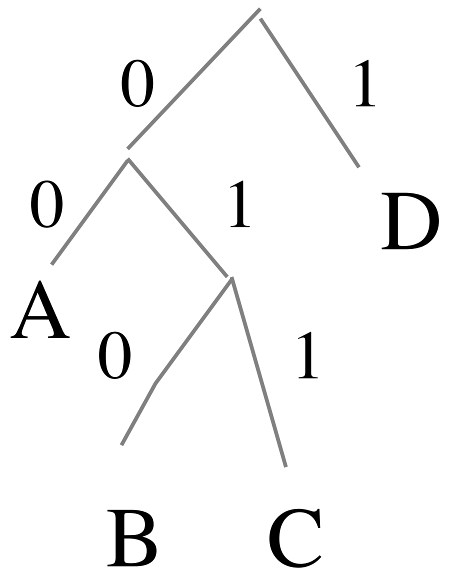


~~Redraw the heap as it would be after all three have been removed.~~

**For the Huffman code represented by the tree below,**

**What is the shortest code for any character?**

**What sequence of characters does 100100011001represent?**

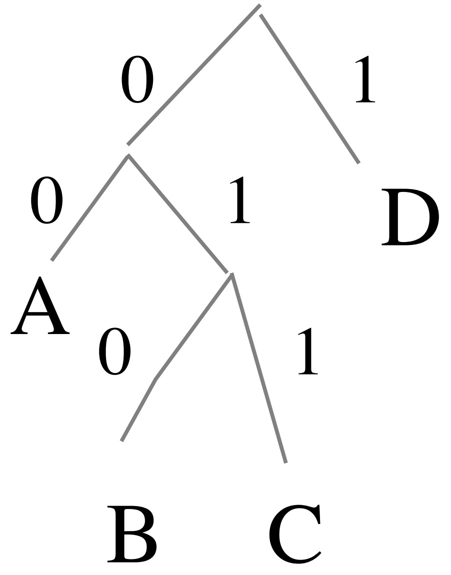


For the Huffman code represented by the tree below,

What is the shortest code for any character? 1 = D

What sequence of characters does 1|001|1|0001|1|001|1represent?

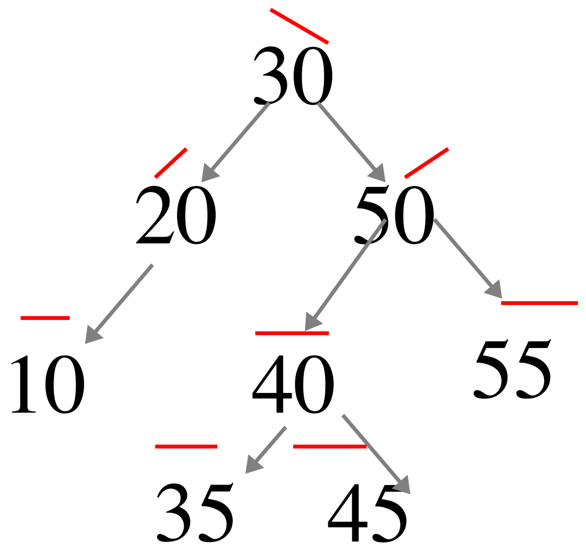
DADACAD



**Mark balance factors in the nodes of the following AVL tree. Then insert 36, and draw the resulting AVL tree.**

$$\begin{tikzcd}
 & & & 30 \arrow[dl] \arrow[dr] & & \\
 & & 20 \arrow[dl] & & 50 \arrow[dl] \arrow[dr] & \\
 & 10 & & 40 \arrow[dl] \arrow[dr] & & 55 \\
 & & 35 & & 45 &
\end{tikzcd}$$

## Mark balance factors in the nodes of the following AVL tree. Then insert 36, and draw the resulting AVL tree.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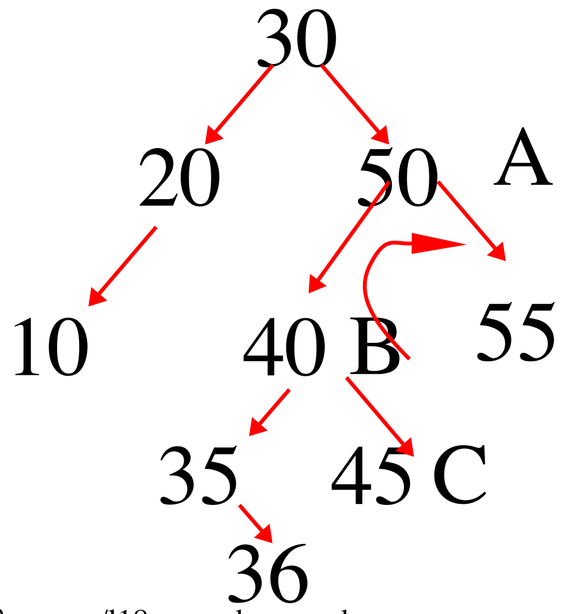
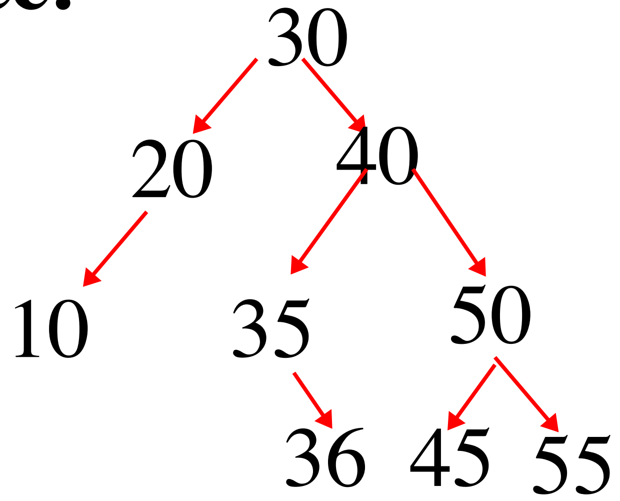

30
20
50
A
10
40 B
55
35
45 C
36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