

Actividad 5.1 - Página 119

2) si $\int_{-2}^3 h(x) dx = 12$ y $\int_0^3 h(x) dx = 3$, hallar el valor de $\int_{-2}^0 h(x) dx$

Aditividad

$$\int_{-2}^3 h(x) dx = \int_{-2}^0 h(x) dx + \int_0^3 h(x) dx =$$

$$12 = \int_{-2}^0 h(x) dx + 3$$

$$12 - 3 = \int_{-2}^0 h(x) dx \rightarrow \boxed{\int_{-2}^0 h(x) dx = 9}$$

1) si $\int_0^9 f(x) dx = 37$ y $\int_0^9 g(x) dx = 16$, encontrar el valor de $\int_0^9 [2f(x) - \frac{1}{4}g(x)] dx$

Linealidad

$$\int_0^9 [2f(x) - \frac{1}{4}g(x)] dx = \int_0^9 2f(x) dx - \int_0^9 \frac{1}{4}g(x) dx = 2 \int_0^9 f(x) dx - \frac{1}{4} \int_0^9 g(x) dx =$$

$$= (2 \cdot 37) - \left(\frac{1}{4} \cdot 16\right) = 74 - 4 = 70 \rightarrow \boxed{\int_0^9 [2f(x) dx - \frac{1}{4}g(x) dx] = 70}$$

3) si $\int_{-1}^3 f(t) dt = 3$ y $\int_{-1}^4 f(t) dt = 7$, determinar el valor de a) $\int_3^4 f(t) dt$ y b) $\int_{-1}^4 f(t) dt$

Aditividad

$$a) \int_{-1}^4 f(t) dt = \int_{-1}^3 f(t) dt + \int_3^4 f(t) dt$$

$$7 = 3 + \int_3^4 f(t) dt$$

$$7 - 3 = \int_3^4 f(t) dt \rightarrow \boxed{\int_3^4 f(t) dt = 4}$$

b) $t = z$ (variable mutua)

$$\int_{-1}^1 f(z) dz = \int_{-1}^1 f(t) dt$$

$$\int_{-1}^1 f(z) dz = ?$$

Combo

- Cuando termino de integrar desaparece $\int dx$, siempre desaparecen juntos.
- Son lo mismo
- $\int dx = \int dx$
- Si no puedo usar el teorema 5.4.1 \rightarrow Resuelvo primero de forma algebraica.
- $\frac{1}{x^2} = x^{-2}$ \rightarrow Propiedad muy útil
- Integral indefinida
- $\int f(x) dx = F(x) + C$
 ↓
 integrando
- $F'(x) = f(x)$
- Integral definida \rightarrow Área definida = suma infinita de rectángulos

Ejercicios 5.7 - Página 136

1) a) $\int_{-2}^3 2x-1 dx \rightarrow$ Encontrar antiderivada

$$\int 2x-1 dx = \int 2x dx - \int 1 dx = 2 \int x dx - \int 1 dx = 2 \left(\frac{x^{1+1}}{1+1} \right) - x = 2 \cdot \frac{x^2}{2} - x = x^2 - x$$

Regla de Barlow

$$\int_{-2}^3 2x-1 dx = (x^2 - x) \Big|_{-2}^3 = (3^2 - 3) - (-2^2 - (-2)) = (9 - 3) - (4 + 2) = 6 - 6 = 0$$

$\int_{-2}^3 2x-1 dx = 0$

b) $\int x^2 + 2x + 8 dx = \int x^2 dx + \int 2x dx + \int 8 dx = \int x^2 dx + 2 \int x dx + 8 \int 1 dx =$

$$= \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 8x + C = \frac{x^3}{3} + x^2 + 8x + C$$

$\int x^2 + 2x + 8 dx = \frac{x^3}{3} + x^2 + 8x + C$

c) $\int_0^{2\pi} \sin(x) + x dx \rightarrow$ Encontrar antiderivada

$$\int \sin(x) + x dx = \int \sin(x) dx + \int x dx = -\cos(x) + \frac{x^2}{2}$$

Regla de Barlow

$$\int_0^{2\pi} \sin(x) + x dx = \left[-\cos(x) + \frac{x^2}{2} \right]_0^{2\pi} = \left(-\cos(2\pi) + \frac{(2\pi)^2}{2} \right) - \left(-\cos(0) + \frac{0^2}{2} \right) =$$

$$= \left(-1 + \frac{4\pi^2}{2} \right) - (-1 + 0) = (-1 + 2\pi^2) - (-1) = -1 + 2\pi^2 + 1 = 2\pi^2$$

$\int_0^{2\pi} \sin(x) + x dx = 2\pi^2$

d) $\int_0^4 2e^x + 3x^4 dx \rightarrow$ Encontrar antiderivada

$$\int 2e^x + 3x^4 dx = 2 \int e^x dx + 3 \int x^4 dx = 2e^x + 3 \cdot \frac{x^5}{5} = 2e^x + \frac{3}{5}x^5 = 2e^x + \frac{3x^5}{5}$$

Regla de Barrow

$$\int_0^4 2e^x + 3x^4 dx = \left(2e^x + \frac{3x^5}{5}\right) \Big|_0^4 = \left(2e^4 + \frac{3 \cdot 4^5}{5}\right) - \left(2e^0 + \frac{3 \cdot 0^5}{5}\right) =$$

$$= \left(2e^4 + \frac{3 \cdot 1024}{5}\right) - 2 \cdot 1 + \frac{3 \cdot 0}{5} = \left(2e^4 + \frac{3072}{5}\right) - (2 \cdot 1 + 0) = 2e^4 + \frac{3072}{5} - 2 =$$

$$= 2e^4 + \frac{3072}{5} - \frac{10}{5} = 2e^4 + \frac{3062}{5}$$

$$\int_0^4 2e^x + 3x^4 dx = 2e^4 + \frac{3062}{5}$$

e) $\int 3\frac{1}{x} + 2e^x dx = 3 \int \frac{1}{x} dx + 2 \int e^x dx = 3 \ln|x| + 2e^x + C$

$$\int 3\frac{1}{x} + 2e^x dx = 3 \ln|x| + 2e^x + C$$

f) $\int \cos(x) + \operatorname{sen}(x) + 2x^{\frac{3}{5}} dx = \int \cos(x) dx + \int \operatorname{sen}(x) dx + 2 \int x^{\frac{3}{5}} dx =$

$$= \operatorname{sen}(x) + (-\cos(x)) + 2 \cdot \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + C = \operatorname{sen}(x) - \cos(x) + 2 \cdot \left(\frac{5}{8} \cdot x^{\frac{8}{5}}\right) + C =$$

$$= \operatorname{sen}(x) - \cos(x) + \frac{5}{4}x^{\frac{8}{5}} + C$$

$$\int \cos(x) + \operatorname{sen}(x) + 2x^{\frac{3}{5}} dx = \operatorname{sen}(x) - \cos(x) + \frac{5}{4}x^{\frac{8}{5}} + C$$

g) $\int_{-5}^1 x^2 + 2x + 8 dx \rightarrow$ Encontrar Primitiva

$$\int x^2 + 2x + 8 dx = \int x^2 dx + \int 2x dx + \int 8 dx = \int x^2 dx + 2 \cdot \int x dx + 8 \int dx =$$

$$= \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 8x + C = \frac{x^3}{3} + x^2 + 8x + C$$

Regla de Barlow

$$\int_{-5}^1 x^2 + 2x + 8 dx = \left(\frac{x^3}{3} + x^2 + 8x + C \right) \Big|_{-5}^1 = \left(\frac{1^3}{3} + 1^2 + 8 \cdot 1 \right) - \left(\frac{(-5)^3}{3} + (-5)^2 + 8 \cdot (-5) \right) =$$

$$= \left(\frac{1}{3} + 1 + 8 \right) - \left(-\frac{125}{3} + 25 - 40 \right) = \left(\frac{1}{3} + 9 \right) - \left(-\frac{125}{3} - 15 \right) = \left(\frac{1}{3} + \frac{27}{3} \right) - \left(-\frac{125}{3} - \frac{45}{3} \right) =$$

$$= \frac{28}{3} - \left(-\frac{170}{3} \right) = \frac{28}{3} + \frac{170}{3} = \frac{198}{3} = 66$$

$$\boxed{\int_{-5}^1 x^2 + 2x + 8 dx = 66}$$

$$\text{b)} \int x - x^{\frac{2}{5}} + 3e^x - \cos(x) dx = \int x dx - \int x^{\frac{2}{5}} dx + 3 \int e^x dx - \int \cos(x) dx =$$

$$= \frac{x^2}{2} - \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 3e^x - \sin(x) + C = \frac{x^2}{2} - \frac{5}{7} \cdot x^{\frac{7}{5}} + 3e^x - \sin(x) + C$$

$$\boxed{\int x - x^{\frac{2}{5}} + 3e^x - \cos(x) dx = \frac{x^2}{2} - \frac{5}{7} \cdot x^{\frac{7}{5}} + 3e^x - \sin(x) + C}$$

2) a) $\int (3x^4 + 5x^2 + 8)^4 \cdot (12x^3 + 10x) dx$ Integración por partes:

$$u \cdot v' + v \cdot u' = (u \cdot v)' \rightarrow \int u \cdot v' dx + \int v \cdot u' dx = u \cdot v \rightarrow$$

$$\rightarrow \int u \cdot dv + \int v \cdot du = u \cdot v \rightarrow \int u \cdot dv = u \cdot v - \int v \cdot du$$

2) a) $\int (3x^4 + 5x^2 + 8)^4 \cdot (12x^3 + 10x) dx = \int (u)^4 \cdot du = \frac{u^{4+1}}{4+1} + C = \frac{u^5}{5} + C$

$$u = 3x^4 + 5x^2 + 8$$

$$du = (12x^3 + 10x) dx$$

$$\int (3x^4 + 5x^2 + 8)^4 \cdot (12x^3 + 10x) dx = \frac{(3x^4 + 5x^2 + 8)^5}{5} + C$$

Sí, la derivada debe darme la función original que se integraba (con regla de la cadena) ↗

b) $\int x \cdot \cos(x) dx$

$$u = x \quad \xrightarrow{\text{Derivo}} \quad du = 1 \cdot dx$$

$$dv = \cos(x) dx \quad \xrightarrow{\text{Integro}} \quad v = \sin(x)$$

$$\int x \cdot \cos(x) dx = x \cdot \sin(x) - \int \sin(x) \cdot 1 dx$$

$$\int x \cdot \cos(x) dx = x \cdot \sin(x) + \cos(x) + C$$

c) $\int x^3 \cdot \ln(x) dx$

$$u = \ln(x) \quad \xrightarrow{\text{Derivo}} \quad du = \frac{1}{x} dx$$

$$dv = x^3 \cdot dx \quad \xrightarrow{\text{Integro}} \quad v = \frac{x^4}{4}$$

$$\int x^3 \cdot \ln(x) dx = \ln(x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

$$\int x^3 \cdot \ln(x) dx = \ln(x) \cdot \frac{x^4}{4} - \int \frac{x^4 + 1}{4x} dx$$

↓

Resolver integral

$$\int \frac{x^3}{4} \cdot \frac{1}{x} dx = \int \frac{x^2}{4} dx = \int \frac{x^3}{4} dx = \frac{1}{4} \cdot \int x^3 dx = \frac{1}{4} \cdot \frac{x^4}{4} = \frac{x^4}{16} + C$$

Reemplazar ↴

$$\boxed{\int x^3 \cdot \ln(x) dx = \ln(x) \cdot \frac{x^4}{4} - \frac{x^4}{16} + C}$$

d) $\int \cos(5x) \cdot 5 dx = \int \cos(u) \cdot du = \sin(u) + C$

$$u = 5x$$

$$du = 5 dx$$

$$\boxed{\int \cos(5x) \cdot 5 dx = \sin(5x) + C}$$

e) $\int \frac{2+e^x}{e^x+2x} dx \rightarrow \int \frac{du}{u} = \int \frac{1}{u} \cdot du = \ln(|u|) + C$

$$u = e^x + 2x$$

Reemplazar ↴

$$\boxed{\int \frac{2+e^x dx}{e^x+2x} = \ln(|e^x+2x|) + C}$$

f) $\int x \cdot \sqrt{x-1} dx = \int u+1 \cdot \sqrt{u} \cdot du = \int u+1 \cdot u^{\frac{1}{2}} \cdot du = \int (u \cdot u^{\frac{1}{2}} + 1 \cdot u^{\frac{1}{2}}) du =$

$$\begin{cases} u = x-1 \\ du = 1 dx \end{cases}$$

para obtener du derivando u → $u' = x-1 \rightarrow du = 1 dx \rightarrow 1-0=1$

$$u = x-1 \rightarrow u+1 = x \rightarrow x = u+1$$

$$= \int (u^{\frac{3}{2}} + u^{\frac{1}{2}}) du =$$

$$= \int u^{\frac{3}{2}} du + \int u^{\frac{1}{2}} du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}} + C$$

Reemplazar

$$\boxed{\int x \sqrt{x-1} dx = \frac{2}{5} (x-1)^{\frac{5}{2}} + \frac{2}{3} (x-1)^{\frac{3}{2}} + C}$$

g) $\int_0^8 \frac{1}{\sqrt{x+1}} dx \quad u = x+1 \quad \text{derivo } u \quad du = 1$
 $| \qquad \qquad \qquad u-1=x \rightarrow x=u-1$
 Despejé u de x

$$\int_0^8 \frac{1}{\sqrt{x+1}} dx = \int_0^8 \frac{1}{\sqrt{u}} du = \int_0^8 u^{-\frac{1}{2}} du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = 2u^{\frac{1}{2}} \circ 2\sqrt{u} \rightarrow 2.(x+1)^{\frac{1}{2}}$$

$$\int_0^8 \frac{1}{\sqrt{x+1}} dx = 2.(x+1)^{\frac{1}{2}} \Big|_0^8 = 2.(8+1)^{\frac{1}{2}} - 2.(0+1)^{\frac{1}{2}} = 2.9^{\frac{1}{2}} - 2.1^{\frac{1}{2}} = 2.3 - 2.1 =$$

$$= 6 - 2 = 4 \quad \text{con regla de Barlow}$$

$$\boxed{\int_0^8 \frac{1}{\sqrt{x+1}} dx = 4}$$

(h) ~~$\int_0^{2\pi} x \cdot \operatorname{sen}(x) dx$~~

~~$u = x \quad \text{Derivo} \quad du = 1 dx$~~

~~$dV = \operatorname{sen}(x) \quad \text{Integro} \quad V = -\operatorname{cos}(x)$~~

~~$\int x \cdot \operatorname{sen}(x) dx = x \cdot -\operatorname{cos}(x) - \int -\operatorname{cos}(x) \cdot 1 dx$~~

3) Hallar el área comprendida

a) $f(x) = x^2 \quad g(x) = -x^2 + 4 \rightarrow f(x) = g(x)$

$$x^2 = -x^2 + 4$$

$$x^2 + x^2 - 4 = 0$$

$$2x^2 - 4 = 0$$

$$2x^2 = 4$$

$$x^2 = \frac{4}{2}$$

se debe calcular el $x_1 = \sqrt{2} \quad x_2 = -\sqrt{2}$

área en el intervalo $[-\sqrt{2}, \sqrt{2}]$

b) $f(x) = x^3 \quad g(x) = x \rightarrow f(x) = g(x)$

$$x^3 = x$$

$$x^3 - x = 0$$

$$x\left(\frac{x^2}{x} - \frac{x}{x}\right) = 0$$

$$1 = \sqrt[3]{1}$$

$$x(x^2 - 1) = 0$$

se debe calcular el área $x=0 \quad 0 \leq x \leq 1$

en el intervalo $[0, 1]$

c) $f(x) = x^3 + \frac{1}{4}x \quad g(x) = \frac{1}{2}x \rightarrow f(x) = g(x)$

$$\frac{x^3}{4} + \frac{1}{4}x = \frac{1}{2}x$$

$$\frac{x^3}{4} + \frac{1}{4}x - \frac{1}{2}x = 0$$

se debe calcular el área

en los intervalos $[-\frac{1}{2}, 0] \cup [0, \frac{1}{2}]$

$$x^3 - \frac{1}{4}x = 0$$

$$x\left(\frac{x^2}{x} - \frac{\frac{1}{4}x}{x}\right) = 0$$

A
 $x=0 \quad x=\frac{1}{2} \quad x=-\frac{1}{2}$

$$x(x^2 - \frac{1}{4}) = 0$$

4) Función 1: $f(x) = -x^2 - 1$
 Función 2: $g(x) = 0$ \rightarrow Intervalo $[-2, 1]$

Identificar Techo y Piso con valores de prueba

	$[-2, 1]$
VP	-1
$f(vP)$	-2
$g(vP)$	0

$$f(-1) = -(-1)^2 - 1 = -1 - 1 = -2$$

$$g(-1) = 0$$

$g(x)$ es TECHO $\rightarrow f(x)$ es PISO en el intervalo $[-2, 1]$

$$A = \int_{-2}^1 [TECHO - PISO] dx = \int_{-2}^1 [g(x) - f(x)] dx = \int_{-2}^1 [(0) - (-x^2 - 1)] dx =$$

$$= \int_{-2}^1 [0 + x^2 + 1] dx = \int_{-2}^1 [x^2 + 1] dx = \left. \frac{x^3}{3} + x \right|_{-2}^1 = \left(\frac{1^3}{3} + 1 \right) - \left(\frac{(-2)^3}{3} + (-2) \right) =$$

$$= \left(\frac{1}{3} + 1 \right) - \left(-\frac{8}{3} - 2 \right) = \left(\frac{1}{3} + \frac{3}{3} \right) - \left(-\frac{8}{3} - \frac{6}{3} \right) = \left(\frac{4}{3} \right) - \left(-\frac{14}{3} \right) = \frac{4}{3} + \frac{14}{3} = \frac{18}{3} = 6$$

$$A = 6$$

5) Función 1: $f(x) = x^2 + 2x - 3$ Intervalo: $[-1, 2]$

Cruces con el eje X

$$x^2 + 2x - 3 = 0 \rightarrow \text{Bhaskara} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2} =$$

$$\frac{-2 \pm \sqrt{4 + 12}}{2} = \frac{-2 \pm \sqrt{16}}{2} \rightarrow x_1 = \frac{-2 + 4}{2} = \frac{2}{2} = 1$$

$$x_2 = \frac{-2 - 4}{2} = \frac{-6}{2} = -3$$

Intervalo 1: $[-1, 1]$

Intervalo 2: $[1, 2]$

Buscar Techo y Piso en ambos intervalos

$$f(0) = 0^2 + 2 \cdot 0 - 3 = -3$$

$$f(1,5) = (1,5)^2 + 2 \cdot 1,5 - 3 = 3 + 3 - 3 = 3$$

Entonces $f(x) < 0$ en $[-1, 1]$ y $f(x) > 0$ en $[1, 2]$

$$\begin{aligned} A = A_1 + A_2 &= - \int_{-1}^1 f(x) dx + \int_1^2 f(x) dx = - \int_{-1}^1 (x^2 + 2x - 3) dx + \int_1^2 (x^2 + 2x - 3) dx = \\ &= - \left[\frac{x^3}{3} + 2 \frac{x^2}{2} - 3x \right]_{-1}^1 + \left[\frac{x^3}{3} + 2 \frac{x^2}{2} - 3x \right]_1^2 = - \left[\frac{x^3}{3} + x^2 - 3x \right]_{-1}^1 + \left[\frac{x^3}{3} + x^2 - 3x \right]_1^2 = \\ &= (- \left[\frac{1^3}{3} + 1^2 - 3 \cdot 1 \right] - \left[\frac{(-1)^3}{3} + (-1)^2 - 3 \cdot (-1) \right]) + \left[\left(\frac{2^3}{3} + 2^2 - 3 \cdot 2 \right) - \left(\frac{1^3}{3} + 1^2 - 3 \cdot 1 \right) \right] = \\ &= - \left[\left(\frac{1}{3} + 1 - 3 \right) - \left(\frac{-1}{3} + 1 + 3 \right) \right] + \left[\left(\frac{8}{3} + 4 - 6 \right) - \left(\frac{1}{3} + 1 - 3 \right) \right] = \\ &= - \left[\left(\frac{1}{3} + \frac{3}{3} - \frac{9}{3} \right) - \left(-\frac{1}{3} + \frac{3}{3} + \frac{9}{3} \right) \right] + \left[\left(\frac{8}{3} - 2 \right) - \left(\frac{1}{3} - 2 \right) \right] = - \frac{5}{3} - \frac{11}{3} + \left[\left(\frac{8}{3} - \frac{6}{3} \right) - \left(\frac{1}{3} - \frac{6}{3} \right) \right] = \\ &= -\left(\frac{16}{3}\right) + \left[\left(\frac{2}{3}\right) - \left(-\frac{5}{3}\right)\right] = \frac{16}{3} + \left(\frac{2}{3} + \frac{5}{3}\right) = \frac{16}{3} + \frac{7}{3} = \frac{23}{3} \end{aligned}$$

$A_1 = \frac{16}{3}$	$A_2 = \frac{7}{3}$	$A_{\text{total}} = \frac{23}{3}$
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6) $f(x) = x^2 + 1 \quad g(x) = 2x^2$ Intervalo $[0, 2]$
general

Buscar puntos de intersección

$$f(x) = g(x)$$

$$x^2 + 1 = 2x^2$$

$$x^2 + 1 - 2x^2 = 0$$

$$-x^2 + 1 = 0$$

$$-x^2 = -1$$

$$x^2 = 1 \quad x = 1 \text{ ó } x = -1$$

Calcular área en los intervalos $[0,1]$ y $[1,2]$

Determinar Techo y Piso

	$[0,1]$	$[1,2]$
vP	0,5	1,5
$f(vP)$	1,25	3,25
$g(vP)$	0,5	4,5

$$f(0,5) = 0,5^2 + 1 = 0,25 + 1 = 1,25$$

$$g(0,5) = 2 \cdot 0,5^2 = 2 \cdot 0,25 = 0,5$$

$$f(1,5) = 1,5^2 + 1 = 2,25 + 1 = 3,25$$

$$g(1,5) = 2 \cdot (1,5)^2 = 2 \cdot 2,25 = 4,5$$

Entonces en el intervalo $[0,1]$ $f(x)$ es TECHO y $g(x)$ PISO, mientras que en el intervalo $[1,2]$ $g(x)$ es TECHO y $f(x)$ PISO.

$$\begin{aligned}
 A &= \int_0^1 (\text{TECHO} - \text{PISO}) dx + \int_1^2 (\text{TECHO} - \text{PISO}) dx = \int_0^1 (f(x) - g(x)) dx + \int_1^2 (g(x) - f(x)) dx = \\
 &= \int_0^1 ((x^2 + 1) - (2x^2)) dx + \int_1^2 ((2x^2) - (x^2 + 1)) dx = \int_0^1 (x^2 + 1 - 2x^2) dx + \int_1^2 (2x^2 - x^2 - 1) dx = \\
 &= \int_0^1 (-x^2 + 1) dx + \int_1^2 (x^2 - 1) dx = \left(-\frac{x^3}{3} + x \right) \Big|_0^1 + \left(\frac{x^3}{3} - x \right) \Big|_1^2 = \\
 &= \left(\left(-\frac{1^3}{3} + 1 \right) - \left(-\frac{0^3}{3} + 0 \right) \right) + \left(\left(\frac{2^3}{3} - 2 \right) - \left(\frac{1^3}{3} - 1 \right) \right) = \left(\left(\frac{1}{3} + 1 \right) - (0) \right) + \left(\left(\frac{8}{3} - 2 \right) - \left(\frac{1}{3} - 1 \right) \right) : \\
 &= \left(\frac{1}{3} + \frac{3}{3} \right) + \left(\frac{8}{3} - \frac{6}{3} \right) - \left(\frac{1}{3} - \frac{3}{3} \right) = \frac{2}{3} + \left(\frac{2}{3} - \left(-\frac{2}{3} \right) \right) = \frac{2}{3} + \left(\frac{2}{3} + \frac{2}{3} \right) : \\
 &= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2
 \end{aligned}$$

Por lo tanto, el área de la región encerrada

por las funciones $f(x)$ y $g(x)$ es $A = A_1 + A_2 = 2$

$\frac{2}{3}$ $\frac{4}{3}$