MATH 354 HW#2 Section 1.2 # 8, 13, 14, 16

(8) Let x = # kilograms of PEST. Let y = # kilograms of BUG. We want to minimize $z = 3x + 2.5y$ subject to the constraints $30x + 40y \geq 120$, $40x + 20y \leq 80$, $x \geq 0$, $y \geq 0$.

In standard form we have

Maximize $w = -3x - 2.5y$

subject to: $-30x - 40y \leq -120$

$40x + 20y \leq 80$, $x \geq 0, y \geq 0$.

In canonical form we have

Maximize $w = -3x - 2.5y$

subject to: $-30x - 40y + u = -120$

$40x + 20y + v = 80$, $x \geq 0, y \geq 0$

The canonical form in matrix notation is

Maximize: $w = \begin{bmatrix} -3 & -2.5 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} = \bar{c}^T \bar{x}$

subject to: $\begin{bmatrix} -30 & -40 & 1 & 0 \\ 40 & 20 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} = \begin{bmatrix} -120 \\ 80 \end{bmatrix}$

or $A\bar{x} = \bar{b}$ $x \geq 0, y \geq 0, u \geq 0, v \geq 0$ or $\bar{x} \geq 0$

Here $\bar{c} = \begin{bmatrix} -3 \\ -2.5 \\ 0 \\ 0 \end{bmatrix}$, $\bar{x} = \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix}$, $A = \begin{bmatrix} -30 & -40 & 1 & 0 \\ 40 & 20 & 0 & 1 \end{bmatrix}$

and $\bar{b} = \begin{bmatrix} -120 \\ 80 \end{bmatrix}$. Written more clearly,

$$\boxed{\begin{array}{ll} \text{Maximize:} & w = \bar{c}^T \bar{x} \\ \text{Subject to:} & A\bar{x} = \bar{b} \\ & \bar{x} \geq 0 \end{array}}$$

(13) We write the standard form problem

Maximize: $z = x + 4y$

subject to: $3x + 4y \le 21$ in canonical form

$x + 2y \le 12$

$x \geq 0, y \geq 0$

Maximize: $z = x + 4y$

Subject to: $3x + 4y + u = 21$

$x + 2y + v = 12$ $\quad x \geq 0, y \geq 0$

$u \geq 0, v \geq 0$

a) Set $u = 3, v = 4$. Is there a feasible solution for these choices for u and v? If so, there must be positive values for x and y for which $3x + 4y + 3 = 21$

$x + 2y + 4 = 12$

That is, find if there are $x \geq 0, y \geq 0$ which are solutions of $3x + 4y = 18$, $x + 2y = 8$ $\rightarrow$

$$\begin{array}{r} 3x + 4y = 18 \\ -3x - 6y = -24 \\ \hline -2y = -6 \end{array}$$

So $y = 3$ and $x = 8 - 2(3) = 2$ gives a solution

Since $y = 3 \geq 0$ and $x \geq 2$, there is a feasible solution

Feasible solution $x = 2, y = 3, u = 3, v = 4$

b) Set $u = 18, v = 10$. Consider $3x + 4y + 18 = 18$

$x + 2y + 10 = 12$

this gives $3x + 4y = 0$, $x + 2y = 2$. The only $x \geq 0, y \geq 0$ for which $3x + 4y = 0$ will be $x = 0, y = 0$. However, these values do not give a solution to $x + 2y = 2$ (substituting $x = 0, y = 0$ gives $0 = 2$)

No feasible solutions for $u = 18, v = 10$.

(14) **Maximize:** $z = 2x + 5y$

subject to:

$$2x + 3y \le 10$$
$$5x + y \le 12$$
$$x + 5y \le 15$$
$$x \ge 0,\ y \ge 0$$

a) Is $\bar{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, that is $x=1, y=2$, a feasible solution?

Set $x=1, y=2$ into $2x+3y$ to get $2+6 = 8 \le 10$

Set $x=1, y=2$ into $5x+y$ to get $5+2 = 7 < 12$

Set $x=1, y=2$ into $x+5y$ to get $1+10 = 11 \le 15$

Yes, $\bar{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ satisfies all the constraints and is, therefore, a feasible solution.

b) Consider the canonical formulation of the above standard form problem

Maximize: $z = 2x + 5y$

Subject to

$$2x + 3y + u = 10$$
$$5x + y + v = 12$$
$$x + 5y + w = 15$$

For $x=1, y=2$, $2x+3y+u = 8 + u = 10$

For $x=1, y=2$, $5x+y+v = 7 + v = 12$

For $x=1, y=2$, $x+5y+w = 11 + v = 15$

Set $u = 2$

Set $v = 5$

Set $w = 4$

So, a feasible solution is: $x=1, y=2, u=2, v=5, w=4$

(16) Since $\bar{x}_1$ and $\bar{x}_2$ are feasible solutions $\bar{x}_1 \ge 0, \bar{x}_2 \ge 0$ and $A\bar{x}_1 \le \bar{b}$, $A\bar{x}_2 \le \bar{b}$. We must show that for $\bar{x} = r\bar{x}_1 + s\bar{x}_2$, $r+s=1$, $\bar{x} \ge 0$ and $A\bar{x} \le \bar{b}$

We have $A\bar{x} = A(r\bar{x}_1 + s\bar{x}_2) = rA\bar{x}_1 + sA\bar{x}_2 \le r\bar{b} + s\bar{b}$ (only if $r \ge 0, s \ge 0$)

So $A\bar{x} \le (r+s)\bar{b} = \bar{b}$ (If $r \ge 0, s \ge 0$)

If $r \ge 0, s \ge 0$, then $\bar{x} = r\bar{x}_1 + s\bar{x}_2 \ge 0 + 0 = 0$.

NOTE: There is an error in the problem statement. It must be assumed that $r \ge 0, s \ge 0$.