

Accumulado 1.13

1)  $f(x) = \begin{cases} -x & \text{si } x \leq 0 \\ x^2 & \text{si } 0 < x \leq 1 \\ 1 & \text{si } x > 1 \end{cases}$

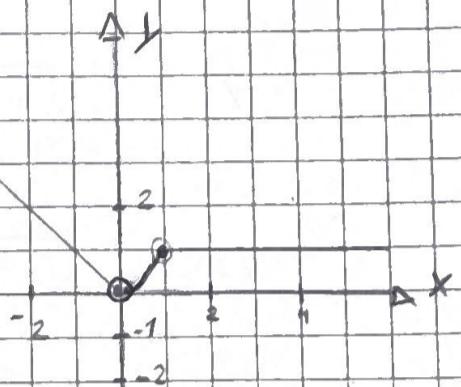
$f(x) = \begin{cases} -x & \text{si } x \leq 0 \\ x^2 & \text{si } 0 < x \leq 1 \\ 1 & \text{si } x > 1 \end{cases}$

Dom  $f(x) = (-\infty, 0] \cup (0, 1] \cup (1, +\infty)$

Dom  $f(x) = \mathbb{R}$

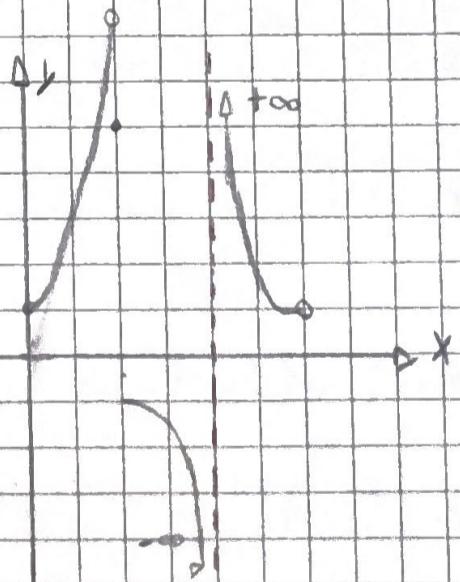
$-(-1) = +1 = 1$

$-(-2) = +2 = 2$



2)  $f(x) = \begin{cases} e^x & \text{si } 0 \leq x < 2 \rightarrow [0, 2) \\ 5 & \text{si } x = 2 \rightarrow \{2\} \\ \frac{2}{x-4} & \text{si } 2 < x < 6 \rightarrow (2, 4) \cup (4, 6) \end{cases}$

Dom  $f(x) = [0, 4) \cup (4, 6)$



## 1.5 Ejercicios - Página 39

D)  $a(x) = x + 1$

• Dom (a) =  $\mathbb{R}$   $(-\infty, +\infty)$  • Im (a) =  $\mathbb{R}$   $(-\infty, +\infty)$

b)  $b(x) = x^2 + 1$

• Dom (b) =  $\mathbb{R}$   $(-\infty, +\infty)$  • Im (b) =  $[1, +\infty)$

$x^2$  nunca es negativo  $\rightarrow 0^2 + 1 = 1$

c)  $c(x) = \frac{1}{x}$

• Dom (c) =  $(-\infty, 0) \cup (0, +\infty)$  • Im (c) =  $\mathbb{R} - \{0\}$

$= \mathbb{R} - \{0\}$  La división por 0 no está permitida

Una división nunca puede dar 0

D)  $d(x) = \frac{2}{x+3}$

• Dom (d) =  $\mathbb{R} - \{-3\}$  • Im (d) =  $\mathbb{R} - \{0\}$

$= (-\infty, -3) \cup (-3, +\infty)$  Nunca puede dar 0 una división

E)  $e(x) = \frac{3}{x^2 - 4}$

• Dom (e) =  $\mathbb{R} - \{-2, 2\}$  • Im (e) =  $(-\infty, -\frac{3}{4}] \cup (0, +\infty)$

$= (-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$   $\frac{1}{4} = -0,75$

F)  $f(x) = \frac{5}{x^2 - 4x + 3}$

• Dom (f) =  $\mathbb{R} - \{1, 3\}$  • Im (f) =  $(-\infty, -5] \cup (0, +\infty)$

$= (-\infty, 1) \cup (1, 3) \cup (3, +\infty)$

Aplique Bhaskata

Bhaskara

$$\frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-(-4) \pm \sqrt{-4^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 - 12}}{2} =$$
$$= \frac{4 \pm \sqrt{4}}{2} \quad \begin{array}{l} \text{---} \\ \text{---} \end{array} \quad \begin{array}{l} x_1 = \frac{4+2}{2} = \frac{6}{2} = 3 \\ x_2 = \frac{4-2}{2} = \frac{2}{2} = 1 \end{array}$$

6)  $g(x) = \frac{x-1}{x^2 - 4x + 3}$

•  $\text{Dom}(g) = \mathbb{R} - \{1, 3\}$

Mismo Dominio que  $f(x)$

•  $\text{Im}(g) = (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, +\infty)$

Cambia la imagen de  $f(x)$

7)  $h(x) = |x| + 1$

•  $\text{Dom}(h) = \mathbb{R} = (-\infty, +\infty)$

•  $\text{Im}(h) = [0, +\infty)$

El resultado de un  $|x|$  siempre  
es mayor o igual a 0.

8)  $i(x) = 2^x$

•  $\text{Dom}(i) = \mathbb{R} = (-\infty, +\infty)$

•  $\text{Im}(i) = [0, +\infty)$

una potencia con base positiva  
siempre da positivo

9)  $j(x) = 2^{x-5}$

•  $\text{Dom}(j) = \mathbb{R} = (-\infty, +\infty)$

•  $\text{Im}(j) = [0, +\infty)$

10)  $k(x) = \ln(x)$

•  $\text{Dom}(k) = (0, +\infty)$

•  $\text{Im}(k) = (-\infty, +\infty)$

✓ No se puede calcular  $\ln$  de 0 o  $< 0$ .

L)  $I(x) = \ln(x+4)$

•  $\text{Dom}(I) = (-4, +\infty)$       •  $\text{Im} = (-\infty, +\infty)$

2)  $f(x) = x+1$       a) Dominio  
 $\hookrightarrow \text{Dom}(f) = \mathbb{R} = (-\infty, +\infty)$

•  $g(x) = \sqrt{x-3}$        $\rightarrow \text{Dom}(g) = [3, +\infty)$  No existe raíz cuadrada de  $-x$

•  $h(x) = \frac{1}{x}$        $\rightarrow \text{Dom}(h) = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$

•  $M(x) = \sin(x)$        $\rightarrow \text{Dom}(M) = (-\infty, +\infty)$

•  $W(x) = \frac{1}{x^2-1}$        $\rightarrow \text{Dom}(W) = \mathbb{R} - \{-1, 1\} = (-\infty, -1) \cup (-1, 1) \cup (1, +\infty)$

b) Expresión y Dominio

i)  $(g+w)(x) \rightarrow (g+w)(x) = \sqrt{x-3} + \frac{1}{x^2-1}$        $\cdot \text{Dom}(g+w) = [3, +\infty)$   
 $[3, +\infty) \cap (\mathbb{R} - \{-1, 1\})$

ii)  $(h \cdot M)(x) \rightarrow (h \cdot M)(x) = \frac{1}{x} \cdot \sin(x)$        $\cdot \text{Dom}(h \cdot M) = \mathbb{R} - \{0\}$   
 $\mathbb{R} - \{0\} \cap \mathbb{R} = (-\infty, 0) \cup (0, +\infty)$

iii)  $\left(\frac{g}{f}\right)(x) \rightarrow \left(\frac{g}{f}\right)(x) = \frac{\sqrt{x-3}}{x+1} \rightarrow \neq 0 \text{ (es división)} \quad \frac{[3, +\infty)}{\mathbb{R} - \{-1\}}$

$\cdot \text{Dom}\left(\frac{g}{f}\right) = [3, +\infty)$

c) Expresión y Dominio

i)  $(f \circ g) = g \text{ compuesta con } f \quad g(x) = \sqrt{x-3} \quad y \quad f(x) = x+1$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x-3}) = \sqrt{x-3} + 1$

$\text{Dom}(f \circ g) = \{x \in \text{Dom}(g) / g(x) \in \text{Dom}(f)\}$

$\cdot \text{Dom}(f \circ g) = \{x \in [3, +\infty) / \sqrt{x-3} \in \mathbb{R}\} = [3, +\infty)$

$$\text{ii)} (h \circ f) = f \text{ compuesta con } h \quad f(x) = x+1 \quad h(x) = \frac{1}{x}$$

$$(h \circ f)(x) = h(f(x)) = h(x+1) = \frac{1}{x+1}$$

$$\text{Dom}(h \circ f) = \{x \in \text{Dom}(f) / f(x) \in \text{Dom}(h)\}$$

$$\cdot \text{Dom}(h \circ f) = \{x \in \mathbb{R} / x+1 \in \mathbb{R} - \{0\}\} = \mathbb{R} - \{-1\} = (-\infty, -1) \cup (-1, \infty)$$

$$\text{iii)} (M \circ h) = h \text{ compuesta con } M \quad h(x) = \frac{1}{x} \quad M(x) = \sin(x)$$

$$(\text{M} \circ h)(x) = M(h(x)) = M\left(\frac{1}{x}\right) = \sin\left(\frac{1}{x}\right) \quad \text{Dom}(M \circ h) = \mathbb{R} - \{0\} \quad \left\{x \in \mathbb{R} \setminus \{0\} / \frac{1}{x} \in \text{Dom}(\text{M}(x))\right\}$$

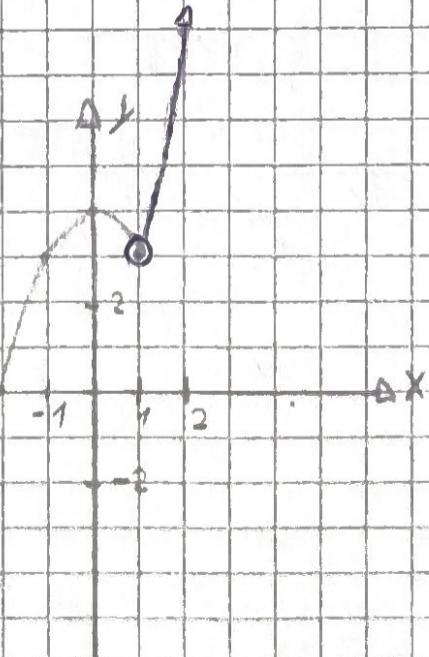
$$3) \text{ a) } \sin x^3 = f(x) : x^3 \quad g(x) = \sin(x) = (f \circ g)$$

$$\text{b) } \sqrt{x^4 + 1} = f(x) = \sqrt{x^4} \quad g(x) = x^4 + 1 = (f \circ g)$$

$$\text{c) } \frac{1}{x^2 + 1} = f(x) = \frac{1}{x} \quad g(x) = x^2 + 1 = (f \circ g)$$

4) a)

$$f_2(x) = \begin{cases} 4-x^2 & \text{si } x \leq 1 \rightarrow \mathbb{R} \\ x^2+2x & \text{si } x > 1 \rightarrow \mathbb{R} \end{cases} \quad \text{Dom}(f_2) = \mathbb{R} = (-\infty, +\infty)$$



b)  $f_3(x) = \begin{cases} \frac{1}{x} & \text{si } x < 0 \\ \ln(x) & \text{si } x > 0 \end{cases}$  Dom ( $f_3$ ) =  $\mathbb{R} - \{0\} = (-\infty, 0) \cup (0, +\infty)$

No hace falta graficar

5)  $f(x) = 2-x \rightarrow \text{Dom}(f) = \mathbb{R}$

$$g(x) = \begin{cases} -x & -2 \leq x < 0 \\ x-1 & 0 \leq x \leq 2 \end{cases}$$

Dom ( $g$ ) =  $[-2, 2]$

a)  $(f \circ g)(x) = f(g(x)) \rightarrow g(0) = x-1 = 0-1 = -1 \rightarrow f(g(0)) = 3$   
 $\rightarrow f(-1) = 2-x = 2-(-1) = 3$

$$(g \circ f)(x) = g(f(x)) \rightarrow f(0) = 2-x = 2-0 = 2 \rightarrow g(f(0)) = 1$$
  
 $\rightarrow g(2) = x-1 = 2-1 = 1$

b)  $(f \circ f)(x) = f(f(x)) \rightarrow f(2) = 2-x = 2-2 = 0 \rightarrow f(f(2)) = 2$   
 $\rightarrow f(0) = 2-x = 2-0 = 2$

$$(g \circ g)(x) = g(g(x)) \rightarrow g(-1) = -x = -(-1) = 1 \rightarrow g(g(-1)) = 0$$
  
 $\rightarrow g(1) = x-1 = 1-1 = 0$

c)  $(f \circ g)(x) = f(g(\frac{1}{2})) \rightarrow g(\frac{1}{2}) = x-1 = \frac{1}{2}-1 = -\frac{1}{2}$   
 $\rightarrow f(-\frac{1}{2}) = 2-x = 2-\left(-\frac{1}{2}\right) = \frac{5}{2} \rightarrow f(g(\frac{1}{2})) = \frac{5}{2} = 2,5$

$$(g \circ f)(x) = g(f(x)) \rightarrow f(3) = 2-x = 2-3 = -1 \rightarrow g(f(3)) = 1$$
  
 $\rightarrow g(-1) = -x = -(-1) = 1$