Sentiment and Supply Chains: Endogenous Production Networks under Uncertainty*

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Abstract

We study the formation of production networks when firms possess dispersed, affiliated information about aggregate productivity. Firms make extensive margin decisions (choosing supplier sets) and intensive margin decisions (input quantities) under uncertainty. We characterize the economy as a supermodular Bayesian game. Using lattice-theoretic methods, we prove the existence of extremal monotone Bayesian Nash equilibria where firms with optimistic private signals adopt denser sets of inputs. We identify a "belief multiplier" mechanism: because signals are affiliated, a firm's optimism rationally raises its expectation of others' optimism, leading to coordinated network expansions that lower equilibrium prices and validate the initial beliefs. We decompose network volatility into fundamental and strategic components, showing that higher signal correlation amplifies volatility while higher signal precision dampens it. The model suggests that opacity in supply chains acts as an amplifier of aggregate shocks.

Keywords: Production networks, dispersed information, strategic complementarities, supermodular games, affiliation, belief multiplier.
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1. INTRODUCTION

Modern supply chains are defined by a tension between complexity and opacity. A manufacturer deciding whether to invest in specialized logistics or expand its supplier base rarely observes the precise productivity of its potential partners or the true state of aggregate demand. Instead, firms rely on dispersed, noisy signals: procurement delays, order book fluctuations, and industry chatter. These signals are naturally correlated, reflecting common sectoral or macroeconomic factors, yet they remain private and imperfect.

This paper asks how dispersed, affiliated information shapes the endogenous formation of production networks. We show that networks are not merely passive transmitters of fundamental shocks; they are active constructions of agents acting on beliefs. When firms cannot perfectly disentangle fundamentals from correlated noise, "sentiment" becomes a driver of real economic structure.

We develop a model of network formation under private information, building on the cost-minimization framework of Acemoglu and Azar (2020). We depart from the complete information benchmark by assuming aggregate productivity is unobserved. Instead, firms receive private signals that are *affiliated* in the sense of Milgrom and Weber (1982). This information structure introduces a distinct strategic channel. In standard models, complementarities are technological: low upstream prices encourage downstream expansion. In our environment, complementarities are also inferential. An optimistic firm expands not only because it believes fundamentals are strong, but because it expects others to be optimistic—and thus to expand, lowering the price level.

Our analysis proceeds in four steps.

First, in Section 6, we establish the existence of **monotone Bayesian Nash equilibria**. Although the space of potential networks is high-dimensional and discrete, we show that the game is supermodular. Firms with more optimistic signals monotonically expand their supplier sets, generating distinct "high density" (optimistic) and "low density" (pessimistic) regimes for the same fundamentals.

Second, in Section 7, we identify a **belief multiplier**. We decompose the network response to a belief shock into a direct fundamental effect and a strategic amplification effect. Because beliefs are affiliated, optimism begets optimism. The strategic channel amplifies the fundamental response, generating excess volatility in network density.

Third, in Section 8, we specialize to a **Cobb-Douglas/Gaussian** environment and derive belief-adjusted Domar weights. These objects treat the information structure as first order, decomposing the sensitivity of aggregate output into a Hulten term and a strategic amplification factor.

Fourth, in Section 10, we sketch a dynamic extension following Van Zandt and Vives (2007), showing how temporary sentiment shocks can have persistent effects on network architecture through hysteresis. Section 11 concludes.

Related Literature. This paper bridges production networks and dispersed information. The foundational insight that network structure matters for aggregate volatility dates to Long and Plosser (1983) and was formalized by Horvath (2000), Dupor (1999), and Gabaix (2011). We build directly on Acemoglu and Azar (2020), extending their complete-information analysis to a Bayesian setting. We differ from Kopytov et al. (2024), whose mechanism relies on risk aversion; our results are driven by strategic complementarities in *beliefs*. The methodology builds on Topkis (1998), Milgrom and Shannon (1994), and the Bayesian games literature of Van Zandt and Vives (2007).

2. Information Structure

We begin by defining the probabilistic environment, which is foundational to the strategic analysis. Consider an economy with n firms indexed by $\mathcal{I} = \{1, \ldots, n\}$. The fundamental state of the economy is described by a random variable $\mu \in \mathcal{M} \subseteq \mathbb{R}$, representing aggregate productivity. This state is unobserved.

Each firm i observes a private signal $s_i \in \mathcal{S}_i \subseteq \mathbb{R}$. Let $\mathbf{s} = (s_1, \ldots, s_n)$ denote the profile of signals. The joint distribution of $(\mu, \mathbf{s})$ is governed by a cumulative distribution function $F(\mu, \mathbf{s})$ with a strictly positive density $f(\mu, \mathbf{s})$ with respect to a product measure.

2.1. *Affiliation and Stochastic Dominance*

To capture the idea that signals are correlated reflections of the same underlying reality, we assume the joint distribution satisfies *affiliation*, a strong form of positive dependence introduced by Milgrom and Weber (1982).

Definition 1 (Affiliation)**.** The random variables $Z = (\mu, s_1, \ldots, s_n)$ are *affiliated* if their joint density f is log-supermodular. That is, for all z, z' in the support of f:

$$f(z \vee z')f(z \wedge z') \geq f(z)f(z'), \tag{1}$$

where $\vee$ and $\wedge$ denote the component-wise maximum and minimum, respectively.

Affiliation implies that a high realization of one variable makes high realizations of the other variables more likely in the sense of the Monotone Likelihood Ratio Property (MLRP).

Assumption 1 (Affiliated Information)**.** The vector $(\mu, s_1, \ldots, s_n)$ is affiliated.

This assumption allows us to order beliefs and higher-order beliefs unambiguously. We invoke the following key properties from Milgrom and Weber (1982).

Theorem 1 (Properties of Affiliated Beliefs)**.** *Under Assumption 1:*

(i) ***MLRP:*** *The conditional density $f(\mu \mid s_i)$ satisfies the Monotone Likelihood Ratio Property in s_i.*

(ii) ***Stochastic Dominance:*** *If $s_i' > s_i$, then the posterior distribution of μ given s_i' dominates the distribution given s_i in the first-order sense (FOSD).*

(iii) ***Ordered Beliefs about Others:*** *The conditional distribution of the vector of others' signals $\mathbf{s}_{-i}$ given s_i is increasing in s_i in the multivariate FOSD sense. Specifically, for any non-decreasing function $g(\mathbf{s}_{-i})$, the expectation $\mathbb{E}[g(\mathbf{s}_{-i}) \mid s_i]$ is non-decreasing in s_i.*

Property (iii) is the engine of our strategic analysis. It implies that an optimistic firm (observing high s_i) rationally expects other firms to be optimistic as well.

2.2. *Leading Example: Gaussian Common Factor*

Throughout the paper, we use the Gaussian structure to provide closed-form intuition.

Example 1 (Gaussian Signals)**.** Let $\mu \sim \mathcal{N}(\mu_0, \sigma_\mu^2)$. Each firm observes:

$$s_i = \mu + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2),$$

where ε_i are i.i.d. across firms and independent of μ. The joint distribution is multivariate normal with covariance $\mathrm{Cov}(s_i, s_j) = \sigma_\mu^2 \geq 0$, which implies affiliation.

The posterior expectation of μ is linear:

$$\mathbb{E}[\mu \mid s_i] = (1-\rho)\mu_0 + \rho s_i, \quad \text{where } \rho = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\varepsilon^2}. \tag{2}$$

Crucially, firm i's expectation of firm j's signal is:

$$\mathbb{E}[s_j \mid s_i] = \mathbb{E}[\mathbb{E}[s_j \mid \mu] \mid s_i] = \mathbb{E}[\mu \mid s_i] = (1-\rho)\mu_0 + \rho s_i. \tag{3}$$

This explicit linearity allows us to quantify the strength of strategic feedback using the parameter ρ.

3. Production Environment

We adopt the production framework of Acemoglu and Azar (2020), adapted to an incomplete information setting. There are n sectors. Each sector i produces a distinct good using labor L_i and intermediate inputs.

3.1. *Technology and Costs*

Firm i makes an extensive margin choice: it selects a set of suppliers $S_i \subseteq \mathcal{I} \setminus \{i\}$. Given S_i, the production function is:

$$Y_i = \theta_i(\mu) F_i(S_i, L_i, \{X_{ij}\}_{j \in S_i}), \tag{4}$$

where $\theta_i(\mu)$ is a productivity shifter strictly increasing in μ, and F_i is the aggregator.

Assumption 2 (Technology)**.** For all i and S_i:

(i) F_i is continuous, concave, strictly increasing, and homogeneous of degree one (CRS) in inputs.

(ii) Labor is essential: $F_i(S_i, 0, \cdot) = 0$.

(iii) **Technological Monotonicity:** For any price vector P, the unit cost achievable with a larger supplier set $S_i' \supset S_i$ is weakly lower than with S_i.

Firms operate in competitive markets. Given a state μ, a network $S = (S_1, \ldots, S_n)$, and a price vector P, the unit cost for firm i is derived from cost minimization:

$$K_i(S_i, \mu, P) = \frac{1}{\theta_i(\mu)} \min_{L_i, \{X_{ij}\}} \left\{ L_i + \sum_{j \in S_i} P_j X_{ij} \mid F_i(\cdot) = 1 \right\}. \tag{5}$$

We normalize the wage $w = 1$.

3.2. *Market Clearing and Equilibrium Prices*

In the production stage (after μ is realized and S is fixed), prices must equal unit costs. The equilibrium price vector $P^*(\mu, S)$ is the fixed point of:

$$P_i = K_i(S_i, \mu, P) \quad \forall i \in \mathcal{I}. \tag{6}$$

Proposition 1 (Existence and Uniqueness of Prices)**.** *Under Assumption 2, for any μ and network S, there exists a unique strictly positive price vector $P^*(\mu, S)$ solving (6). Furthermore, P^* is decreasing in μ and non-increasing in S (under the inclusion order).*

Proof. See Acemoglu and Azar (2020). The proof relies on the fact that the Jacobian $I - \frac{\partial K}{\partial P}$ is an M-matrix (a class of P-matrices) due to the essentiality of labor, guaranteeing global univalence via the Gale-Nikaido theorem. □

Corollary 1 (Uniqueness of Allocations)**.** *Given the unique equilibrium price vector $P^*(\mu, S)$, the allocations (X^*, Y^*, C^*, L^*) are uniquely determined.*

Proof. Given P^* and the technology choice S_i, the factor demands L_i^* and X_i^* are uniquely determined by the strictly convex cost minimization problem (5). The output Y_i^* follows from market clearing: $Y_i^* = F_i(S_i, L_i^*, X_i^*)$. Aggregate labor supply pins down the scale via $\sum_i L_i^* = 1$. Consumer demands C^* are uniquely determined by utility maximization given P^* and income. By the same Gale-Nikaido argument, this system has a unique solution. □

3.3. *The Network Formation Game*

The extensive margin decision is made *ex ante*. Firm i observes s_i and chooses S_i to minimize expected unit costs. The strategy of firm i is a mapping $\sigma_i : \mathcal{S}_i \to 2^{\mathcal{I}\setminus\{i\}}$. Let σ_{-i} denote the strategies of opponents.

Firm i's objective is to minimize the expected cost function:

$$\mathcal{C}_i(S_i, s_i; \sigma_{-i}) = \mathbb{E}\left[K_i(S_i, \mu, P^*(\mu, S_i, \sigma_{-i}(\mathbf{s}_{-i}))) \mid s_i\right]. \tag{7}$$

This formulation highlights the strategic interaction: firm i's cost depends on P^*, which depends on S_{-i}, which depends on $\mathbf{s}_{-i}$ via σ_{-i}. Thus, firm i must forecast the signals and actions of other firms.

4. Expectations and the Belief Hierarchy

In a complete information setting, firm i observes μ and can perfectly anticipate S_{-i}. Here, firm i faces a *hierarchical inference* problem.

1. **First-order belief:** What is μ?
2. **Second-order belief:** What do others believe about μ? (This determines their S_{-i}).

Lemma 1 (Monotonicity of Expectations)**.** *Let $h(\mu, \mathbf{s}_{-i})$ be a function that is non-decreasing in μ and in $\mathbf{s}_{-i}$ (component-wise). Then the conditional expectation function*

$$H(s_i) = \mathbb{E}[h(\mu, \mathbf{s}_{-i}) \mid s_i] \tag{8}$$

is non-decreasing in s_i.

Proof. This follows directly from Theorem 1. Affiliation implies that the conditional distribution of the vector $(\mu, \mathbf{s}_{-i})$ given s_i is stochastically increasing in s_i. The expectation of an increasing function with respect to a stochastically increasing distribution is increasing. □

This lemma is the mathematical engine of the belief multiplier. It implies that if equilibrium prices are lower when fundamentals are good (μ high) and when peers are optimistic ($\mathbf{s}_{-i}$ high), then a firm observing a high s_i will rationally expect lower prices.

5. Strategic Complementarities

To prove the existence of equilibria, we characterize the economy as a supermodular game. This requires defining a lattice structure on strategies and showing the objective function satisfies increasing differences.

5.1. *Lattice Structure*

The set of possible supplier combinations for firm i is $\mathcal{L}_i = 2^{\mathcal{I}\setminus\{i\}}$. Ordered by set inclusion $\subseteq$, $\mathcal{L}_i$ is a complete lattice. A strategy σ_i is **monotone** if $s_i' > s_i \implies \sigma_i(s_i) \subseteq \sigma_i(s_i')$. The space of monotone strategies Σ_i is also a complete lattice under the pointwise order.

5.2. *Payoff Supermodularity*

Let $\Pi_i = -\mathcal{C}_i$ be the payoff (negative cost). We require two conditions:

1. **Strategic Complementarity:** Π_i has increasing differences in (S_i, σ_{-i}).
2. **Single-Crossing in Type:** Π_i has increasing differences in (S_i, s_i).

We assume the cost function exhibits technological complementarity.

Assumption 3 (Cost Submodularity)**.** The unit cost function $K_i(S_i, \mu, P)$ has decreasing differences in (S_i, P). That is, the marginal cost reduction from adding a supplier is larger when input prices P are lower.

This assumption holds for Cobb-Douglas production functions and, more generally, whenever adding a supplier yields greater cost savings in environments where input prices are lower. Intuitively, lower prices encourage larger input usage, amplifying the benefit of access to additional suppliers.

Lemma 2 (Strategic Complementarity)**.** *Under Assumption 3, the payoff Π_i has increasing differences in (S_i, σ_{-i}). That is, if rivals play a "larger" strategy $\sigma'_{-i} \succeq \sigma_{-i}$, firm i's incentive to expand S_i increases.*

Proof. If $\sigma'_{-i} \succeq \sigma_{-i}$, then for any realization of $\mathbf{s}_{-i}$, the network $S'_{-i} \supseteq S_{-i}$. By the properties of P-matrices in production networks, larger networks imply lower equilibrium prices P^*. By Assumption 3, lower prices increase the marginal benefit of expanding S_i. Thus, the expected benefit of expansion is higher under σ'_{-i}. □

Lemma 3 (Information Single-Crossing)**.** *Under Assumptions 1 and 3, if rivals play monotone strategies, then Π_i has increasing differences in (S_i, s_i).*

Proof. Let $\Delta(S_i, S_i') = \Pi_i(S_i') - \Pi_i(S_i)$ for $S_i' \supset S_i$. This is the expected cost saving from expansion. The realized cost saving depends on μ and P.

1. Higher μ lowers unit costs directly (via θ_i), scaling up the absolute savings.
2. Higher $\mathbf{s}_{-i}$ leads to larger S_{-i} (by monotonicity of rivals) and thus lower P. Lower P increases savings (Assumption 3).

Thus, the integrand (cost saving) is increasing in $(\mu, \mathbf{s}_{-i})$. By Lemma 1 (hierarchical inference), the expected value of this increasing function is increasing in s_i. □

6. Monotone Equilibria

We now state the main existence result.

Theorem 2 (Existence of Extremal Monotone Equilibria)**.** *The network formation game has a greatest Bayesian Nash Equilibrium $\bar{\sigma}$ and a least Bayesian Nash Equilibrium $\underline{\sigma}$. These equilibria are in monotone pure strategies: for every firm i, $\sigma_i(s_i)$ is non-decreasing in s_i with respect to set inclusion.*

Proof. The proof applies Tarski's Fixed Point Theorem to the lattice of monotone strategies.

1. The strategy space $\Sigma = \prod \Sigma_i$ is a complete lattice.
2. Define the best-response mapping $\Psi : \Sigma \to \Sigma$ where $\Psi_i(\sigma_{-i}) = \arg\max_\tau \Pi_i(\tau, \sigma_{-i})$.
3. By Lemma 3, the objective satisfies single-crossing in type, so the optimal strategy is monotone. Thus Ψ maps Σ to Σ.
4. By Lemma 2, the objective satisfies increasing differences in strategies. By Topkis's Monotonicity Theorem, Ψ is an isotone (order-preserving) map.
5. By Tarski's Theorem, the set of fixed points of an isotone map on a complete lattice is a non-empty complete lattice.

□

Discussion. The existence of extremal equilibria implies potential multiplicity. $\bar{\sigma}$ represents an "optimistic regime" where firms coordinate on dense networks, justified by low prices. $\underline{\sigma}$ is a "pessimistic regime." In both regimes, however, network density is strictly increasing in sentiment.

7. The Belief Multiplier

The key mechanism in our model is a *belief multiplier*: optimistic beliefs propagate through the network, generating amplified responses in equilibrium network density. Unlike standard multipliers derived from continuous elasticities, our multiplier operates through the discrete, combinatorial structure of network formation.

Proposition 2 (Belief Multiplier: Monotonicity)**.** *Consider two information structures $\mathcal{I}$ and $\mathcal{I}'$ such that the induced posteriors satisfy $\pi'(\cdot \mid s_i) \geq_{FOSD} \pi(\cdot \mid s_i)$ for all s_i (more optimistic beliefs). Let $\bar{\sigma}$ and $\bar{\sigma}'$ denote the greatest monotone equilibria under $\mathcal{I}$ and $\mathcal{I}'$ respectively. Then:*

$$\bar{\sigma}'(s_i) \supseteq \bar{\sigma}(s_i) \quad \textit{for all } s_i.$$

That is, more optimistic beliefs lead to (weakly) denser equilibrium networks at every signal realization.

Proof. The proof proceeds by monotone comparative statics on the best-response correspondence.

Step 1: Under more optimistic beliefs $\mathcal{I}'$, for any fixed opponent strategy σ_{-i}, firm i's expected cost of expansion decreases. This follows from FOSD: if μ is expected to be higher, then $\theta_i(\mu)$ is higher and unit costs K_i are lower. Moreover, if others' signals are expected to be higher (via affiliation), expected prices $\mathbb{E}[P^* \mid s_i]$ are lower.

Step 2: By the single-crossing property (Lemma 3), the best-response correspondence $\mathrm{BR}_i(\sigma_{-i}; \mathcal{I}')$ is pointwise greater than $\mathrm{BR}_i(\sigma_{-i}; \mathcal{I})$.

Step 3: The greatest equilibrium is the limit of iterated best responses starting from the maximal strategy. Since the best-response operator shifts upward under $\mathcal{I}'$, the fixed point $\bar{\sigma}'$ is pointwise greater than $\bar{\sigma}$. □

This proposition captures the *belief multiplier* without assuming differentiability or continuous actions. The amplification arises because:

1. **Direct channel:** A higher signal s_i raises $\mathbb{E}[\mu \mid s_i]$, directly increasing the expected benefit of expansion.

2. **Strategic channel:** A higher signal s_i raises $\mathbb{E}[s_j \mid s_i]$ for $j \neq i$ (by affiliation). This leads firm i to expect that others will expand, lowering expected prices, further increasing the benefit of expansion.

The combinatorial nature of network formation means this multiplier operates through *set inclusion*: the equilibrium supplier set $S_i^*(s_i)$ expands discretely as beliefs become more optimistic. The strategic channel reinforces the direct channel because the lattice of supplier sets is closed under union—if both firm i and its peers expand, the intersection benefits compound.

8. Cobb-Douglas Example and Belief-Adjusted Domar Weights

We now specialize to the **Cobb-Douglas/Gaussian** case to obtain explicit formulas for how beliefs enter aggregate productivity. This allows us to define belief-adjusted Domar weights that treat the information structure as first order.

8.1. *Cobb-Douglas Production and Gaussian Signals*

Assume Cobb-Douglas technology:

$$Y_i = \theta_i(\mu) L_i^{\alpha_i} \prod_{j \in S_i} X_{ij}^{\beta_{ij}}, \quad \text{with } \alpha_i + \sum_{j \in S_i} \beta_{ij} = 1. \tag{9}$$

Productivity is log-linear in the common factor:

$$\theta_i(\mu) = \exp(\varphi_i \mu + \eta_i),$$

where $\varphi_i > 0$ measures sector i's exposure to aggregate conditions and η_i is an idiosyncratic component.

Signals are Gaussian as in Example 1: $s_i = \mu + \varepsilon_i$ with $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ independent across i and of μ.

8.2. *Signal-Conditioned Domar Weights*

Let the equilibrium mapping from the signal profile $\mathbf{s} = (s_1, \ldots, s_n)$ to allocations be

$$\mathbf{s} \mapsto \big(P(\mathbf{s}), Y(\mathbf{s}), C(\mathbf{s}), S(\mathbf{s})\big),$$

where $S(\mathbf{s})$ is the endogenous network and (P, Y, C) are induced prices, outputs, and final demands.

Definition 2 (Signal-Conditioned Domar Weight)**.** The **signal-conditioned Domar weight** of sector i is:

$$D_i(\mathbf{s}) \equiv \frac{P_i(\mathbf{s})Y_i(\mathbf{s})}{\sum_{k=1}^{n} P_k(\mathbf{s})C_k(\mathbf{s})} = \frac{P_i(\mathbf{s})Y_i(\mathbf{s})}{\mathrm{GDP}(\mathbf{s})}. \tag{10}$$

This object is a function of signals because signals determine networks and thus prices. It summarizes which sectors are systemically important in the equilibrium induced by the belief state $\mathbf{s}$.

8.3. *Interim Domar Weights*

From the perspective of agent i, who observes only s_i, the relevant object is the expected Domar weight.

Definition 3 (Interim Domar Weight)**.** Agent i's **interim Domar weight** for sector j is:

$$D_j^i(s_i) \equiv \mathbb{E}\left[D_j(\mathbf{s}) \mid s_i\right]. \tag{11}$$

This is what firm i *believes* the Domar weight to be, given its information. Under affiliation, these interim expectations satisfy monotonicity:

Lemma 4 (Monotonicity of Interim Domar Weights)**.** *If the network is monotone in signals (Theorem* **??***) and larger networks increase sector j's output share, then $D_j^i(s_i)$ is non-decreasing in s_i.*

8.4. *Belief-Adjusted Domar Elasticities*

The Hulten/Domar logic says that a productivity change in sector i moves aggregate output by that sector's Domar weight. In our setting, we differentiate with respect to the *belief state*, allowing networks to adjust.

Define the posterior mean belief:

$$\hat{\mu}(\mathbf{s}) \equiv \mathbb{E}[\mu \mid \mathbf{s}], \qquad \hat{\theta}_i(\mathbf{s}) \equiv \exp(\varphi_i \hat{\mu}(\mathbf{s}) + \eta_i).$$

Definition 4 (Belief-Adjusted Domar Elasticity)**.** The **belief-adjusted Domar elasticity** of sector i is:

$$\Lambda_i(\mathbf{s}) \equiv \frac{\partial \log \mathrm{GDP}(\mathbf{s})}{\partial \log \hat{\theta}_i(\mathbf{s})}. \tag{12}$$

The **aggregate belief-Domar loading** is:

$$\Lambda(\mathbf{s}) \equiv \frac{\partial \log \mathrm{GDP}(\mathbf{s})}{\partial \hat{\mu}(\mathbf{s})} = \sum_{i=1}^{n} \Lambda_i(\mathbf{s}) \cdot \varphi_i. \tag{13}$$

This $\Lambda(\mathbf{s})$ is the summary statistic that treats beliefs as first order: it captures how a small belief shift about μ changes aggregate output through *both* the direct fundamental channel and the strategic network channel.

8.5. Decomposition: Hulten Term and Strategic Amplification

Proposition 3 (Belief-Adjusted Domar Decomposition)**.** *In the Cobb-Douglas/Gaussian economy with interior equilibrium, the aggregate belief-Domar loading decomposes as:*

$$\Lambda(\mathbf{s}) = \underbrace{\sum_{i=1}^{n} D_i(\mathbf{s}) \cdot \varphi_i}_{\textit{Hulten/Domar term}} \times \underbrace{\frac{1}{1-\theta(\mathbf{s})}}_{\textit{strategic amplification}}, \tag{14}$$

where $\theta(\mathbf{s}) \in (0,1)$ is an equilibrium feedback index measuring the strength of the network spillover.

Proof. (Sketch) The proof follows standard Leontief-inverse algebra. In the Cobb-Douglas case, log-linearizing around the equilibrium yields:

$$d \log \mathrm{GDP} = \sum_i D_i \cdot d \log \theta_i + \text{(price adjustment terms)}.$$

The price adjustment terms arise because network expansion lowers input costs, which raises output. Collecting terms, the strategic complementarity contributes a geometric series that sums to $(1-\theta)^{-1}$, where θ depends on the spectral radius of the input-output matrix weighted by belief correlations. □

Interpretation. In optimistic belief states, the endogenous network is denser, which pushes $\theta(\mathbf{s})$ up and makes the multiplier larger. This is precisely the "belief-adjusted Domar weight" story: beliefs enter first order not just through the direct productivity channel but through the network channel that amplifies shocks.

When the network is exogenous (fixed S), $\theta = 0$ and we recover Hulten's theorem: $\Lambda = \sum_i D_i \varphi_i$. Endogenous networks under dispersed information add the amplification factor.

9. Comparative Statics

We now examine how the information structure affects equilibrium network density. The comparative statics are derived using Topkis's monotone comparative statics theorem, without assuming differentiability.

9.1. Ordering Information Structures

We define an order on information structures based on their induced posteriors.

Definition 5 (More Informative Signals)**.** An information structure $\mathcal{I}'$ is *more optimistic* than $\mathcal{I}$ if, for all s_i, the posterior under $\mathcal{I}'$ first-order stochastically dominates the posterior under $\mathcal{I}$: $\pi'(\cdot \mid s_i) \geq_{\text{FOSD}} \pi(\cdot \mid s_i)$.

Definition 6 (More Correlated Signals)**.** An information structure $\mathcal{I}'$ has *more correlated signals* than $\mathcal{I}$ if, for all s_i, the conditional distribution of others' signals $\mathbf{s}_{-i}$ satisfies $\pi'(\mathbf{s}_{-i} \mid s_i) \geq_{\text{FOSD}} \pi(\mathbf{s}_{-i} \mid s_i)$.

9.2. Main Comparative Statics Results

Theorem 3 (Monotonicity in Signal Precision)**.** *Consider two information structures $\mathcal{I}$ and $\mathcal{I}'$ where signals under $\mathcal{I}'$ are more precise (lower noise variance). If higher precision induces more optimistic posteriors on average, then the greatest equilibrium network $\bar{\sigma}'$ is weakly denser than $\bar{\sigma}$.*

Proof. By Topkis's Monotone Comparative Statics Theorem. Higher signal precision increases $\mathbb{E}[\mu \mid s_i]$ for high signals and decreases it for low signals (signals become more informative about the true μ). For agents with high signals, the expected benefit of network expansion increases (via lower expected unit costs). By the single-crossing property (Lemma 3), best responses shift upward for optimistic types. The greatest equilibrium, being the limit of iterated best responses from the maximal strategy, increases. □

Theorem 4 (Monotonicity in Signal Correlation)**.** *Consider two information structures with identical marginal signal distributions, but $\mathcal{I}'$ has higher correlation between s_i and $\mathbf{s}_{-i}$. Then the greatest equilibrium network $\bar{\sigma}'$ is weakly denser than $\bar{\sigma}$.*

Proof. Higher correlation means that $\mathbb{E}[\mathbf{s}_{-i} \mid s_i]$ is larger for high s_i. By affiliation, this increases the expected network expansion by peers. Since larger peer networks lower expected prices (Proposition 1), and lower prices increase the marginal benefit of own expansion (Assumption 3), the best-response correspondence shifts upward. By Topkis's theorem, the greatest fixed point of the best-response operator increases. □

Economic Interpretation. These results have a common structure: changes in the information environment that increase the *expected actions of others* lead to denser equilibrium networks. This occurs through the strategic channel—the lattice of supplier sets expands monotonically as firms become more confident that their peers are expanding.

The policy implication is that **correlated information sources** (e.g., reliance on common public signals) amplify network volatility relative to **dispersed, idiosyncratic information**. Transparency policies that reduce correlation in forecast errors can stabilize supply chain formation.

10. Dynamic Extension

We extend the static model to a dynamic setting following Van Zandt and Vives (2007). The key insight is that the lattice-theoretic structure of the static game extends naturally to dynamic environments, yielding existence of monotone Markov perfect equilibria.

10.1. Dynamic Bayesian Game

Time is discrete, $t = 0, 1, 2, \ldots$. Each period, nature draws a productivity shock μ_t from a stationary distribution. Firms observe private signals s_{it} correlated with μ_t and with each other's signals (affiliation). The network choice S_{it} is made at the beginning of period t after observing s_{it}.

A firm's **state** at time t is its current belief about fundamentals and peers' actions, which we summarize by the pair $(s_{it}, S_{i,t-1})$—the current signal and the inherited network. The payoff is:

$$u_i(S_{it}, S_{-i,t}, \mu_t) - c(S_{it}, S_{i,t-1}),$$

where u_i is the period payoff from production (decreasing in unit cost) and $c(\cdot)$ is an adjustment cost that penalizes changes in the network.

10.2. Monotone Markov Strategies

Following Van Zandt and Vives (2007), we restrict attention to **Markov strategies** that depend only on the current state $(s_{it}, S_{i,t-1})$, not on the full history. A Markov strategy is **monotone** if $\sigma_i(s_{it}, S_{i,t-1})$ is non-decreasing in s_{it} (with respect to set inclusion) for each $S_{i,t-1}$.

The space of monotone Markov strategies forms a complete lattice under the pointwise order.

Theorem 5 (Existence of Monotone Markov Equilibria)**.** *Under Assumptions* **??**–*3, the dynamic game possesses a greatest and a least monotone Markov perfect equilibrium. In these equilibria, network expansion is monotone in the current signal: optimistic firms expand, pessimistic firms contract.*

Proof. (Sketch) The proof follows Van Zandt and Vives (2007). The best-response operator maps monotone strategies to monotone strategies (by the single-crossing property in the static game). The space of monotone strategies is a complete lattice. By Tarski's fixed point theorem, extremal fixed points exist. Discounting ensures that the dynamic best-response is a contraction in an appropriate metric, guaranteeing uniqueness of the value function for each strategy profile. □

10.3. Dynamics of Beliefs and Networks

A key feature of the dynamic model is **belief updating**. As firms observe their signals and the evolution of aggregate prices, they update their beliefs about the persistent component of μ. This creates a natural source of persistence: a sequence of positive signals leads to increasingly optimistic beliefs, which leads to denser networks, which lowers prices, which reinforces the expansion.

Remark 1 (Hysteresis)*.* If adjustment costs are asymmetric ($c(S', S) > c(S, S')$ for $S' \supset S$), the economy may exhibit hysteresis. A temporary negative shock can push the economy to the sparse equilibrium, from which it does not return even when signals recover. This is a dynamic manifestation of the equilibrium multiplicity in the static game.

11. Conclusion

This paper has integrated dispersed information into the theory of endogenous production networks. We showed that the formation of supply chains is driven by a **belief multiplier** arising from the interaction of affiliated beliefs and strategic complementarities.

Our results imply that supply chain volatility is not merely a reflection of fundamental TFP shocks but is endogenous to the information structure. Policies that improve transparency—such as standardized reporting of supply chain stress or public data on aggregate input flows—can reduce the correlation of belief errors and dampen the belief multiplier. Conversely, reliance on opaque, correlated signals exacerbates instability.

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A. APPENDIX: OMITTED PROOFS

A.1. Lattice Theory Preliminaries

We utilize the following definitions and theorems.

Definition 7 (Complete Lattice)**.** A partially ordered set $(L, \preceq)$ is a *complete lattice* if every subset $S \subseteq L$ has both a supremum (least upper bound) $\bigvee S$ and an infimum (greatest lower bound) $\bigwedge S$.

Theorem 6 (Tarski's Fixed Point Theorem)**.** *Let L be a complete lattice and $f : L \to L$ be an isotone (order-preserving) map. Then the set of fixed points of f is a non-empty complete lattice. In particular, there exist greatest and least fixed points.*

Our strategy space Σ_i is the set of monotone functions from $\mathbb{R}$ to the power set $2^{\mathcal{I}\setminus\{i\}}$. The power set is a complete lattice under inclusion. The function space is a complete lattice under the pointwise order.

Definition 8 (Supermodularity)**.** A function $f : L \to \mathbb{R}$ on a lattice is *supermodular* if for all $x, y \in L$:

$$f(x \vee y) + f(x \wedge y) \geq f(x) + f(y).$$

Definition 9 (Increasing Differences)**.** A function $f : L \times T \to \mathbb{R}$ has *increasing differences* in (x, t) if for all $x' \succeq x$ and $t' \succeq t$:

$$f(x', t') - f(x, t') \geq f(x', t) - f(x, t).$$

Theorem 7 (Topkis's Monotonicity Theorem)**.** *If $f : L \times T \to \mathbb{R}$ is supermodular in x and has increasing differences in (x, t), then $\arg\max_x f(x, t)$ is isotone in t.*

A.2. Proof of Lemma 3 (Single-Crossing)

We must show $\Delta(s_i) = \mathbb{E}[\Pi(S') - \Pi(S) \mid s_i]$ is increasing in s_i. Let $g(\mu, \mathbf{s}_{-i}) = K(S, \mu, P(\mu, \sigma_{-i}(\mathbf{s}_{-i}))) - K(S', \mu, P(\mu, \sigma_{-i}(\mathbf{s}_{-i})))$. We need to show g is increasing in its arguments.

Step 1: Monotonicity in μ. The unit cost is $K_i \propto 1/\theta_i(\mu)$. Since $\theta_i(\mu)$ is increasing in μ, higher μ lowers unit costs. The cost difference $K(S) - K(S')$ scales with $1/\theta_i$. Assuming costs are convex in S (diminishing returns), the *benefit* of expansion (negative cost difference) scales positively with productivity.

Step 2: Monotonicity in $\mathbf{s}_{-i}$. Since σ_{-i} is monotone, higher $\mathbf{s}_{-i}$ implies larger S_{-i}. By the P-matrix property (Proposition 1), larger S_{-i} implies lower P. By Assumption 3 (decreasing differences in (S, P)), lower P increases the benefit of expansion.

Step 3: Applying Lemma 1. Since g is increasing in $(\mu, \mathbf{s}_{-i})$, by Lemma 1, $\Delta(s_i) = \mathbb{E}[g(\mu, \mathbf{s}_{-i}) \mid s_i]$ is increasing in s_i. $\square$

A.3. *Remark: Linear Approximation in Gaussian Case*

In the Gaussian limit with continuous actions, one can approximate the belief multiplier using a linear best-response. Let y denote network density and consider:

$$y_i = \alpha \mathbb{E}[\mu | s_i] + \beta \mathbb{E}[y_{-i} | s_i].$$

In a symmetric equilibrium $y(s) = cs$, substituting Gaussian posteriors yields $c = \frac{\alpha\rho}{1-\beta\rho}$. The term $(1 - \beta\rho)^{-1}$ captures the amplification from iterated expectations. However, this continuous approximation abstracts from the discrete, combinatorial nature of the supplier set S_i. The monotone comparative statics in Proposition 2 provides a more general characterization that does not require differentiability or continuous actions. $\square$