# Chaotic Dynamical Systems

Contents

(a) **Part 1. Introduction.** *(1 week).*

(b) **Part 2. The Logistic Map.** *(7 weeks).*

In this chapter we introduce the basic concepts of a dynamical system defined in the unit interval or in the real line. We split the contents of this part into seven pieces, each one corresponds to one week. The logistic map is given by the iteration of the map, given by

$$f_\mu(x) = \mu x(1 - x)$$

where $\mu$ is a real parameter. There are many references to the dynamics of this map for concrete values of the parameter $\mu$. You can visualize these examples using the applet *The Logistic Map* in the web page http://deim.urv.cat/∼antonio.garijo.

- **Orbits**. Contents of Chapter 1, sec. 1.3 and sec.1.4.

  The author introduces the concept of an orbit, and the different types of orbits. Using the graph of the map, the graphical analysis is a very useful tool to visualize the iteration process. Periodic orbits play a fundamental role in the study on the long term behavior. Given a periodic point it is crucial to know if this orbit is attracting, repelling or indifferent.

- **The logistic family.** Contents of sec. 1.3 Example 4.10 and sec. 1.5

  In this section the author presents a preliminary study of the dynamics of the map $f_\mu(x) = \mu x(1 - x)$.

- **Chaos.** Contents of sec. 1.7 and 1.8

  In the sec. 1.8 the concept of Chaos is introduced. Example 8.9 is about the Logistic map.

- **Sarkovskii's Theorem.** Contents of sec. 1.10

  This fundamental Theorem explains basically that having a periodic orbit of some period "forces" the existence of periodic orbits of other periods. Thus, for example if you have a periodic orbits of period three, forces the map to have periodic orbits of all the periods. You can try to understand this proof from sec. 1.10. Also it is possible to read the paper "Period Three Implies Chaos" from the web site http://faculty.washington.edu/joelzy/LiYorke_period3.pdf.

- **Bifurcation theory** Contents of sec. 1.12 and sec. 1.16

  Basically we present the way that real dynamical systems change. The easiest case is the different types of bifurcations can occur in the behavior of a fixed point. Two basics bifurcations are presented the saddle-node and the period doubling.

- **The period-doubling route to Chaos** Contents of sec. 1.17

  In this section is presented a route to arrive to Chaos starting from a hyperbolic behavior. This process is given doubling the period of the attracting cycle. In this section this process is explained in detail and taking as a basic example the Logistic map. Could be useful to visualize this process in the applet *The Logistic Map* in the web page http://deim.urv.cat/~antonio.garijo.

- **Bifurcation diagram for the logistic map.** Contents of sec. 1.13 and sec. 1.17

  The bifurcation diagram of the logistic map is a simple way to visualize how the dynamics of the Logistic map changes when we move the parameter $\mu$.

**Reference Book:** *An introduction to Chaotic Dynamical Systems.* R. L. Devaney. Addison-Wesley Studies in Nonlinearity. 1989.