

Introduction. Exercises

1. I recommend you the following mathematical web <http://mathworld.wolfram.com>, you can use this web site when you like to remember some mathematical concept. For example you can remember the basics about the complex numbers or the unit circle.
2. Before doing the exercise see the video <https://www.youtube.com/watch?v=-RdOwhmqP5s>. In this video it is explained graphically Newton's method.

Consider the polynomial $p(x) = x^5 + x^2 - x + 1$. Newton's method applied to the polynomial p is given by

$$\begin{array}{ll} x_0 & \text{initial seed} \\ x_{n+1} = & x_n - \frac{p(x_n)}{p'(x_n)} \end{array}$$

- (a) Write the expression of the Newton's method applied to the polynomial p .
- (b) Using different initial seeds x_0 find an approximation of the five roots of p . You must use real and complex initial seeds, since the polynomial p has only one real root.
- (c) Divide the complex plane in a fine grid and using the approximation of the five roots and five different colors plot each point in the grid depending on the root which it converges.

You could use the following algorithm:

We consider the square $Q := \{z = x + iy, x \in [-2, 2], y \in [-2, 2]\}$. Let $N > 0$ and consider the grid of initial conditions $z_{\ell, m} = -2 + \frac{4\ell}{N} + i(-2 + \frac{4m}{N})$ for $\ell = 0, \dots, N$ and $m = 0, \dots, N$. Now, for each point $z_{\ell, m}$ compute Newton's iterates and depending on the root which it converges plot $z_{\ell, m}$ with a different color.