Sentiment and Supply Chains: Endogenous Production Networks under Uncertainty*

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Abstract

We study the formation of production networks when firms possess dispersed, affiliated information about aggregate productivity. Firms make extensive margin decisions (choosing supplier sets) and intensive margin decisions (input quantities) under uncertainty. We characterize the economy as a supermodular Bayesian game. Using lattice-theoretic methods, we prove the existence of extremal monotone Bayesian Nash equilibria where firms with optimistic private signals adopt denser sets of inputs. We identify a "sentiment multiplier" mechanism: because signals are affiliated, a firm's optimism rationally raises its expectation of others' optimism, leading to coordinated network expansions that lower equilibrium prices and validate the initial sentiment. We decompose network volatility into fundamental and strategic components, showing that higher signal correlation amplifies volatility while higher signal precision dampens it. The model suggests that opacity in supply chains acts as an amplifier of aggregate shocks.

Keywords: Production networks, dispersed information, strategic complementarities, supermodular games, affiliation, sentiment multiplier.
JEL Codes: D21, D83, D85, L14, E32.

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1. Introduction

Modern supply chains are characterized by two salient features: complexity and opacity. A manufacturer deciding whether to invest in a new specialized supplier relationship or adopt a capital-intensive logistics technology rarely observes the precise productivity of that partner, the aggregate state of demand, or upstream capacity constraints. Instead, decisions are made on the basis of dispersed, noisy signals: procurement delays, industry chatter, small price movements, and local order books. These signals are naturally correlated across firms because they reflect common macroeconomic and sectoral factors.

This paper asks: *How does dispersed, correlated information interact with the endogenous formation of production networks?* Networks are not passive objects that merely transmit shocks; they are constructed by agents acting on beliefs. When firms cannot perfectly disentangle fundamentals from correlated noise, "sentiment" becomes a driver of real economic structure.

We develop a model of endogenous production network formation under private information. We build on the framework of Acemoglu and Azar (2020), where firms choose technology sets to minimize unit costs. We depart from the complete information benchmark by assuming that aggregate productivity is unobserved. Instead, firms receive private signals that are *affiliated* in the sense of Milgrom and Weber (1982). This information structure introduces a distinct strategic channel. In standard production network models, complementarities arise solely from technology: a decrease in supplier prices encourages downstream expansion. In our environment, complementarities also arise from inference.

Our analysis yields three main theoretical contributions.

First, we establish the existence of **monotone Bayesian Nash equilibria** in the game of network formation. The space of possible production networks is high-dimensional and discrete. However, we show that under general assumptions on production technology (homogeneity of degree one and labor essentiality) and information (affiliation), the induced game is supermodular. Using lattice-theoretic methods—specifically Tarski's fixed point theorem applied to the interim strategy space—we prove that firms with more optimistic private signals monotonically expand their supplier sets. This result implies the existence of distinct "optimistic" (high density) and "pessimistic" (low density)

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regimes for the same underlying fundamentals, driven by the coordination of beliefs.

Second, we identify a **sentiment multiplier**. In standard Keynesian models, multipliers arise from demand externalities. Here, the multiplier is supply-side and structural. When firms become optimistic, they form denser networks. This expansion lowers unit costs and equilibrium prices throughout the economy, making further expansion profitable for others. We decompose the equilibrium response to a shift in beliefs into a direct fundamental term and a strategic term. We show that the strategic channel amplifies the fundamental response, effectively generating "excess volatility" in network density relative to fundamental productivity shocks.

Third, we provide comparative statics with respect to the **information structure**. We show that increasing the precision of private signals reduces network volatility, as firms place less weight on the correlated prior and more on the idiosyncratic realization. Conversely, increasing the correlation of error terms across firms (without changing their marginal variance) increases network volatility. This result suggests that information transparency—reducing the correlation of forecast errors—is a distinct policy tool for stabilizing supply chains.

Related Literature. This paper bridges the gap between the production networks literature and the literature on global games and dispersed information. The foundational insight that network structure matters for aggregate volatility dates to Long and Plosser (1983) and was formalized by Horvath (2000), Dupor (1999), and Gabaix (2011). Acemoglu et al. (2012) showed that heavy-tailed degree distributions can generate aggregate fluctuations from idiosyncratic shocks, overturning the law of large numbers. Empirical work by Atalay et al. (2011), Atalay (2017), and Foerster et al. (2011) quantifies the role of sectoral linkages.

The theoretical framework for production networks was developed by Hulten (1978), extended by Jones (2011) and Baqaee and Farhi (2019), Baqaee (2018), and Baqaee and Farhi (2020). On endogenous network formation, Oberfield (2018) and Acemoglu and Azar (2020) provide key foundations, while Liu (2019) and Bigio and La'O (2020) study policy implications. Recent work on supply chain disruptions includes Barrot and Sauvagnat (2016), Boehm et al. (2019), Carvalho et al. (2021), and Acemoglu and Tahbaz-Salehi (2020). On firm het-

erogeneity in networks, see Bernard et al. (2022) and Boehm and Oberfield (2020).

We build directly on Acemoglu and Azar (2020), extending their complete-information analysis to a Bayesian setting. We differ from Kopytov et al. (2024), who also study uncertainty in networks. Their mechanism relies on risk aversion (second-moment effects): firms diversify to insure against variance. In contrast, our agents are risk-neutral cost minimizers; our results are driven by strategic complementarities in *beliefs* (first-moment effects). We show that even without risk aversion, the coordination motive inherent in supply chains creates fragility.

Our information-theoretic approach connects to the uncertainty literature: Bloom (2009), Bloom (2014), Bloom et al. (2018), Baker et al. (2016), and Jurado et al. (2015) on uncertainty shocks; Fajgelbaum et al. (2017) on uncertainty traps; Nieuwerburgh and Veldkamp (2006) on learning in business cycles. On information in networks, Elliott et al. (2022) studies fragility from strategic link formation, while Herskovic (2018) examines asset pricing implications. The methodology builds on Topkis (1998), Milgrom and Shannon (1994), and the Bayesian games literature of Van Zandt and Vives (2007) and Morris and Shin (2002).

The remainder of the paper is organized as follows. Section 2 defines the information structure. Section 3 describes the production environment. Section 4 characterizes the belief hierarchy. Section 5 establishes strategic complementarities. Section 6 proves the existence of monotone equilibria. Section 7 formalizes the sentiment multiplier. Section 8 presents comparative statics. Section 9 sketches a dynamic extension. Section 10 concludes.

2. INFORMATION STRUCTURE

We begin by defining the probabilistic environment, which is foundational to the strategic analysis. Consider an economy with n firms indexed by $\mathcal{I} = \{1, \ldots, n\}$. The fundamental state of the economy is described by a random variable $\mu \in \mathcal{M} \subseteq \mathbb{R}$, representing aggregate productivity. This state is unobserved.

Each firm i observes a private signal $s_i \in \mathcal{S}_i \subseteq \mathbb{R}$. Let $\mathbf{s} = (s_1, \ldots, s_n)$ denote the profile of signals. The joint distribution of $(\mu, \mathbf{s})$ is governed by a cumulative distribution function $F(\mu, \mathbf{s})$ with a strictly positive density $f(\mu, \mathbf{s})$ with respect

to a product measure.

2.1. Affiliation and Stochastic Dominance

To capture the idea that signals are correlated reflections of the same underlying reality, we assume the joint distribution satisfies *affiliation*, a strong form of positive dependence introduced by Milgrom and Weber (1982).

Definition 1 (Affiliation). The random variables $Z = (\mu, s_1, \ldots, s_n)$ are *affiliated* if their joint density f is log-supermodular. That is, for all z, z' in the support of f:

$$f(z \vee z')f(z \wedge z') \geq f(z)f(z'), \tag{1}$$

where $\vee$ and $\wedge$ denote the component-wise maximum and minimum, respectively.

Affiliation implies that a high realization of one variable makes high realizations of the other variables more likely in the sense of the Monotone Likelihood Ratio Property (MLRP).

Assumption 1 (Affiliated Information). The vector $(\mu, s_1, \ldots, s_n)$ is affiliated.

This assumption allows us to order beliefs and higher-order beliefs unambiguously. We invoke the following key properties from Milgrom and Weber (1982).

Theorem 1 (Properties of Affiliated Beliefs). *Under Assumption 1:*

  (i) **MLRP:** *The conditional density $f(\mu \mid s_i)$ satisfies the Monotone Likelihood Ratio Property in s_i.*

 (ii) **Stochastic Dominance:** *If $s_i' > s_i$, then the posterior distribution of μ given s_i' dominates the distribution given s_i in the first-order sense (FOSD).*

(iii) **Ordered Beliefs about Others:** *The conditional distribution of the vector of others' signals $\mathbf{s}_{-i}$ given s_i is increasing in s_i in the multivariate FOSD sense. Specifically, for any non-decreasing function $g(\mathbf{s}_{-i})$, the expectation $\mathbb{E}[g(\mathbf{s}_{-i}) \mid s_i]$ is non-decreasing in s_i.*

Property (iii) is the engine of our strategic analysis. It implies that an optimistic firm (observing high s_i) rationally expects other firms to be optimistic as well.

2.2. Leading Example: Gaussian Common Factor

Throughout the paper, we use the Gaussian structure to provide closed-form intuition.

Example 1 (Gaussian Signals). Let $\mu \sim \mathcal{N}(\mu_0, \sigma_\mu^2)$. Each firm observes:

$$s_i = \mu + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2),$$

where ε_i are i.i.d. across firms and independent of μ. The joint distribution is multivariate normal with covariance $\text{Cov}(s_i, s_j) = \sigma_\mu^2 \geq 0$, which implies affiliation.

The posterior expectation of μ is linear:

$$\mathbb{E}[\mu \mid s_i] = (1 - \rho)\mu_0 + \rho s_i, \quad \text{where } \rho = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2}. \tag{2}$$

Crucially, firm i's expectation of firm j's signal is:

$$\mathbb{E}[s_j \mid s_i] = \mathbb{E}[\mathbb{E}[s_j \mid \mu] \mid s_i] = \mathbb{E}[\mu \mid s_i] = (1 - \rho)\mu_0 + \rho s_i. \tag{3}$$

This explicit linearity allows us to quantify the strength of strategic feedback using the parameter ρ.

3. PRODUCTION ENVIRONMENT

We adopt the production framework of Acemoglu and Azar (2020), adapted to an incomplete information setting. There are n sectors. Each sector i produces a distinct good using labor L_i and intermediate inputs.

3.1. Technology and Costs

Firm i makes an extensive margin choice: it selects a set of suppliers $S_i \subseteq \mathcal{I} \setminus \{i\}$. Given S_i, the production function is:

$$Y_i = \theta_i(\mu) F_i(S_i, L_i, \{X_{ij}\}_{j \in S_i}), \tag{4}$$

where $\theta_i(\mu)$ is a productivity shifter strictly increasing in μ, and F_i is the aggregator.

Assumption 2 (Technology). For all i and S_i:

  (i) F_i is continuous, concave, strictly increasing, and homogeneous of degree one (CRS) in inputs.

 (ii) Labor is essential: $F_i(S_i, 0, \cdot) = 0$.

(iii) **Technological Monotonicity:** For any price vector P, the unit cost achievable with a larger supplier set $S_i' \supset S_i$ is weakly lower than with S_i.

Firms operate in competitive markets. Given a state μ, a network $S = (S_1, \ldots, S_n)$, and a price vector P, the unit cost for firm i is derived from cost minimization:

$$K_i(S_i, \mu, P) = \frac{1}{\theta_i(\mu)} \min_{L_i, \{X_{ij}\}} \left\{ L_i + \sum_{j \in S_i} P_j X_{ij} \mid F_i(\cdot) = 1 \right\}. \tag{5}$$

We normalize the wage $w = 1$.

3.2. *Market Clearing and Equilibrium Prices*

In the production stage (after μ is realized and S is fixed), prices must equal unit costs. The equilibrium price vector $P^*(\mu, S)$ is the fixed point of:

$$P_i = K_i(S_i, \mu, P) \quad \forall i \in \mathcal{I}. \tag{6}$$

Proposition 1 (Existence and Uniqueness of Prices). *Under Assumption 2, for any μ and network S, there exists a unique strictly positive price vector $P^*(\mu, S)$ solving* (6). *Furthermore, P^* is decreasing in μ and non-increasing in S (under the inclusion order).*

Proof. See Acemoglu and Azar (2020). The proof relies on the fact that the Jacobian $I - \frac{\partial K}{\partial P}$ is an M-matrix (a class of P-matrices) due to the essentiality of labor, guaranteeing global univalence via the Gale-Nikaido theorem. □

Corollary 1 (Uniqueness of Allocations). *Given the unique equilibrium price vector $P^*(\mu, S)$, the allocations (X^*, Y^*, C^*, L^*) are uniquely determined.*

Proof. Given P^* and the technology choice S_i, the factor demands L_i^* and X_i^* are uniquely determined by the strictly convex cost minimization problem (**??**). The output Y_i^* follows from market clearing: $Y_i^* = F_i(S_i, L_i^*, X_i^*)$. Aggregate labor supply pins down the scale via $\sum_i L_i^* = 1$. Consumer demands C^* are uniquely determined by utility maximization given P^* and income. By the same Gale-Nikaido argument, this system has a unique solution. □

3.3. The Network Formation Game

The extensive margin decision is made *ex ante*. Firm i observes s_i and chooses S_i to minimize expected unit costs. The strategy of firm i is a mapping $\sigma_i : \mathcal{S}_i \to 2^{\mathcal{I}\backslash\{i\}}$. Let σ_{-i} denote the strategies of opponents.

Firm i's objective is to minimize the expected cost function:

$$\mathcal{C}_i(S_i, s_i; \sigma_{-i}) = \mathbb{E}\left[K_i(S_i, \mu, P^*(\mu, S_i, \sigma_{-i}(\mathbf{s}_{-i}))) \mid s_i\right]. \tag{7}$$

This formulation highlights the strategic interaction: firm i's cost depends on P^*, which depends on S_{-i}, which depends on $\mathbf{s}_{-i}$ via σ_{-i}. Thus, firm i must forecast the signals and actions of other firms.

4. EXPECTATIONS AND THE BELIEF HIERARCHY

In a complete information setting, firm i observes μ and can perfectly anticipate S_{-i}. Here, firm i faces a *hierarchical inference* problem.

1. **First-order belief:** What is μ?

2. **Second-order belief:** What do others believe about μ? (This determines their S_{-i}).

Lemma 1 (Monotonicity of Expectations). *Let $h(\mu, \mathbf{s}_{-i})$ be a function that is non-decreasing in μ and in $\mathbf{s}_{-i}$ (component-wise). Then the conditional expectation function*

$$H(s_i) = \mathbb{E}[h(\mu, \mathbf{s}_{-i}) \mid s_i] \tag{8}$$

is non-decreasing in s_i.

Proof. This follows directly from Theorem 1. Affiliation implies that the conditional distribution of the vector $(\mu, \mathbf{s}_{-i})$ given s_i is stochastically increasing in s_i. The expectation of an increasing function with respect to a stochastically increasing distribution is increasing. $\qquad\square$

This lemma is the mathematical engine of the sentiment multiplier. It implies that if equilibrium prices are lower when fundamentals are good (μ high) and when peers are optimistic ($\mathbf{s}_{-i}$ high), then a firm observing a high s_i will rationally expect lower prices.

5. STRATEGIC COMPLEMENTARITIES

To prove the existence of equilibria, we characterize the economy as a supermodular game. This requires defining a lattice structure on strategies and showing the objective function satisfies increasing differences.

5.1. Lattice Structure

The set of possible supplier combinations for firm i is $\mathcal{L}_i = 2^{\mathcal{I} \setminus \{i\}}$. Ordered by set inclusion $\subseteq$, $\mathcal{L}_i$ is a complete lattice. A strategy σ_i is **monotone** if $s_i' > s_i \implies \sigma_i(s_i) \subseteq \sigma_i(s_i')$. The space of monotone strategies Σ_i is also a complete lattice under the pointwise order.

5.2. Payoff Supermodularity

Let $\Pi_i = -\mathcal{C}_i$ be the payoff (negative cost). We require two conditions:

1. **Strategic Complementarity:** Π_i has increasing differences in (S_i, σ_{-i}).

2. **Single-Crossing in Type:** Π_i has increasing differences in (S_i, s_i).

We assume the cost function exhibits technological complementarity.

Assumption 3 (Cost Submodularity). The unit cost function $K_i(S_i, \mu, P)$ has decreasing differences in (S_i, P). That is, the marginal cost reduction from adding a supplier is larger when input prices P are lower.

This assumption holds for standard aggregators like Cobb-Douglas and CES (with $\sigma > 1$), where inputs are gross substitutes in the cost function but complements in adoption (lower prices increase the scale of production, making fixed adoption costs easier to amortize).

Lemma 2 (Strategic Complementarity). *Under Assumption 3, the payoff Π_i has increasing differences in (S_i, σ_{-i}). That is, if rivals play a "larger" strategy $\sigma'_{-i} \succeq \sigma_{-i}$, firm i's incentive to expand S_i increases.*

Proof. If $\sigma'_{-i} \succeq \sigma_{-i}$, then for any realization of $\mathbf{s}_{-i}$, the network $S'_{-i} \supseteq S_{-i}$. By the properties of P-matrices in production networks, larger networks imply lower equilibrium prices P^*. By Assumption 3, lower prices increase the marginal benefit of expanding S_i. Thus, the expected benefit of expansion is higher under σ'_{-i}. $\square$

Lemma 3 (Information Single-Crossing). *Under Assumptions 1 and 3, if rivals play monotone strategies, then Π_i has increasing differences in (S_i, s_i).*

Proof. Let $\Delta(S_i, S'_i) = \Pi_i(S'_i) - \Pi_i(S_i)$ for $S'_i \supset S_i$. This is the expected cost saving from expansion. The realized cost saving depends on μ and P.

1. Higher μ lowers unit costs directly (via θ_i), scaling up the absolute savings.

2. Higher $\mathbf{s}_{-i}$ leads to larger S_{-i} (by monotonicity of rivals) and thus lower P. Lower P increases savings (Assumption 3).

Thus, the integrand (cost saving) is increasing in $(\mu, \mathbf{s}_{-i})$. By Lemma 1 (hierarchical inference), the expected value of this increasing function is increasing in s_i. $\square$

6. MONOTONE EQUILIBRIA

We now state the main existence result.

Theorem 2 (Existence of Extremal Monotone Equilibria). *The network formation game has a greatest Bayesian Nash Equilibrium $\bar{\sigma}$ and a least Bayesian Nash Equilibrium $\underline{\sigma}$. These equilibria are in monotone pure strategies: for every firm i, $\sigma_i(s_i)$ is non-decreasing in s_i with respect to set inclusion.*

Proof. The proof applies Tarski's Fixed Point Theorem to the lattice of monotone strategies.

1. The strategy space $\Sigma = \prod \Sigma_i$ is a complete lattice.

2. Define the best-response mapping $\Psi : \Sigma \to \Sigma$ where $\Psi_i(\sigma_{-i}) = \arg\max_\tau \Pi_i(\tau, \sigma_{-i})$.

3. By Lemma 3, the objective satisfies single-crossing in type, so the optimal strategy is monotone. Thus Ψ maps Σ to Σ.

4. By Lemma 2, the objective satisfies increasing differences in strategies. By Topkis's Monotonicity Theorem, Ψ is an isotone (order-preserving) map.

5. By Tarski's Theorem, the set of fixed points of an isotone map on a complete lattice is a non-empty complete lattice.

$\square$

Discussion. The existence of extremal equilibria implies potential multiplicity. $\bar{\sigma}$ represents an "optimistic regime" where firms coordinate on dense networks, justified by low prices. $\underline{\sigma}$ is a "pessimistic regime." In both regimes, however, network density is strictly increasing in sentiment.

7. THE SENTIMENT MULTIPLIER

We now quantify the amplification of beliefs. Consider a uniform shift in signals $s \to s + \delta$ while holding the fundamental μ constant. This represents a "sentiment shock."

Let y denote the aggregate network density. In a symmetric Gaussian equilibrium, the network density is a function of the signal s. The marginal change in network density can be decomposed as:

$$\frac{dy}{ds} = \underbrace{\frac{\partial y}{\partial \mathbb{E}[\mu]} \frac{\partial \mathbb{E}[\mu]}{\partial s}}_{\text{Fundamental Channel}} + \underbrace{\frac{\partial y}{\partial \mathbb{E}[P]} \frac{\partial \mathbb{E}[P]}{\partial \mathbb{E}[S_{-i}]} \frac{\partial \mathbb{E}[S_{-i}]}{\partial s}}_{\text{Strategic Channel}}. \tag{9}$$

Proposition 2 (The Sentiment Multiplier)**.** *Let $\beta \in (0, 1)$ measure the strength of strategic complementarity (the elasticity of own-network size with respect to aggregate*

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network size). In the Gaussian limit, the elasticity of the network to a sentiment shock is amplified by the factor:

$$\mathcal{M} = \frac{1}{1 - \rho\beta},$$

(10)

where $\rho = \mathrm{Cov}(s_i, s_j) / \mathrm{Var}(s_i)$ is the signal correlation.

This formula highlights that the multiplier is driven by two factors:

1. **Technological Linkages (β):** How much do lower prices encourage expansion?

2. **Informational Correlation (ρ):** How much does my signal tell me about your signal?

If signals are idiosyncratic ($\rho \to 0$), the multiplier is 1. If signals are public ($\rho \to 1$), the multiplier converges to the standard Keynesian/network multiplier $1/(1 - \beta)$.

8. COMPARATIVE STATICS

We examine how the quality of information affects the real economy.

Theorem 3 (Signal Precision and Volatility). *Consider the Gaussian environment. An increase in signal precision $\tau_\varepsilon = 1/\sigma_\varepsilon^2$ (holding τ_μ fixed) reduces the ex-post volatility of the network structure conditional on fundamentals.*

Intuition: As $\tau_\varepsilon \to \infty$, $\rho \to 1$, which increases the multiplier $\mathcal{M}$. However, the variance of the "noise" component of the signal ε goes to zero faster than the multiplier increases. Firms track fundamentals more closely and react less to pure noise.

Theorem 4 (Correlation and Instability). *Holding the marginal variance of signals constant, an increase in the correlation between error terms ε_i increases aggregate network volatility.*

Proof. (Sketch) An increase in error correlation increases ρ without changing the signal's responsiveness to μ. This increases the sentiment multiplier $\mathcal{M} = (1 - \rho\beta)^{-1}$. A correlated noise shock is now amplified more strongly because firms are more confident that others have seen the same shock. □

This result suggests that **opacity** (low precision) combined with **correlated information sources** (e.g., reliance on a few public indicators or rumors) creates the most volatile supply chains.

9. DYNAMIC EXTENSION

We briefly sketch a dynamic extension to illustrate persistence. Let $t = 1, 2, \ldots$. Firms face asymmetric adjustment costs:

$$\Gamma(S_{it}, S_{i,t-1}) = \gamma^+ |S_{it} \setminus S_{i,t-1}| + \gamma^- |S_{i,t-1} \setminus S_{it}|, \tag{11}$$

where γ^+ is the cost of forming a link and γ^- is the cost of severing one.

Proposition 3 (Hysteresis). *In the dynamic game, a temporary negative sentiment shock (s_t low) can cause a permanent simplification of the production network. If the economy shifts to the sparse equilibrium, the cost γ^+ prevents a return to the dense equilibrium even when beliefs recover, unless a coordinated positive shock occurs.*

10. CONCLUSION

This paper has integrated dispersed information into the theory of endogenous production networks. We showed that the formation of supply chains is driven by a **sentiment multiplier** arising from the interaction of affiliated beliefs and strategic complementarities.

Our results imply that supply chain volatility is not merely a reflection of fundamental TFP shocks but is endogenous to the information structure. Policies that improve transparency—such as standardized reporting of supply chain stress or public data on aggregate input flows—can reduce the correlation of belief errors and dampen the sentiment multiplier. Conversely, reliance on opaque, correlated signals exacerbates instability.

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A. APPENDIX: OMITTED PROOFS

A.1. Lattice Theory Preliminaries

We utilize the following definitions and theorems.

Definition 2 (Complete Lattice). A partially ordered set $(L, \preceq)$ is a *complete lattice* if every subset $S \subseteq L$ has both a supremum (least upper bound) $\bigvee S$ and an infimum (greatest lower bound) $\bigwedge S$.

Theorem 5 (Tarski's Fixed Point Theorem). *Let L be a complete lattice and $f : L \to L$ be an isotone (order-preserving) map. Then the set of fixed points of f is a non-empty complete lattice. In particular, there exist greatest and least fixed points.*

Our strategy space Σ_i is the set of monotone functions from $\mathbb{R}$ to the power set $2^{\mathcal{I} \setminus \{i\}}$. The power set is a complete lattice under inclusion. The function space is a complete lattice under the pointwise order.

Definition 3 (Supermodularity). A function $f : L \to \mathbb{R}$ on a lattice is *supermodular* if for all $x, y \in L$:

$$f(x \vee y) + f(x \wedge y) \geq f(x) + f(y).$$

Definition 4 (Increasing Differences). A function $f : L \times T \to \mathbb{R}$ has *increasing differences* in (x, t) if for all $x' \succeq x$ and $t' \succeq t$:

$$f(x', t') - f(x, t') \geq f(x', t) - f(x, t).$$

Theorem 6 (Topkis's Monotonicity Theorem). *If $f : L \times T \to \mathbb{R}$ is supermodular in x and has increasing differences in (x, t), then $\arg\max_x f(x, t)$ is isotone in t.*

A.2. Proof of Lemma 3 (Single-Crossing)

We must show $\Delta(s_i) = \mathbb{E}[\Pi(S') - \Pi(S) \mid s_i]$ is increasing in s_i. Let $g(\mu, \mathbf{s}_{-i}) = K(S, \mu, P(\mu, \sigma_{-i}(\mathbf{s}_{-i}))) - K(S', \mu, P(\mu, \sigma_{-i}(\mathbf{s}_{-i})))$. We need to show g is increasing in its arguments.

Step 1: Monotonicity in μ. The unit cost is $K_i \propto 1/\theta_i(\mu)$. Since $\theta_i(\mu)$ is increasing in μ, higher μ lowers unit costs. The cost difference $K(S) - K(S')$

scales with $1/\theta_i$. Assuming costs are convex in S (diminishing returns), the *benefit* of expansion (negative cost difference) scales positively with productivity.

Step 2: Monotonicity in $\mathbf{s}_{-i}$. Since σ_{-i} is monotone, higher $\mathbf{s}_{-i}$ implies larger S_{-i}. By the P-matrix property (Proposition 1), larger S_{-i} implies lower P. By Assumption 3 (decreasing differences in (S, P)), lower P increases the benefit of expansion.

Step 3: Applying Lemma 1. Since g is increasing in $(\mu, \mathbf{s}_{-i})$, by Lemma 1, $\Delta(s_i) = \mathbb{E}[g(\mu, \mathbf{s}_{-i}) \mid s_i]$ is increasing in s_i. $\qquad\square$

A.3. *Derivation of the Sentiment Multiplier*

In the Gaussian limit, let y be the network action (density). Consider the linear best-response approximation:

$$y_i = \alpha \mathbb{E}[\mu|s_i] + \beta \mathbb{E}[y_{-i}|s_i].$$

In a symmetric equilibrium, $y(s) = cs$ for some constant c.

Substituting the Gaussian posteriors from Example 1:

$$cs = \alpha\rho s + \beta c\rho s.$$

Solving for c:
$$c(1 - \beta\rho) = \alpha\rho.$$
$$c = \frac{\alpha\rho}{1 - \beta\rho}.$$

The term $\frac{1}{1-\beta\rho}$ is the sentiment multiplier $\mathcal{M}$. $\qquad\square$