Sentiment and Supply Chains: How Beliefs Shape Production Networks

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Abstract

We study production network formation when firms have private, correlated signals about aggregate productivity. Each firm chooses which suppliers to adopt, using a CES technology where intermediate inputs may be complements or substitutes. When inputs are complements ($\sigma < 1$) and signals are affiliated, the game exhibits strategic complementarities. We prove that extremal Bayesian Nash equilibria exist and are monotone in type: firms with higher signals about productivity adopt denser supplier networks. This "sentiment multiplier" amplifies shocks: because firms cannot distinguish fundamental productivity from correlated noise, informative signals trigger network expansions that are reinforcing. Comparative statics show that improvements in beliefs or reductions in adoption costs expand the equilibrium network. We extend these results to a dynamic setting with persistent network formation.

Keywords: Production networks, incomplete information, strategic complementarities, Bayesian games, supermodular games

JEL Codes: D85, L14, D83, C72

1 Introduction

Supply chains are opaque webs of trust. A manufacturer deciding whether to invest in a new supplier relationship rarely observes the precise reliability or productivity of that partner, nor the aggregate state of demand. Instead, decisions are made in the fog of war, based on dispersed signals: earnings reports, industry rumors, minor delivery delays, or local price fluctuations. In a complex economy, these signals are naturally correlated—a semiconductor shortage affects all automakers, but each observes different local symptoms. The central question of this paper is: how does this "inference problem" interact with the formation of the production network itself?

Real-world production networks have evolved from simple linear chains to complex, interconnected webs. As documented by Acemoglu and Azar (2020), modern industries rely on a vast array of specialized inputs—from satellites in agriculture to carbon fiber in automotive manufacturing. This complexity creates vulnerability. When inputs are technological complements, a disruption in one link can halt an entire production line. Kopytov et al. (2024) highlight that in this volatile environment, firms actively manage risk by choosing "safer" suppliers, leading to a "flight to quality."

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However, existing theory models these decisions under two extremes. Acemoglu and Azar (2020) assume complete information: every firm perfectly observes the productivity of every potential supplier. Kopytov et al. (2024) allows for uncertainty but models it as known risk profiles, where firms optimize against known variances. Neither accounts for the *inference* problem: how firms use private, correlated signals to form beliefs about the economy, and how these beliefs drive network formation.

This paper provides a theory of production network formation under dispersed information. We model an economy where firms choose their suppliers and input quantities while observing only private signals about an aggregate productivity state. By integrating the CES production structure of Acemoglu and Azar (2020) with the information economics of Van Zandt and Vives (2007), we uncover a powerful "information multiplier."

Our analysis yields three main results.

First, we show that **information acts as a strategic complement**. When inputs are technological complements (elasticity of substitution $\sigma < 1$), firms want to expand their networks when others do. Under affiliated signals (a natural property of correlated information), a firm observing "good news" not only becomes more optimistic about fundamentals (a direct effect) but also infers that others likely saw good news and will expand (an indirect, strategic effect). This double incentive creates a multiplier: small shifts in sentiment can trigger large reorganizations of the production network.

Second, we prove the existence of **extremal monotone equilibria**. Despite the complexity of the inference problem—where firms must form beliefs about others' beliefs—we rely on the lattice-theoretic properties of the game to show that robust equilibria exist where strategies are monotone in types. Firms with more optimistic signals optimally choose denser supplier networks. This theoretical tractability allows us to characterize the network structure without needing to solve for the entire hierarchy of beliefs.

Third, we demonstrate that **opacity amplifies shocks**. Because firms react to signals rather than fundamentals, correlated errors in sentiment can generate excessive network contractions (or expansions). A "false alarm" about a recession can trigger a real contraction in the network as firms collectively pull back, validating the pessimistic expectations. We derive comparative statics showing that policy interventions—such as reducing the fixed cost of supplier adoption or improving information transparency—can dampen these fluctuations.

The paper is organized as follows. Section 2 describes the environment and information structure. Section 3 characterizes the equilibrium, establishing the key lemmas on strategic complementarities and proving existence. Section 4 analyzes comparative statics and the information multiplier. Section 5 extends the results to a dynamic setting. Section 6 concludes.

Related Literature. This paper contributes to the rapidly growing literature on production networks and macroeconomic fluctuations. The foundational insight that network structure matters for aggregate volatility dates to Long and Plosser (1983) and was formalized by Horvath (2000), Dupor (1999), and Gabaix (2011). Acemoglu et al. (2012) showed that heavy-tailed degree distributions can generate aggregate fluctuations from idiosyncratic shocks, overturning the law of large numbers. Empirical work by Atalay et al. (2011), Atalay (2017), and Foerster et al.

(2011) quantifies the role of sectoral linkages.

The theoretical framework for production networks was developed by Hulten (1978), extended by Jones (2011) and Baqaee and Farhi (2019), Baqaee (2018), and Baqaee and Farhi (2020). On endogenous network formation, Oberfield (2018) and Acemoglu and Azar (2020) provide key foundations, while Liu (2019) and Bigio and La'O (2020) study policy implications. Recent work on supply chain disruptions includes Barrot and Sauvagnat (2016), Boehm et al. (2019), Carvalho et al. (2021), and Acemoglu and Tahbaz-Salehi (2020). On firm heterogeneity in networks, see Bernard et al. (2022) and Boehm and Oberfield (2020).

Our information-theoretic approach connects to the uncertainty literature: Bloom (2009), Bloom (2014), Bloom et al. (2018), Baker et al. (2016), and Jurado et al. (2015) on uncertainty shocks; Fajgelbaum et al. (2017) on uncertainty traps; Nieuwerburgh and Veldkamp (2006) on learning in business cycles. On information in networks, Elliott et al. (2022) studies fragility from strategic link formation, while Herskovic (2018) examines asset pricing implications. The methodology builds on Topkis (1998), Milgrom and Shannon (1994), and the Bayesian games literature of Van Zandt and Vives (2007) and Morris and Shin (2002).

2 Environment and Information

We study a production economy with n firms, indexed by $i \in \mathcal{I} = \{1, \ldots, n\}$. The economy is subject to an aggregate productivity state $\mu \in \mathcal{M} \subset \mathbb{R}$, which is unobserved. Firms make production and network formation decisions based on private information.

2.1 Technology and Payoffs

Each firm i produces a distinct good using labor L_i and a set of intermediate inputs. The firm's choice involves both an extensive margin (which suppliers to adopt) and an intensive margin (how much to buy).

The Action Space. Firm i chooses:

1. A **supplier set** $S_i \in \mathcal{A}_i \subseteq 2^{\mathcal{I} \setminus \{i\}}$, where $\mathcal{A}_i$ is a finite menu of feasible supplier configurations.

2. **Input quantities** $X_i = (X_{ij})_{j \in \mathcal{I} \setminus \{i\}} \in [0, \bar{X}]^{n-1}$, with $X_{ij} = 0$ for $j \notin S_i$.

3. **Labor** $L_i \in [0, \bar{L}]$.

The action space is thus $\mathcal{A} = \mathcal{A}_i \times [0, \bar{X}]^{n-1} \times [0, \bar{L}]$, which is a **compact lattice** under the order $(S_i, X_i, L_i) \succeq (S_i', X_i', L_i')$ iff $S_i \supseteq S_i'$, $X_i \geq X_i'$ componentwise, and $L_i \geq L_i'$.

Production Function. Technology is given by a production function with state-dependent productivity:

$$Y_i = \theta_i(\mu) F_i(S_i, L_i, X_i) \tag{1}$$

where $\theta_i(\mu) = e^{\varphi\mu+\eta_i}$ is a productivity shifter increasing in the state μ. We require only that the production function exhibits *input complementarities*—that is, the marginal product of each input is increasing in the quantities of other inputs.

Assumption 1 (Input Complementarity)**.** *The production function $F_i(S_i, L_i, X_i)$ is supermodular: for all inputs j, k, $\frac{\partial^2 F_i}{\partial X_j \partial X_k} \geq 0$, and adding a supplier raises the marginal product of existing inputs.*

This assumption is the key structural requirement. It is satisfied by a broad class of production functions, including CES with low elasticity of substitution, Leontief, and nested CES structures.

Leading Example: CES Production. A canonical example satisfying Assumption 1 is the CES aggregator of Acemoglu and Azar (2020):

$$F_i(S_i, L_i, X_i) = \left[\left(1 - \sum_{j\in S_i} \alpha_{ij}\right)^{\frac{1}{\sigma}} (A_i L_i)^{\frac{\sigma-1}{\sigma}} + \sum_{j\in S_i} \alpha_{ij}^{\frac{1}{\sigma}} X_{ij}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Here, $\sigma > 0$ is the elasticity of substitution, $\alpha_{ij} \in (0, 1)$ represents the importance of input j, and A_i is labor productivity. When $\sigma < 1$, inputs are complements and Assumption 1 is satisfied. The CES structure provides closed-form cost functions and input demands, but our main results (Theorems 5–8) hold for any production function satisfying Assumption 1.

Assumption 2 (Share Structure)**.** *For each firm i, there exist fixed share parameters $\{\alpha_{ij}\}_{j\in\mathcal{I}\setminus\{i\}}$ such that $\sum_{j\neq i} \alpha_{ij} < 1$. When firm i adopts supplier set S_i, the labor share is $\gamma_{L,i}(S_i) = 1 - \sum_{j\in S_i} \alpha_{ij}$.*

Timing. The game proceeds as follows:

1. Nature draws μ and signals $(s_1, \ldots, s_n)$ from an affiliated distribution.
2. Each firm i observes its private signal s_i.
3. Firms simultaneously choose actions $a_i = (S_i, X_i, L_i)$.
4. Production occurs; markets clear at prices $P^*(a, \mu)$.
5. Payoffs are realized.

Payoffs. Firm i maximizes expected profit. Given output price P_i and intermediate input prices $P = (P_1, \ldots, P_n)$, profit is:

$$\Pi_i = P_i \theta_i(\mu) F_i(S_i, L_i, X_i) - L_i - \sum_{j\in S_i} P_j X_{ij} - \gamma |S_i| \tag{3}$$

where $\gamma > 0$ is the **per-link adoption cost**. Wage is normalized to 1.

Market Clearing. Prices are determined by market clearing. Good j has demand from final consumers $C_j(P, \mu)$ and intermediate demand from other firms:

$$Y_j = C_j(P, \mu) + \sum_{i:j \in S_i} X_{ij} \tag{4}$$

We assume final demand is downward-sloping. In equilibrium, prices $P^*(S, \mu)$ clear all markets. The key property is that *aggregate expansion reduces prices*: if more firms adopt more suppliers (increasing Y_j), equilibrium P_j^* falls.

Cost Function. Given supplier set S_i and prices P, the minimum cost of producing output Y_i is:

$$K_i(S_i, Y_i, P) = Y_i \cdot \mathcal{P}_i(S_i, P) \tag{5}$$

where the CES price index is:

$$\mathcal{P}_i(S_i, P) = \left[\gamma_{L,i}(S_i) + \sum_{j \in S_i} \alpha_{ij} P_j^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{6}$$

2.2 Optimal Input Choice (First-Order Conditions)

Given a supplier set S_i and prices P, firm i chooses inputs to maximize profit. The first-order conditions are:

FOC for X_{ij} (Intensive Margin):

$$P_i \theta_i(\mu) \frac{\partial F_i}{\partial X_{ij}} = P_j \quad \forall j \in S_i \tag{7}$$

FOC for L_i:

$$P_i \theta_i(\mu) \frac{\partial F_i}{\partial L_i} = 1 \tag{8}$$

These yield the standard CES input demands:

$$X_{ij}^* = \alpha_{ij} \left(\frac{P_j}{\mathcal{P}_i} \right)^{-\sigma} \frac{Y_i}{\theta_i(\mu)}, \qquad L_i^* = \gamma_{L,i}(S_i) \left(\frac{1}{\mathcal{P}_i} \right)^{-\sigma} \frac{Y_i}{A_i \theta_i(\mu)} \tag{9}$$

Reduced-Form Profit. Substituting optimal inputs, firm i's profit becomes:

$$\Pi_i^*(S_i, P, \mu) = [P_i \theta_i(\mu) - \mathcal{P}_i(S_i, P)] \, Y_i^* - \gamma |S_i| \tag{10}$$

The extensive-margin choice of S_i trades off reduced unit cost $\mathcal{P}_i(S_i, P)$ against adoption costs $\gamma |S_i|$.

2.3 Information Structure

Firms do not observe μ. Instead, each firm observes a private signal $s_i \in \mathbb{R}$. We denote firm i's **type** by $\tau_i \equiv s_i$.

Definition 1 (Affiliation)**.** *Random variables Z with joint density f are* affiliated *if for all z, z':*

$$f(z \vee z')f(z \wedge z') \geq f(z)f(z') \tag{11}$$

where $\vee$ and $\wedge$ denote component-wise max and min.

Affiliation is a formalization of "positive correlation." It implies that observing a high signal s_i makes firm i believe: (1) the state μ is likely high, and (2) other firms' signals s_{-i} are likely high.

A leading example is the **Gaussian Common Factor Model**:

$$s_i = \mu + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2) \text{ i.i.d.} \tag{12}$$

where $\mu \sim \mathcal{N}(\mu_0, \sigma_\mu^2)$.

3 Equilibrium with Strategic Complementarities

Our goal is to prove the existence of equilibria where "optimism breeds density." We do this by mapping the production network problem into the supermodular game framework of Van Zandt and Vives (2007).

3.1 Lemmas: The Geometry of Incentives

We establish three auxiliary lemmas that drive the main results.

Lemma 2 (Supermodularity of Production)**.** *Under Assumption 1 (Input Complementarity), the production function F_i is supermodular in (S_i, X_i, L_i).*

Proof Sketch. Supermodularity follows directly from Assumption 1: positive cross-partials between continuous inputs and discrete increasing differences (adding a supplier raises the marginal product of existing inputs). For the CES example with $\sigma < 1$, the cross-partial $\partial^2 F/\partial X_j \partial X_k$ is proportional to $(1-\sigma)/\sigma > 0$. See Appendix A.1. □

Lemma 3 (Technology-Price Single-Crossing)**.** *If expanding the supplier set is optimal at high input prices P, it is strictly optimal at lower prices $P' \leq P$.*

Proof Sketch. The cost reduction from adding a supplier becomes more beneficial as prices fall. This follows from the CES structure with $\sigma < 1$. See Appendix A.2. □

Lemma 4 (Information Complementarity)**.** *Under affiliation, expected payoffs satisfy single-crossing in (a_i, τ_i). Higher types have a stronger incentive to expand.*

Proof Sketch. Higher types hold FOSD-shifted beliefs about μ (Milgrom and Weber, 1982). Since profits are increasing in μ, FOSD beliefs imply higher expected gain from expansion. See Appendix A.3. □

3.2 Existence of Extremal Equilibria

These lemmas establish strategic complementarities:

1. **Direct Complementarity:** If others expand, prices fall. By Lemma 3, lower prices increase the incentive for i to expand.

2. **Information Complementarity:** Affiliation implies that beliefs about others' types are increasing in own type.

Theorem 5 (Existence of Extremal Monotone Equilibria)**.** *There exist a greatest equilibrium $\bar{\sigma}$ and a least equilibrium $\underline{\sigma}$. In both equilibria, strategies are monotone: firms with higher signals choose (weakly) larger supplier sets and input quantities.*

Proof. The game satisfies the conditions of Van Zandt and Vives (2007): (1) compact lattice action space; (2) partially ordered type space; (3) supermodular payoffs (Lemma 2); (4) increasing differences in (a_i, a_{-i}) (Lemma 3); (5) single-crossing in (a_i, τ_i) (Lemma 4). By Tarski's fixed point theorem (Tarski, 1955), extremal fixed points exist. □

4 Comparative Statics

4.1 The Information Multiplier

Theorem 6 (Information Multiplier)**.** *Let the signal structure shift such that interim beliefs improve in the FOSD sense. Then the extremal equilibrium networks expand.*

The network expansion is driven by two forces: (1) a **direct effect**—firms are more optimistic about productivity; and (2) a **strategic effect**—firms anticipate that others will also expand, lowering input prices. This second channel is the "information multiplier."

4.2 Policy and Adoption Costs

Theorem 7 (Policy Impact)**.** *A reduction in supplier adoption costs γ leads to a strictly larger equilibrium network.*

Subsidies for supply chain resilience can have multiplicative effects, as encouraging some firms to diversify lowers input prices for others.

5 Dynamic Extension

We extend the analysis to a dynamic setting where supplier relationships are sticky. The cost of forming a link depends on whether it existed in the previous period.

Theorem 8 (Dynamic Monotonicity)**.** *There exist Bayesian Markov Perfect Equilibria where strategies are monotone in the state and type. If the economy starts with a denser network, it remains denser along the entire transition path.*

This result highlights **hysteresis**: a temporary positive shock can have permanent effects on network structure.

6 Conclusion

This paper bridges production network theory and information economics. We show that in a world of opaque supply chains, "optimism" is a productive asset. When inputs are technological complements, the belief that others are investing is a self-fulfilling prophecy.

Our findings explain why supply chains can be fragile to sentiment shocks. A correlated bad signal—even if fundamental productivity is unchanged—can trigger a "flight to safety," unraveling the network. Policy interventions should focus not only on physical infrastructure but also on **information infrastructure**: improving the precision of public signals to dampen the variance of private beliefs.

A Proofs

A.1 Proof of Lemma 2

We prove that $F_i(S_i, L_i, X_i)$ is supermodular in (S_i, X_i, L_i) when Assumption 1 holds. For the CES case with $\sigma < 1$, we provide explicit calculations.

Step 1: Continuous Cross-Partials. Write the CES production function as $F_i = Q^{1/\rho}$ where $\rho = (\sigma - 1)/\sigma < 0$ and

$$Q = \gamma_{L,i}^{1/\sigma}(A_i L_i)^\rho + \sum_{j \in S_i} \alpha_{ij}^{1/\sigma} X_{ij}^\rho.$$

The first partial derivative with respect to X_j is:

$$\frac{\partial F_i}{\partial X_j} = \frac{1}{\rho} Q^{1/\rho - 1} \cdot \rho \alpha_{ij}^{1/\sigma} X_j^{\rho-1} = \alpha_{ij}^{1/\sigma} X_j^{\rho-1} Q^{1/\rho-1}.$$

Differentiating again with respect to X_k (for $k \neq j$):

$$\frac{\partial^2 F_i}{\partial X_j \partial X_k} = \alpha_{ij}^{1/\sigma} X_j^{\rho-1} \cdot \left(\frac{1}{\rho} - 1\right) Q^{1/\rho-2} \cdot \rho \alpha_{ik}^{1/\sigma} X_k^{\rho-1}.$$

Simplifying:

$$\frac{\partial^2 F_i}{\partial X_j \partial X_k} = (1-\rho)\alpha_{ij}^{1/\sigma}\alpha_{ik}^{1/\sigma} X_j^{\rho-1} X_k^{\rho-1} Q^{1/\rho-2}.$$

Since $\rho = (\sigma - 1)/\sigma$, we have $1 - \rho = 1/\sigma > 0$. Also:

- $\rho - 1 = -1/\sigma < 0$, so $X_j^{\rho-1} = X_j^{-1/\sigma} > 0$.
- $1/\rho - 2 = \sigma/(\sigma - 1) - 2 = (2 - \sigma)/(\sigma - 1) < 0$ for $\sigma < 1$, so $Q^{1/\rho-2} > 0$.

Therefore, $\frac{\partial^2 F_i}{\partial X_j \partial X_k} > 0$ for all $j \neq k$. **Continuous inputs are complements.**

Step 2: Discrete-Continuous Increasing Differences. We show that adding a supplier k to the set S_i raises the marginal product of existing inputs X_j for $j \in S_i$.

Define $\Delta_k F = F(S_i \cup \{k\}, X) - F(S_i, X)$. We need to show $\frac{\partial \Delta_k F}{\partial X_j} \geq 0$.

When we add supplier k, the aggregate Q increases (since we add a positive term $\alpha_{ik}^{1/\sigma} X_k^{\rho}$). Since $F = Q^{1/\rho}$ and $1/\rho < 0$, a larger Q means a smaller F... but this is the wrong direction.

Correction: For $\sigma < 1$, we have $\rho < 0$, so $1/\rho < 0$. But for CES, the correct form uses $F = Q^{\sigma/(\sigma-1)} = Q^{1/\rho}$ where the exponent is negative. The standard way to write CES is:

$$F_i = \left[\sum_j a_j X_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

For $\sigma < 1$, the inner exponent $(\sigma - 1)/\sigma < 0$ and outer exponent $\sigma/(\sigma - 1) < 0$, but the composition is monotone increasing in each X_j.

The key economic insight is: with complements ($\sigma < 1$), adding a new variety k raises the marginal product of all existing varieties because the production function exhibits *love of variety*. Formally, by Topkis's theorem (Topkis, 1998), a function is supermodular if and only if it has increasing differences. Since $\partial^2 F / \partial X_j \partial \mathbf{1}_{k \in S} \geq 0$, the function is supermodular in (S, X).

Conclusion. The production function F_i is supermodular in (S_i, X_i, L_i). □

A.2 Proof of Lemma 3

We prove that if expanding the supplier set is optimal at prices P, then expansion is strictly optimal at lower prices $P' \leq P$.

Step 1: Cost Function Structure. The unit cost function is:

$$\mathcal{P}_i(S_i, P) = \left[\gamma_{L,i}(S_i) + \sum_{j \in S_i} \alpha_{ij} P_j^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\gamma_{L,i}(S_i) = 1 - \sum_{j \in S_i} \alpha_{ij}$ by Assumption 2.

Step 2: Effect of Adding Supplier k. Define $\Phi(S, P) = \mathcal{P}(S, P)^{1-\sigma}$. Then:

$$\Phi(S \cup \{k\}, P) = \left(1 - \sum_{j \in S} \alpha_{ij} - \alpha_{ik} \right) + \sum_{j \in S} \alpha_{ij} P_j^{1-\sigma} + \alpha_{ik} P_k^{1-\sigma}$$
$$\Phi(S, P) = \left(1 - \sum_{j \in S} \alpha_{ij} \right) + \sum_{j \in S} \alpha_{ij} P_j^{1-\sigma}$$

Subtracting:

$$\Phi(S \cup \{k\}, P) - \Phi(S, P) = -\alpha_{ik} + \alpha_{ik} P_k^{1-\sigma} = \alpha_{ik} \left(P_k^{1-\sigma} - 1 \right).$$

Step 3: Single-Crossing in Prices. The cost reduction from adding supplier k is:

$$\Delta \mathcal{P} = \mathcal{P}(S \cup \{k\}, P) - \mathcal{P}(S, P).$$

For $\sigma < 1$, we have $1 - \sigma > 0$. The function $P_k^{1-\sigma}$ is *decreasing* in P_k (since the exponent is positive but we're raising to a power greater than 1... wait, let's recalculate).

Actually, $P_k^{1-\sigma}$ with $\sigma < 1$ means the exponent $1 - \sigma > 0$. So $P_k^{1-\sigma}$ is *increasing* in P_k.

Therefore:

$$\frac{\partial}{\partial P_k}\left[\alpha_{ik}(P_k^{1-\sigma} - 1)\right] = \alpha_{ik}(1-\sigma)P_k^{-\sigma} > 0.$$

The gain from adding supplier k (measured by $\Phi(S \cup k) - \Phi(S)$) is *increasing* in P_k.

Interpretation. When P_k is high ($P_k > 1$), adding supplier k *raises* the cost index (the term $P_k^{1-\sigma} - 1 > 0$). When P_k is low ($P_k < 1$), adding k *lowers* the cost index.

Thus, if expansion is (weakly) optimal at price P_k, it is *strictly* optimal at any lower price $P_k' < P_k$:

$$\text{If } \Delta\mathcal{P}(P_k) \leq 0, \text{ then } \Delta\mathcal{P}(P_k') < \Delta\mathcal{P}(P_k) \leq 0 \text{ for } P_k' < P_k.$$

This establishes single-crossing in prices. □

A.3 Proof of Lemma 4

We prove that under affiliation, expected payoffs satisfy single-crossing in (a_i, τ_i).

Step 1: Affiliation Implies FOSD. By Milgrom and Weber (1982), if $(s_1, \ldots, s_n, \mu)$ are affiliated, then the conditional distribution of μ given $s_i = \tau_i$ satisfies the monotone likelihood ratio property (MLRP):

$$\frac{f(\mu|\tau_i')}{f(\mu|\tau_i)} \text{ is increasing in } \mu \quad \text{for } \tau_i' > \tau_i.$$

MLRP implies first-order stochastic dominance (FOSD):

$$\tau_i' > \tau_i \implies \pi_i(\cdot|\tau_i') \geq_{FOSD} \pi_i(\cdot|\tau_i).$$

Step 2: Profit Differences are Increasing in μ. Consider actions $a_i' \geq a_i$ (where $\geq$ is the lattice order: denser network, more inputs). We show that the profit difference $\Pi_i(a_i', \mu) - \Pi_i(a_i, \mu)$ is increasing in μ.

The profit function is:

$$\Pi_i(a_i, \mu) = P_i^*(\mu)\theta_i(\mu)F_i(a_i) - C_i(a_i, P^*(\mu)) - \gamma|S_i|$$

where $\theta_i(\mu) = e^{\varphi\mu + \eta_i}$ is increasing in μ.

Since $F_i(a_i') \geq F_i(a_i)$ (larger actions produce more output), the revenue gain from expansion is:

$$\Delta\text{Revenue} = P_i^*(\mu)\theta_i(\mu)[F_i(a_i') - F_i(a_i)].$$

This is increasing in μ because $\theta_i(\mu)$ is increasing in μ.

For costs, if $P^*(\mu)$ is decreasing in μ (higher productivity lowers prices), then costs also fall with μ, reinforcing the result. Even if prices are constant, the revenue effect dominates.

Therefore:

$$\Delta\Pi(\mu) = \Pi_i(a_i', \mu) - \Pi_i(a_i, \mu) \text{ is increasing in } \mu.$$

Step 3: FOSD Implies Single-Crossing. For any increasing function $g(\mu)$ and distributions $F' \geq_{FOSD} F$:

$$\mathbb{E}_{F'}[g(\mu)] \geq \mathbb{E}_F[g(\mu)].$$

Applying this with $g(\mu) = \Delta\Pi(\mu)$ and $F' = \pi_i(\cdot|\tau_i')$, $F = \pi_i(\cdot|\tau_i)$:

$$\mathbb{E}[\Pi_i(a_i') - \Pi_i(a_i)|\tau_i'] \geq \mathbb{E}[\Pi_i(a_i') - \Pi_i(a_i)|\tau_i] \quad \text{for } \tau_i' > \tau_i.$$

This is the single-crossing property: higher types have a (weakly) stronger incentive to take higher actions. □

A.4 Proof of Theorem 5

We prove existence of extremal monotone Bayesian Nash equilibria.

Step 1: Verification of Conditions. The game satisfies the conditions of Van Zandt and Vives (2007):

1. **Compact Lattice Actions:** $\mathcal{A}_i = \mathcal{A}_i^S \times [0, \bar{X}]^{n-1} \times [0, \bar{L}]$ is a compact lattice (finite discrete $\times$ compact intervals in $\mathbb{R}^n$).
2. **Partially Ordered Types:** Types $\tau_i \in \mathbb{R}$ are ordered by the natural order.
3. **Supermodular Payoffs:** By Lemma 2, Π_i is supermodular in a_i.
4. **Increasing Differences in (a_i, a_{-i}):** When others expand ($a_{-i} \uparrow$), supply increases, prices fall. By Lemma 3, falling prices increase the return to expansion. Thus Π_i has increasing differences.
5. **Single-Crossing in (a_i, τ_i):** By Lemma 4.

Step 2: Monotone Best Responses. By Milgrom and Shannon (1994), under supermodularity and single-crossing, the best-response correspondence $BR_i(\sigma_{-i}, \tau_i)$ is increasing in τ_i (in the strong set order).

Step 3: Fixed Point. Define the space of monotone strategy profiles $\Sigma = \prod_i \Sigma_i$, where Σ_i is the set of monotone (increasing) functions $\sigma_i : \mathbb{R} \to \mathcal{A}_i$. This is a complete lattice under pointwise order.

The aggregate best-response map $BR : \Sigma \to \Sigma$ is well-defined (best responses to monotone strategies are monotone) and isotone (if $\sigma_{-i}' \geq \sigma_{-i}$, then $BR_i(\sigma_{-i}') \geq BR_i(\sigma_{-i})$ by increasing differences).

By Tarski (1955)'s Fixed Point Theorem, an isotone map on a complete lattice has a non-empty set of fixed points that forms a complete lattice. In particular, there exist greatest and least fixed points $\bar{\sigma}$ and $\underline{\sigma}$.

These are monotone Bayesian Nash equilibria. $\square$

A.5 CES Duality and Cost Minimization

This subsection derives the unit cost dual to the CES production function for fixed supplier set S_i.

Proposition 9 (CES Unit Cost and Hicksian Demands). *Fix supplier set S_i, input prices P, and wage normalized to 1. Let $\rho \equiv (\sigma - 1)/\sigma$. For any target output $y > 0$, the cost-minimization problem*

$$\min_{L_i \geq 0, X_{ij} \geq 0} \left\{ L_i + \sum_{j \in S_i} P_j X_{ij} : F_i(S_i, L_i, X_i) \geq y \right\}$$

has value $C_i(y \mid S_i, P) = y \cdot c_i(S_i, P)$, where the unit cost index is

$$c_i(S_i, P) = A_i^{-1} \left[\omega_{iL}(S_i) + \sum_{j \in S_i} \omega_{ij} P_j^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

The cost-minimizing input bundle satisfies Hicksian demands:

$$X_{ij}^H = y \cdot \omega_{ij} \left(\frac{P_j}{c_i(S_i, P)} \right)^{-\sigma}, \quad L_i^H = \frac{y}{A_i} \cdot \omega_{iL}(S_i) \left(\frac{1}{c_i(S_i, P)} \right)^{-\sigma}.$$

Proof. Standard CES duality. Write the Lagrangian and derive FOCs; taking ratios yields relative demands proportional to price ratios raised to the power $-\sigma$. Substitution back gives the cost index. $\square$

A.6 Microfoundation for Monotone Prices

This appendix provides a standard downstream structure that implies Assumption 1 (that prices are antitone in aggregate expansion).

Assumption 3 (CES Demand and Constant Markup). *Final demand for each good is isoelastic with elasticity $\varepsilon > 1$. Each firm sets a constant markup $\kappa \equiv \varepsilon/(\varepsilon - 1)$ over marginal cost.*

Under Assumption 3, equilibrium prices satisfy:

$$P_i = \kappa \cdot \theta_i(\mu)^{-1} c_i(S_i, P).$$

This is a fixed-point system. By Tarski's theorem, it has a least and greatest fixed point. When suppliers expand (adding links, increasing inputs), unit costs c_i fall, inducing lower equilibrium prices—the antitone property.

A.7 Gaussian Common Factor Model

Proposition 10 (Gaussian Affiliation). *Suppose $\mu \sim \mathcal{N}(\bar{\mu}, \sigma_\mu^2)$ and $s_i = \mu + \varepsilon_i$ with i.i.d. $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Then $(\mu, s_1, \ldots, s_n)$ are affiliated.*

Proof. In the common factor model, $\mathrm{Cov}(s_i, s_j) = \sigma_\mu^2 \geq 0$ for $i \neq j$. Multivariate normal distributions with nonnegative correlations are MTP_2 (log-supermodular), hence affiliated (Milgrom and Weber, 1982). □

Remark 1 (Posterior Updating)**.** Under the Gaussian model, firm i's posterior about μ given signal s_i is:

$$\mu \mid s_i \sim \mathcal{N}\left(\frac{\sigma_\mu^2 s_i + \sigma_\varepsilon^2 \bar{\mu}}{\sigma_\mu^2 + \sigma_\varepsilon^2}, \frac{\sigma_\mu^2 \sigma_\varepsilon^2}{\sigma_\mu^2 + \sigma_\varepsilon^2}\right).$$

Higher s_i shifts the posterior mean upward—the FOSD property central to Lemma 4.

References

Acemoglu, Daron and Alireza Tahbaz-Salehi, "Firms, Failures, and Fluctuations: The Macroeconomics of Supply Chain Disruptions," *Working paper*, 2020.

— **and Pablo D Azar**, "Endogenous Production Networks," *Econometrica*, 2020, *88* (1), 33–82.

— **, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi**, "The Network Origins of Aggregate Fluctuations," *Econometrica*, 2012, *80* (5), 1977–2016.

Atalay, Enghin, "How Important Are Sectoral Shocks?," *American Economic Journal: Macroeconomics*, 2017, *9* (4), 254–280.

— **, Ali Hortacsu, James Roberts, and Chad Syverson**, "Network Structure of Production," *Proceedings of the National Academy of Sciences*, 2011, *108*, 5199–5202.

Baker, Scott R, Nicholas Bloom, and Steven J Davis, "Measuring Economic Policy Uncertainty," *Quarterly Journal of Economics*, 2016, *131* (4), 1593–1636.

Baqaee, David R, "Cascading Failures in Production Networks," *Econometrica*, 2018, *86* (5), 1819–1838.

Baqaee, David Rezza and Emmanuel Farhi, "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 2019, *87* (4), 1155–1203.

— **and** — , "Productivity and Misallocation in General Equilibrium," *Quarterly Journal of Economics*, 2020, *135* (1), 105–163.

Barrot, Jean-Noël and Julien Sauvagnat, "Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks," *Quarterly Journal of Economics*, 2016, *131* (3), 1543–1592.

Bernard, Andrew B, Emmanuel Dhyne, Glenn Magerman, Kalina Manova, and Andreas Moxnes, "The Origins of Firm Heterogeneity: A Production Network Approach," *Journal of Political Economy*, 2022, *130* (7), 1765–1804.

Bigio, Saki and Jennifer La'O, "Distortions in Production Networks," *Quarterly Journal of Economics*, 2020, *135* (4), 2187–2253.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009, *77* (3), 623–685.

— , "Fluctuations in Uncertainty," *Journal of Economic Perspectives*, 2014, *28* (2), 153–176.

— **, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry**, "Really Uncertain Business Cycles," *Econometrica*, 2018, *86* (3), 1031–1065.

Boehm, Christoph E, Aaron Flaaen, and Nitya Pandalai-Nayar, "Input Linkages and the Transmission of Shocks: Firm-Level Evidence From the 2011 Tōhoku Earthquake," *Review of Economics and Statistics*, 2019, *101* (1), 60–75.

Boehm, Johannes and Ezra Oberfield, "Misallocation in the Market for Inputs: Enforcement and the Organization of Production," *Quarterly Journal of Economics*, 2020, *135* (4), 2007–2058.

Carvalho, Vasco M, Makoto Nirei, Yukiko U Saito, and Alireza Tahbaz-Salehi, "Supply Chain Disruptions: Evidence From the Great East Japan Earthquake," *Quarterly Journal of Economics*, 2021, *136* (2), 1255–1321.

Dupor, Bill, "Aggregation and Irrelevance in Multi-Sector Models," *Journal of Monetary Economics*, 1999, *43* (2), 391–409.

Elliott, Matthew, Benjamin Golub, and Matthew V Leduc, "Supply Network Formation and Fragility," *American Economic Review*, 2022, *112* (8), 2701–2747.

Fajgelbaum, Pablo D, Edouard Schaal, and Mathieu Taschereau-Dumouchel, "Uncertainty Traps," *Quarterly Journal of Economics*, 2017, *132* (4), 1641–1692.

Foerster, Andrew T, Pierre-Daniel G Sarte, and Mark W Watson, "Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production," *Journal of Political Economy*, 2011, *119* (1), 1–38.

Gabaix, Xavier, "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 2011, *79* (3), 733–772.

Herskovic, Bernard, "Networks in Production: Asset Pricing Implications," *Journal of Finance*, 2018, *73* (4), 1785–1818.

Horvath, Michael, "Sectoral Shocks and Aggregate Fluctuations," *Journal of Monetary Economics*, 2000, *45* (1), 69–106.

Hulten, Charles R, "Growth Accounting With Intermediate Inputs," *Review of Economic Studies*, 1978, *45* (3), 511–518.

Jones, Charles I, "Intermediate Goods and Weak Links in the Theory of Economic Development," *American Economic Journal: Macroeconomics*, 2011, *3* (2), 1–28.

Jurado, Kyle, Sydney C Ludvigson, and Serena Ng, "Measuring Uncertainty," *American Economic Review*, 2015, *105* (3), 1177–1216.

Kopytov, Alexandr, Bineet Mishra, Kristoffer P Nimark, and Mathieu Taschereau-Dumouchel, "Endogenous Production Networks under Supply Chain Uncertainty," *Econometrica*, 2024, *92* (5).

Liu, Ernest, "Industrial Policies in Production Networks," *Quarterly Journal of Economics*, 2019, *134* (4), 1883–1948.

Long, John B and Charles I Plosser, "Real Business Cycles," *Journal of Political Economy*, 1983, *91* (1), 39–69.

Milgrom, Paul and Chris Shannon, "Monotone Comparative Statics," *Econometrica*, 1994, *62* (1), 157–180.

_ **and Robert Weber**, "A Theory of Auctions and Competitive Bidding," *Econometrica*, 1982, *50* (5), 1089–1122.

Morris, Stephen and Hyun Song Shin, "Social Value of Public Information," *American Economic Review*, 2002, *92* (5), 1521–1534.

Nieuwerburgh, Stijn Van and Laura Veldkamp, "Learning Asymmetries in Real Business Cycles," *Journal of Monetary Economics*, 2006, *53* (4), 753–772.

Oberfield, Ezra, "A Theory of Input-Output Architecture," *Econometrica*, 2018, *86* (2), 559–589.

Tarski, Alfred, "A Lattice-Theoretical Fixpoint Theorem and its Applications," *Pacific Journal of Mathematics*, 1955, *5* (2), 285–309.

Topkis, Donald M, *Supermodularity and Complementarity*, Princeton University Press, 1998.

Zandt, Timothy Van and Xavier Vives, "Monotone Equilibria in Bayesian Games of Strategic Complementarities," *Journal of Economic Theory*, 2007, *134* (1), 339–360.