Delegation and Learning

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Very Preliminary

Abstract

I study a dynamic delegation model in which the principal can delegate the decision of implementing or rejecting an investment project to the agent. The agent is new to the organization, and the principal is better than the agent at recognizing bad projects. Still, the agent can become an expert and perfectly recognizes bad projects if he gains experience. The principal faces a cost if the agent implements a bad project, while the agent is protected by limited liability. The principal faces a tradeoff: more delegation implies improving the agent, but also more bad projects implemented before he gains enough experience. The optimal delegation rule is a Balanced Delegation rule, in which the principal cannot delegate as often as desired to incentive good behavior of the agent. This rule combines periods of reward and punishment that are path-dependent until the agent reaches his full potential.

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1 Introduction

In any productive organization it is fundamental to incorporate new members with new skills that increase the productivity of the organization. This poses a challenge to the organization because of the learning curve: new members are less productive when they first join an organization, but they become more productive over time. A new member cannot show her or his full potential from the beginning; first she or he needs to get used to the specific environment of the organization. This learning curve is costly for the organization, and generates incentives to people who are in charge to not assign activities to new employees because of their lack of expertise.

I analyze an infinite discrete time delegation model where at the beginning of each period a new project of unknown quality arrives. A good quality project generates a positive profit and a bad quality one generates a negative profit. The quality is not directly observable, but it is common knowledge that bad projects arrive more often than good projects. The principal decides whether to delegate the evaluation and implementation of the project to the agent, or doing it by herself. When the agent first joins the organization, his ability to identify bad projects is worse than the principal's. The person who is put in charge of the project decides whether to implement it, and be subject to the random profit, or reject it and get zero profit.

The ability of recognize bad project is given by the precision of a signal. The principal receives a more precise signal than the agent at the beginning of the game, but after each delegation and implementation decision, the agent become an expert with some probability. That is, he receives perfectly informative signal in the following periods. The incentives are not perfectly aligned, while the principal faces a cost if a bad project is implemented, the agent is protected by limited liability. The principal's tradeoff is: delegating more often allows the agent to become an expert more quickly and achieve perfect discrimination project quality; however, very frequent delegation imposes a cost to the principal given that more bad projects will be implemented.

I show that the optimal delegation rule that maximizes the principal's profit uses two instruments: (1) Reward periods (or no-questions-asked periods), where the principal rewards the agent after a rejection with periods with sure-delegation in which the implementation decision does not affect future delegation decisions. And (2) punishment periods (or probation periods), where the principal punishes the agent after every implementation with no-delegation periods.

The optimal rule starts with delegation. After an implementation, the principal punishes the agent by not delegating for τ periods. After a rejection, the principal rewards the agent with T periods of delegation. T periods of delegation are interpreted as $T-1$ no-question-asked periods. This induces the agent to accept any project which generates a negative expected profit to the principal. However, this increases the chances of the agent becoming an expert and it provides a positive expected profit to the principal. The value of T and τ are a function of the value of the relationship that the principal promises to the agent.

This solution is optimal for some values of the discount factor. Delegation can be seen as an investment: more delegation means a higher cost today for the principal, but given the probability of the agent becoming an expert, it increases the expected value of the relationship tomorrow. If the discount factor is too low, the principal prefers never to delegate because the investment is costlier than the return. On the other hand, if the discount factor is too high, the principal prefers always to delegate because she values future return more than the cost of today's investment.

The reason why the principal uses these two instruments in the optimal contract is because her cost is compensated by profit gaining. When she is punishing the agent, she can also work on the projects and get positive profit. When she is rewarding the agent, she is faced with the cost of bad projects being implemented but still getting a positive expected value given the probability that the agent becomes an expert.

The novel feature of this optimal rule is the flexibility that principal has over agent's continuation value provided by the two instruments defined above. This is different from the models of Lipnowski and Ramos (2019) and Guo and Horner (2018). In their models (under some adaptations), if the principal does not delegate she is not getting any payoff and if she always delegates she only gets negative payoff. In my model, I consider the case where the principal faces a tradeoff given the possibility of positive (but lower in expectation) profit if no delegation exists.

This paper joins to the literature of dynamic delegation without monetary transfers. The two closed relater papers are Lipnowski and Ramos (2019), and Guo and Horner (2018), both of them study a dynamic delegation problem where the agent has better information than the principal. In Lipnowski and Ramos (2019) the principal cannot commit to a delegation rule, and in model of Guo and Horner (2018), principal has full commitment power. They characterized the optimal dynamic mechanisms. As the previous paragraph explains, my model differs from theirs because the principal can also work on the project and agent can become expert which aligns the incentives of both players.

Li, Matouschek, and Powell (2017) study a repeated game of project selection where, at each period, a biased agent selects a project from a set of possible projects that might not include the preferred by the principal. In my model, the optimal option for the principal always exists.

Outline: I present the model in Section 2. I solve the complete information case as a benchmark to obtain the first best rule in Section 3 and I focus in the private signal case in Section 4, presenting the main result of the paper. Section 5 is the analysis and conclusion. All the proofs are in the Appendix (to be completed).

2 Model

I consider a repeated and infinite discrete time relationship between an agent (he) and a principal (she). The agent starts the game being a *junior*, but he can become *expert* in later periods. Being expert is an absorbing state. A new and perishable investment project arrives every period of unknown quality θ. The project can be good ($\theta = G$) or bad ($\theta = B$). I assume it is more probable the project to be bad $P[\theta = G] = \alpha < 1/2$.

The principal decides who is going to be in charge of the project; being in charge implies making the decision of implement the project or reject it. I say the principal *delegates* if she decides that the agent is in charge. The player who is in charge receives a private informative signal about the quality of the project. The signal that the principal receives is more informative than the signal the agent receives when he is a junior. I consider a binary signal $s \in \{g, b\}$ such that the signal match the quality of the project with higher probability if it is received by the principal than by the junior agent. Formally:

$$P(s = g \mid \theta = G) = P(s = b \mid \theta = B) = \begin{cases} p^P & \text{if principal is in charge} \\ p^A & \text{if Junior agent is in charge} \end{cases}$$

where $1/2 < p^A < p^P < 1$.

The player who is in charge of the project decides whether to *implement* or *reject* the project. If the project is rejected both players get zero that period. If the project is accepted, the principal's profit depends on the quality of the project in the following way:

$$\pi = \begin{cases} 1 & \text{if } \theta = G \\ -1 & \text{if } \theta = B \end{cases}$$

The agent gets utility equal to zero if there is no delegation, and in case there is delegation he is protected with limited liability:

$$u = \begin{cases} 1 & \text{if } \theta = G \\ 0 & \text{if } \theta = B \end{cases}$$

this utility function implies the agent is willing to accept any project when he is junior in the one shot game. This is because $1 \cdot P(\theta = G \mid s) + 0 \cdot P(\theta = B \mid s) > 0$.

Every time the agent implements a project he can learn and become expert. Formally; after every implementation the agent becomes expert with probability ϕ. When the agent becomes expert the signal that he receives is perfectly informative, that means the signal matches the quality of the project with probability 1.

Some important assumptions I make are:

Assumption 1: I consider monetary transfers are not allowed to focus exclusively in the provision of incentives trough the delegation rule.

Assumption 2: I assume principal can observe whether agent becomes an expert or not.

The timing of each period is:

1. The principal decides who is in charge of the project.
2. The player who is in charge receives a private signal.
3. The player in charge decides whether to *implement* or to *reject* the project.
4. The junior agent becomes expert with probability ϕ if he takes the decision of implementing the project.

Both players share a common discount factor $\delta \in (0, 1)$. The implementation decision at period t is denoted by $x_t \in \{0, 1\}$ where $x_t = 1$ represents the project was implemented. The delegation decision at period t is denoted by $d_t \in \{0,1\}$ where $d_t = 1$ represents the principal delegated to the agent. The principal's discounted realized profit is:

$$W = (1-\delta)\sum_{t=0}^{\infty} \delta^t x_t \pi_t$$

and the agent's discounted realized utility is:

$$V = (1-\delta)\sum_{t=0}^{\infty} \delta^t d_t x_t u_t$$

Both players are profit and utility maximizer.

2.1 Discussion of the Assumptions

The two important assumptions I make seem restrictive, although they ensure an interesting and tractable model. The first assumption has become common in the literature of delegation in which is better restrict the attention only to payoffs in continuation value to isolate the effects of monetary transfer in the incentives.

The second assumption ensure the private information is restricted to the information the agent receives each period, and not the type of the agent.

2.2 Preliminaries

In this section I provide some useful notation for future analysis.

- Probability or receiving a good signal:
  - h: Probability Junior agent receives a good signal

    $$h = \alpha p^A + (1-\alpha)(1-p^A)$$

  - h_P: Probability principal receives a good signal.

    $$h_P = \alpha p^P + (1-\alpha)(1-p^P)$$

- Junior agent's expected payoff:

- $\bar{\theta}$: Agent's expected payoff for the project after good signal if he is taking the decision.
$$\bar{\theta} = \frac{\alpha p^A}{h}$$

- Principal's expected payoff:
  - $\bar{\theta}_P^P$: Principal's expected payoff for the project after good signal if she is taking the decision.
$$\bar{\theta}_P^P = \frac{\alpha p^P}{h_P} - \left(1 - \frac{\alpha p^P}{h_P}\right)$$
  - $\bar{\theta}_P^A$: Principal's expected payoff for the project after good signal if delegation.
$$\bar{\theta}_P^A = \frac{\alpha p^A}{h} - \left(1 - \frac{\alpha p^A}{h}\right)$$

3 Complete Information Benchmark

To have a benchmark analysis and calculate the First Best I consider signals are public information so the agent follows the recommendation of the principal.

Note the principal will never prefer to delay learning (delegation) if she is planning to delegate at some point in the future. It means she starts delegating in the first period or she never delegates at all.

There are two possible optimal ways to delegate depending on δ: (1) let the agent implement all the projects he receives until he fully learns, and (2) make agent implement projects only after good signals.

I call W and V the expected value of the relationship for principal and agent respectively.

3.1 No Delegation (ND)

Under this rule there is no learning. Note principal only implements projects after a good signal. Expected profit for the principal and expected utility for the agent are:

$$\begin{aligned} W_{ND} = &\quad h_P \bar{\theta}_P^P \\ V_{ND} = &\quad 0 \end{aligned}$$

3.2 Full Delegation (FD)

Agent implements all the projects. Agent become expert with probability ϕ in each period after the first one. Expected profit for the principal and expected utility for the agent are:

$$\begin{aligned} W_{FD} = &\quad (1-\delta)[\alpha-(1-\alpha)]+\delta[\phi\alpha+(1-\phi)W_{FD}] \\ V_{FD} = &\quad \alpha \end{aligned}$$

This implies:

$$\begin{aligned} W_{FD} = &\quad \frac{(1-\delta)\big(\alpha-(1-\alpha)\big)+\delta\phi\alpha}{1-\delta(1-\phi)} \\ V_{FD} = &\quad \alpha \end{aligned}$$

3.3 Balanced Delegation (BD)

Agent implements only good-signal projects. Expected profit for the principal is:

$$\begin{aligned} W_{BD} = &\quad h\Big\{(1-\delta)\bar{\theta}_P^A+\delta[\phi\alpha+(1-\phi)W_{BD}]\Big\}+(1-h)\delta W_{BD} \\ V_{BD} = &\quad h\Big\{(1-\delta)\bar{\theta}+\delta[\phi\alpha+(1-\phi)V_{BD}]\Big\}+(1-h)\delta V_{BD} \end{aligned}$$

This implies:

$$\begin{aligned} W_{BD} = &\quad \frac{h\big[(1-\delta)\bar{\theta}_P^A+\delta\phi\alpha\big]}{1-\delta(1-\phi h)} \\ V_{BD} = &\quad \frac{h\big[(1-\delta)\bar{\theta}+\delta\phi\alpha\big]}{1-\delta(1-\phi h)} \end{aligned}$$

Lemma 1 *The optimal delegation strategy for the principal depends on the value of δ in the following way:*

- *If $\delta \in [0, \underline{\delta})$ the optimal strategy is No delegation.*
- *If $\delta \in [\underline{\delta}, \bar{\delta}]$ the optimal strategy is Balanced Delegation.*
- *If $\delta \in (\bar{\delta}, 1]$ the optimal strategy is Fully Delegation.*

were

$$\underline{\delta} = \frac{h\bar{\theta}_P^A - h_P\bar{\theta}_P^P}{h\bar{\theta}_P^A - h_P\bar{\theta}_P^P + h\phi[h_P\bar{\theta}_P^P - \alpha]}$$
$$\bar{\delta} = \frac{h\bar{\theta}_P^A - (\alpha - (1-\alpha))}{h\bar{\theta}_P^A - (\alpha - (1-\alpha)) + \phi[\alpha - h((1-\alpha) + \bar{\theta}_P^A)]}$$

Remark: Note that with private signals the optimal equilibrium when $\delta < \underline{\delta}$ or $\delta > \bar{\delta}$ are the optimal strategy in the complete information benchmark case. Given that, I only focus in the case $\delta \in [\underline{\delta}, \bar{\delta}]$.

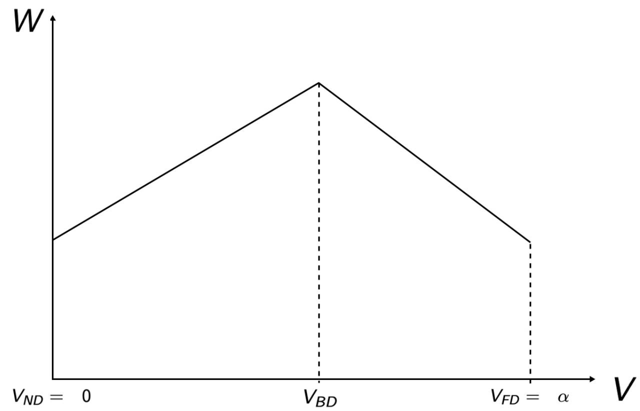

Figure 1: Balanced Delegation is the optimal delegation in this figure.

4 Private Signals

I consider now signals are private information. An optimal delegation rule for the principal assigns probabilities on the probability of delegation after every history of events. I characterize the problem using continuation values and using the promised value of the relationship after any history as state variable.

Define the values $V_I^G(V)$, $V_I^B(V)$ and $V_N(V)$ as the continuation value of the agent after an implementation with good project realization, an implementation with bad project realization, and a no-implementation respectively.

Given the signals are private, it is necessary to add incentives constraints to the problem of the principal. Below ICs ensure agent only implements good-signal projects when principal decides to enforce IC.

$$(1-\delta)\Big[1 \cdot P(G \mid g)\Big] + \delta\Big[V_I^G \cdot P(G \mid g) + V_I^B \cdot P(B \mid g)\Big] \geq \delta V_N \quad \text{(IC-good-signal)}$$

$$(1-\delta)\Big[1 \cdot P(G \mid b)\Big] + \delta\Big[V_I^G \cdot P(G \mid b) + V_I^B \cdot P(B \mid b)\Big] \leq \delta V_N \quad \text{(IC-bad-signal)}$$

Lemma 2 *It is without loss of generality to consider $V_I^G = V_I^B = V_I$.*

Lemma 2 gives the intuition that principal only cares about the action taken by the agent $a \in \{\text{Implement}, \text{No Implement}\}$ and not in the outcome of the action. This considerable reduces the number of parameters.

The intuition of the lemma 2 is the principal cannot provide additional incentives to the agent by selection a different action depending on the outcome of the implementation. Therefore, it is enough to focus on the action rather than the results. The incentive constraints can be written as:

$$(1-\delta)\bar{\theta} + \delta V_I \geq \delta V_N \quad \text{(IC-good-signal)}$$

$$(1-\delta)\underline{\theta} + \delta V_I \leq \delta V_N \quad \text{(IC-bad-signal)}$$

Any optimal rule for the principal requires to make IC-bad-signal binding. This implies $V_N \geq V_I$.

The promise keeping constraint, i.e., the value that principal promises to the agent when there is delegation and principal is enforcing IC is:

$$V = h\Big[(1-\delta)\bar{\theta} + \delta V_I\Big] + (1-h)\delta V_N \quad \text{(PK)}$$

Note that IC-bad-signal binding and PK implies:

$$V = (1-\delta)h(\bar{\theta} - \underline{\theta}) + \delta V_N = (1-\delta)\alpha + \delta V_I$$

It means the continuation values V_I and V_N are a function of V in the following way:

$$
\begin{aligned}
V_I = & \quad \frac{V-(1-\delta)\alpha}{\delta} \\
V_N = & \quad \frac{V-(1-\delta)h(\bar{\theta}-\underline{\theta})}{\delta}
\end{aligned} \tag{1}
$$

This implies that $V_I < V$ always, and $V < V_N$ if $V > h(\bar{\theta}-\underline{\theta})$. In other words, as long as $V > h(\bar{\theta}-\underline{\theta})$, the principal needs to decreases agent's continuation value after an implementation and increases the continuation value after a rejection in order to provide incentives.

Note also that equation (1) represents the continuation values that principal optimally offers to the agent in order to make IC-bad-signal binding. These values do not provide the way to achieve them, this is going to depend on the optimal rule by the principal.

4.1 Principal's Problem

Once the IC are characterized, it is possible to write down the principal's problem. The first step is to write down the specific values of the continuation values after any decision take for the agent. These values need to be equal to the values $v_I(V)$ and $V_N(V)$ calculated in equation (1). To do this I consider a set of probabilities $q = \{q^{I,(1-\phi)}, q^{I,\phi}, q^N, q^P\}$ that represent the probability of delegation after the agent implements the project and he does not become expert, the agent implements the project and becomes expert, the agent does not implement the project, and the principal does not delegates respectively.

$$
\begin{aligned}
V_I(V) =&(1-\phi)\Bigg\{q^{I,(1-\phi)}\Big[h\Big((1-\delta)\bar{\theta}+\delta V_I(V_I(V))\Big)+(1-h)\delta V_N(V_I(V))\Big] \\
&+\Big(1-q^{I,(1-\phi)}\Big)\Big[h_P\delta V_I^P(V_I(V))+(1-h_P)\delta V_N^P(V_I(V))\Big]\Bigg\} \\
&+\phi\Bigg\{q^{I,\phi}\cdot\alpha+\Big(1-q^{I,\phi}\Big)\Big[h_P\delta V_I^P(V_I(V))+(1-h_P)\delta V_N^P(V_I(V))\Big]\Bigg\}
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
W(V_I(V)) =&(1-\phi)\Bigg\{q^{I,(1-\phi)}\Big[h\Big((1-\delta)\bar{\theta}_P^A+\delta W(V_I(V_I(V)))\Big)+(1-h)\delta W(V_N(V_I(V)))\Big]\\
&+\Big(1-q^{I,(1-\phi)}\Big)\Big[h_P\Big((1-\delta)\bar{\theta}_P^P+\delta W(V_I^P(V_I(V)))\Big)+(1-h_P)\delta W(V_N^P(V_I(V)))\Big]\Bigg\}\\
&+\phi\Bigg\{q^{I,\phi}\cdot\alpha+\Big(1-q^{I,\phi}\Big)\Big[h_P\Big((1-\delta)\bar{\theta}_P^P+\delta W(V_I^P(V_I(V)))\Big)\\
&+(1-h_P)\delta W(V_N^P(V_I(V)))\Big]\Bigg\}
\end{aligned}
\tag{3}
$$

There are some important remarks to do. First, if the agent become expert then the incentives are aligned, and given there is not full commitment the optimal decision is to delegate forever immediately after becoming expert. This implies $q^\phi = 1$. To simplify notation $q^{I,(1-\phi)} = q^I$.

Second, from an strategic point of view, it is without loss to set $V^{I,P} = V^{N,P} = V^P$. The reason is the agent is not getting any payoff in the period where there is no delegation, therefore a value of $V^{I,P}$ different than $V^{N,P}$ does not generates extra incentives.

This implies equations (2) and (3) can be written as:

$$
\begin{aligned}
V_I(V) =&(1-\phi)\Bigg\{q^I\Big[h\Big((1-\delta)\bar{\theta}+\delta V_I(V_I(V))\Big)+(1-h)\delta V_N(V_I(V))\Big]\\
&+\Big(1-q^I\Big)\delta V_I^P(V_I(V))\Bigg\}+\phi\cdot\alpha
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
W(V_I(V)) =&(1-\phi)\Bigg\{q^I\Big[h\Big((1-\delta)\bar{\theta}_P^A+\delta W(V_I(V_I(V)))\Big)+(1-h)\delta W(V_N(V_I(V)))\Big]\\
&+\Big(1-q^I\Big)\Big[(1-\delta)h_P\bar{\theta}_P^P+\delta W(V_I^P(V_I(V)))\Big]\Bigg\}+\phi\cdot\alpha
\end{aligned}
\tag{5}
$$

Now let's consider V_N.

$$V_N(V) = q^N\Big[h\Big((1-\delta)\bar{\theta} + \delta V_I(V_N(V))\Big) + (1-h)\delta V_N(V_N(V))\Big] + \Big(1-q^N\Big)\delta V_N^P(V_N(V)) \tag{6}$$

$$\begin{aligned} W(V_N(V)) = & q^N\Big[h\Big((1-\delta)\bar{\theta}_P^A + \delta W(V_I(V_N(V)))\Big) + (1-h)\delta W(V_N(V_N(V)))\Big] \\ & + \Big(1-q^N\Big)\Big[(1-\delta)h_P\bar{\theta}_P^P + \delta W(V_N^P(V_N(V)))\Big] \end{aligned} \tag{7}$$

Now let's consider V_I^P and V_N^P (principal does not delegate after an implementation and after a rejection respectively).

$$V_I^P(V) = q^P\Big[h\Big((1-\delta)\bar{\theta} + \delta V_I(V_I(V))\Big) + (1-h)\delta V_N(V_I(V))\Big] + \Big(1-q^P\Big)\delta V_I^P(V_I(V)) \tag{8}$$

$$V_N^P(V) = q^P\Big[h\Big((1-\delta)\bar{\theta} + \delta V_I(V_N(V))\Big) + (1-h)\delta V_N(V_N(V))\Big] + \Big(1-q^P\Big)\delta V_I^P(V_N(V)) \tag{9}$$

$$\begin{aligned} W(V_I^P(V)) = & q^P\Big[h\Big((1-\delta)\bar{\theta}_P^A + \delta W(V_I(V_I(V)))\Big) + (1-h)\delta W(V_N(V_I(V)))\Big] \\ & + \Big(1-q^P\Big)\Big[(1-\delta)h_P\bar{\theta}_P^P + \delta W(V_I^P(V_I(V)))\Big] \end{aligned} \tag{10}$$

$$\begin{aligned} W(V_N^P(V)) = & q^P\Big[h\Big((1-\delta)\bar{\theta}_P^A + \delta W(V_I(V_N(V)))\Big) + (1-h)\delta W(V_N(V_N(V)))\Big] \\ & + \Big(1-q^P\Big)\Big[(1-\delta)h_P\bar{\theta}_P^P + \delta W(V_N^P(V_N(V)))\Big] \end{aligned} \tag{11}$$

Now the strategy of the principal corresponds to assign probabilities of delegation q^I, q^N, q^P after an Implementation, a Rejection and a No Delegation. These probabilities are not necessarily stationary given they can take in account the history of events. The following lemma simplifies the analysis reducing the problem from three to two parameters.

Lemma 3 *It is enough to consider two parameters $\{\tau, T\}$ where τ represents a number of periods of no delegation and T a number of extra period of delegation ($T = 0$ means one period of delegation).*

To get intuition for lemma 2, note the values of V_I and V_N are a function of V. Then, the strategy of the principal is maximize her profit subject to $\tilde{V}_I = V_I$. The problem reduces to find the optimal way to get $\tilde{V}_I = V_I$. Consider equation (4) in the following way:

$$V_I(V) = (1-\phi)\left\{ q^I V_I' + \left(1 - q^I\right)\delta V_I^P(V) \right\} + \phi \cdot \alpha$$

where:

$$V_I' = h\Big((1-\delta)\bar{\theta} + \delta V_I(V_I(V))\Big) + (1-h)\delta V_N(V_I(V))$$

Then the principal needs to find q^I and q^P. The optimal rule uses the *maximum delegation principle* which states that it is optimal for the principal to minimize the number of no delegation periods. The proof in the appendix shows that it is optimal $q^P = 1$. If setting $q^P = 1$ implies $q^I > 1$ then set $q^I = 1$ and set the shortest sequence of $q^P = \{0, 0, ..., 0, q\}$, with $q \in (0, 1]$.

Note this is equivalent to choose the minimum number of No-Delegation periods to get the value $V_I(V)$. Then, $V_I(V)$ can be written as:

$$V_I(V) = (1-\phi)\delta^\tau V_I' + \phi \cdot \alpha$$

A similar analysis applies to the value of V_N. The derivation is in the Appendix.

$$V_N(V) = \frac{1 - \tilde{\delta}^T}{1 - \tilde{\delta}}\Big[(1-\delta)\alpha + \delta\phi\alpha\Big] + \tilde{\delta}^T V_N'$$

where:

$$\begin{aligned} V_N' =& h\Big((1-\delta)\bar{\theta} + \delta V_I(V_N(V))\Big) + (1-h)\delta V_N(V_N(V)) \\ \tilde{\delta} =& \delta(1-\phi) \end{aligned}$$

This reduces the problem to two parameters: $T(V)$ and $\tau(V)$. Then, the problem that the principal solves is choosing V such that maximizes her profit.

$$W(V) = \max_{\{V\}} \left\{ h\Big[(1-\delta)\bar{\theta}_P^A + \delta \tilde{W}_I(V)\Big] + (1-h)\Big[\delta \tilde{W}_N(V)\Big] \right\} \tag{P}$$

$$\begin{aligned}
\text{Where:} \quad & \tilde{W}_I(V) = \phi\alpha + (1-\alpha)\Big[(1-\delta^{\tau(V)})h_P\bar{\theta}_P^P + \delta^{\tau(V)}W(V_I(V))\Big] \\
& \tilde{W}_N(V) = \frac{1-\tilde{\delta}^{T(V)}}{1-\tilde{\delta}}\Big[(1-\delta)(\alpha-(1-\alpha)) + \delta\phi\alpha\Big] + \tilde{\delta}^{T(V)}W(V_N) \\
T(V) \text{ and } \tau(V) \text{ are given by:} \quad & \phi\alpha + (1-\phi)\delta^{\tau(V)}V_I' = \frac{V-(1-\delta)\alpha}{\delta} \\
& \frac{1-\tilde{\delta}^{T(V)}}{1-\tilde{\delta}}\Big[(1-\delta)\alpha + \delta\phi\alpha\Big] + \tilde{\delta}^{T(V)}V_N' = \frac{V-(1-\delta)h(\bar{\theta}-\underline{\theta})}{\delta}
\end{aligned}$$

4.2 Optimal Rule

Program (P) shows that the problem that the principal faces can be reduced to find the optimal promised expected value V of the relationship. As the complete information benchmark suggest, the principal benefits for offering a positive value of the relationship to the agent.

Second, the program (P) is silent about the optimal rule that principal must follow. The potential optimal delegation rule might restrict the movement of the continuation values promised to the agent; keeping them closer to the original promise utility. The reason of this is because $W(V)$ is a concave function (see next lemma). This is optimal only if the cost for the principal associated to restrict the movement of V is compensated by the gaining in profit.

Lemma 4 *The function $W(V)$ is strictly concave.*

Note that the values of $V_I(V)$ and $V_N(V)$ are deterministic under the maximizer rule that set IC-bad-signal binding. Then the important question is how to get $V_I(V)$ and $V_N(V)$ in an optimal way for the principal.

To get intuition call V^* to the maximizer of $W(V)$. Suppose principal delegates and agent implements the project. The promised value for the agent is $V_I(V)$. That promised value can be seen as a convex combination of α and $\delta^\tau V_I$

$$V_I = \phi \cdot \alpha + (1-\phi) \cdot \delta^\tau V_I'$$

Given α is fixed, the only value principal can manipulate is δ^τ and V_I' under the restriction that $\delta^\tau V_I$ remains constant. What is the best strategy for the principal?

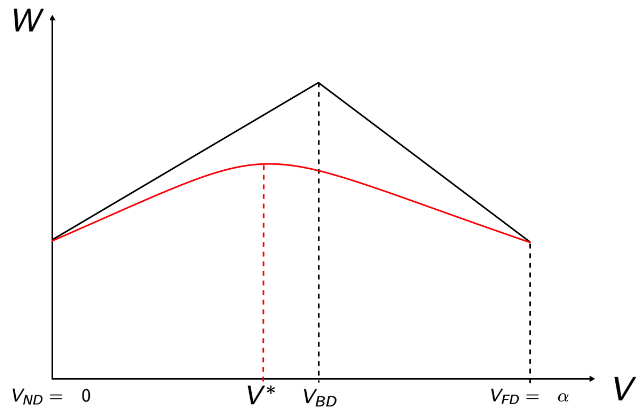

Figure 2: $W(V)$ is a convex function and V^* is the maximizer in this example.

Note if τ increases then V_I' also increases. Then, if agent does not become expert principal will have $W(V_I'(\tau > 0)) > W(V_I'(\tau = 0))$. It means principal is better off promising a higher V_I' to the agent, however this has the cost of no delegating during τ periods.

Optimal rule shows that $\tau > 0$ is optimal if $V_I(V) < \bar{V}_I \equiv \phi\alpha + (1-\phi)V^*$.

Now suppose agent choose to reject the project. The promised value is:

$$V_N(V) = \frac{1-\tilde{\delta}^T}{1-\tilde{\delta}}\Big[(1-\delta)\alpha + \delta\phi\alpha\Big] + \tilde{\delta}^T V_N'$$

Similarly as before, if T increases V_N' has to decrease to keep $V_N(V)$ constant. $T > 0$ when $V_N(V) > V^*$.

Proposition 1 *The optimal rule is as follows:*

- *Delegate at $t = 0$*
- *If agent implements the project and does not become expert:*
  - *If $V_I < \bar{V}_I$: not delegate for $\tau(V)$ periods.*
  - *If $V_I \geq \bar{V}_I$: delegate next period.*
- *If agent implements the project and becomes expert: delegate forever.*
- *If agent rejects the project:*
  - *If $V_N > V^*$: delegate for sure $T(V) + 1$ periods.*

- *If $V_N \leq V^*$: delegate for sure only next period.*

Note $T(V)$ and $\tau(V)$ can be a no natural number. In that case the way to implement this optimal contract is with a lottery of no-delegation between $\lfloor T \rfloor$ and $\lceil T \rceil$ such that $\mathbb{E}T = T$. and delegation between $\lfloor \tau \rfloor$ and $\lceil \tau \rceil$ such that $\mathbb{E}\tau = \tau$.

The T periods of sure delegation imply there is a no-question-asked phase after each no-delegation in order to induce the agent to implement every project and increase the rate of learning. The τ periods of no-delegation is a punishment phase to discipline the agent to satisfies IC.

The following figure shows that intervals in which there is rewards and punishment.

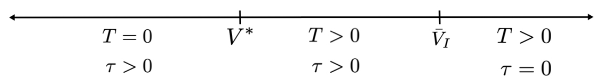

Figure 3: If $V < v^*$ there is only punishment for implementing projects.
If $V \in [v^*, \bar{V}_I]$ there is punishment for implementing projects and reward for rejecting them.
If $V < \bar{V}_I$ there is only reward for implementing projects.

5 Analysis and Conclusion

The optimal rule shows that the dynamic of the relationship can be decomposed in three stages after period $t = 0$.

- Punishment stage: if the promised continuation value after the agent's action is low, the principal would want to increase the future continuation value not delegating for τ periods.

- Punishment and Reward stage: if the agent's promised value is medium, the principal would want to keep in that neighborhood, to do that the principal will punish after an implementation and reward after a rejection.

- Reward stage: when the promised value is high, the principal would want to decrease it by rewarding the agent after a rejection.

The intuition behind this policy is keep the promised continuation value to the agent as near to V^* as possible. This ensure the principal is maximizing her profit.

The reason why principal use a combination of both instrument is because she receives positive expected profit when she is punishing and rewarding the agent. When she is punishing him, she can work by herself with even higher expected payoff conditional to not become expert. When she is rewarding the agent, she is having the cost of bad projects which is compensating by the probability of agent becoming expert.

Appendix

To be completed.

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