Actividad 5.1 - Página 119

2) Si $\int_{-2}^{3} h(x)\,dx = 12$ y $\int_{0}^{3} h(x)\,dx = 3$, hallar el valor de $\int_{-2}^{0} h(x)\,dx$

Aditividad

$$\int_{-2}^{3} h(x)\,dx = \int_{-2}^{0} h(x)\,dx + \int_{0}^{3} h(x)\,dx =$$

$$12 = \int_{-2}^{0} h(x)\,dx + 3$$

$$12 - 3 = \int_{-2}^{0} h(x)\,dx \longrightarrow \boxed{\int_{-2}^{0} h(x)\,dx = 9}$$

1) Si $\int_{0}^{9} f(x)\,dx = 37$ y $\int_{0}^{9} g(x)\,dx = 16$, encontrar el valor de $\int_{0}^{9} \left[2\,f(x) - \frac{1}{4} g(x)\right] dx$

Linealidad | Linealidad

$$\int_{0}^{9} \left[2\,f(x) - \frac{1}{4} g(x)\right] dx = \int_{0}^{9} 2\,f(x)\,dx - \int_{0}^{9} \frac{1}{4} g(x)\,dx = 2\int_{0}^{9} f(x)\,dx - \frac{1}{4}\int_{0}^{9} g(x)\,dx =$$

$$= (2 \cdot 37) - \left(\frac{1}{4} \cdot 16\right) = 74 - 4 = 70 \longrightarrow \boxed{\int_{0}^{9} \left[2\,f(x)\,dx - \frac{1}{4} g(x)\,dx\right] = 70}$$

3) Si $\int_{-1}^{3} f(t)\,dt = 3$ y $\int_{-1}^{4} f(t)\,dt = 7$, determinar el valor de a) $\int_{3}^{4} f(t)\,dt$ y

b) $\int_{-1}^{4} f(z)\,dz$

Aditividad

a) $$\int_{-1}^{4} f(t)\,dt = \int_{-1}^{3} f(t)\,dt + \int_{3}^{4} f(t)\,dt$$

$$7 = 3 + \int_{3}^{4} f(t)\,dt$$

$$7 - 3 = \int_{3}^{4} f(t)\,dt \longrightarrow \boxed{\int_{3}^{4} f(t)\,dt = 4}$$

HOJA Nº

FECHA

b) $t = z$ (variable muda)

$$\int_{-1}^{4} f(z)\,dz = \int_{-1}^{4} f(t)\,dt$$

$$\boxed{\int_{-1}^{4} f(z)\,dz = 7}$$

- Cuando termino de integrar desaparece $\overbrace{\int \text{ y}}^{\text{combo}}$ dx, siempre desaparecen juntos.
- $\overbrace{\int 1 dx = \int dx}^{\text{son lo mismo}}$
- Si no puedo usar el teorema 5.4.1 → resuelvo primero de forma algebraica.
- $\frac{1}{x^2} = x^{-2}$ → propiedad muy útil
- Integral indefinida

$$\int \underbrace{f(x)}_{\text{integrando}} dx = F(x) + C$$

$$F'(x) = f(x)$$

- Integral definida → Área definida = suma infinita de rectángulos

NOTA

Ejercicios 5.7 - Página 136

1) a) $\int_{-2}^{3} 2x-1\,dx \Rightarrow$ Encontrar antiderivada

$$\int 2x-1\,dx = \int 2x\,dx - \int 1\,dx = 2\int x\,dx - \int 1\,dx = 2\left(\frac{x^{1+1}}{1+1}\right) - x = 2\cdot\frac{x^2}{2} - x = x^2 - x$$

Regla de Barrow

$$\int_{-2}^{3} 2x-1\,dx = (x^2-x)\Big|_{-2}^{3} = (3^2-3) - (-2^2-(-2)) = (9-3) - (4+2) = 6-6 = 0$$

$$\boxed{\int_{-2}^{3} 2x-1\,dx = 0}$$

b) $\int x^2+2x+8\,dx = \int x^2 dx + \int 2x\,dx + \int 8\,dx = \int x^2\,dx + 2\int x\,dx + 8\int 1\,dx =$

$$= \frac{x^3}{3} + 2\cdot\frac{x^2}{2} + 8x + C = \frac{x^3}{3} + x^2 + 8x + C$$

$$\boxed{\int x^2+2x+8\,dx = \frac{x^3}{3} + x^2 + 8x + C}$$

c) $\int_{0}^{2\pi} \sin(x) + x\,dx \rightarrow$ Encontrar antiderivada

$$\int \sin(x) + x\,dx = \int \sin(x)\,dx + \int x\,dx = -\cos(x) + \frac{x^2}{2}$$

Regla de Barrow

$$\int_{0}^{2\pi} \sin(x)+x\,dx = \left(-\cos(x) + \frac{x^2}{2}\right)\Big|_{0}^{2\pi} = \left(-\cos(2\pi) + \frac{(2\pi)^2}{2}\right) - \left(-\cos(0) + \frac{0^2}{2}\right) =$$

$$= \left(-(1) + \frac{4\pi^2}{2}\right) - \left(-(1) + 0\right) = (-1 + 2\pi^2) - (-1) = -1 + 2\pi^2 + 1 = 2\pi^2$$

$$\boxed{\int_{0}^{2\pi} \sin(x) + x\,dx = 2\pi^2}$$

HOJA Nº

FECHA

d) $\int_0^4 2e^x + 3x^4 \, dx \rightarrow$ Encontrar antiderivada

$\int 2e^x + 3x^4 \, dx = 2\int e^x \, dx + 3\int x^4 \, dx = 2.e^x + 3.\frac{x^5}{5} = 2e^x + \frac{3}{1}.\frac{x^5}{5} = 2e^x + \frac{3x^5}{5}$

Regla de Barrow

$\int_0^4 2e^x + 3x^4 \, dx = \left(2e^x + \frac{3x^5}{5}\right)\Big|_0^4 = \left(2e^4 + \frac{3(4)^5}{5}\right) - \left(2e^0 + \frac{3(0)^5}{5}\right) =$

$= \left(2e^4 + \frac{3.1024}{5}\right) - 2.1 + \frac{3.0}{5} = \left(2e^4 + \frac{3072}{5}\right) - (2.1 + 0) = 2e^4 + \frac{3072}{5} - 2 =$

$= 2e^4 + \frac{3072}{5} - \frac{10}{5} = 2e^4 + \frac{3062}{5}$

$\boxed{\int_0^4 2e^x + 3x^4 \, dx = 2e^4 + \frac{3062}{5}}$

e) $\int 3\frac{1}{x} + 2e^x \, dx = 3\int \frac{1}{x} \, dx + 2\int e^x \, dx = 3\ln|x| + 2e^x + C$

$\boxed{\int 3\frac{1}{x} + 2e^x \, dx = 3\ln|x| + 2e^x + C}$

f) $\int \cos(x) + \text{sen}(x) + 2x^{\frac{3}{5}} \, dx = \int \cos(x) \, dx + \int \text{sen}(x) \, dx + 2\int x^{\frac{3}{5}} \, dx =$

$= \text{sen}(x) + (-\cos(x)) + 2.\frac{x^{\frac{8}{5}}}{\frac{8}{5}} + C = \text{sen}(x) - \cos(x) + 2.\left(\frac{5}{8}.x^{\frac{8}{5}}\right) + C =$

$= \text{sen}(x) - \cos(x) + \frac{5}{4}x^{\frac{8}{5}} + C$

$\boxed{\int \cos(x) + \text{sen}(x) + 2x^{\frac{3}{5}} \, dx = \text{sen}(x) - \cos(x) + \frac{5}{4}x^{\frac{8}{5}} + C}$

g) $\int_{-5}^{1} x^2 + 2x + 8 \, dx \longrightarrow$ Encontrar primitiva

$\int x^2 + 2x + 8 \, dx = \int x^2 \, dx + \int 2x \, dx + \int 8 \, dx = \int x^2 \, dx + 2.\int x \, dx + 8\int dx =$

NOTA

$= \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 8x + C = \frac{x^3}{3} + x^2 + 8x + C$

Regla de Barrow

$\int_{-5}^{1} x^2 + 2x + 8\, dx = \left(\frac{x^3}{3} + x^2 + 8x + C\right)\Big|_{-5}^{1} = \left(\frac{1^3}{3} + 1^2 + 8.1\right) - \left(\frac{(-5)^3}{3} + (-5)^2 + 8.(-5)\right) =$

$= \left(\frac{1}{3} + 1 + 8\right) - \left(\frac{-125}{3} + 25 - 40\right) = \left(\frac{1}{3} + 9\right) - \left(\frac{-125}{3} - 15\right) = \left(\frac{1}{3} + \frac{27}{3}\right) - \left(-\frac{125}{3} - \frac{45}{3}\right) =$

$= \frac{28}{3} - \left(-\frac{170}{3}\right) = \frac{28}{3} + \frac{170}{3} = \frac{198}{3} = 66$

$\boxed{\int_{-5}^{1} x^2 + 2x + 8\, dx = 66}$

h) $\int x - x^{\frac{2}{5}} + 3e^x - \cos(x)\, dx = \int x\, dx - \int x^{\frac{2}{5}}\, dx + 3 \int e^x\, dx - \int \cos(x)\, dx =$

$= \frac{x^2}{2} - \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 3e^x - \operatorname{sen}(x) + C = \frac{x^2}{2} - \frac{5}{7} \cdot x^{\frac{7}{5}} + 3e^x - \operatorname{sen}(x) + C$

$\boxed{\int x - x^{\frac{2}{5}} + 3e^x - \cos(x)\, dx = \frac{x^2}{2} - \frac{5}{7} \cdot x^{\frac{7}{5}} + 3e^x - \operatorname{sen}(x) + C}$

HOJA Nº

FECHA

~~2) a) $\int (3x^4+5x^2+8)^4 \cdot (12x^3+10x)\,dx$~~ Integración por partes:

$u \cdot v' + v \cdot u' = (u \cdot v)' \rightarrow \int u \cdot v'dx + \int v \cdot u'dx = u \cdot v \rightarrow$

$\rightarrow \int u \cdot dv + \int v \cdot du = u \cdot v \rightarrow \boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$

2) a) $\int (3x^4+5x^2+8)^4 \cdot (12x^3+10x)\,dx = \int (u)^4 \cdot du = \frac{u^{4+1}}{4+1} + C = \frac{u^5}{5} + C$

$u = 3x^4+5x^2+8$

$du = (12x^3+10x)\,dx$

$$\boxed{\int (3x^4+5x^2+8)^4 \cdot (12x^3+10x)\,dx = \frac{(3x^4+5x^2+8)^5}{5} + C}$$

Si la derivo debe darme la función original que se integraba (con regla de la cadena)

b) $\int x \cdot \cos(x)\,dx$

$u = x \xrightarrow{\text{Derivo}} du = 1 \cdot dx$

$dv = \cos(x) \cdot dx \xrightarrow{\text{Integro}} v = \operatorname{sen}(x)$

$\int x \cdot \cos(x)\,dx = x \cdot \operatorname{sen}(x) - \int \operatorname{sen}(x) \cdot 1\,dx$

$$\boxed{\int x \cdot \cos(x)\,dx = x \cdot \operatorname{sen}(x) + \cos(x) + C}$$

c) $\int x^3 \cdot \ln(x)\,dx$

$u = \ln(x) \xrightarrow{\text{Derivo}} du = \frac{1}{x}\,dx$

$dv = 1x^3 \cdot dx \xrightarrow{\text{Integro}} v = \frac{x^4}{4}$

$\int x^3 \cdot \ln(x)\,dx = \ln(x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx$

NOTA

$\int x^3 . \ln(x)\, dx = \ln(x) . \frac{x^4}{4} - \int \frac{x^4}{4} . \frac{1}{x}\, dx$

Resolver integral

$\int \frac{x^4}{4} . \frac{1}{x}\, dx = \int \frac{x^4}{4x}\, dx = \int \frac{x^3}{4}\, dx = \frac{1}{4} . \int x^3\, dx = \frac{1}{4} . \frac{x^4}{4} = \frac{x^4}{16} + c$

Reemplazar

$$\boxed{\int x^3 . \ln(x)\, dx = \ln(x) . \frac{x^4}{4} - \frac{x^4}{16} + c}$$

d) $\int \cos(5x) . 5\, dx = \int \cos(u) . du = \operatorname{sen}(u) + C$

$u = 5x$

$du = 5\, dx$

$$\boxed{\int \cos(5x) . 5\, dx = \operatorname{sen}(5x) + C}$$

e) $\int \frac{2 + e^x}{e^x + 2x}\, dx \longrightarrow \int \frac{du}{u} = \int \frac{1}{u} . du = \ln(|u|) + c$

$u = e^x + 2x$

$du = (2 + e^x)\, dx$

Reemplazar

$$\boxed{\int \frac{2 + e^x\, dx}{e^x + 2x} = \ln(|e^x + 2x|) + c}$$

f) $\int x . \sqrt{x-1}\, dx = \int u+1 . \sqrt{u} . du = \int u+1 . u^{\frac{1}{2}} . du = \int (u . u^{\frac{1}{2}} + 1 . u^{\frac{1}{2}})\, du =$

$u = x - 1$

$du = 1 dx$ para obtener du derivo $u \rightarrow u' = x - 1 \rightarrow du = 1 - 0 = 1$

$u = x - 1 \rightarrow u + 1 = x \rightarrow x = u + 1$

$= \int (u^{\frac{3}{2}} + u^{\frac{1}{2}})\, du =$

$= \int u^{\frac{3}{2}}\, du + \int u^{\frac{1}{2}}\, du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}} + C$

HOJA N°

FECHA

Reemplazar

$$\boxed{\int x\sqrt{x-1}\,dx = \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C}$$

g) $\int_0^8 \frac{1}{\sqrt{x+1}}\,dx \longrightarrow u = x+1 \xrightarrow{\text{derivo } u} du = 1$

Despejo x $\longrightarrow u-1 = x \rightarrow x = u-1$

$$\int_0^8 \frac{1}{\sqrt{x+1}}\,dx = \int_0^8 \frac{1}{\sqrt{u}}\,du = \int_0^8 u^{-\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = 2u^{\frac{1}{2}} \text{ o } 2\sqrt{u} \longrightarrow 2.(x+1)^{\frac{1}{2}}$$

$$\int_0^8 \frac{1}{\sqrt{x+1}}\,dx = 2.(x+1)^{\frac{1}{2}}\Big|_0^8 = 2.(8+1)^{\frac{1}{2}} - 2.(0+1)^{\frac{1}{2}} = 2.9^{\frac{1}{2}} - 2.1^{\frac{1}{2}} = 2.3 - 2.1 =$$

$= 6 - 2 = 4$ con regla de Barrow

$$\boxed{\int_0^8 \frac{1}{\sqrt{x+1}}\,dx = 4}$$

~~h) $\int_0^{2\pi} x.\operatorname{sen}(x)\,dx$~~

~~$u = x \xrightarrow{\text{Derivo}} du = 1dx$~~

~~$dv = \operatorname{sen}(x) \xrightarrow{\text{Integro}} V = -\cos(x)$~~

~~$\int x.\operatorname{Sen}(x)\,dx = x.-\cos(x) - \int -\cos(x).1\,dx$~~

NOTA

3) Hallar el área comprendida

a) $f(x) = x^2$ y $g(x) = -x^2+4$ $\rightarrow$

$$f(x) = g(x)$$
$$x^2 = -x^2+4$$
$$x^2 + x^2 - 4 = 0$$
$$2x^2 - 4 = 0$$
$$2x^2 = 4$$
$$x^2 = \frac{4}{2}$$

se debe calcular el área en el intervalo $[-\sqrt{2}, \sqrt{2}]$ $\leftarrow$ $x_1 = \sqrt{2}$ y $x_2 = -\sqrt{2}$

b) $f(x) = x^4$ y $g(x) = x$ $\rightarrow$

$$f(x) = g(x)$$
$$x^4 = x$$
$$x^4 - x = 0$$
$$x\left(\frac{x^4}{x} - \frac{x}{x}\right) = 0$$
$$x(x^3 - 1) = 0$$

Se debe calcular el área en el intervalo $[0, 1]$ $\leftarrow$ $x = 0$ ó $x = 1$

c) $f(x) = x^3 + \frac{1}{4}x$ y $g(x) = \frac{1}{2}x$ $\rightarrow$

$$f(x) = g(x)$$
$$x^3 + \frac{1}{4}x = \frac{1}{2}x$$
$$x^3 + \frac{1}{4}x - \frac{1}{2}x = 0$$
$$x^3 - \frac{1}{4}x = 0$$
$$x\left(\frac{x^3}{x} - \frac{\frac{1}{4}x}{x}\right) = 0$$
$$x\left(x^2 - \frac{1}{4}\right) = 0$$

$x = 0$ $x = \frac{1}{2}$ $x = -\frac{1}{2}$

se debe calcular el área en los intervalos $\left[-\frac{1}{2}, 0\right]$ y $\left[0, \frac{1}{2}\right]$

HOJA N°

FECHA

4) Función 1: $f(x) = -x^2 - 1$

Función 2: $g(x) = 0$

Intervalo $[-2, 1]$

Identificar Techo y Piso con valores de prueba

	$[-2,1]$
VP	-1
f(VP)	-2
g(VP)	0

$f(-1) = -(-1)^2 - 1 = -1 - 1 = -2$

$g(-1) = 0$

g(x) es TECHO y f(x) es PISO en el intervalo $[-2, 1]$

$$A = \int_{-2}^{1} [\text{TECHO} - \text{PISO}]\,dx = \int_{-2}^{1} [g(x) - f(x)]\,dx = \int_{-2}^{1} [(0) - (-x^2 - 1)]\,dx =$$

$$= \int_{-2}^{1} [0 + x^2 + 1]\,dx = \int_{-2}^{1} [x^2 + 1]\,dx = \frac{x^3}{3} + x \Big|_{-2}^{1} = \left(\frac{(1)^3}{3} + 1\right) - \left(\frac{(-2)^3}{3} + (-2)\right) =$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{-8}{3} - 2\right) = \left(\frac{1}{3} + \frac{3}{3}\right) - \left(\frac{-8}{3} + \frac{6}{3}\right) = \left(\frac{4}{3}\right) - \left(\frac{-14}{3}\right) = \frac{4}{3} + \frac{14}{3} = \frac{18}{3} = 6$$

$= 6$ $\boxed{A = 6}$

5) Función 1: $f(x) = x^2 + 2x - 3$ Intervalo $= [-1, 2]$

Cruces con el eje x

$x^2 + 2x - 3 = 0 \rightarrow$ Bhaskara $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} =$

$\dfrac{-2 \pm \sqrt{4 + 12}}{2} = \dfrac{-2 \pm \sqrt{16}}{2} \rightarrow x_1 = \dfrac{-2 + 4}{2} = \dfrac{2}{2} = 1$

$x_2 = \dfrac{-2 - 4}{2} = \dfrac{-6}{2} = -3$

Intervalo 1 $= [-1, 1]$ Intervalo 2 $= [1, 2]$

NOTA

Buscar Techo y Piso en ambos intervalos

$f(0) = 0^2 + 2.0 - 3 = -3$ $\qquad$ $f(1,5) = (1,5)^2 + 2.1,5 - 3 = 3+3-3 = 3$

Entonces $f(x) < 0$ en $[-1,1]$ y $f(x) >$ en $[1,2]$

$$A = A_1 + A_2 = -\int_{-1}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx = -\int_{-1}^{1} (x^2+2x-3)\,dx + \int_{1}^{2} (x^2+2x-3)\,dx =$$

$$= -\left[\frac{x^3}{3} + 2\frac{x^2}{2} - 3x\right]_{-1}^{1} + \left[\frac{x^3}{3} + 2\frac{x^2}{2} - 3x\right]_{1}^{2} = -\left[\frac{x^3}{3} + x^2 - 3x\right]_{-1}^{1} + \left[\frac{x^3}{3} + x^2 - 3x\right]_{1}^{2} =$$

$$= -\left[\left(\frac{1^3}{3} + 1^2 - 3.1\right) - \left(\frac{(-1)^3}{3} + (-1)^2 - 3.(-1)\right)\right] + \left[\left(\frac{2^3}{3} + 2^2 - 3.2\right) - \left(\frac{1^3}{3} + 1^2 - 3.1\right)\right] =$$

$$= -\left[\left(\frac{1}{3} + 1 - 3\right) - \left(\frac{-1}{3} + 1 + 3\right)\right] + \left[\left(\frac{8}{3} + 4 - 6\right) - \left(\frac{1}{3} + 1 - 3\right)\right] =$$

$$= -\left[\left(\frac{1}{3} + \frac{3}{3} - \frac{9}{3}\right) - \left(-\frac{1}{3} + \frac{3}{3} + \frac{9}{3}\right)\right] + \left[\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 2\right)\right] = -\left[-\frac{5}{3} - \frac{11}{3}\right] + \left[\left(\frac{8}{3} - \frac{6}{3}\right) - \left(\frac{1}{3} - \frac{6}{3}\right)\right] =$$

$$= -\left(-\frac{16}{3}\right) + \left[\left(\frac{2}{3}\right) - \left(\frac{5}{3}\right)\right] = \frac{16}{3} + \left(\frac{2}{3} + \frac{5}{3}\right) = \frac{16}{3} + \frac{7}{3} = \frac{23}{3}$$

$$\boxed{A_1 = \frac{16}{3} \qquad A_2 = \frac{7}{3} \qquad A_{Total} = \frac{23}{3}}$$

6) $f(x) = x^2 + 1$ y $g(x) = 2x^2$ $\qquad$ Intervalo general $[0,2]$

Buscar puntos de intersección $\qquad f(x) = g(x)$

$$x^2 + 1 = 2x^2$$

$$x^2 + 1 - 2x^2 = 0$$

$$-x^2 + 1 = 0$$

$$-x^2 = -1$$

$$x^2 = 1 \qquad x = 1 \text{ ó } x = -1$$

HOJA Nº
FECHA

Calcular área en los intervalos $[0,1]$ y $[1,2]$

Determinar Techo y Piso

	$[0,1]$	$[1,2]$
vP	0,5	1,5
f(vP)	1,25	3,25
g(vP)	0,5	4,5

$f(0,5) = 0,5^2 + 1 = 0,25 + 1 = 1,25$

$g(0,5) = 2 \cdot 0,5^2 = 2 \cdot 0,25 = 0,5$

$f(1,5) = 1,5^2 + 1 = 2,25 + 1 = 3,25$

$g(1,5) = 2 \cdot (1,5)^2 = 2 \cdot 2,25 = 4,5$

Entonces en el intervalo $[0,1]$ $f(x)$ es TECHO y $g(x)$ PISO, mientras que en el intervalo $[1,2]$ $g(x)$ es TECHO y $f(x)$ PISO.

$$A = \int_0^1 (\text{TECHO} - \text{PISO})\,dx + \int_1^2 (\text{TECHO} - \text{PISO})\,dx = \int_0^1 (f(x) - g(x))\,dx + \int_1^2 (g(x) - f(x))\,dx =$$

$$= \int_0^1 ((x^2+1) - (2x^2))\,dx + \int_1^2 ((2x^2) - (x^2+1))\,dx = \int_0^1 (x^2+1-2x^2)\,dx + \int_1^2 (2x^2 - x^2 - 1)\,dx =$$

$$= \int_0^1 (-x^2+1)\,dx + \int_1^2 (x^2-1)\,dx = \left(-\frac{x^3}{3} + x\right)\Big|_0^1 + \left(\frac{x^3}{3} - x\right)\Big|_1^2 =$$

$$= \left(\left(-\frac{(1)^3}{3} + 1\right) - \left(-\frac{(0)^3}{3} + 0\right)\right) + \left(\left(\frac{2^3}{3} - 2\right) - \left(\frac{1^3}{3} - 1\right)\right) = \left(\left(-\frac{1}{3} + 1\right) - (0)\right) + \left(\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right)\right) =$$

$$= \left(-\frac{1}{3} + \frac{3}{3}\right) + \left(\left(\frac{8}{3} - \frac{6}{3}\right) - \left(\frac{1}{3} - \frac{3}{3}\right)\right) = \left(\frac{2}{3}\right) + \left(\left(\frac{2}{3}\right) - \left(-\frac{2}{3}\right)\right) = \frac{2}{3} + \left(\frac{2}{3} + \frac{2}{3}\right) =$$

$$= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$$

Por lo tanto, el área de la región encerrada por las funciones $f(x)$ y $g(x)$ es $A = \underset{\frac{2}{3}}{A_1} + \underset{\frac{4}{3}}{A_2} = 2$

NOTA