Práctico 4: deducción natural en PROP

1. Realice los agregados que estime pertinentes para que los siguientes árboles sean elementos de DER. Asimismo, provea juicios que correspondan con dichas pruebas i.e. dé $\Gamma \subseteq \texttt{PROP}$ y $\varphi \in \texttt{PROP}$ tales que cada razonamiento (árbol) pruebe que $\Gamma \vdash \varphi$.

(a)

$$\dfrac{\dfrac{\dfrac{[g]}{b \to g} \quad \dfrac{[a \leftrightarrow b \to g]}{a} \quad \dfrac{[\neg a \wedge b]}{\neg a}}{\dfrac{\bot}{\neg g}}}{\dfrac{\neg a \wedge b \to \neg g}{(a \leftrightarrow b \to g) \to \neg a \wedge b \to \neg g}}$$

(b)

$$\dfrac{\dfrac{\dfrac{[c]}{d \to c}}{(d \to c) \vee a} \quad \neg((d \to c) \vee a)}{\dfrac{\bot}{\neg c}}}{\neg c \vee \neg b}$$

(c)

$$\dfrac{\dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a} \quad \dfrac{[a \to b]}{b} \quad \dfrac{[a \wedge \neg b]}{\neg b}}{\dfrac{\bot}{\neg(a \to b)}}}{a \wedge \neg(a \to b)} \quad \neg(a \wedge \neg(a \to b))}{\dfrac{\bot}{\neg(a \wedge \neg b)}}}{\neg(a \wedge \neg b)}$$

(d)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[f] \quad [f \to p]}{p}}{p \vee s}}{\neg f \vee p \vee s} \quad [\neg(\neg f \vee p \vee s)]}{\dfrac{\bot}{\neg(f \to p)}}}{\neg(f \to p) \to s}}{\dfrac{\dfrac{s}{p \vee s}}{\neg f \vee p \vee s}}}{\dfrac{\dfrac{\bot}{\neg f}}{\neg f \vee p \vee s}} \quad [\neg(\neg f \vee p \vee s)]}{\dfrac{\bot}{\neg f \vee p \vee s}}}{\neg f \vee p \vee s}$$

(e)

$$\dfrac{\dfrac{\dfrac{[p] \quad \dfrac{[p] \quad p \leftrightarrow \neg p}{\neg p}}{\bot}}{\neg p} \quad p \leftrightarrow \neg p}{p} \quad \dfrac{\dfrac{\dfrac{[\neg p] \quad p \leftrightarrow \neg p}{p} \quad [\neg p]}{\bot}}{p} \quad p \leftrightarrow \neg p}{\neg p}}{\bot}$$

2. Construya derivaciones que justifiquen los siguientes juicios.

   (a) $\vdash \varphi \to \varphi$
   (b) $\vdash \bot \to \varphi$
   (c) $\vdash \neg(\varphi \wedge \neg\varphi)$
   (d) **[PP2007]** $\varphi \vee \varphi \vdash \varphi$
   (e) $\alpha \vee \beta \vdash \beta \vee \alpha$
   (f) $\neg\varphi \vdash \varphi \to (\bot \vee \neg\bot)$ [1]
   (g) **[ED2005]** $\vdash \varphi \vee \neg\varphi$ $(*)$
   (h) $\neg(\varphi \to \psi) \to \sigma \vdash \neg\varphi \vee (\psi \vee \sigma)$ **[EF2014]**
   (i) $\clubsuit \vdash \neg\bot \leftrightarrow \neg\neg\neg\bot$

3. Demuestre que:

   (a) Si $\vdash \varphi$ entonces $\vdash \psi \vee \varphi$.
   (b) Si $\vdash \varphi$ entonces $\vdash \psi \to \varphi$.
   (c) Si $\vdash \varphi$ y $\vdash \psi$ entonces $\vdash \varphi \wedge (\psi \vee \sigma)$
   (d) $\clubsuit \vdash \varphi$ si y solamente si $\vdash \varphi \leftrightarrow \neg\bot$

Ejercicios integradores.
Construya derivaciones que justifiquen los siguientes juicios.

1. **[PP1998]** $(\varphi \to \psi) \vdash ((\psi \to \sigma) \wedge \neg\sigma) \to \neg\varphi)$
2. **[PP1999]** $\varphi \wedge \sigma \to \psi, \quad \varphi \to \sigma \vdash \neg\psi \to \neg\varphi$
3. **[PP2000]** $\varphi \to (\sigma \vee \psi) \vdash \neg\sigma \to (\varphi \to \psi)$
4. **[PP2001]** $(\neg\beta \to \neg\alpha) \wedge \alpha \vdash \beta$
5. **[PP2002]** $\neg\psi \to \neg\varphi \vdash \varphi \to ((\psi \to \neg\varphi) \to \sigma)$
6. **[PP2003]** $\neg(\alpha \wedge \neg(\alpha \to \beta)) \vdash \neg(\alpha \wedge \beta)$
7. **[PP2004]** $\vdash \neg(\alpha \vee \beta) \leftrightarrow (\neg\alpha \wedge \neg\beta)$ [Ley de DeMorgan]
8. **[PP2005]** $(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta) \vdash \neg(\alpha \leftrightarrow \beta)$
9. **[PP2006]** $\neg\beta \to \gamma, \quad \alpha \vee \beta \vdash g$ $(*)$
10. **[PP2007]** $\vdash (\neg\sigma \to (\neg\varphi \wedge \neg\psi)) \to ((\varphi \vee \psi) \to \sigma)$
11. **[PP2008]** $\vdash (a \to b) \wedge (c \to b) \to (\neg\neg a \vee c) \to b$
12. **[PP2009]** $\vdash \neg\alpha \wedge (\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma)$
13. **[PP2010]** $\neg((\psi \to \varphi) \vee \alpha) \vdash \neg\varphi \vee \neg\beta$

[1] Sugerencia: notar que $\vdash \neg\bot$.

14. **[PP2010]** $((\alpha \vee \beta) \to (\alpha \wedge \beta)) \leftrightarrow (\alpha \leftrightarrow \beta)$
15. **[PP2011]** $\vdash ((\alpha \vee \beta) \to (\beta \wedge \neg\alpha)) \to \neg\alpha$
16. **[PP2011]** $\neg(\varphi \wedge \psi) \leftrightarrow (\neg\varphi \vee \neg\neg\psi)$
17. **[PP2012]** $(q \to \neg p) \to p \vdash \neg(p \to \neg q) \vee p$
18. **[PP2012]** $\vdash ((p \to q) \to p) \to \neg(\neg p \wedge \neg(p \wedge q))$
19. **[PP2013]** $\varphi \to \psi \vdash \neg\neg\psi \vee \neg\varphi$
20. **[PP2013]** $\neg\varphi, \quad \neg\alpha \vee \neg\beta \vdash \sigma \quad \Rightarrow \quad \neg\varphi \vdash \neg(\alpha \wedge \beta) \to \sigma$
21. **[PP2013]** $(p_0 \to p_1) \to \neg p_1 \vdash \neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$
22. **[PP2014]** $(\alpha \to \beta) \to \gamma \vdash \gamma \vee \neg\beta$
23. **[PP2014]** $\vdash (\alpha \leftrightarrow (\beta \to \gamma)) \to (\neg\alpha \wedge \beta \to \neg\gamma)$
24. **[PP2015]** $\vdash ((\alpha \to \gamma) \wedge (\beta \to \gamma)) \leftrightarrow (\alpha \vee \beta \to \gamma)$
25. **[PP2015]** $\vdash (\alpha \to \beta \to \gamma) \to \neg(\neg\gamma \vee \neg\beta) \to \gamma$
26. **[PP2016]** $\neg(\beta \vee \alpha) \vdash \alpha \wedge \beta$
27. **[PP2016]** $(\varphi \vee \psi) \leftrightarrow (\varphi \vee (\neg\varphi \wedge \psi))$
28. **[PP2017]** $\gamma \to \alpha, \quad \neg \to (\neg\beta \vee \alpha) \vdash \neg\alpha \to (\neg\beta \wedge \neg\gamma)$
29. **[PP2017]** $p_2 \to \neg p_1, \quad p_3 \to p_1, \quad \neg p_1 \to (\neg p_2 \vee p_1) \vdash \neg p_1 \to (\neg p_2 \wedge \neg p_3)$
30. **[PP2017]** $p_3 \to p_1, \quad \neg p_1 \to (\neg p_2 \vee p_1) \vdash (p_2 \vee p_3) \to p_1$ [Riesgo de salud]
31. **[PP2018]** $p \leftrightarrow \neg p \vdash \bot$
32. **[PP2018]** $p \vee q, \quad p \to r, \quad q \to s \vdash t \to \neg(\neg r \wedge \neg s)$

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33. **[ED2008]** $\neg p \wedge q \to r, \quad r \vee s \to \neg p, \quad s \vdash \neg q \vee r$
34. **[ED2010]** $\alpha \vee \beta \leftrightarrow \beta \vdash \alpha \leftrightarrow \alpha \wedge \beta$
35. **[ED2011]** $\neg p \wedge \neg q \to r \vdash (p \vee q) \vee r$
36. **[EF2012]** $(\neg\alpha \vee \beta) \vee \gamma \vdash \neg\gamma \wedge \alpha \to \neg\neg\beta$

Soluciones propuestas al Práctico 4

A continuación se presenta un esbozo de las soluciones de los ejercicios, es decir, las mismas no están completas: falta agregar el inciso indicador de la cancelación de las hipótesis destacadas entre corchetes. Se alienta al lector a realizar tales agregados a efectos de profundizar la comprensión de dichas pruebas.

Asimismo, a efectos de no sobrecargar la notación se ha optado por emplear letras del alfabeto latino en lugar de aquellas del alfabeto griego.

1. Vistos en clase.

2. (i)

$$\dfrac{\dfrac{\dfrac{[\bot]\quad[\neg\bot]}{\bot}E\neg}{\neg\bot}RAA \qquad \dfrac{\dfrac{\dfrac{[\bot]\quad[\neg\bot]}{\bot}E\neg}{\neg\bot}RAA}{\neg\bot}I\neg \qquad \dfrac{\dfrac{[\neg(\neg\bot)]}{\bot}E\neg}{\neg(\neg(\neg\bot))}I\neg}{\neg\bot\leftrightarrow\neg(\neg(\neg\bot))}I\leftrightarrow$$

Otra forma más sencilla:

$$\dfrac{\dfrac{[\bot]}{\neg\bot}I\neg \qquad \dfrac{\dfrac{[\neg\bot]\quad[\neg(\neg\bot)]}{\bot}E\neg}{\neg(\neg(\neg\bot))}I\neg}{\neg\bot\leftrightarrow\neg(\neg(\neg\bot))}I\leftrightarrow$$

3. Vistos en clase.

Ejercicios integradores.

1.

$$\dfrac{\dfrac{\dfrac{[a]\quad a\to b}{b}E\to \qquad \dfrac{[(b\to c)\wedge\neg c]}{b\to c}E\wedge_1}{c}E\to \qquad \dfrac{[(b\to c)\wedge\neg c]}{\neg c}E\wedge_2}{\dfrac{\dfrac{\bot}{\neg a}I\neg}{(b\to c)\wedge\neg c\to\neg a}I\to}E\neg$$

2.

$$\dfrac{\dfrac{[a]\quad\dfrac{\dfrac{[a]\quad a\to b}{b}E\to}{a\wedge b}I\wedge \qquad a\wedge b\to c}{c}E\to \qquad [\neg c]}{\dfrac{\dfrac{\bot}{\neg a}I\neg}{\neg c\to\neg a}I\to}E\neg$$

3.

$$\dfrac{\dfrac{\dfrac{[a] \quad a \to b \vee c}{b \vee c}\, E\to \qquad \dfrac{\dfrac{[b] \quad [\neg b]}{\frac{\perp}{c}\, E\perp}\, E\neg \qquad \dfrac{[c] \quad [\neg b]}{c \wedge \neg b}\, I\wedge}{c}\, E\vee}{\dfrac{c}{a \to c}\, I\to}}{\neg b \to a \to c}\, I\to$$

4.

$$\dfrac{\dfrac{\dfrac{(\neg b \to \neg a) \wedge a}{a}\, E\wedge_2 \qquad \dfrac{[\neg b] \qquad \dfrac{(\neg b \to \neg a) \wedge a}{\neg b \to \neg a}\, E\wedge_1}{\neg a}\, E\to}{\frac{\perp}{b}\, E\neg}}{}\, RAA$$

5.

$$\dfrac{\dfrac{[b] \qquad \dfrac{\dfrac{\dfrac{[a] \quad [a \to \neg b]}{\neg b}\, E\to}{\frac{\perp}{\neg a}\, I\neg} \qquad \neg a \to \neg b}{\neg b}\, E\to}{\frac{\perp}{c}\, E\perp}}{(a \to \neg b) \to c}\, I\to}{b \to (a \to \neg b) \to c}\, I\to$$

6.

$$\dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a}\, E\wedge_1}{a \wedge \neg(a \to b)}\, I\wedge \qquad \dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a}\, E\wedge_1 \qquad \dfrac{[a \to b] \qquad \dfrac{[a \wedge \neg b]}{\neg b}\, E\wedge_2}{b}\, E\to}{\frac{\perp}{\neg(a \to b)}\, I\neg}}{a \wedge \neg(a \to b)}\, I\wedge}{\dfrac{\neg(a \wedge \neg(a \to b)) \qquad \dfrac{\perp}{\neg(a \wedge \neg b)}\, I\neg}{}\, E\neg}$$

10.

$$\dfrac{\dfrac{\dfrac{[b \vee c] \qquad \dfrac{[b]}{\perp}\, E\neg \qquad \dfrac{[c]}{\perp}\, E\neg}{\perp}\, E\vee}{\frac{\perp}{a}\, RAA}}{b \vee c \to a}\, I\to}{(\neg a \to \neg b \wedge \neg c) \to b \vee c \to a}\, I\to$$

7.

$$\dfrac{\dfrac{[a\vee b]\quad \dfrac{[a]\quad \dfrac{\dfrac{[\neg a\wedge\neg b]}{\neg a}E\wedge_1}{\dfrac{\bot}{b}E\bot}E\neg \qquad [b]}{b}\qquad \dfrac{[\neg a\wedge\neg b]}{\neg b}E\wedge_2}{\dfrac{\bot}{\neg(a\vee b)}I\neg}}{\neg(a\vee b)\leftrightarrow\neg a\wedge\neg b}I\leftrightarrow \qquad \dfrac{\dfrac{[\neg a\wedge\neg b]}{\neg b}E\wedge_2}{\dfrac{\bot}{b}E\neg}\qquad \dfrac{\dfrac{[b]}{a\vee b}I\vee_2 \qquad [\neg(a\vee b)]}{\dfrac{\bot}{\neg b}I\neg}E\neg}{\neg a\wedge\neg b}I\wedge$$

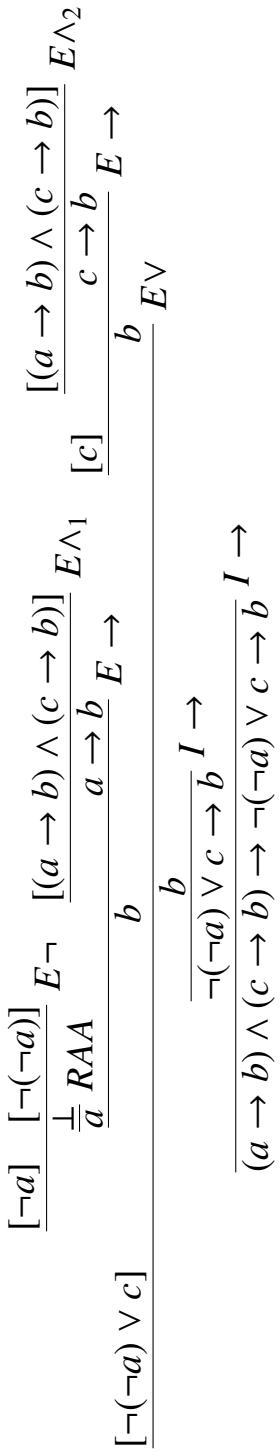
8.

$$\dfrac{\dfrac{\dfrac{[a\wedge\neg b]}{a}E\wedge_1 \qquad [a\leftrightarrow b]}{b}E\leftrightarrow_1 \qquad \dfrac{[a\wedge\neg b]}{\neg b}E\wedge_2}{\dfrac{\bot}{\neg(a\leftrightarrow b)}I\neg}}{(a\wedge\neg b)\vee\neg a\wedge b}E\vee \qquad \dfrac{\dfrac{[\neg a\wedge b]}{b}E\wedge_2 \qquad [a\leftrightarrow b]}{a}E\leftrightarrow_1 \qquad \dfrac{[\neg a\wedge b]}{\neg a}E\wedge_1}{\dfrac{\bot}{\bot}E\neg}E\vee$$

9.

$$\dfrac{\dfrac{[\neg b]\quad \dfrac{\dfrac{[g]}{g\vee\neg g}I\vee_1 \qquad [\neg(g\vee\neg g)]}{\dfrac{\bot}{\neg g}I\neg}E\neg}{g\vee\neg g}I\vee_2}{\dfrac{\bot}{g\vee\neg g}RAA}\qquad \dfrac{[g]\quad \dfrac{[\neg g]}{g\wedge\neg g}I\wedge}{\neg g}E\wedge_2 \qquad [\neg g]}{\dfrac{\bot}{g}RAA}E\vee}{\dfrac{\bot}{b}E\neg}\qquad \dfrac{[\neg b]\quad \dfrac{\dfrac{[\neg a\wedge b]}{b}E\wedge_2 \qquad [a\leftrightarrow b]}{a}E\leftrightarrow_1 \qquad \dfrac{[\neg a\wedge b]}{\neg a}E\wedge_1}{\dfrac{\bot}{\bot}E\neg}E\vee}{\dfrac{\bot}{a\vee b}RAA}\qquad \dfrac{[\neg g]}{\neg g}E\neg}{\dfrac{\bot}{g}RAA}$$

$$\dfrac{\dfrac{\dfrac{[\neg b]\quad \neg b\to g}{g}E\to \qquad \dfrac{\dfrac{[\neg g]}{g}E\to \qquad a\vee b\to g}{g}E\to}{\dfrac{\bot}{g}RAA}}{a\vee b}I\vee_2}{\dfrac{\bot}{g}RAA}E\neg$$



12.

$$\dfrac{\dfrac{[a\vee b]\qquad \dfrac{\dfrac{[\neg a\wedge(b\to g)]}{\neg a}E\wedge_1\quad [a]}{\dfrac{\bot}{b}E\bot}E\neg}{b}E\vee\qquad [b]}{\dfrac{\dfrac{g}{a\vee b\to g}I\to}{\neg a\wedge(b\to g)\to a\vee b\to g}I\to}\qquad \dfrac{[\neg a\wedge(b\to g)]}{b\to g}E\wedge_2\ E\to$$

13.

$$\dfrac{\dfrac{\dfrac{[c]}{d\to c}I\to}{(d\to c)\vee a}I\vee_1\qquad \neg((d\to c)\vee a)}{\dfrac{\dfrac{\bot}{\neg c}I\neg}{\neg c\vee\neg b}I\vee_1}E\neg$$

15.

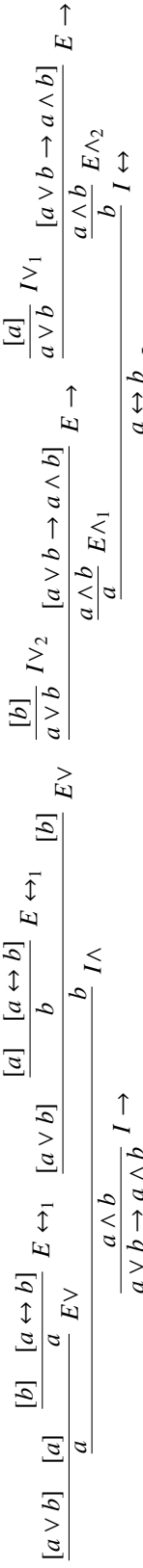
$$\dfrac{\dfrac{\dfrac{[a]}{a\vee b}I\vee_1\qquad [a\vee b\to b\wedge\neg a]}{b\wedge\neg a}E\to\qquad [a]}{\dfrac{\dfrac{\bot}{\neg a}I\neg}{(a\vee b\to b\wedge\neg a)\to\neg a}I\to}E\neg$$

17.

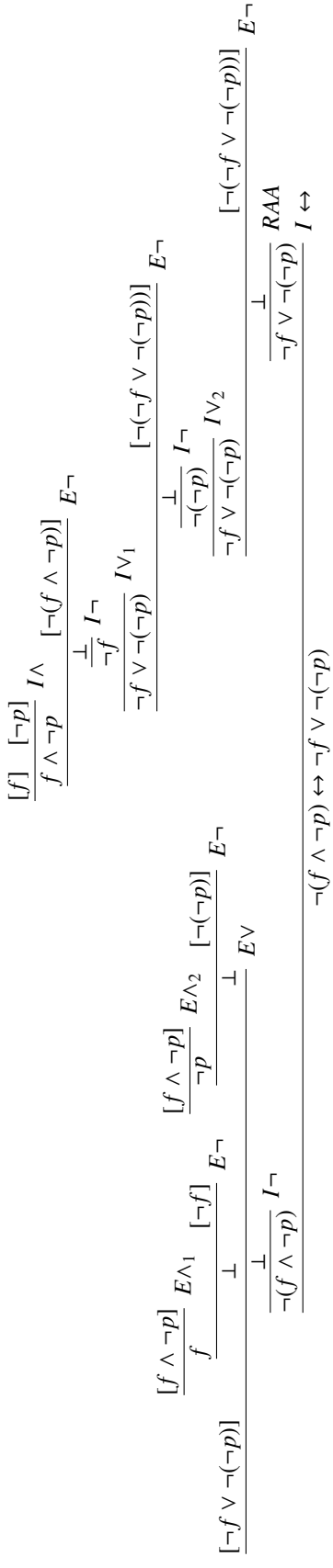
$$\dfrac{\dfrac{\dfrac{[\neg p]}{q\to\neg p}I\to\qquad (q\to\neg p)\to p}{p}E\to\qquad [\neg p]}{\dfrac{\dfrac{\bot}{p}RAA}{\neg(p\to\neg q)\vee p}I\vee_2}E\neg$$

18.

$$\dfrac{\dfrac{\dfrac{\dfrac{[p]\qquad \dfrac{[\neg p\wedge\neg(p\wedge q)]}{\neg p}E\wedge_1}{\dfrac{\bot}{q}E\bot}E\neg}{p\to q}I\to\qquad [(p\to q)\to p]}{p}E\to\qquad \dfrac{[\neg p\wedge\neg(p\wedge q)]}{\neg p}E\wedge_1}{\dfrac{\dfrac{\bot}{\neg(\neg p\wedge\neg(p\wedge q))}I\neg}{((p\to q)\to p)\to\neg(\neg p\wedge\neg(p\wedge q))}I\to}E\neg$$



14.



19.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[f] \quad f \to p}{p} E\to \qquad [\neg p]}{\bot} E\neg}{\neg f} I\neg}{\neg(\neg p) \vee \neg f} I\vee_2 \qquad [\neg(\neg(\neg p) \vee \neg f)]}{\bot} E\neg}{\neg(\neg p)} I\neg}{\neg(\neg p) \vee \neg f} I\vee_1 \qquad [\neg(\neg(\neg p) \vee \neg f)]}{\bot} E\neg}{\neg(\neg p) \vee \neg f} RAA$$

20.

$$\neg\varphi \quad \dfrac{\dfrac{[\neg(\alpha \wedge \beta)]^{(1)} \qquad \neg(\alpha \wedge \beta) \to \neg\alpha \vee \neg\beta \text{ (tautología dem. en [PP2014])}}{\neg\alpha \vee \neg\beta} E\to}{\neg(\alpha \wedge \beta) \to \sigma} I\to (1)$$

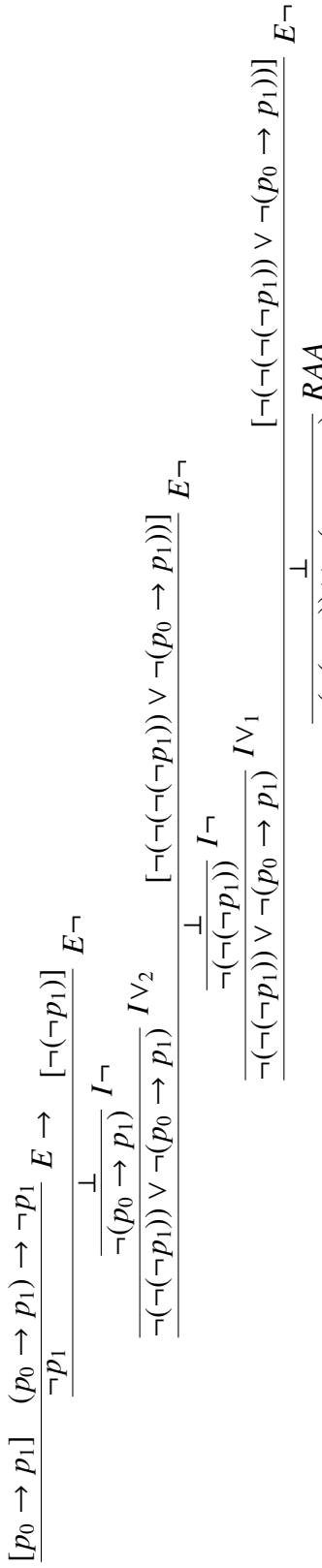
22.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg b]}{g \vee \neg b} I\vee_2 \qquad [\neg(g \vee \neg b)]}{\bot} E\neg}{b} RAA}{a \to b} I\to \qquad (a \to b) \to g}{g} E\to}{g \vee \neg b} I\vee_1 \qquad [\neg(g \vee \neg b)]}{\bot} E\neg}{g \vee \neg b} RAA$$

23.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[g]}{b \to g} I\to \qquad [a \leftrightarrow b \to g]}{a} E\leftrightarrow_1 \qquad \dfrac{[\neg a \wedge b]}{\neg a} E\wedge_1}{\bot} E\neg}{\neg g} I\neg}{\neg a \wedge b \to \neg g} I\to}{(a \leftrightarrow b \to g) \to \neg a \wedge b \to \neg g} I\to$$

21.


$[p_0 \to p_1]$
$(p_0 \to p_1) \to \neg p_1$
$\neg p_1$
$E\to$
$[\neg(\neg p_1)]$
$\bot$
$\neg(p_0 \to p_1)$
$I\neg$
$\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$
$I\vee_2$
$[\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)]$
$E\neg$
$\neg(\neg p_1)$
$I\neg$
$\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$
$I\vee_1$
$[\neg(\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1))]$
$E\neg$
$\bot$
$\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$
RAA

24.

$$\dfrac{\dfrac{\dfrac{[a]}{a \vee b}\, I\vee_1 \quad \dfrac{[a \vee b \to g]}{g}\, E\to}{a \to g}\, I\to \quad \dfrac{\dfrac{[b]}{a \vee b}\, I\vee_2 \quad \dfrac{[a \vee b \to g]}{g}\, E\to}{b \to g}\, I\to}{(a \to g) \wedge (b \to g)}\, I\wedge \qquad \dfrac{\dfrac{\dfrac{[a \vee b]}{\dfrac{[a]}{\dfrac{[(a \to g) \wedge (b \to g)]}{a \to g}\, E\wedge_1}\, E\to \quad \dfrac{[b]}{\dfrac{[(a \to g) \wedge (b \to g)]}{b \to g}\, E\wedge_2}\, E\to}{g}\, E\vee}{a \vee b \to g}\, I\to}{(a \to g) \wedge (b \to g) \leftrightarrow a \vee b \to g}\, I\leftrightarrow$$

25.

$$\dfrac{\dfrac{\dfrac{\dfrac{[\neg g]}{\neg g \vee \neg b}\, I\vee_1 \quad [\neg(\neg g \vee \neg b)]}{\dfrac{\bot}{g}\, RAA}\, E\neg}{\neg(\neg g \vee \neg b) \to g}\, I\to}{(a \to b \to g) \to \neg(\neg g \vee \neg b) \to g}\, I\to$$

26.

$$\dfrac{\dfrac{[a \vee b] \quad \dfrac{[a]}{b \vee a}\, I\vee_2 \quad \dfrac{[b]}{b \vee a}\, I\vee_1}{b \vee a}\, E\vee \quad \neg(b \vee a)}{\dfrac{\dfrac{\bot}{a \wedge b}\, E\bot}{a \vee b \to a \wedge b}\, I\to}\, E\neg$$

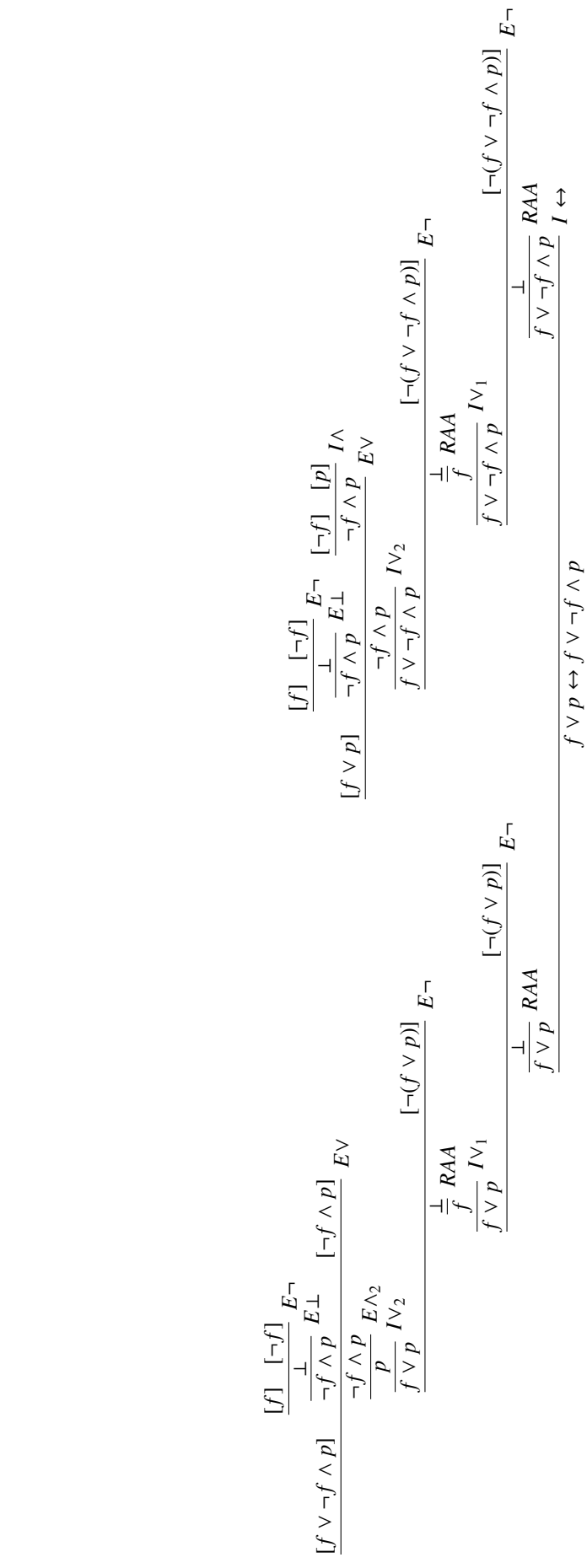
28.

$$\dfrac{\dfrac{[\neg a] \quad \neg a \to \neg b \vee a}{\neg b \vee a}\, E\to \quad \dfrac{\dfrac{[a] \quad [\neg a]}{\bot}\, E\neg}{\neg b}\, E\bot \quad [\neg b]}{\dfrac{\neg b \quad \dfrac{\dfrac{[g] \quad g \to a}{a}\, E\to \quad [\neg a]}{\bot}\, E\neg}{\neg g}\, I\neg}{\neg b \wedge \neg g}\, I\wedge}{\neg a \to \neg b \wedge \neg g}\, I\to$$

32.

$$\dfrac{\dfrac{\dfrac{p \vee q \quad \dfrac{\dfrac{[p] \quad p \to r}{r}\, E\to \quad \dfrac{[\neg r \wedge \neg s]}{\neg r}\, E\wedge_1}{\bot}\, E\neg}{\neg p}\, I\neg \quad [p]}{\bot}\, E\neg}{q}\, E\bot \quad [q]}{\dfrac{\dfrac{s}{\bot}\, E\vee \quad q \to s}{\neg(\neg r \wedge \neg s)}\, E\to \quad \dfrac{[\neg r \wedge \neg s]}{\neg s}\, E\wedge_2}{t \to \neg(\neg r \wedge \neg s)}\, I\neg\, E\neg\, I\to$$

27.



29.

$$\dfrac{[p_2] \quad \dfrac{\dfrac{\dfrac{[p_2] \quad p_2 \to \neg p_1}{\neg p_1} E\to \quad \neg p_1 \to \neg p_2 \vee p_1}{\neg p_2 \vee p_1} E\to \quad [\neg p_2] \quad \dfrac{\dfrac{[p_1] \quad [\neg p_1]}{\bot} E\neg}{\neg p_2} E\bot}{\neg p_2} E\vee}{\dfrac{\bot}{\neg p_2} I\neg} E\neg$$

$$\dfrac{\dfrac{\bot}{\neg p_2} I\neg \qquad \dfrac{\dfrac{\dfrac{[p_3] \quad p_3 \to p_1}{p_1} E\to \quad [\neg p_1]}{\bot} E\neg}{\neg p_3} I\neg}{\dfrac{\neg p_2 \wedge \neg p_3}{\neg p_1 \to \neg p_2 \wedge \neg p_3} I\to} I\wedge$$

30.

$$\dfrac{[p_2 \vee p_3] \quad [p_2] \quad \dfrac{\dfrac{[p_3] \quad \dfrac{\dfrac{\dfrac{[p_3] \quad p_3 \to p_1}{p_1} E\to \quad [\neg p_1]}{\bot} E\neg}{\neg p_3} I\neg}{\bot} E\neg}{p_2} E\bot}{p_2} E\vee$$

$$\dfrac{\dfrac{\dfrac{[\neg p_1] \quad \neg p_1 \to \neg p_2 \vee p_1}{\neg p_2 \vee p_1} E\to \quad [\neg p_2] \quad \dfrac{\dfrac{[p_1] \quad [\neg p_1]}{\bot} E\neg}{\neg p_2} E\bot}{\neg p_2} E\vee \qquad \dfrac{\dfrac{\dfrac{[p_3] \quad p_3 \to p_1}{p_1} E\to \quad [\neg p_1]}{\bot} E\neg}{\neg p_3} I\neg}{\dfrac{\neg p_2 \wedge \neg p_3}{\neg p_1 \to \neg p_2 \wedge \neg p_3} I\to} I\wedge$$

$$\dfrac{p_2 \qquad \dfrac{\dfrac{[\neg p_1] \quad \neg p_1 \to \neg p_2 \wedge \neg p_3}{\neg p_2 \wedge \neg p_3} E\to}{\neg p_2} E\wedge_1}{\dfrac{\dfrac{\bot}{p_1} RAA}{p_2 \vee p_3 \to p_1} I\to} E\neg$$

31.

$$\dfrac{\dfrac{\dfrac{\dfrac{[p] \quad \dfrac{[p] \quad p \leftrightarrow \neg p}{\neg p} E\leftrightarrow_1}{\bot} E\neg}{\neg p} I\neg \quad p \leftrightarrow \neg p}{p} E\leftrightarrow_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{[\neg p] \quad p \leftrightarrow \neg p}{p} E\leftrightarrow_1 \quad [\neg p]}{\bot} E\neg}{p} RAA \quad p \leftrightarrow \neg p}{\neg p} E\leftrightarrow_1}{\bot} E\neg$$

33.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{s}{r \vee s}\, I\vee_2 \quad r \vee s \to \neg p}{\neg p}\, E\to \quad [q]}{\neg p \wedge q}\, I\wedge \quad \neg p \wedge q \to r}{r}\, E\to}{\neg q \vee r}\, I\vee_2 \quad [\neg(\neg q \vee r)]}{\bot}\, E\neg}{\neg q}\, I\neg}{\neg q \vee r}\, I\vee_1 \quad [\neg(\neg q \vee r)]}{\bot}\, E\neg}{\neg q \vee r}\, RAA$$

34.

$$\dfrac{\dfrac{[a \wedge b]}{a}\, E\wedge_1 \quad \dfrac{[a] \quad \dfrac{\dfrac{[a]}{a \vee b}\, I\vee_1 \quad a \vee b \leftrightarrow b}{b}\, E\leftrightarrow_1}{a \wedge b}\, I\wedge}{a \leftrightarrow a \wedge b}\, I\leftrightarrow$$

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35.

$$\dfrac{\dfrac{\dfrac{[p]}{p \vee q}\, I\vee_1 \quad [\neg(p \vee q)]}{\dfrac{\bot}{\neg p}\, I\neg}\, E\neg \quad \dfrac{\dfrac{[q]}{p \vee q}\, I\vee_2 \quad [\neg(p \vee q)]}{\dfrac{\bot}{\neg q}\, I\neg}\, E\neg}{\dfrac{\neg p \wedge \neg q}{\dfrac{r}{(p \vee q) \vee r}\, I\vee_2}\, I\wedge}\, E\to \qquad [\neg((p \vee q) \vee r)]}{\dfrac{\dfrac{\bot}{p \vee q}\, RAA}{(p \vee q) \vee r}\, I\vee_1}\, E\neg \qquad [\neg((p \vee q) \vee r)]}{\dfrac{\bot}{(p \vee q) \vee r}\, RAA}\, E\neg$$

36.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg g \wedge a]}{a}\, E\wedge_2 \quad [\neg a]}{\bot}\, E\neg \quad \dfrac{[b] \quad [\neg b]}{\bot}\, E\neg}{[\neg a \vee b]}\, E\vee \quad [g]}{\bot}\, E\vee \quad \dfrac{[\neg g \wedge a]}{\neg g}\, E\wedge_1}{(\neg a \vee b) \vee g}\, E\neg$$

$$\dfrac{\dfrac{\bot}{\neg(\neg b)}\, I\neg}{\neg g \wedge a \to \neg(\neg b)}\, I\to$$