

1. Estructura óptima del problema

$$e[i, j] = \begin{cases} q_{i-1} & ; \quad j = i - 1 \\ \min_{i \leq r \leq j} \{e[i, r - 1] + e[r + 1, j] + w(i, j)\} & ; \quad i \leq j \end{cases}$$

$$w(i, j) = \sum_{l=i}^j p_l + \sum_{l=i-1}^j q_l$$

2. Algoritmo recurrente

```
1: procedure BUILD_OPTBINTREE( $P, Q$ )
2:   return BUILD_OPTBINTREE_REC( $P, Q, 1, |P|$ )
3: end procedure
```

```
1: procedure BUILD_OPTBINTREE_REC( $P, Q, i, j$ )
2:   if  $j = i - 1$  then
3:     return  $Q[i - 1]$ 
4:   else
5:      $w \leftarrow$  DUMMY_WEIGHT( $P, Q, i, j$ )
6:      $e \leftarrow \infty$ 
7:     for  $r \leftarrow i$  to  $j$  do
8:        $v_l \leftarrow$  BUILD_OPTBINTREE_REC( $P, Q, i, r - 1$ )
9:        $v_r \leftarrow$  BUILD_OPTBINTREE_REC( $P, Q, r + 1, j$ )
10:       $v \leftarrow v_l + v_r + w$ 
11:      if  $v < e$  then
12:         $e \leftarrow v$ 
13:      end if
14:    end for
15:    return  $e$ 
16:   end if
17: end procedure
```

```
1: procedure DUMMY_WEIGHT( $P, Q, i, j$ )
2:    $w \leftarrow Q[i - 1]$ 
3:   for  $l \leftarrow i$  to  $j$  do
4:      $w \leftarrow w + P[l] + Q[l]$ 
5:   end for
6:   return  $w$ 
7: end procedure
```

3. Algoritmo recorrente «memoizado»

```
1: procedure BUILD_OPTBINTREE( $P, Q$ )
2:   let  $W$  be a matrix  $[1..|P|] \times [1..|P|]$ 
3:   let  $M$  be a matrix  $[0..|P|] \times [0..|P|]$ 
4:    $W \leftarrow 0$ 
5:    $M \leftarrow 0$ 
6:   for  $i \leftarrow 1$  to  $|P|$  do
7:      $W[i, i] \leftarrow Q[i - 1] + P[i] + Q[i]$ 
8:      $M[i, i] \leftarrow Q[i - 1]$ 
9:     for  $j \leftarrow i + 1$  to  $|P|$  do
10:       $W[i, j] \leftarrow W[i, j - 1] + P[j] + Q[j]$ 
11:       $M[i, j] \leftarrow \infty$ 
12:    end for
13:  end for
14:  return BUILD_OPTBINTREE_REC( $P, Q, 1, |P|, M, W$ )
15: end procedure
```

```
1: procedure BUILD_OPTBINTREE_REC( $P, Q, i, j, M, W$ )
2:   if  $M[i, j] = \infty$  then
3:     if  $j = i - 1$  then
4:        $M[i, j] \leftarrow Q[i - 1]$ 
5:     else
6:       for  $r \leftarrow i$  to  $j$  do
7:          $v_l \leftarrow \text{BUILD\_OPTBINTREE\_REC}(P, Q, i, r - 1, M, W)$ 
8:          $v_r \leftarrow \text{BUILD\_OPTBINTREE\_REC}(P, Q, r + 1, j, M, W)$ 
9:          $v \leftarrow v_l + v_r + W[i, j]$ 
10:        if  $v < M[i, j]$  then
11:           $M[i, j] \leftarrow v$ 
12:        end if
13:      end for
14:    end if
15:  end if
16:  return  $M[i, j]$ 
17: end procedure
```

La tabla de memoización tiene la siguiente forma:

$M[i, j]$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	$Q[0]$	$M[1, 1]$	$M[1, 2]$	$M[1, 3]$	$M[1, 4]$	$M[1, 5]$
2	0	$Q[1]$	$M[2, 2]$	$M[2, 3]$	$M[2, 4]$	$M[2, 5]$
3	0	0	$Q[2]$	$M[3, 3]$	$M[3, 4]$	$M[3, 5]$
4	0	0	0	$Q[3]$	$M[4, 4]$	$M[4, 5]$
5	0	0	0	0	$Q[4]$	$M[5, 5]$

Y w puede ser pre-calculado:

$W[i, j]$	1	2	3	4	5
1	$Q[0] + P[1] + Q[1]$	$W[1, 1] + P[2] + Q[2]$	$W[1, 2] + P[3] + Q[3]$	$W[1, 3] + P[4] + Q[4]$	$W[1, 4] + P[5] + Q[5]$
2	0	$Q[1] + P[2] + Q[2]$	$W[2, 2] + P[3] + Q[3]$	$W[2, 3] + P[4] + Q[4]$	$W[2, 4] + P[5] + Q[5]$
3	0	0	$Q[2] + P[3] + Q[3]$	$W[3, 3] + P[4] + Q[4]$	$W[3, 4] + P[5] + Q[5]$
4	0	0	0	$Q[3] + P[4] + Q[4]$	$W[4, 4] + P[5] + Q[5]$
5	0	0	0	0	$Q[4] + P[5] + Q[5]$

4. Algoritmo «bottom-up» con la solución

```

1: procedure BUILD_OPTBINTREE( $P, Q$ )
2:   let  $W$  be a matrix  $[1..|P|] \times [1..|P|]$ 
3:   let  $M$  be a matrix  $[0..|P|] \times [0..|P|]$ 
4:   let  $R$  be a matrix  $[1..|P|] \times [1..|P|]$ 
5:    $W \leftarrow 0$ 
6:    $M \leftarrow 0$ 
7:    $R \leftarrow 0$ 
8:   for  $i \leftarrow 1$  to  $|P|$  do
9:      $W[i, i] \leftarrow Q[i - 1] + P[i] + Q[i]$ 
10:     $M[i, i] \leftarrow Q[i - 1]$ 
11:    for  $j \leftarrow i + 1$  to  $|P|$  do
12:       $W[i, j] \leftarrow W[i, j - 1] + P[j] + Q[j]$ 
13:       $M[i, j] \leftarrow \infty$ 
14:    end for
15:  end for
16:  for  $l \leftarrow 1$  to  $|P|$  do
17:    for  $i \leftarrow 1$  to  $|P| - l + 1$  do
18:       $j \leftarrow i + l - 1$ 
19:      for  $r \leftarrow i$  to  $j$  do
20:         $v \leftarrow M[i, r - 1] + M[r + 1, j] + W[i, j]$ 
21:        if  $v < M[i, j]$  then
22:           $M[i, j] \leftarrow v$ 
23:           $R[i, j] \leftarrow r$ 
24:        end if
25:      end for
26:    end for
27:  end for
28:  return  $R$ 
29: end procedure

```
