Práctico 4: deducción natural en PROP

1. Realice los agregados que estime pertinentes para que los siguientes árboles sean elementos de DER. Asimismo, provea juicios que correspondan con dichas pruebas i.e. dé $\Gamma \subseteq \mathtt{PROP}$ y $\varphi \in \mathtt{PROP}$ tales que cada razonamiento (árbol) pruebe que $\Gamma \vdash \varphi$.

(a)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[g]}{b \to g} \quad [a \leftrightarrow b \to g]}{a} \quad \dfrac{[\neg a \wedge b]}{\neg a}}{\bot}}{\neg g}}{\neg a \wedge b \to \neg g}}{(a \leftrightarrow b \to g) \to \neg a \wedge b \to \neg g}$$

(b)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[c]}{d \to c}}{(d \to c) \vee a} \quad \neg((d \to c) \vee a)}{\bot}}{\neg c}}{\neg c \vee \neg b}$$

(c)

$$\dfrac{\dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a} \quad \dfrac{\dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a} \quad [a \to b]}{b} \quad \dfrac{[a \wedge \neg b]}{\neg b}}{\bot}}{\neg(a \to b)}}{a \wedge \neg(a \to b)} \quad \neg(a \wedge \neg(a \to b))}{\bot}}{\neg(a \wedge \neg b)}$$

(d)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[f] \quad [f \to p]}{p}}{p \vee s}}{\neg f \vee p \vee s} \quad [\neg(\neg f \vee p \vee s)]}{\bot}}{\neg(f \to p)} \quad \neg(f \to p) \to s}{s}}{p \vee s}}{\neg f \vee p \vee s} \quad [\neg(\neg f \vee p \vee s)]}{\bot}}{\neg f}}{\neg f \vee p \vee s} \quad [\neg(\neg f \vee p \vee s)]}{\bot}}{\neg f \vee p \vee s}$$

(e)

$$\dfrac{\dfrac{\dfrac{\dfrac{[p] \quad \dfrac{[p] \quad p \leftrightarrow \neg p}{\neg p}}{\bot}}{\neg p} \quad p \leftrightarrow \neg p}{p} \quad \dfrac{\dfrac{\dfrac{\dfrac{[\neg p] \quad p \leftrightarrow \neg p}{p} \quad [\neg p]}{\bot}}{p} \quad p \leftrightarrow \neg p}{\neg p}}{\bot}$$

2. Construya derivaciones que justifiquen los siguientes juicios.

   (a) $\vdash \varphi \to \varphi$

   (b) $\vdash \bot \to \varphi$

   (c) $\vdash \neg(\varphi \wedge \neg\varphi)$

   (d) **[PP2007]** $\varphi \vee \varphi \vdash \varphi$

   (e) $\alpha \vee \beta \vdash \beta \vee \alpha$

   (f) $\neg\varphi \vdash \varphi \to (\bot \vee \neg\bot)$ [1]

   (g) **[ED2005]** $\vdash \varphi \vee \neg\varphi$ $(*)$

   (h) $\neg(\varphi \to \psi) \to \sigma \vdash \neg\varphi \vee (\psi \vee \sigma)$ **[EF2014]**

   (i) ♣ $\vdash \neg\bot \leftrightarrow \neg\neg\neg\bot$

3. Demuestre que:

   (a) Si $\vdash \varphi$ entonces $\vdash \psi \vee \varphi$.

   (b) Si $\vdash \varphi$ entonces $\vdash \psi \to \varphi$.

   (c) Si $\vdash \varphi$ y $\vdash \psi$ entonces $\vdash \varphi \wedge (\psi \vee \sigma)$

   (d) ♣ $\vdash \varphi$ si y solamente si $\vdash \varphi \leftrightarrow \neg\bot$

Ejercicios integradores.
Construya derivaciones que justifiquen los siguientes juicios.

1. **[PP1998]** $(\varphi \to \psi) \vdash ((\psi \to \sigma) \wedge \neg\sigma) \to \neg\varphi)$
2. **[PP1999]** $\varphi \wedge \sigma \to \psi, \quad \varphi \to \sigma \vdash \neg\psi \to \neg\varphi$
3. **[PP2000]** $\varphi \to (\sigma \vee \psi) \vdash \neg\sigma \to (\varphi \to \psi)$
4. **[PP2001]** $(\neg\beta \to \neg\alpha) \wedge \alpha \vdash \beta$
5. **[PP2002]** $\neg\psi \to \neg\varphi \vdash \varphi \to ((\psi \to \neg\varphi) \to \sigma)$
6. **[PP2003]** $\neg(\alpha \wedge \neg(\alpha \to \beta)) \vdash \neg(\alpha \wedge \beta)$
7. **[PP2004]** $\vdash \neg(\alpha \vee \beta) \leftrightarrow (\neg\alpha \wedge \neg\beta)$ [Ley de DeMorgan]
8. **[PP2005]** $(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta) \vdash \neg(\alpha \leftrightarrow \beta)$
9. **[PP2006]** $\neg\beta \to \gamma, \quad \alpha \vee \beta \vdash g$ $(*)$
10. **[PP2007]** $\vdash (\neg\sigma \to (\neg\varphi \wedge \neg\psi)) \to ((\varphi \vee \psi) \to \sigma)$
11. **[PP2008]** $\vdash (a \to b) \wedge (c \to b) \to (\neg\neg a \vee c) \to b$
12. **[PP2009]** $\vdash \neg\alpha \wedge (\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma)$
13. **[PP2010]** $\neg((\psi \to \varphi) \vee \alpha) \vdash \neg\varphi \vee \neg\beta$

[1] Sugerencia: notar que $\vdash \neg\bot$.

14. **[PP2010]** $((\alpha \vee \beta) \to (\alpha \wedge \beta)) \leftrightarrow (\alpha \leftrightarrow \beta)$
15. **[PP2011]** $\vdash ((\alpha \vee \beta) \to (\beta \wedge \neg\alpha)) \to \neg\alpha$
16. **[PP2011]** $\neg(\varphi \wedge \psi) \leftrightarrow (\neg\varphi \vee \neg\neg\psi)$
17. **[PP2012]** $(q \to \neg p) \to p \vdash \neg(p \to \neg q) \vee p$
18. **[PP2012]** $\vdash ((p \to q) \to p) \to \neg(\neg p \wedge \neg(p \wedge q))$
19. **[PP2013]** $\varphi \to \psi \vdash \neg\neg\psi \vee \neg\varphi$
20. **[PP2013]** $\neg\varphi, \quad \neg\alpha \vee \neg\beta \vdash \sigma \quad \Rightarrow \quad \neg\varphi \vdash \neg(\alpha \wedge \beta) \to \sigma$
21. **[PP2013]** $(p_0 \to p_1) \to \neg p_1 \vdash \neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$
22. **[PP2014]** $(\alpha \to \beta) \to \gamma \vdash \gamma \vee \neg\beta$
23. **[PP2014]** $\vdash (\alpha \leftrightarrow (\beta \to \gamma)) \to (\neg\alpha \wedge \beta \to \neg\gamma)$
24. **[PP2015]** $\vdash ((\alpha \to \gamma) \wedge (\beta \to \gamma)) \leftrightarrow (\alpha \vee \beta \to \gamma)$
25. **[PP2015]** $\vdash (\alpha \to \beta \to \gamma) \to \neg(\neg\gamma \vee \neg\beta) \to \gamma$
26. **[PP2016]** $\neg(\beta \vee \alpha) \vdash \alpha \wedge \beta$
27. **[PP2016]** $(\varphi \vee \psi) \leftrightarrow (\varphi \vee (\neg\varphi \wedge \psi))$
28. **[PP2017]** $\gamma \to \alpha, \quad \neg \to (\neg\beta \vee \alpha) \vdash \neg\alpha \to (\neg\beta \wedge \neg\gamma)$
29. **[PP2017]** $p_2 \to \neg p_1, \quad p_3 \to p_1, \quad \neg p_1 \to (\neg p_2 \vee p_1) \vdash \neg p_1 \to (\neg p_2 \wedge \neg p_3)$
30. **[PP2017]** $p_3 \to p_1, \quad \neg p_1 \to (\neg p_2 \vee p_1) \vdash (p_2 \vee p_3) \to p_1$ [Riesgo de salud]
31. **[PP2018]** $p \leftrightarrow \neg p \vdash \perp$
32. **[PP2018]** $p \vee q, \quad p \to r, \quad q \to s \vdash t \to \neg(\neg r \wedge \neg s)$

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33. **[ED2008]** $\neg p \wedge q \to r, \quad r \vee s \to \neg p, \quad s \vdash \neg q \vee r$
34. **[ED2010]** $\alpha \vee \beta \leftrightarrow \beta \vdash \alpha \leftrightarrow \alpha \wedge \beta$
35. **[ED2011]** $\neg p \wedge \neg q \to r \vdash (p \vee q) \vee r$
36. **[EF2012]** $(\neg\alpha \vee \beta) \vee \gamma \vdash \neg\gamma \wedge \alpha \to \neg\neg\beta$

SOLUCIONES PROPUESTAS AL PRÁCTICO 4

A continuación se presenta un esbozo de las soluciones de los ejercicios, es decir, las mismas no están completas: falta agregar el inciso indicador de la cancelación de las hipótesis destacadas entre corchetes. Se alienta al lector a realizar tales agregados a efectos de profundizar la comprensión de dichas pruebas.

Asimismo, a efectos de no sobrecargar la notación se ha optado por emplear letras del alfabeto latino en lugar de aquellas del alfabeto griego.

1. Vistos en clase.

2. (g)

$$\dfrac{\dfrac{\dfrac{\dfrac{[\bot]\quad[\neg\bot]}{\bot}\,E\neg}{\bot}\,RAA}{\neg\bot}\,I\neg \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\bot]\quad[\neg\bot]}{\bot}\,E\neg}{\bot}\,RAA}{\neg\bot}\,I\neg \quad [\neg(\neg\bot)]}{\bot}\,E\neg}{\neg(\neg(\neg\bot))}\,I\neg}{\neg\bot \leftrightarrow \neg(\neg(\neg\bot))}\,I\leftrightarrow$$

3. Vistos en clase.

Ejercicios integradores.

1.

$$\dfrac{\dfrac{\dfrac{\dfrac{[a]\quad a\to b}{b}\,E\to \quad \dfrac{[(b\to c)\wedge\neg c]}{b\to c}\,E\wedge_1}{c}\,E\to \quad \dfrac{[(b\to c)\wedge\neg c]}{\neg c}\,E\wedge_2}{\dfrac{\bot}{\neg a}\,I\neg}\,E\neg}{(b\to c)\wedge\neg c\to\neg a}\,I\to$$

2.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[a]\quad \dfrac{[a]\quad a\to b}{b}\,E\to}{a\wedge b}\,I\wedge \quad a\wedge b\to c}{c}\,E\to \quad [\neg c]}{\bot}\,E\neg}{\neg a}\,I\neg}{\neg c\to\neg a}\,I\to$$

3.

$$\dfrac{\dfrac{\dfrac{\dfrac{[a]\quad a\to b\vee c}{b\vee c}\,E\to \quad \dfrac{\dfrac{[b]\quad[\neg b]}{\bot}\,E\neg}{c}\,E\bot \quad \dfrac{\dfrac{[c]\quad[\neg b]}{c\wedge\neg b}\,I\wedge}{c}\,E\wedge_1}{c}\,E\vee}{a\to c}\,I\to}{\neg b\to a\to c}\,I\to$$

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4.

$$\dfrac{\dfrac{\dfrac{(\neg b \to \neg a) \wedge a}{a}\,E\wedge_2 \qquad \dfrac{[\neg b] \qquad \dfrac{(\neg b \to \neg a) \wedge a}{\neg b \to \neg a}\,E\wedge_1}{\neg a}\,E\to}{\bot}\,E\neg}{b}\,RAA$$

5.

$$\dfrac{\dfrac{\dfrac{\dfrac{[b] \qquad \dfrac{\dfrac{\dfrac{[b] \qquad \dfrac{[a] \qquad [a \to \neg b]}{\neg b}\,E\to}{\bot}\,E\neg}{\neg a}\,I\neg \qquad \neg a \to \neg b}{\neg b}\,E\to}{\bot}\,E\neg}{c}\,E\bot}{(a \to \neg b) \to c}\,I\to}{b \to (a \to \neg b) \to c}\,I\to$$

6.

$$\dfrac{\dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a}\,E\wedge_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a}\,E\wedge_1 \qquad [a \to b]}{b}\,E\to \qquad \dfrac{[a \wedge \neg b]}{\neg b}\,E\wedge_2}{\bot}\,E\neg}{\neg(a \to b)}\,I\neg}{a \wedge \neg(a \to b)}\,I\wedge \qquad \neg(a \wedge \neg(a \to b))}{\bot}\,E\neg}{\neg(a \wedge \neg b)}\,I\neg$$

10.

$$\dfrac{\dfrac{\dfrac{[b \vee c] \qquad \dfrac{[b] \qquad \dfrac{\dfrac{[\neg a] \qquad [\neg a \to \neg b \wedge \neg c]}{\neg b \wedge \neg c}\,E\to}{\neg b}\,E\wedge_1}{\bot}\,E\neg \qquad \dfrac{[c] \qquad \dfrac{\dfrac{[\neg a] \qquad [\neg a \to \neg b \wedge \neg c]}{\neg b \wedge \neg c}\,E\to}{\neg c}\,E\wedge_2}{\bot}\,E\neg}{\dfrac{\bot}{a}\,RAA}\,E\vee}{b \vee c \to a}\,I\to}{(\neg a \to \neg b \wedge \neg c) \to b \vee c \to a}\,I\to$$

12.

$$\dfrac{\dfrac{\dfrac{\dfrac{[a \vee b] \qquad \dfrac{\dfrac{[a] \qquad \dfrac{[\neg a \wedge (b \to g)]}{\neg a}\,E\wedge_1}{\bot}\,E\neg}{b}\,E\bot \qquad [b]}{b}\,E\vee \qquad \dfrac{[\neg a \wedge (b \to g)]}{b \to g}\,E\wedge_2}{g}\,E\to}{a \vee b \to g}\,I\to}{\neg a \wedge (b \to g) \to a \vee b \to g}\,I\to$$

7.

$$\dfrac{\dfrac{\dfrac{\dfrac{[a \vee b] \quad \dfrac{\dfrac{[a] \quad \dfrac{[\neg a \wedge \neg b]}{\neg a}\,E\wedge_1}{\bot}\,E\neg}{b}\,E\bot \quad \dfrac{\dfrac{[b] \quad \dfrac{[\neg a \wedge \neg b]}{\neg b}\,E\wedge_2}{\bot}\,E\neg}{b}\,E\bot}{b}\,E\vee \quad \dfrac{[\neg a \wedge \neg b]}{\neg b}\,E\wedge_2}{\bot}\,E\neg}{\neg(a \vee b)}\,I\neg \quad \dfrac{\dfrac{\dfrac{\dfrac{[a]}{a \vee b}\,I\vee_1 \quad [\neg(a \vee b)]}{\bot}\,E\neg}{\neg a}\,I\neg \quad \dfrac{\dfrac{\dfrac{[b]}{a \vee b}\,I\vee_2 \quad [\neg(a \vee b)]}{\bot}\,E\neg}{\neg b}\,I\neg}{\neg a \wedge \neg b}\,I\wedge}{\neg(a \vee b) \leftrightarrow \neg a \wedge \neg b}\,I\leftrightarrow$$

8.

$$\dfrac{\dfrac{(a \wedge \neg b) \vee \neg a \wedge b \quad \dfrac{\dfrac{\dfrac{[a \wedge \neg b]}{a}\,E\wedge_1 \quad [a \leftrightarrow b]}{b}\,E\leftrightarrow_1 \quad \dfrac{[a \wedge \neg b]}{\neg b}\,E\wedge_2}{\bot}\,E\neg \quad \dfrac{\dfrac{\dfrac{[\neg a \wedge b]}{b}\,E\wedge_2 \quad [a \leftrightarrow b]}{a}\,E\leftrightarrow_1 \quad \dfrac{[\neg a \wedge b]}{\neg a}\,E\wedge_1}{\bot}\,E\neg}{\bot}\,E\vee}{\neg(a \leftrightarrow b)}\,I\neg$$

9.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg b] \quad \neg b \to g}{g}\,E\to \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[g]}{g \vee \neg g}\,I\vee_1 \quad [\neg(g \vee \neg g)]}{\bot}\,E\neg}{\neg g}\,I\neg}{g \vee \neg g}\,I\vee_2 \quad [\neg(g \vee \neg g)]}{\bot}\,E\neg}{\bot}\,RAA}{g \vee \neg g}\,RAA \quad \dfrac{\dfrac{[g] \quad [\neg g]}{g \wedge \neg g}\,I\wedge}{\neg g}\,E\wedge_2 \quad [\neg g]}{\neg g}\,E\vee}{\bot}\,E\neg}{b}\,RAA}{a \vee b}\,I\vee_2 \quad a \vee b \to g}{g}\,E\to \quad [\neg g]}{\bot}\,E\neg}{g}\,RAA$$

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11.

$$
\begin{prooftree}
\AxiomC{$[\neg(\neg a)\vee c]$}
\AxiomC{$[\neg a]$}
\AxiomC{$[\neg(\neg a)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\perp$}
\RightLabel{RAA}
\UnaryInfC{a}
\AxiomC{$[(a\to b)\wedge(c\to b)]$}
\RightLabel{$E\wedge_1$}
\UnaryInfC{$a\to b$}
\RightLabel{$E\to$}
\BinaryInfC{b}
\AxiomC{$[c]$}
\AxiomC{$[(a\to b)\wedge(c\to b)]$}
\RightLabel{$E\wedge_2$}
\UnaryInfC{$c\to b$}
\RightLabel{$E\to$}
\BinaryInfC{b}
\RightLabel{$E\vee$}
\TrinaryInfC{b}
\RightLabel{$I\to$}
\UnaryInfC{$\neg(\neg a)\vee c\to b$}
\RightLabel{$I\to$}
\UnaryInfC{$(a\to b)\wedge(c\to b)\to\neg(\neg a)\vee c\to b$}
\end{prooftree}
$$

13.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[c]}{d \to c}\,I\to}{(d \to c) \vee a}\,I\vee_1 \qquad \neg((d \to c) \vee a)}{\bot}\,E\neg}{\neg c}\,I\neg}{\neg c \vee \neg b}\,I\vee_1$$

15.

$$\dfrac{\dfrac{\dfrac{[a] \qquad \dfrac{\dfrac{\dfrac{[a]}{a \vee b}\,I\vee_1 \qquad [a \vee b \to b \wedge \neg a]}{b \wedge \neg a}\,E\to}{\neg a}\,E\wedge_2}{\bot}\,E\neg}{\neg a}\,I\neg}{(a \vee b \to b \wedge \neg a) \to \neg a}\,I\to$$

17.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg p]}{q \to \neg p}\,I\to \qquad (q \to \neg p) \to p}{p}\,E\to \qquad [\neg p]}{\bot}\,E\neg}{p}\,RAA}{\neg(p \to \neg q) \vee p}\,I\vee_2$$

18.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[p] \qquad \dfrac{[\neg p \wedge \neg(p \wedge q)]}{\neg p}\,E\wedge_1}{\bot}\,E\neg}{q}\,E\bot}{p \to q}\,I\to \qquad [(p \to q) \to p]}{p}\,E\to \qquad \dfrac{[\neg p \wedge \neg(p \wedge q)]}{\neg p}\,E\wedge_1}{\bot}\,E\neg}{\neg(\neg p \wedge \neg(p \wedge q))}\,I\neg}{((p \to q) \to p) \to \neg(\neg p \wedge \neg(p \wedge q))}\,I\to$$

19.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[f] \qquad f \to p}{p}\,E\to \qquad [\neg p]}{\bot}\,E\neg}{\neg f}\,I\neg}{\neg(\neg p) \vee \neg f}\,I\vee_2 \qquad [\neg(\neg(\neg p) \vee \neg f)]}{\bot}\,E\neg}{\neg(\neg p)}\,I\neg}{\neg(\neg p) \vee \neg f}\,I\vee_1 \qquad [\neg(\neg(\neg p) \vee \neg f)]}{\bot}\,E\neg}{\neg(\neg p) \vee \neg f}\,RAA$$

14.

$$
\dfrac{
\dfrac{
\dfrac{[a \lor b] \quad [a] \quad \dfrac{[b] \quad [a \leftrightarrow b]}{a}\, E\leftrightarrow_1}{a}\, E\lor
\qquad
\dfrac{[a \lor b] \quad \dfrac{[a] \quad [a \leftrightarrow b]}{b}\, E\leftrightarrow_1 \quad [b]}{b}\, E\lor
}{\dfrac{a \land b}{a \lor b \to a \land b}\, I\to}\, I\land
\qquad
\dfrac{
\dfrac{\dfrac{\dfrac{[b]}{a \lor b}\, I\lor_2 \quad [a \lor b \to a \land b]}{a \land b}\, E\to}{a}\, E\land_1
\qquad
\dfrac{\dfrac{\dfrac{[a]}{a \lor b}\, I\lor_1 \quad [a \lor b \to a \land b]}{a \land b}\, E\to}{b}\, E\land_2
}{a \leftrightarrow b}\, I\leftrightarrow
}{(a \lor b \to a \land b) \leftrightarrow a \leftrightarrow b}\, I\leftrightarrow
$$

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16.

$$\dfrac{\dfrac{\dfrac{[\neg f \vee \neg(\neg p)] \quad \dfrac{\dfrac{[f \wedge \neg p]}{f}\,E\wedge_1 \quad [\neg f]}{\bot}\,E\neg \quad \dfrac{\dfrac{[f \wedge \neg p]}{\neg p}\,E\wedge_2 \quad [\neg(\neg p)]}{\bot}\,E\neg}{\bot}\,E\vee}{\neg(f \wedge \neg p)}\,I\neg \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[f] \quad [\neg p]}{f \wedge \neg p}\,I\wedge \quad [\neg(f \wedge \neg p)]}{\bot}\,E\neg}{\neg f}\,I\neg}{\neg f \vee \neg(\neg p)}\,I\vee_1 \quad [\neg(\neg f \vee \neg(\neg p))]}{\bot}\,E\neg}{\neg(\neg p)}\,I\neg}{\neg f \vee \neg(\neg p)}\,I\vee_2 \quad [\neg(\neg f \vee \neg(\neg p))]}{\bot}\,E\neg}{\neg f \vee \neg(\neg p)}\,RAA}{\neg(f \wedge \neg p) \leftrightarrow \neg f \vee \neg(\neg p)}\,I\leftrightarrow$$

20.

$$\dfrac{\begin{array}{c} \neg\varphi \qquad \dfrac{[\neg(\alpha\wedge\beta)]^{(1)} \qquad \begin{array}{c}\text{(tautología dem. en [PP2014])}\\ \vdots \\ \neg(\alpha\wedge\beta)\to\neg\alpha\vee\neg\beta\end{array}}{\neg\alpha\vee\neg\beta}\,E\to \\ \vdots \\ \sigma\end{array}}{\neg(\alpha\wedge\beta)\to\sigma}\,I\to(1)$$

22.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg b]}{g\vee\neg b}\,I\vee_2 \qquad [\neg(g\vee\neg b)]}{\bot}\,E\neg}{b}\,RAA}{a\to b}\,I\to \qquad (a\to b)\to g}{g}\,E\to}{g\vee\neg b}\,I\vee_1 \qquad [\neg(g\vee\neg b)]}{\dfrac{\bot}{g\vee\neg b}\,RAA}\,E\neg$$

23.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[g]}{b\to g}\,I\to \qquad [a\leftrightarrow b\to g]}{a}\,E\leftrightarrow_1 \qquad \dfrac{[\neg a\wedge b]}{\neg a}\,E\wedge_1}{\bot}\,E\neg}{\neg g}\,I\neg}{\neg a\wedge b\to\neg g}\,I\to}{(a\leftrightarrow b\to g)\to\neg a\wedge b\to\neg g}\,I\to$$

25.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg g]}{\neg g\vee\neg b}\,I\vee_1 \qquad [\neg(\neg g\vee\neg b)]}{\bot}\,E\neg}{g}\,RAA}{\neg(\neg g\vee\neg b)\to g}\,I\to}{(a\to b\to g)\to\neg(\neg g\vee\neg b)\to g}\,I\to$$

26.

$$\dfrac{\dfrac{\dfrac{\dfrac{[a\vee b] \qquad \dfrac{[a]}{b\vee a}\,I\vee_2 \qquad \dfrac{[b]}{b\vee a}\,I\vee_1}{b\vee a}\,E\vee \qquad \neg(b\vee a)}{\bot}\,E\neg}{a\wedge b}\,E\bot}{a\vee b\to a\wedge b}\,I\to$$

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21.

$$
\begin{prooftree}
\AxiomC{$[p_0 \to p_1]$}
\AxiomC{$(p_0 \to p_1) \to \neg p_1$}
\RightLabel{$E\to$}
\BinaryInfC{$\neg p_1$}
\AxiomC{$[\neg(\neg p_1)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$I\neg$}
\UnaryInfC{$\neg(p_0 \to p_1)$}
\RightLabel{$I\vee_2$}
\UnaryInfC{$\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$}
\AxiomC{$[\neg(\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1))]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$I\neg$}
\UnaryInfC{$\neg(\neg(\neg p_1))$}
\RightLabel{$I\vee_1$}
\UnaryInfC{$\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$}
\AxiomC{$[\neg(\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1))]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{RAA}
\UnaryInfC{$\neg(\neg(\neg p_1)) \vee \neg(p_0 \to p_1)$}
\end{prooftree}
$$

24.

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{[a]}{a \vee b}\, I\vee_1 \qquad [a \vee b \to g]
}{g}\, E\to
}{a \to g}\, I\to
\qquad
\dfrac{
\dfrac{
\dfrac{[b]}{a \vee b}\, I\vee_2 \qquad [a \vee b \to g]
}{g}\, E\to
}{b \to g}\, I\to
}{(a \to g) \wedge (b \to g)}\, I\wedge
$$

$$
\dfrac{
\dfrac{
[a \vee b] \qquad
\dfrac{[a] \qquad \dfrac{[(a \to g) \wedge (b \to g)]}{a \to g}\, E\wedge_1}{g}\, E\to
\qquad
\dfrac{[b] \qquad \dfrac{[(a \to g) \wedge (b \to g)]}{b \to g}\, E\wedge_2}{g}\, E\to
}{g}\, E\vee
}{a \vee b \to g}\, I\to
$$

$$
\dfrac{(a \to g) \wedge (b \to g) \qquad a \vee b \to g}{(a \to g) \wedge (b \to g) \leftrightarrow a \vee b \to g}\, I\leftrightarrow
$$

27.

$$
\begin{prooftree}
\AxiomC{$[f \vee \neg f \wedge p]$}
\AxiomC{$[f]$}
\AxiomC{$[\neg f]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$E\bot$}
\UnaryInfC{$\neg f \wedge p$}
\AxiomC{$[\neg f \wedge p]$}
\RightLabel{$E\vee$}
\TrinaryInfC{$\neg f \wedge p$}
\RightLabel{$E\wedge_2$}
\UnaryInfC{p}
\RightLabel{$I\vee_2$}
\UnaryInfC{$f \vee p$}
\AxiomC{$[\neg(f \vee p)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{RAA}
\UnaryInfC{f}
\RightLabel{$I\vee_1$}
\UnaryInfC{$f \vee p$}
\AxiomC{$[\neg(f \vee p)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{RAA}
\UnaryInfC{$f \vee p$}
\AxiomC{$[f \vee p]$}
\AxiomC{$[f]$}
\AxiomC{$[\neg f]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$E\bot$}
\UnaryInfC{$\neg f \wedge p$}
\AxiomC{$[\neg f]$}
\AxiomC{$[p]$}
\RightLabel{$I\wedge$}
\BinaryInfC{$\neg f \wedge p$}
\RightLabel{$E\vee$}
\TrinaryInfC{$\neg f \wedge p$}
\RightLabel{$I\vee_2$}
\UnaryInfC{$f \vee \neg f \wedge p$}
\AxiomC{$[\neg(f \vee \neg f \wedge p)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{RAA}
\UnaryInfC{f}
\RightLabel{$I\vee_1$}
\UnaryInfC{$f \vee \neg f \wedge p$}
\AxiomC{$[\neg(f \vee \neg f \wedge p)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{RAA}
\UnaryInfC{$f \vee \neg f \wedge p$}
\RightLabel{$I\leftrightarrow$}
\BinaryInfC{$f \vee p \leftrightarrow f \vee \neg f \wedge p$}
\end{prooftree}
$$

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28.

$$
\begin{prooftree}
\AxiomC{$[\neg a]$}
\AxiomC{$\neg a \to \neg b \vee a$}
\RightLabel{$E \to$}
\BinaryInfC{$\neg b \vee a$}
\AxiomC{$[\neg b]$}
\AxiomC{$[a]$}
\AxiomC{$[\neg a]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$E\bot$}
\UnaryInfC{$\neg b$}
\RightLabel{$E\vee$}
\TrinaryInfC{$\neg b$}
\AxiomC{$[g]$}
\AxiomC{$g \to a$}
\RightLabel{$E \to$}
\BinaryInfC{a}
\AxiomC{$[\neg a]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$I\neg$}
\UnaryInfC{$\neg g$}
\RightLabel{$I\wedge$}
\BinaryInfC{$\neg b \wedge \neg g$}
\RightLabel{$I \to$}
\UnaryInfC{$\neg a \to \neg b \wedge \neg g$}
\end{prooftree}
$$

32.

$$
\begin{prooftree}
\AxiomC{$p \vee q$}
\AxiomC{$[p]$}
\AxiomC{$[p]$}
\AxiomC{$p \to r$}
\RightLabel{$E \to$}
\BinaryInfC{r}
\AxiomC{$[\neg r \wedge \neg s]$}
\RightLabel{$E\wedge_1$}
\UnaryInfC{$\neg r$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$I\neg$}
\UnaryInfC{$\neg p$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$E\bot$}
\UnaryInfC{q}
\AxiomC{$[q]$}
\RightLabel{$E\vee$}
\TrinaryInfC{q}
\AxiomC{$q \to s$}
\RightLabel{$E \to$}
\BinaryInfC{s}
\AxiomC{$[\neg r \wedge \neg s]$}
\RightLabel{$E\wedge_2$}
\UnaryInfC{$\neg s$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$I\neg$}
\UnaryInfC{$\neg(\neg r \wedge \neg s)$}
\RightLabel{$I \to$}
\UnaryInfC{$t \to \neg(\neg r \wedge \neg s)$}
\end{prooftree}
$$

33.

$$
\begin{prooftree}
\AxiomC{s}
\RightLabel{$I\vee_2$}
\UnaryInfC{$r \vee s$}
\AxiomC{$r \vee s \to \neg p$}
\RightLabel{$E \to$}
\BinaryInfC{$\neg p$}
\AxiomC{$[q]$}
\RightLabel{$I\wedge$}
\BinaryInfC{$\neg p \wedge q$}
\AxiomC{$\neg p \wedge q \to r$}
\RightLabel{$E \to$}
\BinaryInfC{r}
\RightLabel{$I\vee_2$}
\UnaryInfC{$\neg q \vee r$}
\AxiomC{$[\neg(\neg q \vee r)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{$I\neg$}
\UnaryInfC{$\neg q$}
\RightLabel{$I\vee_1$}
\UnaryInfC{$\neg q \vee r$}
\AxiomC{$[\neg(\neg q \vee r)]$}
\RightLabel{$E\neg$}
\BinaryInfC{$\bot$}
\RightLabel{RAA}
\UnaryInfC{$\neg q \vee r$}
\end{prooftree}
$$

34.

$$
\begin{prooftree}
\AxiomC{$[a \wedge b]$}
\RightLabel{$E\wedge_1$}
\UnaryInfC{a}
\AxiomC{$[a]$}
\AxiomC{$[a]$}
\RightLabel{$I\vee_1$}
\UnaryInfC{$a \vee b$}
\AxiomC{$a \vee b \leftrightarrow b$}
\RightLabel{$E\leftrightarrow_1$}
\BinaryInfC{b}
\RightLabel{$I\wedge$}
\BinaryInfC{$a \wedge b$}
\RightLabel{$I \leftrightarrow$}
\BinaryInfC{$a \leftrightarrow a \wedge b$}
\end{prooftree}
$$

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29.

$$\dfrac{\dfrac{\dfrac{[p_2] \quad \dfrac{\dfrac{\dfrac{[p_2] \quad p_2 \to \neg p_1}{\neg p_1}\,E\to \quad \neg p_1 \to \neg p_2 \vee p_1}{\neg p_2 \vee p_1}\,E\to \quad [\neg p_2] \quad \dfrac{\dfrac{[p_1] \quad [\neg p_1]}{\bot}\,E\neg}{\neg p_2}\,E\bot}{\neg p_2}\,E\vee}{\bot}\,E\neg}{\neg p_2}\,I\neg \quad \dfrac{\dfrac{\dfrac{[p_3] \quad p_3 \to p_1}{p_1}\,E\to \quad [\neg p_1]}{\bot}\,E\neg}{\neg p_3}\,I\neg}{\dfrac{\neg p_2 \wedge \neg p_3}{\neg p_1 \to \neg p_2 \wedge \neg p_3}\,I\to}\,I\wedge$$

30.

$$\dfrac{\dfrac{\dfrac{\dfrac{[p_2 \vee p_3] \quad [p_2] \quad \dfrac{\dfrac{[p_3] \quad \dfrac{\dfrac{\dfrac{[p_3] \quad p_3 \to p_1}{p_1}\,E\to \quad [\neg p_1]}{\bot}\,E\neg}{\neg p_3}\,I\neg}{\bot}\,E\neg}{p_2}\,E\bot}{p_2}\,E\vee \quad \dfrac{\dfrac{[\neg p_1] \quad \dfrac{\dfrac{\dfrac{\dfrac{[\neg p_1] \quad \neg p_1 \to \neg p_2 \vee p_1}{\neg p_2 \vee p_1}\,E\to \quad [\neg p_2] \quad \dfrac{\dfrac{[p_1] \quad [\neg p_1]}{\bot}\,E\neg}{\neg p_2}\,E\bot}{\neg p_2}\,E\vee \quad \dfrac{\dfrac{\dfrac{[p_3] \quad p_3 \to p_1}{p_1}\,E\to \quad [\neg p_1]}{\bot}\,E\neg}{\neg p_3}\,I\neg}{\neg p_2 \wedge \neg p_3}\,I\wedge}{\neg p_1 \to \neg p_2 \wedge \neg p_3}\,I\to}{\neg p_2 \wedge \neg p_3}\,E\to}{\neg p_2}\,E\wedge_1}{\bot}\,E\neg}{\dfrac{p_1}{p_2 \vee p_3 \to p_1}\,I\to}\,RAA$$

31.

$$\dfrac{\dfrac{\dfrac{\dfrac{[p] \quad \dfrac{[p] \quad p \leftrightarrow \neg p}{\neg p}\,E\leftrightarrow_1}{\bot}\,E\neg}{\neg p}\,I\neg \quad p \leftrightarrow \neg p}{p}\,E\leftrightarrow_1 \quad \dfrac{\dfrac{\dfrac{\dfrac{[\neg p] \quad p \leftrightarrow \neg p}{p}\,E\leftrightarrow_1 \quad [\neg p]}{\bot}\,E\neg}{p}\,RAA \quad p \leftrightarrow \neg p}{\neg p}\,E\leftrightarrow_1}{\bot}\,E\neg$$

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35.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[p]}{p \vee q}\,I\vee_1 \quad [\neg(p \vee q)]}{\bot}\,E\neg}{\neg p}\,I\neg \qquad \dfrac{\dfrac{\dfrac{[q]}{p \vee q}\,I\vee_2 \quad [\neg(p \vee q)]}{\bot}\,E\neg}{\neg q}\,I\neg}{\neg p \wedge \neg q}\,I\wedge \qquad \neg p \wedge \neg q \to r}{r}\,E\to}{(p \vee q) \vee r}\,I\vee_2 \qquad [\neg((p \vee q) \vee r)]}{\dfrac{\bot}{p \vee q}\,RAA}\,E\neg}{(p \vee q) \vee r}\,I\vee_1 \qquad [\neg((p \vee q) \vee r)]}{\dfrac{\bot}{(p \vee q) \vee r}\,RAA}\,E\neg$$

36.

$$\dfrac{\dfrac{(\neg a \vee b) \vee g \qquad \dfrac{[\neg a \vee b] \qquad \dfrac{\dfrac{[\neg g \wedge a]}{a}\,E\wedge_2 \quad [\neg a]}{\bot}\,E\neg \qquad \dfrac{[b] \quad [\neg b]}{\bot}\,E\neg}{\bot}\,E\vee \qquad \dfrac{[g] \quad \dfrac{[\neg g \wedge a]}{\neg g}\,E\wedge_1}{\bot}\,E\neg}{\dfrac{\bot}{\neg(\neg b)}\,I\neg}\,E\vee}{\neg g \wedge a \to \neg(\neg b)}\,I\to$$

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