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Herramientas básicas - Actividad 1.1

1.1 In the area of Fig 1.4,

- **Which vertices are leaves and which are interior vertices?**

Leaves: 2,4,9,8,6. Interior vertices:1,3,5,7

- **Which vertices are the sons of 5?**

Sons of 5: 7 and 8

- **Which vertex is the father of 5?**

Father of 5: 3

- **What is the length of the path from 1 to 9?**

The length is 5

- **Which vertex is the root?**

The root is the number 1

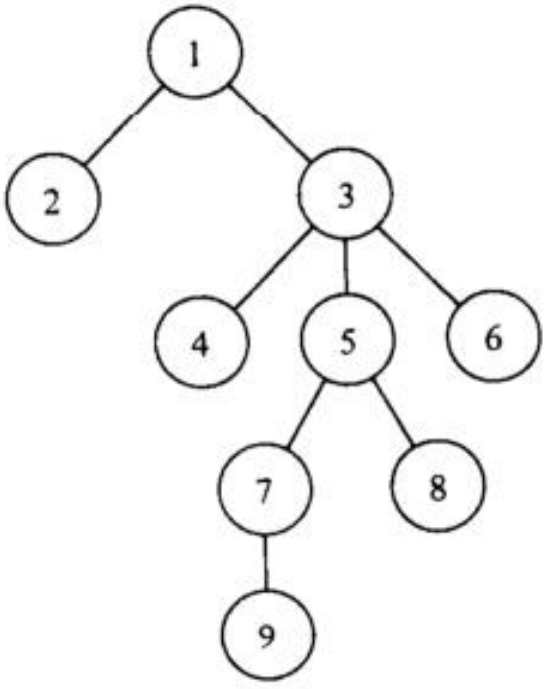

Fig. 1.4 A tree.

1.2 Prove by induction on n that

a) $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ b) $\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$i=0 \longrightarrow 0 = \frac{0(0+1)}{2} = 0$ TRUE $i=0$

$i=1 \longrightarrow 1 = \frac{1(2)}{2} = 1$ TRUE $i=1$

$i=2 \longrightarrow 1+2 = \frac{2(2+1)}{2} \longrightarrow 3 = 3$ TRUE $i=2$

Es verdadero para $0 \leq K \leq 2$, probar para $K+1$

$$\sum_{i=0}^{K} i + (K+1) = \frac{(K+1)(K+2)}{2}$$ ← queremos llegar a esta expresión

$$\sum_{i=0}^{K} i + (K+1) = \frac{(K)(K+1)}{2} + (K+1)$$

$$\frac{K^2+K}{2} + \frac{K+1}{1} = \frac{K^2+K+2K+2}{2} = \frac{K^2+3K+2}{2}$$

$$\frac{(K+1)(K+2)}{2}$$

Como son iguales se prueba por inducción que la formula es verdadera para $\forall n \in \mathbb{N}$

* Nomás se hizo la primera ya que se nos pidió solo hacer una de las dos demostraciones.