Threat, Commitment and Brinkmanship in Adversarial Bargaining

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Abstract

We study commitment strategies in adversarial bargaining, situations in which the proposer demands payment from the responder under the threat of a conflict. The bargaining environment is risky; a welfare-destroying conflict might randomly start during the negotiation, but it also can be induced by the proposer. If the conflict starts, the responder's loss depends on the scale of the conflict that the proposer chooses, and it is maximized for a larger scale than the one that maximizes the proposer's benefit. The proposer has pre-commitment power to a scale given by audience costs. We described under which conditions the proposer commits to start the conflict to threaten the responder and when she relies on the risky environment as a threat. We also show that having high pre-commitment power can be detrimental for the proposer because of a commitment trap. The proposer prefers a risky environment if she has high pre-commitment and a safer one in the opposite case.

Keywords: Bargaining, Commitment, Brinkmanship, Threat.
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1 Introduction

"...the negotiators reduce the scope of their own authority, and confront the management with the threat of a strike that the union itself cannot avert, even though it was the union's own action that eliminated its power to prevent the strike."

An Essay on Bargaining (1956)
Thomas C. Schelling

In adversarial bargaining—situations where an agent demands payment from another under the threat of a conflict—the ability to commit to a threat plays an important role. For example, in an international conflict between two countries, one country demands territory from the other under the threat of starting a war. If the threat of war is credible, it increases the demanding country's bargaining power.

But the demanding party will not necessarily be committed to the war. If the threatened country decides not to satisfy the demand, will the demanding country indeed start the war? A war is a welfare-destroying activity, and even if the demanding country has some benefits from starting it, if the losses of the other country are higher than the benefits, there are incentives to continue negotiating instead of actually starting a war. Furthermore, if the demanding party starts the war, will the intensity be enough to persuade the threatened country to accept a demand? The same problem is found in many different situations. A prosecutor offers a plea deal to a defendant under the threat of taking him to a trial if the deal is not accepted. A worker's union threat to start a strike if the employer does not raise their salaries.

The objective of this paper is to study the commitment strategies of the demanding party. We consider an adversarial bargaining situation in which there is a risk that a welfare-destroying conflict will exogenously start. The demanding party can choose the scale of the conflict during the negotiation as a way to threaten the other party—for example, mobilization of troops or charges before plea bargaining. If an agreement is not reached, the demanding party can choose to start the conflict, or it exogenously starts with some probability. The demanding party has a source of pre-commitment power that depends on how costly it is to scale down the scale of the conflict. In the international conflict case, that cost takes the form of audience costs, which refers to "*domestic political costs a leader may pay for escalating an international dispute, or for making implicit or explicit threats, and then backing down...*" [1] A similar form of reputation costs applies to plea bargaining and

[1] James Fearon, "Credibility is not everything but it's not nothing either." September 2013. URL: https://themonkeycage.org/2013/09/credibility-is-not-everything-either-but-its-not-nothing-either/

union-employer negotiations.

We consider a multiperiod adversarial bargaining model between a proposer (she) and a responder (he). We model the conflict as a reduced form function that assigns payoffs. At each negotiation period, the demanding party chooses the scale of the conflict as a threat to the responder. She then offers a deal that ends the game if accepted. The scale affects the payoff at the conflict. If the offer is rejected, the proposer decides whether to start the conflict. Given that the environment is risky, even if the proposer decides not to start the conflict, it exogenously starts with some probability at the end of each period.

If the conflict starts, the proposer has the last opportunity to revisit the scale before the payoffs are realized. The scale that maximizes the responder's loss at the conflict is higher than the one that maximizes the proposer's benefit. It implies the proposer has incentives to threaten the responder with a large scale but scales it down if the conflict starts. There is a scaling-down cost of reducing the scale, which is higher as the magnitude of the reduction is higher.

Whether the proposer commits to start the resolution stage depends on the probability that it exogenously starts and the pre-commitment power. Given that the scale of the conflict that maximizes the responder loss is higher than the one that maximizes the proposer's payoff, the proposer promises a high scale during the negotiation and then scales it down to a lower one if the conflict starts.

If the probability that the conflict exogenously start is high, the proposer relies on it and decides not to start the resolution stage if the responder rejects the offer. As long as the pre-commitment allows it, the proposer chooses a scale that maximizes the responder's loss at the conflict. On the contrary, if the probability that the conflict exogenously start is low, the proposer does not rely on it and commits to starting the conflict in the first period. To do so, the proposer chooses a scale such that, if her offer is rejected, she would weakly prefer to start the resolution stage than to continue negotiating. With that scale, the proposer induces herself to start the conflict, and in that way, the proposer increases her bargaining power.

In the middle case, if the probability that the conflict exogenously start takes intermediate values, whether the proposer commits to start the resolution stage depends on the pre-commitment power level. If the pre-commitment is high, the proposer does not start the resolution stage and chooses the scale that maximizes the responder's loss. On the contrary, if the pre-commitment power level is low, the proposer credible commits to start the resolution stage by promising a lower scale of the conflict.

This result formalizes the definitions that Thomas Schelling provides for the types of threats. He defines a *probabilistic threat* as one that "*the threatener may carry out, or maybe* *not, if the second party fails to comply... The motive may be that what is threatened is of enormous size...*"[2] The case in which the proposer relies on the probability of the exogenous shock corresponds to a probabilistic threat. Furthermore, the equilibrium corresponds to a *brinkmanship equilibrium* if the pre-commitment power is high, as the *enormous size* puts the proposer in a position of having a loss. Thomas Schelling refers to brinkmanship as "*exploiting the danger that somebody may inadvertently go over the brink, dragging the other with him.*"[3] We refer to the case in which the proposer credible commits to start the resolution stage as a *deterministic threat*.

In cases where the probability that the conflict exogenously starts is either low or high, the proposer is better off with high pre-commitment power. It allows her to commit to generating a high loss to the responder if there is a conflict, which maximizes the offer the responder is willing to accept. However, for intermediate values of the probability that the conflict exogenously starts, the proposer is better off with low pre-commitment power. She faces a commitment trap for high pre-commitment power: she would like to commit to a low scale because the offer that the responder is willing to accept is higher if the proposer commits to start the resolution stage, but it is not credible.

Suppose the proposer has high pre-commitment power. Because the probability that the conflict exogenously starts is not high enough, the offer the responder is willing to accept is higher under a deterministic threat with a low scale than under a probabilistic threat with the scale that maximizes the responder's loss. Therefore, if the proposer has high pre-commitment power, she prefers to choose a low scale to start the conflict if the responder rejects an offer. However, starting the conflict is not credible. If the responder rejects an offer, the proposer will prefer to choose not to start the conflict and increase the scale to the one that maximizes the responder's loss because, given that the probability that the conflict exogenously starts is also not low enough, the offer that the responder is willing to accept under a probabilistic threat is higher than the proposer's payoff of the conflict.

When instead, the proposer has low pre-commitment power, she cannot credibly commit to the scale that maximizes the responder's loss. Therefore, she has no possible deviation when choosing a low scale to start the conflict. It makes it credible for the proposer with low pre-commitment power to commit to starting the conflict following a rejection, which induces the responder to accept a higher offer.

Our model delivers the following comparative statics. If the proposer has high pre-commitment power, she is better off in a negotiation in a risky environment than in a safer one. She can induce brinkmanship by putting herself in a position in which, if there is

[2]See Schelling (2006).
[3]See Schelling (1966).

conflict, she gets a loss, but that also maximizes the loss for the responder which increases her bargaining power. If the proposer has low pre-commitment power, she is better off negotiating in a safer environment than in a risky one. In that case, the proposer cannot commit to a scale that maximizes the responder's loss. So, she prefers a safer environment to credibly commit to starting the conflict. The responder is better off with the lowest risk that induces a probabilistic threat.

Lastly, we extend our model to one in which the proposer is privately informed about her commitment power. We introduce a rational full-commitment proposer, who rationally chooses the scale at the first negotiation period and does not modify it. We show that the partial-commitment proposer only benefits by mimicking the full-commitment type if the pre-commitment power is low.

Related Literature: The present paper is related to the pre-commitment literature, which starts with Schelling (1956) and Schelling (1960), who viewed the bargaining process as an attempt of players to commit themselves to a position, and credible convince the others that conceding is not possible. Crawford (1982) formalizes Schelling's ideas with a two-period bilateral bargaining model, in which players attempt to tie their hand in the first period, and in the second period they decide whether to back down from their commitment. Muthoo (1992) and Muthoo (1996) formalize the scaling down cost, making it proportional to the size of the concession.

Dutta (2012) extends Muthoo's model to have arbitrary scaling-down costs to concede. Dutta (2021) extends Dutta (2012) to characterize sequential concessions in an infinite horizon model. Miettinen and Perea (2015) study an infinity horizon bargaining model in which the proposer can commit in each round, but commitment is costly and last only one round.[4]

Basak and Deb (2020) and Basak (2021) applied the pre-commitment model to political competition in which public opinion plays a role in the concession decision. Levenotoğlu and Tarar (2005) and Tarar and Leventoğlu (2009) apply the model to international conflict negotiations.

There are two main differences between the previous literature and our paper: First, in previous models, the players first commit to a demand, and then there is a negotiation that can either last infinitely or for one period. Although we allow concessions during the negotiation, in our paper, we include the conflict as the last stage, which allows us to study the commitment decision to a threat of a potentially harmful action for everyone if a demand is not satisfied. It allows us to analyze the credibility of the threat, meaning what is the

[4]Papers that study commitment previous to the negotiation, but in which the commitment can fail, are: Ellingsen and Miettinen (2008), Li (2011), Chung and Wood (2019), Ellingsen and Miettinen (2014), Miettinen and Vanberg (2020).

scale and whether the proposer will rely on the risky environment or commit to starting the conflict.

The second difference is that we focus on adversarial bargaining. Instead of the mutual benefits of an agreement, we consider that one player demands a payoff from another. This difference changes the incentives for agreement; in bargaining with mutual benefits, all players prefer no delay, but in adversarial bargaining, the player facing a loss prefers to delay the agreement.

Among papers focusing on starting a conflict rather than pre-commitment to a position, Schwarz and Sonin (2008) present a bilateral conflict model in which one party can start a conflict that ends the game with a negative payoff for both players. They show that if the proposer can divide the conflict into small 'attacks' that do not end the game, the proposer gains bargaining power and can extract all the responder surplus as a sequence of transfers on the equilibrium path, and conflict is avoided. In the present paper, we focus on the commitment strategies that derive from the interaction between pre-commitment power and the scale of the threat. Also, we focus on a more general and realistic setting in which a player can have benefits from the conflict, although the total welfare of the economy is reduced.

Our analysis of the inclusion of a full-commitment type is related to the reputational literature, in which there is no pre-commitment stage. Instead, one of the players can be, with a small probability, a behavioral type that never changes her demand. The type is private information; therefore, a rational player can pretend to be the behavioral type. Abreu and Gul (2000) present the canonical model, and several extensions have been made, among others: Kambe (1999), Abreu and Sethi (2003), Wolitzky (2012), Atakan and Ekmekci (2014), Sanktjohanser (2020), and Ekmekci and Zhang (2021).[5]

Sanktjohanser (2020) considers that the behavioral type is a rational player with full commitment. The behavioral type optimally chooses the initial demand and cannot change it in later rounds. We consider a similar full-commitment type, who behaves rationally but cannot modify her scale of the threat.

Outline: The plan of the paper is as follows. Section 2 introduces the model. Section 3 analyzes the equilibrium and Section 4 introduces private information and the presence of a full-commitment type. Section 5 concludes. Appendix A contains extensions, and Appendix B has all the proofs.

[5]In Ekmekci and Zhang (2021), the rational type can make an ultimatum to start a conflict (e.g., a war), but in their framework, the outcome of the conflict depends solely on the players' type and not on the threats.

2 Model

There are two players: a proposer (she) and a responder (he). They play an adversarial bargaining game in which the proposer demands a payment from the responder. The game is composed of two phases: *negotiation* and *resolution stage*.

The game starts with the multiperiod negotiation stage, in which the proposer and responder negotiate an agreement. At the end of period $t \in \mathbb{N} \equiv \{1, 2, 3, ...\}$ in the negotiation stage, if the proposer and responder do not reach an agreement, the proposer decides whether to start the resolution stage. If the proposer does not start the resolution stage, with probability p a shock exogenously starts the resolution stage, and with probability $(1 - p)$, a new period of negotiation starts.

Negotiation stage: In each period $t \in \mathbb{N} \equiv \{1, 2, 3, ...\}$, the proposer chooses a *intended scale of the conflict* $x_I^t \in [0, \bar{x}]$, which affects the payoffs at the resolution stage. After she chooses the *intended scale*, the proposer offers a deal y to the responder. A deal is a payment from the responder to the proposer. The game ends if the responder accepts the deal. In that case, the proposer gets a payoff of y and the responder a loss of y. If the offer is rejected, a new negotiation period starts with probability $(1-p)$ if the proposer decides not to start the resolution stage. For each period $t \geq 2$, the proposer has a scaling-down cost $kc(x_I^{t-1}, x_I^t)$ of decreasing the intended scale from x_I^{t-1} to x_I^t, in which $c(x_I^{t-1}, x_I^t)$ is an increasing and convex function in the difference $x_I^{t-1} - x_I^t$, with $c(x_I^{t-1}, x_I^t) = 0$ if $x_I^{t-1} \leq x_I^t$, and $c(x_I^{t-1}, x_I^t) > 0$ if $x_I^{t-1} > x_I^t$. We call *pre-commitment power* to the parameter $k \geq 0$.

Resolution stage: This stage is a reduced form function that assigns benefit and loss depending on the proposer's *realized scale of the conflict*. Before it begins, the proposer has the last chance to change her intended scale x_I^t to a realized scale x_F at a cost $kc(x_I^t, x_F)$ as described above. The responder's loss function is $u_R(x)$, and the proposer's benefit function is $u_P(x)$. Benefit and loss are not symmetric, as the resolution stage is a welfare-destroying activity. We discuss this assumption in Section 2.1. Both functions are continuous and concave over $[0, \bar{x}]$ with a unique maximizer. The unique maximizers $x^P = \arg\max_{x \in [0,\bar{x}]} u_P(x)$ and $x^R = \arg\max_{x \in [0,\bar{x}]} u_R(x)$ are related in the following way:[6]

$$0 < x^P < x^R < \bar{x}.$$

The maximum of each function, $u_P(x^P)$ and $u_R(x^R)$, are strictly higher than zero. We restrict attention to $u_P(x) < u_R(x)$ for $x \in (0, \bar{x}]$, as shown in Figure 1. That is, there is

[6]In the main part of the paper we focus on the case $x^R < \bar{x}$, and relegate to Appendix A.1 the case $x^R = \bar{x}$.

welfare destruction in the resolution stage. For example, in an international conflict, the invading country gets new territory but loses troops and faces international sanctions; the prosecutor can damage her reputation for losing at the trial, and the union leader might face retaliation in the future.[7]

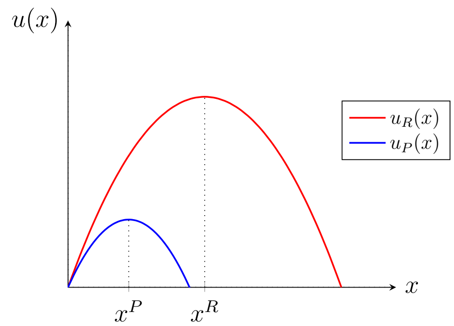


Figure 1: Benefit and loss.

Both players discount the future with a discount factor $\delta \in [\underline{\delta}, 1)$.

Parametric assumptions: We assume $u_P(x^R) < 0$. That is the scale that maximizes the responder's loss and also generates a loss to the proposer. We also assume the discount factor is bounded below by $\underline{\delta} = \frac{u_P(x^P)}{u_R(x^P)}$.

2.1 Discussion of assumptions

1. The benefit and loss function: The model captures the conflict between two parties. It highlights how the proposer uses the pre-commitment power to threaten the responder and how it affects outcomes. Considering $u_R(x) > u_P(x)$ for all x means that the proposer prefers to induce the responder to accept the offer instead of directly starting the resolution stage. Further, considering $x^R < x^P$ means that the proposer has incentives to choose a large intended scale in the negotiation but a more conservative scale at the resolution stage.

If $x^R = x^P$, there is no trade-off in choosing the scale, and if $u_R(x) < u_P(x)$ for all x the proposer prefers to directly start the resolution stage. Finally, if $x^R > x^P$, under slight modifications, the same intuition applies in the opposite direction.

There is welfare destruction at the resolution stage, as the proposer cannot capture all the responder losses. For example, after invading a country and successfully annexing new

[7]We relegate to Appendix A.1 the case $u_R(x^P) < u_P(x^P) \leq u_R(x^R)$. In that case, there is no welfare destruction for all scale but rather for only some values. For example, in the international conflict case, the invading country is demanding a territory with a strategic resource for them but less appreciated by the country that initially has it. In the plea bargaining case, the prosecutor might have high-powered private benefits (for example, media exposition) of "winning" at trial.

territories, the demanding country faces international sanctions and a loss of credibility. A proposer can *win* at a trial but also *lose*, affecting her reputation and career concerns.

2. Scaling-down costs: Scaling-down costs refer to audience or reputation costs of reducing the intended scale. Fearon (1994) introduces audience costs in international conflict, and we extend its intuition to any situation in which a leader faces electoral costs, as in the case of a union leader. Reputation costs are similar. For example, reducing charges before trial might be interpreted as bad prosecutorial practice and hurt the prosecutor's future career.

3. Probability that a shock exogenously starts the resolution stage: The resolution stage might exogenously begin in some cases. For example, a general on the border might trigger an attack without authorization, which starts a war; and the judge rejects postponing the trial (with probability p), which ends the plea bargaining. Not considering the external shock is equivalent to set $p = 0$, a particular case of our model.

3 Analysis

In this Section, we show that the proposer's credibility regarding starting the resolution stage is determined by the value of p and k. If p is low enough or takes intermediate values, but k is low enough, the proposer can promise an appropriate scale that makes credible the threat of starting the resolution stage if the responder rejects an offer. In the opposite case, the proposer does not commit to starting it and relies on the probability that the exogenous shock will start it. We also show that having a high pre-commitment power might be detrimental for the proposer for intermediate values of p, as the proposer faces a high commitment trap.

To formalize Schelling's definitions regarding the nature of a threat, we divide the threat into probabilistic and deterministic:

- *Probabilistic Threat:* The proposer does not start the resolution stage after a rejection of the offer. Instead, chooses a scale and the threat is given by the chance that the resolution stage exogenous starts.

- *Deterministic Threat:* The proposer starts the resolution stage after a rejection of the offer, and it is credibly communicated to the responder.

The equilibrium concept is Subgame Nash Perfect Equilibrium (SNPE). We say that an equilibrium is *Brinkmanship Equilibrium* if it features a probabilistic threat, and the proposer chooses, during the bargaining, an intended scale such that the realized scale generates the largest loss to the responder.

We show the results and describe the equilibrium details following the backward induction form. We start with the resolution stage, and then we discuss the election of the intended scale.

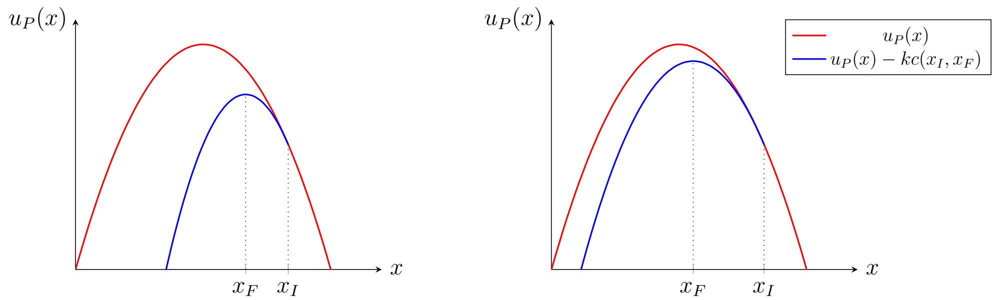
3.1 Realized scale at the resolution stage

Suppose the proposer chose x_I as the intended scale in the last period before the resolution stage started. We define $x(x_I, k)$ as the realized scale given x_I and k. That is:

$$x(x_I, k) \equiv \arg\max_{x\in[0,\bar{x}]} u_P(x) - kc(x, x_I). \tag{1}$$

Lemma 1 *$x(x_I, k)$ is strictly increasing in x_I for $x_I \in [0, \bar{x}]$, and strictly increasing in k for $x_I \in [x^P, \bar{x}]$.*

Figure 2 shows the function $u_P(x) - kc(x, x_I)$. Note that if $x_I > x^P$, then $x(x_I, k) < x_I$ for any x_I and $k > 0$. If $x_I = x^P$, then $x(x_I, k) = x_I$ for any $k \geq 0$. And if $x_I < x^P$, then $x(x_I, k) = x^P$ for any $k \geq 0$, as there is no cost of increasing the scale.



(a) High k (b) Low k

Figure 2: Proposer's payoff at resolution stage after choosing x_I.

The realized scale $x(x_I, k)$ is increasing in both parameters, as if the intended scale is higher, for any k, the realized scale will be higher. Also, for any intended scale, the realized scale is increasing in the cost k; for a larger k, any decrease in the scale is more expensive. The proposer optimally never increases the scale in the resolution stage if $x_I \geq x^P$, as x^P is the optimal scale without considering the scaling-down cost. Increasing the scale has no cost for any $x_I < x^P$; therefore, the realized scale will be x^P.

Figure 3 shows the realized scale $x_F = x(x_I, k)$ as a function of x_I for several k. In the figure, to get a realized scale $x_F = x^R$, the optimal promised threat is x^R if $k \to \infty$, x^1 if $k = k_1$, $\bar{x}$ if $k = \bar{k}$, and it is not feasible for $k = k_2$ and $k = 0$.

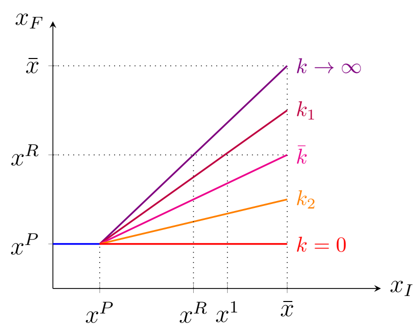


Figure 3: realized scale depending on the intended scale.

We define $X_F(k)$ as the set of feasible realized scale at the resolution stage for pre-commitment power k:

$$X_F(k) \equiv [x^P, x(\bar{x}, k)].$$

3.2 Decisions at the negotiation stage

We first analyze the highest offer the responder is willing to accept, which depends on the realized scale and credibility of the threat. Then we analyze under which conditions the proposer decides whether to start the resolution stage and the optimal x_I that she chooses.

3.2.1 Responder's willingness to accept the offer

The responder benefits from delaying an agreement. He is getting a loss; therefore, not reaching an immediate deal is good for him. However, in equilibrium, the responder accepts the offer in the first period.

If the proposer chooses x_I at t, for any k, the responder's continuation value if he rejects the offer at period t is:

$$V_R(x_I, k) = \begin{cases} pu_R(x(x_I, k)) + (1-p)\delta V_R^{t+1}(x_I^{t+1}, k) & \text{if proposer does not start the resolution stage,} \\ u_R(x(x_I, k)) & \text{if proposer starts the resolution stage,} \end{cases}$$

in which $V_R^{t+1}(x_I^{t+1}, k)$ is the responder's continuation value if there is a new negotiation period $t+1$. If the proposer chooses a stationary x_I at t, that is, she does not change it during the negotiation, the above expression becomes:

$$V_R(x_I, k) = \begin{cases} \tilde{p}u_R(x(x_I, k)) & \text{if proposer does not start the resolution stage,} \\ u_R(x(x_I, k)) & \text{if proposer starts the resolution stage.} \end{cases}$$

In that case, $\tilde{p}$ represents the *composed probability of the resolution stage* in a stationary equilibrium, which takes high values if p is high. It is defined as:

$$\tilde{p} = \frac{p}{1-(1-p)\delta} \ .$$

Then, the optimal offer that the proposer makes is equal to the continuation value, and it depends on whether the proposer starts the resolution stage.

3.2.2 Proposer's decision whether to start resolution stage

The proposer decides to start the resolution stage following a rejection of the offer if two conditions are satisfied:

1. Proposer's payoff at the resolution stage is higher than waiting one more period under the same scale x_I, and

2. Proposer's payoff at the resolution stage is higher than waiting one more period with a different scale x'_I.

Provided that the proposer does not change x_I, the first condition is if the payoff of the resolution stage evaluated at the realized scale $x(x_I, k)$ is higher that deciding not to do it:

$$u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x(x_I, k)). \tag{2}$$

The second condition is the proposer does not have incentives to change the intended scale:

$$u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta V_P(x_I^{t+1}, k), \tag{3}$$

in which $V_P(x_I^{t+1}, k)$ represents the proposer continuation value of choosing x_I^{t+1} in case she decides not to start the resolution stage and it is not started by the shock. Note that during the negotiation, the proposer does not scale down the scale because it is costly. The proposer is better off directly choosing a lower x_I than choosing a higher one and scaling it down in subsequent periods. However, the proposer might have incentives to increase the scale in subsequent periods because it has no cost.

Conditions (2) and (3) are satisfied depending on the values $\tilde{p}$, k, and the optimal selection of x_I.

3.2.3 Probabilistic and deterministic threat

Whether the equilibrium is a probabilistic or deterministic threat depends on the pre-commitment power and $\tilde{p}$. We define two cutoffs, $\tilde{p}_H$ and $\tilde{p}_L$, for the value $\tilde{p}$:

$$\tilde{p}_H = \frac{u_P(x^P)}{\delta u_R(x^P)} \quad \text{and} \quad \tilde{p}_L = \frac{u_P(x^P)}{\delta u_R(x^R)} \ .$$

We also define k^* as the k such that:

$$u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta \tilde{p} u_R(x(\bar{x}, k)) \ .$$

Proposition 1 *The equilibrium depends on $\tilde{p}$ and k as follows:*

- *The equilibrium features a probabilistic threat for any k if $\tilde{p} > \tilde{p}_H$, and for $k > k^*$ if $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$. The proposer chooses an optimal intended threat $x_I(k, \tilde{p})$, and offers $y = \tilde{p} u_R(x_F(k, \tilde{p}))$ which is accepted by the proposer at $t = 1$.*

- *The equilibrium features a deterministic threat for any k if $\tilde{p} < \tilde{p}_L$, and for $k \leq k^*$ if $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$. The proposer chooses an optimal intended threat $x_I(k, \tilde{p})$, and offers $y = u_R(x_F(k, \tilde{p}))$ which is accepted by the proposer at $t = 1$.*

In what follows we described the optimal $x_I(k, \tilde{p})$ for each case of the value $\tilde{p}$, as well as the equilibrium details.

If $\tilde{\mathbf{p}}$ is high. If $\tilde{p}$ is high enough ($\tilde{p} > \tilde{p}_H$), there is no x_I and it respective $x_F \in [x^P, \bar{x}]$ given by (1) that satisfy condition (2). Therefore, the proposer cannot commit to starting the resolution stage for any k.

Intuitively, the proposer's highest payoff of starting the resolution stage is given by choosing $x_I = x^P$, which realized scale is $x_F = x^P$. In that case, at the moment of deciding whether to start the resolution stage, the proposer is better off not starting it, as the responder's loss of rejecting any offer (assuming the proposer never starts the resolution stage) is $\tilde{p} u_R(x^P)$, and:

$$u_P(x^P) < p u_P(x^P) + (1-p)\delta \tilde{p} u_R(x^P) \quad \iff \quad u_P(x^P) < \delta \tilde{p} u_R(x^P),$$

which holds for $\tilde{p} > \tilde{p}_H$. Therefore, for any x_I and k, it is not sequentially rational for the proposer to start the resolution stage.

The value $\tilde{p}_H$ is the $\tilde{p}$ that makes the proposer indifferent between starting the resolution stage or waiting one more period if $x_I = x^P$. If $x_F > x^P$, then $u_P(x_F) < \delta \tilde{p}_H u_R(x_F)$. Therefore, for any k the proposer decides not to start the resolution stage. The optimal offer the proposer makes to the responder is $y = \tilde{p}u_R(x)$, which is accepted. The off the path of equilibrium strategy of not starting the resolution stage, but which exogenously starts with a high probability, supports the strategy of accepting the offer.

The proposer uses the probability that an exogenous shock starts the resolution stage to threaten the responder. The proposer maximizes her payoff by choosing at $t = 1$ a threat x_I that maximizes the offer $y = \tilde{p}u_R(x(x_I, k))$. The optimal $x_I(k)$ the proposer chooses at period $t = 1$ is:

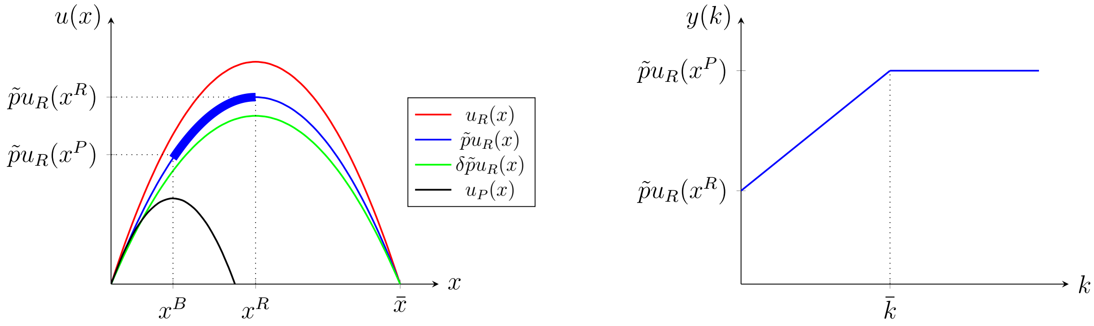
$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \bar{k} \\ x_I \text{ such that } x(x_I, k) = x^R & \text{if } k > \bar{k} \end{cases}$$

and the proposer does not change the intended scale at any period $t \geq 2$ in the negotiation stage.

For $\tilde{p} > \tilde{p}_H$ the proposer maximizes the offer the responder is willing to accept using a probabilistic threat. Furthermore, if $k > \bar{k}$ the equilibrium is brinkmanship, as the threat is probabilistic, and the realized scale if the resolution stage starts maximizes the responder loss and gives a negative payoff to the proposer.

The proposer maximizes the offer the responder is willing to accept by choosing a x_I such that $x(x_I, k) = x^R$ as long as $k \geq \bar{k}$ because $x^R \in X_F(k)$. To achieve x^R as realized scale, the proposer chooses $x_I > x^R$ such that at the resolution stage she scales it down to $x(x_I, k) = x^R$. The optimal x_I is increasing in k. The proposer offers $y = \tilde{p}u_R(x^R)$ to the responder. The responder accepts $\tilde{p}u_R(x^R)$ because it is equal to his continuation value evaluated at x^R.

If k is lower ($k < \bar{k}$), the scale that maximizes the responder's loss does not belong to the feasible set of realized scale: $x^R \notin X_F(k)$. The best the proposer can do is to choose $\bar{x}$ and offer $y = \tilde{p}u_R(x(\bar{x}, k))$ to the responder, and the responder accepts it because it is equal to his continuation loss evaluated at $x(\bar{x}, k)$. In this case, the proposer gets a larger payoff at the resolution stage compared to $k \geq \bar{k}$. Figure 4 Panel (a) shows the responder's induced continuation loss, which translates into the optimal offer in Figure 4 Panel (b).



(a) Responder's induced continuation loss

(b) Optimal Offer

Figure 4: Payoff, loss and offers if $\bar{p} > \bar{p}_H$. The set of optimal offers is highlighted in bold in Panel (a).

If $\tilde{\mathbf{p}}$ is low. If $\tilde{p} \leq \tilde{p}_L$, the proposer can choose an appropriate x_I to induce herself to start the resolution stage after a rejection of the offer for any k. For any k, the proposer can choose $x_I = x^P$ to satisfy conditions (2) and (3). If $x_I = x^P$, condition (2) is satisfied because:

$$u_P(x^P) > pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^P) \qquad \Longleftrightarrow \qquad u_P(x^P) > \delta\tilde{p}u_R(x^P),$$

which is true for $\tilde{p} < \tilde{p}_H$. Condition (3) is satisfied because:

$$u_P(x^P) > pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^R) \qquad \Longleftrightarrow \qquad u_P(x^P) > \delta\tilde{p}u_R(x^R),$$

which is true for $\tilde{p} < \tilde{p}_L$. In this case, the best deviation $x'_I > x_I$ that the proposer can choose after a rejection if the resolution stage does not start is x_I such that $x(x_I, k) = x^R$ if $k \geq \bar{k}$. Therefore, if $\tilde{p} \leq \tilde{p}_L$, for any k there is x_I that induces the proposer to start the resolution stage.

Indeed, in this case, the proposer prefers to start the resolution stage. She chooses the optimal x_I that satisfies conditions (2) and (3) because the offer that the proposer can induce the responder to accept under a deterministic threat is higher than under a probabilistic threat. The lowest offer that the proposer can induce the responder to accept under deterministic threat is $u_R(x^P)$, which is higher than the highest offer that the proposer can induce to accept under probabilistic threat $\tilde{p}u_R(x^R)$ because $\tilde{p}$ is small.[8]

Therefore, the proposer chooses an appropriate x_I to credible commit to starting the resolution stage if the offer is rejected. In this case, the proposer targets a more conservative realized scale than in the case $\tilde{p} > \tilde{p}_H$ because a necessary condition to start the resolution

[8] In this case $u_R(x^P) > \delta^{-1}u_P(x^P)$ by the parametric assumption, and $\delta^{-1}u_P(x^P) > \tilde{p}u_R(x^R)$ because $\tilde{p} < \tilde{p}_L$.

stage is to get a positive payoff of it. The optimal x_I that the proposer chooses at the beginning of $t = 1$ is:

$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta \tilde{p} u_R(x(\bar{x}, k)) & \text{if } k \in [\underline{k}, \bar{k}] \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta \tilde{p} u_R(x^R) & \text{if } k > \bar{k} \end{cases}$$

and the proposer does not change the intended scale at any period $t \geq 2$ in the negotiation stage.

If the proposer's pre-commitment power is very high, $k \to \infty$, the highest scale that satisfies conditions (2) and (3), which we denote by x^B and it is the x_I that satisfies:

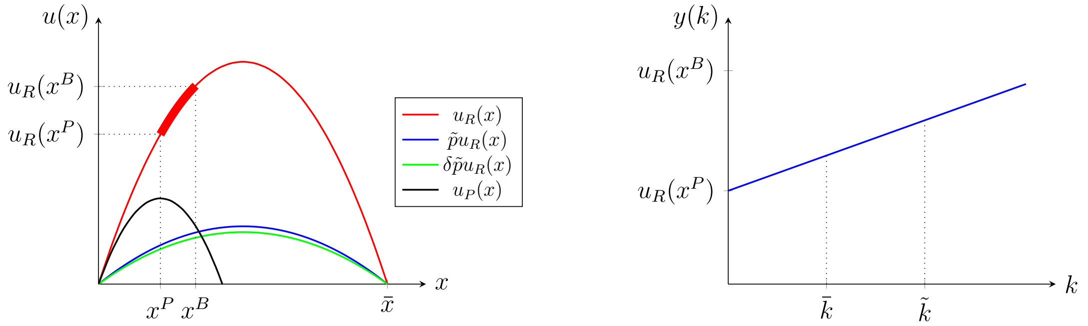
$$u_P(x_I) = \delta \tilde{p} u_R(x^R).$$

A lower scale induces the responder to accept a lower offer, and a higher one does not satisfy (3). Suppose the proposer chooses a higher scale $x'_I > x^B$ such that $u_P(x'_I) > \delta \tilde{p} u_R(x'_I)$ but $u_P(x'_I) < \delta \tilde{p} u_R(x^R)$. In that case, the proposer satisfies condition (2), in the sense that the proposer is better off starting the resolution stage than not doing it, considering that x'_I is stationary. However x'_I is not stationary, as the proposer is better off by choosing not to start the resolution stage after a rejection of the offer and increase the scale to $x_I = x^R$ because the expected payoff of increasing it is $pu_P(x'_I) + (1-p)\delta \tilde{p} u_P(x^R)$ which is higher than $u_P(x'_I)$ if $u_P(x'_I) < \delta \tilde{p} u_R(x^R)$.

The proposer chooses the closest value to x^B that satisfies (2) and (3), that is, for any k, the realized scale is $x(x_I, k) \in (x^P, x^B)$. If k is low enough, the best the proposer can do is to choose $\bar{x}$, which satisfies (2) because the scaling-down costs are low, and (3) because there is no option of increasing the scale. Denote by $\underline{k}$ the k that satisfies (2) with equality by choosing $x_I = \bar{x}$.[9]

Figure 5 Panel (a) shows the value x^B. The proposer's realized scale off the path of equilibrium is given by $x(x_I(k), k) = x(k)$, which is increasing and $x(0) = x^P$ and $\lim_{k \to \infty} x_F(k) = x^B$.

[9] $\underline{k}$ is the k such that $u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta \tilde{p} u_R(x(\bar{x}, k))$.



(a) Responder's induced continuation loss

(b) Optimal Offer

Figure 5: Continuation value for $\bar{p} < \bar{p}^*$ and optimal intended scale. The set of optimal offers is highlighted in black.

If $\tilde{\mathbf{p}}$ takes an intermediate value. If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$, the optimal strategy of the proposer regarding the intended scale depends on the pre-commitment power. In this case, if k is high enough, the only feasible decision for the proposer is not to commit to starting the resolution stage. In the opposite case, if k is low enough, the only feasible option for the proposer is to start the resolution stage.

As discussed above, to commit to starting the resolution stage the proposer needs to choose x_I that satisfies conditions (2) and (3). Condition (2) can be satisfied by choosing x_I such that $x(x_I, k)$ is low, for example, $x_I = x^P$.

However, it is not possible to satisfy (3) if k is high. Suppose k is high enough that $x^R \in X_F(k)$, that is, the proposer can choose an x_I such that the realized scale is x^R. If the proposer wants to commit to starting the resolution stage and chooses $x_I = x^P$, which is the scale that maximizes her payoff at the resolution stage, after the responder rejects an offer, she will have incentives to increase the scale. If she starts the resolution stage, she gets a payoff of $u_P(x^P)$. However, if she decides to increase the scale to one that delivers x^R as realized scale, her expected payoff is:

$$pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^R)$$

which is larger than $u_P(x^P)$ for $\tilde{p} > \tilde{p}_L$.

Intuitively, starting the resolution stage is not sequentially rational because the proposer would have incentives to not do it and increase the scale given that the probability that the resolution stage is both: not very high so the risk of the resolution stage exogenously starts is not that high, also not very low, so the probabilistic threat is more attractive than the payoff at the resolution stage.

On the contrary, if k is low enough, such that even by choosing $x_I = \bar{x}$ the realized scale is low, then (3) is possible to be satisfied. Intuitively, if the responder rejects an offer and the responder wants to choose a higher x_I, the best she can do is $x_I = \bar{x}$. However, by choosing $x_I = \bar{x}$ the equilibrium still features a deterministic threat as $x(\bar{x}, k)$ is low and satisfied (2). Therefore increasing x_I following a rejection is not optimal which implies condition (3) is satisfied.

Denote by $\tilde{k}$ to the k such that $u_P(x^P) = \delta\tilde{p}u_R((x(\bar{x}, k))$, that is, the k for which condition (3) is satisfied by choosing $x_I = x^P$ if $\tilde{p} \geq \tilde{p}_L$.

Lemma 2 *The following results hold:*

1. *For $\tilde{p} < \tilde{p}_H$, if $k \leq \underline{k}$, the proposer starts the resolution stage following a rejection.*
2. *For $\tilde{p} > \tilde{p}_L$, if $k \geq \tilde{k}$, the proposer does not start the resolution stage following a rejection.*

Following Lemma 2, for $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$, if $k \in [\underline{k}, \tilde{k}]$ the proposer chooses an x_I to induce herself to start the resolution stage if:

$$u_R(x(x_{DT}, k)) \geq \tilde{p}u_R(x(\bar{x}, k)),$$

in which the value x_{DT} is the intended scale that maximizes the offer the responder is willing to accept if the proposer commits to start the resolution stage. The value x_{DT} is the x_I such that:

$$u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k)).$$

We denote for k^* the k that solves: $u_R(x(x_{DT}, k)) \geq \tilde{p}u_R(x(\bar{x}, k))$. Therefore, in period $t = 1$ the proposer chooses:

$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k)) & \text{if } k \in [\underline{k}, k^*] \\ \bar{x} & \text{if } k \in [k^*, \bar{k}] \\ x_I \text{ such that } x(x_I, k) = x^R & \text{if } k > \bar{k} \end{cases}$$

and the proposer does not change the initial threat at any period $t \geq 2$ in the negotiation stage.[10]

The intuition of the described optimal intended scale can be divided in two. If $k \geq k^*$, the equilibrium threat is a probabilistic threat, and if $k < k^*$, the threat is deterministic. In each of the described equilibrium types, the proposer targets the value x that maximizes

[10]Note that $\tilde{k} \in [k^*, \bar{k}]$.

her payoff as realized scale. If $k \geq \tilde{k}$ that value correspond to x^R, and following the same intuition than $\tilde{p} > \tilde{p}_H$, the proposer chooses x_I such that $x(x_I, k) = x^R$ if k is high enough, and $x = \bar{x}$ if $k \leq \bar{k}$. If $k < \tilde{k}$, the proposer targets to get the realized scale that satisfies conditions (2) and (3). If $k < \underline{k}$, the best the proposer can do is to choose $\bar{x}$.

3.3 High pre-commitment power can be detrimental

For $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$, depending on the value $\tilde{p}$, the offer that the proposer induces the responder to accept can be higher under probabilistic or deterministic threat. If $\tilde{p}$ takes a high value, then a probabilistic threat in which the proposer can induce the responder to accept $\tilde{p}u_R(x^R)$ maximizes the proposer's payoff. However, if the $\tilde{p}$ takes a low value, then a deterministic threat in which the proposer induces the responder to accept $u_R(x^P)$ is larger than $\tilde{p}u_R(x^R)$. We define $\tilde{p}_M$ as the $\tilde{p}$ such that:

$$u_R(x^P) = \tilde{p}_M u_R(x^R),$$

that is, if $\tilde{p} \leq \tilde{p}_M$ the proposer is better off by inducing the responder to accept $u_R(x^P)$ than $\tilde{p}u_R(x^R)$, which is possible with a deterministic threat.

However, as discussed above, the proposer can only credibly choose a deterministic threat to start the resolution stage following a rejection if k is low.

Proposition 2 *If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, the offer the proposer induces the responder to accept is higher for any $k < \tilde{k}$ than for any $k \geq \tilde{k}$.*

Proposition 2 shows that the proposer is better off with low commitment power k if the value of $\tilde{p}$ takes intermediate values. As Figure 6 shows, if $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, the proposer is better off by committing to start the resolution stage. However, it is not credible for the proposer to choose that scale if the pre-commitment power is high.

If the pre-commitment is high, the proposer cannot satisfy condition (3) because she is incentivized not to start the resolution stage and choose a higher x_I following a rejection.

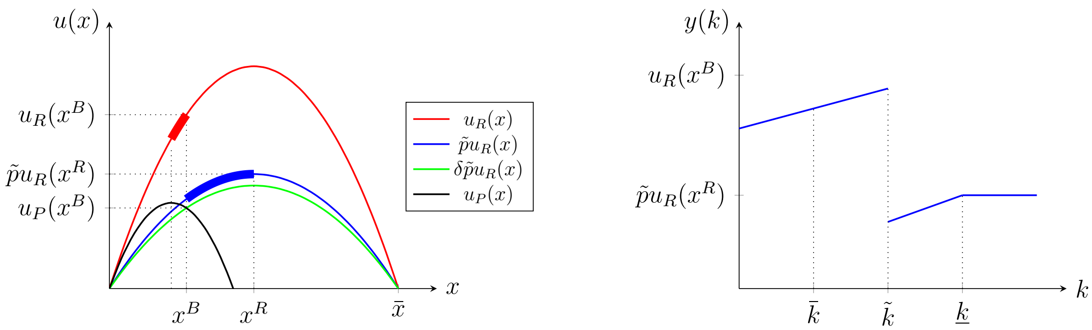
Suppose the extreme case in which $k \to \infty$. Suppose further the proposer chooses a $x_I = x^P$ which satisfies (2) because $u_P(x^P) > \delta\tilde{p}u_R(x^P)$ for $\tilde{p} < \tilde{p}_H$. The scale $x_I = x^P$ provides the highest payoff to the proposer at the resolution stage. Even for that x_I, if the responder rejects an offer and the proposer has to decide whether to start the resolution stage or continue negotiating, she would prefer to continue negotiating and choosing $x_I = x^R$ instead of starting the resolution stage because the expected payoff is higher:

$$pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^R) > u_P(x^P),$$

as $u_P(x^P) > \delta\tilde{p}u_R(x^R)$ because $\tilde{p} > \tilde{p}_L$.

Therefore, a deterministic threat is non credible for the proposer if the pre-commitment is high, which generates a *Commitment Trap*.

We define the *Commitment Trap* as the situation in which the proposer with high pre-commitment power would like to choose a low scale to induce a deterministic threat, but the high pre-commitment power makes it non credible. She would like to choose a low scale because the responder accepts a higher offer if the proposer starts the resolution stage following a rejection. However, if there is a rejection, the proposer prefers to choose a larger realized scale to induce a probabilistic threat. Note that if the pre-commitment is low, she would also like to choose x^R as realized scale instead of starting the resolution stage, but she cannot do it as her pre-commitment does not support x^R as a feasible scale.



(a) Responder's induced continuation loss

(b) Optimal Offer

Figure 6: Payoff, loss and offers if $\bar{p} \in [\bar{p}_L, \bar{p}_M]$. The set of optimal offers is highlighted in bold in Panel (a).

3.4 Riskier of safer environment?

The probability of the exogenous shock p represents how risky the negotiation environment is. That is, how likely it is that the resolution stage exogenously starts and destroys welfare. We showed the proposer takes advantage of a risky environment ($\tilde{p} > \tilde{p}_H$) by generating a brinkmanship equilibrium if k is high. If the environment is less risky ($\tilde{p} < \tilde{p}_L$) the responder is willing to accept $u_R(x)$, which is higher than $\tilde{p}u_R(x)$.

If $k < \underline{k}$, the proposer is better off in a safer environment (low p) than in a risky one (except if $\tilde{p} = 0$). And if $k > \bar{k}$ the proposer can take advantage of the risky environment, therefore she is better off with high p. Figure 7 shows the highest offer that the responder is willing to accept as a function of $\tilde{p}$.

If $k < \underline{k}$, the proposer payoff is $u_R(x(\bar{x}, k))$ for any $\tilde{p} < \tilde{p}_H$. If $\tilde{p} \geq \tilde{p}_H$, her payoff is $\tilde{p}u_R(x(\bar{x}, k))$, which is lower than $u_R(x(\bar{x}, k))$ for $\tilde{p} < 1$. If $k > \bar{k}$, the proposer targets x^R

as the realized scale for any $\tilde{p} \geq \tilde{p}_L$, and the highest x_I that satisfy conditions (2) and (3) if $\tilde{p} > \tilde{p}_L$. Therefore, if $\tilde{p} \geq \tilde{p}_L$ her payoff is $\tilde{p}u_R(x^R)$ and if $\tilde{p} < \tilde{p}_L$ the payoff is $u_R(x_F)$ in which $x_F < x^R$.

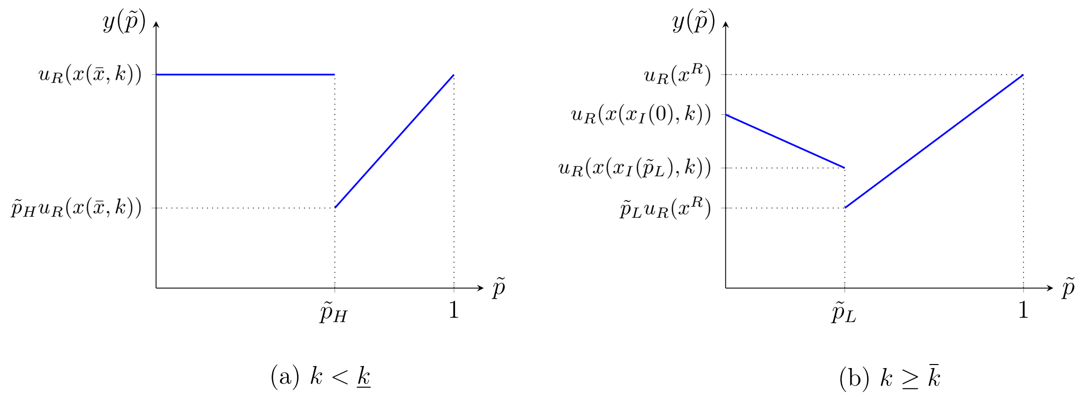


Figure 7: Highest offer the responder is willing to accept as a function of $\tilde{p}$ for $k < \underline{k}$ and $k > \bar{k}$

3.5 Wrapping up

The equilibrium takes the form of deterministic threat equilibrium if the risk that the conflict exogenously starts is low because relying on that risk to threaten the proposer is not effective. In that case, the proposer prefers to start the resolution stage. If it is high, the proposer can rely on it, as it generates enough loss for the responder to accept a high offer. In this case, if the pre-commitment power is high enough for the proposer to generate the highest loss to the responder, the equilibrium is brinkmanship; the proposer puts herself in a position in which if the resolution stage exogenously starts she will get a loss.

If the probability that a shock starts the resolution stage takes intermediate values, the proposer relies on the risky environment if the pre-commitment is high. In this case, there might exist a commitment trap; if the pre-commitment is high, the proposer would like to choose a more conservative scale and commit to starting the resolution stage, as it generates a higher payoff. However, it is not sequentially rational as the proposer is better off not starting the resolution stage and increasing the threat if an offer is rejected.

Lastly, if the proposer has low pre-commitment power, she prefers to negotiate in a safer environment, and if she has high commitment power, she prefers a risky environment to carry out the negotiation.

4 Private information

In this Section, we show that for each type of equilibrium threat, probability or deterministic, the proposer gets a higher payoff under the presence of private information if the commitment power k is low.

Following the reputational bargaining literature, we introduce a full-commitment proposer type who behaves rationally but cannot change her intended scale. The two proposer types that we consider are:

- *Full-commitment type*: does not change the intended scale $x_I = x_F$, and
- *Partial-commitment type*: the commitment power is given by k.

Both players discount the future with the same discount factor $\delta \in [\underline{\delta}, 1)$. The proposer's type α is the proposer's private information. We denote the full commitment type as $\alpha = H$ and the partial commitment type as $\alpha = L$. The responder's prior belief about the proposer being the full-commitment type is $P(\alpha = H) = \theta \in [0, 1]$.

The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Depending on the specification of the responder's beliefs, there are potentially many equilibria. In order to analyze under which condition the partial-commitment benefits from the presence of the full-commitment type, we restrict attention to the equilibrium that maximizes the partial-commitment-type expected payoff.

We define $x_I^H(k, \tilde{p})$ as the optimal intended scale that the full-commitment chooses, and by $x_I^L(k, \tilde{p})$ and $x_F^L(x_I^L, k)$ the intended scale and the realized scale of the partial-commitment type respectively

4.1 Equilibrium under high $\tilde{\mathbf{p}}$

We start the analysis with the case $\tilde{p} > \tilde{p}_M$. Under public information, the proposer targets x^R as realized scale.

Proposition 3 *In the partial-commitment type payoff maximizing equilibrium, for $\tilde{p} > \tilde{p}_M$, there is a k^{Pool-H} such that the equilibrium is pooling $x_I^H(k) = x_I^L(k)$ if $k \leq k^{Pool-H}$ and separating $x_I^L(k) > x_I^H(k)$ if $k > k^{Pool-H}$.*

To sustain the equilibrium described, we consider the following responder's beliefs regarding the proposer type after observing the proposer's intended scale x_I:

$$\theta' = P(\alpha = H \mid x_I) = \begin{cases} 1 & \text{if } x_I \leq x^* \\ \theta & \text{if } x_I \in (x^*, x^{**}) \\ 0 & \text{if } x_I \geq x^{**}. \end{cases} \tag{4}$$

In which $x^* \leq x^R$ and x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$. The value of x^* depends on $\tilde{p}$ and k. We relegate to the Appendix the specific values of x^*.

If $\tilde{p} > \tilde{p}_M$, the full-commitment type proposer chooses $x_I^H = x^R$ if $k > \bar{k}$, and the partial commitment proposer chooses x_I such that $x_F^L(x_I, k) = x^R$. That is, she can induce the responder to accept the offer $\tilde{p}u_R(x^R)$ by herself by choosing the appropriate x_I^L described in the previous Section. Therefore the equilibrium is separating.

If $k \leq \bar{k}$ and $\tilde{p} > \tilde{p}_H$, the partial commitment proposer does not maximize by herself the offer that the responder is willing to accept. Nevertheless, her payoff in pooling equilibrium is not necessarily better than separating. If the partial-commitment type chooses $x_I^L = x^R$, the offer that the responder is willing to accept (assuming the proposer knows she is the partial-commitment type) is $\tilde{p}u_R(x(x^R, k))$ which is lower than $\tilde{p}u_R(x(x_I^*, k))$, in which x_I^* is the optimal intended scale described in Section 3.

Therefore, the partial-commitment proposer's payoff maximizing equilibrium is pooling if the cost of choosing a sub-optimal intended scale is compensated by the pooling offer that the responder is willing to accept. For example, if $\tilde{p} > \tilde{p}_H$:

$$\theta\tilde{p}u_R(x^R) + (1-\theta)\tilde{p}u_R(x(x^R, k)) > \tilde{p}u_R(x(\bar{x}, k)).$$

The value k^{Pool-H} is the k such that the above equation holds with equality, which depends on the value of θ as well. If $k < k^{Pool-H}$, both the partial and full commitment type choose x^R as intended scale.

If $k \in [k^{Pool-H}, \bar{k}]$, the partial-commitment type is better off in separating equilibrium by choosing $x_I^L = \bar{x}$ than pooling with the full-commitment type. However, for $x_I^L = \bar{x}$ being an equilibrium, it must be that the partial-commitment proposer has no incentives to deviate to a threat that puts probability 1 to $\alpha = H$.

For example, if the responder posterior belief after observing $x_I = x^R$ is $\theta' = 1$ (or if $x_I = [x^R - \Delta, x^R + \Delta]$), the partial-commitment type would deviate to $x_I^L = x^R$ (or a value in $[x^R - \Delta, x^R + \Delta]$) and get a higher payoff pretending to be the full-commitment type. Therefore is not an equilibrium.

The beliefs specified above support the equilibrium. The responder's belief are such that $\theta' = \theta$ if $x_I = (x_F^L(\bar{x}, k), x^{**})$, which implies that deviating to the neighborhood of x^R makes the partial commitment proposer worse-off. The equilibrium is such that the full-commitment type chooses $x_I^H = x_F^L(\bar{x}, k)$ and the partial-commitment type chooses $x_I^H = \bar{x}$. If the partial-commitment proposer deviates to $x_I^L = x_F^L(\bar{x}, k)$ she gets the same payoff as no deviating. Note that, given that the full-commitment proposer is rational, her best response is to choose $x_I^H = x_F^L(\bar{x}, k)$. Figure 8 shows the equilibrium intended scale for each k if $\tilde{p} > \tilde{p}_M$.

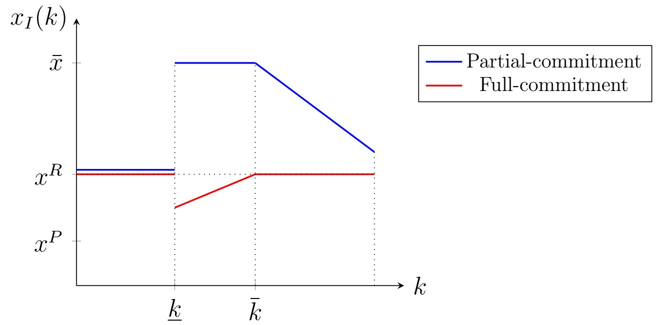


Figure 8: scale for each proposer's type for $\tilde{p} > \tilde{p}_H$.

For $\tilde{p} \in (\tilde{p}_M, \tilde{p}_H]$ and $k < \bar{k}$ the analysis is similar than $\tilde{p} > \tilde{p}_H$. However, in this case, the partial-commitment proposer's payoff maximizing equilibrium is pooling if:

$$\theta \tilde{p} u_R(x^R) + (1 - \theta)\tilde{p} u_R(x(x^R, k)) > u_R(x(x_I^*, k)),$$

in which x_I^* is the optimal intended scale described in the previous Section.

In this case, the off the path of equilibrium strategy is not to start the resolution stage, even though under public information x_I^* induces the proposer to start the resolution stage. Following a rejection, the partial commitment payoff at the resolution stage is lower than pooling the decision of not starting the resolution stage with the full-commitment proposer.

4.2 Equilibrium under low p̃

For $\tilde{p} \leq \tilde{p}_L$, the analysis is similar than for $\tilde{p} > \tilde{p}_H$. The difference is the full-commitment type chooses x^{B2} as scale, in which x^{B2} is the highest scale that satisfies condition (2), and therefore she starts the resolution stage following a rejection. the value x^{B2} is given by the x that:

$$u_P(x) = \delta \tilde{p} u_R(x^R).$$

Proposition 4 *In the partial-commitment type payoff maximizing equilibrium, for $\tilde{p} < \tilde{p}_L$, there is a k^{Pool-L} such that the equilibrium is pooling $x_I^H(k) = x_I^L(k)$ if $k < k^{Pool-L}$, and* *separating $x_I^L(k) > x_I^H(k)$ otherwise.*

In a pooling equilibrium, both proposer types choose x^{B2}, which does not satisfy condition (3) for high values of k. Therefore, if k is high, the partial-commitment proposer does not start the resolution stage following a rejection of the offer. In that case, the offer includes that possibility. Therefore, for the set of k such that (3) is not satisfied, the equilibrium is pooling if:

$$\theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta p u_R(x^R)\Big] \geq u_R(x(x_I^*, k)), \tag{5}$$

in which x_I^* is the optimal intended scale for the partial commitment proposer under public information.

Note that it can be that no k satisfies the above equation. If indeed exists k^{Pool-L} that satisfies the above equation, then the equilibrium is pooling for any k lower than k^{Pool-L}. For the set of k that $x_I = x^B$ satisfy (3), the equilibrium is polling if:

$$\theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k)) \geq u_R(x(x_I^*, k)), \tag{6}$$

which is satisfy for any $k < k^{Pool-L}$. If no k satisfy equation (5), then k^{Pool-L} is the k that satisfy equation (6) with equality.

The analysis of the beliefs that sustain the equilibrium is similar to $\tilde{p} > \tilde{p}_H$, and we relegate it to the Appendix.

4.3 Equilibrium if $\tilde{p}$ takes intermediate values

If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, there is a commitment trap under public information. Under private information, the commitment trap still exists for high values of k.

Proposition 5 *In the partial-commitment type payoff maximizing equilibrium, for $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, there are a k^{Pool-L} and k^{Pool-H} such that:*

- *For $k < k^*$, the equilibrium is pooling if $k \leq k^{Pool-L}$, and separating if $k \in (k^{Pool-L}, k^*)$, and*
- *For $k \geq k^*$, the equilibrium is pooling if $k \in [k^*, k^{Pool-H}]$, and separating if $k > k^{Pool-H}$.*

In this case, in pooling equilibrium, both proposer types choose x^{B2}. Proposition 5 shows that the equilibrium is pooling for low values of k within each type of threat.

Suppose $k < k^*$, in this case, the equilibrium features a deterministic treat. The pre-commitment value is low enough ($k < k^*$), which implies that condition (3) is satisfied for any k in this range of values. Therefore, the equilibrium is pooling if:

$$\theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k)) \geq u_R(x(x_I^*, k)),$$

and the value k^{Pool-L} is the one that satisfies the above equation with equality.

Suppose $k \geq k^*$, in this case, the equilibrium features a probabilistic threat. Choosing $x_I = x^B$ does not satisfy condition (3) for the partial-commitment proposer for any $k \geq k^*$. Therefore, the pooling offer that the responder is willing to accept includes the possibility that the partial-commitment type chooses not to start the resolution stage and the full-commitment type will start it. The equilibrium is pooling if:

$$\theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k)) \geq u_R(x(x_I^*, k)).$$

The value k^{Pool-H} is the one that satisfies the above equation with equality. In this case, the responder's off the path of equilibrium belief if there is a new period of negotiation after rejecting an offer, is that the proposer is partial-commitment type.

The discussion regarding the beliefs that sustain the equilibrium is similar to $\tilde{p} > \tilde{p}_H$, and we relegate it to the Appendix.

4.4 Wrapping up

If there is no commitment trap, the possible presence of the full-commitment type benefits the partial commitment proposer if the commitment power is low, as the equilibrium is pooling. In the opposite case, the partial-commitment proposer is better off by not pooling with the full-commitment type.

Although the partial-commitment proposer might not reach the highest payoff by herself, she still is better off in a separating equilibrium as the trade-off of pooling is the proposer choosing a lower intended scale to do so. Therefore, reducing the expected payoff. So, if k is high, the proposer is better off in a separating equilibrium.

If the commitment power of the proposer is high, and the proposer faces a commitment trap, the partial-commitment proposer with high commitment power also benefits from the presence of the full-commitment type. In this case, the partial-commitment type pools with the full commitment type in a low intended scale.

5 Concluding remarks

The proposer's ability to induce the responder to accept a high demand in adversarial bargaining depends on the threat and the credibility of carrying out that threat. This paper studies the effect of a risky environment and scale-down cost on the optimal threat and the credibility of honoring it. Honoring the threat has two dimensions; how much to scale it down in the resolution stage and whether to start the conflict voluntarily.

We show that committing to start the threat is not necessarily the best strategy as long as the proposer can rely on the riskier environment to threaten the proposer. We also show that high pre-commitment power can be detrimental to the proposer, as it induces a commitment trap if the probability that the conflict exogenously starts is neither low nor high.

Appendices

A Extensions

A.1 High incentives to start the resolution stage

If $u_R(x^R) > u_P(x^P) > u_R(x^P)$, the same results apply. The only difference is that there is a $\underline{x}$, which is the $x = x(x_I, k)$ that:

$$u_P(x^P) = y(x(x_I, k)),$$

that is, the x that makes indifferent the proposer to get a payoff equal to the offer $y(x(x_I, K))$ or going to the resolution stage choosing x^P. If $x(x_I, k) < \underline{x}$, then the proposer prefers to choose x^P, makes an offer that is rejected for sure, and then starts the resolution stage.

If $u_P(x^P) > u_R(x^R)$, the proposer prefers to choose x^P, makes an offer that is rejected for sure, and then starts the resolution stage.

A.2 Strictly increasing responder's loss

If the responder's loss is strictly increasing, the maximizer is $x^R = \bar{x}$ as seen in Figure 8.

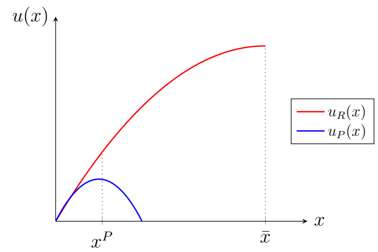


Figure 9: Benefit and loss.

The same qualitative results hold in this context. The difference is that for $\tilde{p} > \tilde{p}_L$, instead of targeting x^R by choosing x_I such that $x(x_I, k) = x^R$, the proposer chooses $x_I = x^R$ for high k. All the results remain the same.

A.3 Strictly decreasing proposer's payoff

If the proposer's benefit is strictly decreasing, such that the proposer's benefit maximizer is $x^P = 0$ as seen in Figure 9, is equivalent to considering the case $\tilde{p}_H = 0$. That is, the equilibrium is probabilistic brinkmanship for each k. The same result hold.

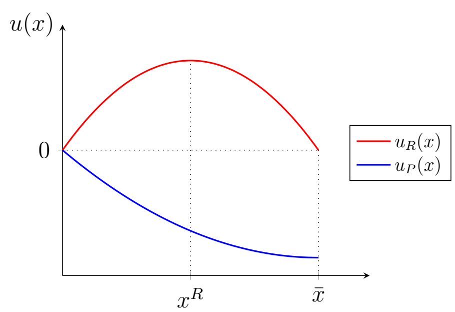


Figure 10: Benefit and loss.

B Proofs

Preliminary definitions and results:

Define $v_P(x, x_I, k) \equiv u_P(x) - kc(x, x_I)$, and $V_P(x_I, k) \equiv u_P(x(x_I, k)) - kc(x(x_I, k), x_I)$. Also define $x_I(k) \equiv \arg\max_{x_I \in [0,\bar{x}]} v_P(x(x_I, k), x_I, k)$ as the optimal intended scale given k, and $x(k) \equiv x(x_I(k), k)$ as the optimal realized scale given the proposer chooses the optimal intended scale $x_I(k)$.

Lemma B1 *For any x_I, $v_P(x, x_I, k)$ is strictly convex and differentiable*

Proof. As $u_P(x)$ is convex and differentiable, and $-kc(x, x_I)$ is strictly convex and differentiable. Note that if $x \geq x_I$, then $v(x, x_I, k) = u_P(x)$, and if $x < x_I$, then $v(x, x_I, k) < u_P(x)$ for all $k > 0$, because $v(x, x_I, k) < v(x, x_I, k) + kc(x, x_I) = u_P(x)$. It implies $v(x, x_I, k)$ is a convex function always bounded by $u_P(x)$. ■

Lemma B2 *$V_P(x_I, k)$ is strictly decreasing in x_I and k for $x_I \in [x^P, \bar{x}]$.*

Proof. Using the envelope theorem: $\frac{\partial V_P(x_I,k)}{\partial x_I} = -k\frac{\partial c(x(x_I,k),x_I)}{\partial x_I} < 0$ because $\frac{\partial c(x(x_I,k),x_I)}{\partial x_I} > 0$ by assumptions on $c(x(x_I, k), x_I)$. $\frac{\partial V_P(x_I,k)}{\partial K} = -kc(x(x_I, k), x_I) < 0$. ■

B.1 Proof of Lemma 1

For x_I: Take $x_I' > x_I$. Suppose $x(x_I', k) \leq x(x_I, k)$, then $c(x(x_I', k), x_I') > c(x(x_I', k), x_I)$, and given that $u_P(x(x_I', k)) - kc(x(x_I', k), x_I')$ is optimal, then $V(x_I, k)$ is strictly higher with $x(x_I', k)$ instead of $x(x_I, k)$, because is the same $u_P(x)$ but lower $c(x, x_I)$. Therefore $x(x_I, k)$ is not the maximizer. Contradiction.

For k: Take $k' > k$. Suppose $x(x_I, k') \leq x(x_I, k)$, then $c(x(x_I, k'), x_I) > c(x(x_I, k), x_I)$, and given that $u_P(x(x_I, k')) - k'c(x(x_I, k'), x_I)$ is optimal, then $V(x_I, k)$ is strictly higher with $x(x_I, k')$ instead of $x(x_I, k)$, because is the same $u_P(x)$ but lower $c(x, x_I)$ that is scaled lower with $k < k'$. Therefore $x(x_I, k)$ is not the maximizer. Contradiction.

B.2 Proof of Proposition 1

For $\tilde{p} > \tilde{p}_H$, $\delta\tilde{p}u_R(x) > u_P(x)$ for all $x \in [x^P, \bar{x}]$. Therefore, $V_P(x_I, k) < \delta\tilde{p}u_R(x(k))$ for all k. The proposer does not start the resolution stage following a rejection. The offer the proposer makes is equal to the responder's continuation loss $y = \tilde{p}u_R(x(k))$. For each $k \geq \bar{k}$, the offer $u_R(x(k))$ is maximized by choosing x_I such that $x(x_I, k) = x^R$, that is, the optimal offer is $y = \tilde{p}u_R(x^R)$ which is accepted by the responder. For $k \in (0, \bar{k})$, the offer is maximized by $x_I = \bar{x}$ and the optimal offer is $y = \tilde{p}u_R(x(\bar{x}, k))$, which is accepted by the responder.

Off the path of equilibrium, if the responder rejects the offer, the proposer does not modify x_I because a probabilistic threat is the only feasible option and for that threat, the proposer is choosing the optimal scale.

For $\tilde{p} < \tilde{p}_L$, then, for any k there is x_I such that the threat is deterministic. For each $k \geq \bar{k}$, there is a $x_I^*(k)$ such that $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x^R)$, for any $k \geq \bar{k}$, the value $x_I \in [x^P, x_I^*(k)]$ induces a deterministic threat. Condition (2) is satisfies because for any x_I, $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x^R) \geq \delta\tilde{p}u_R(x(x_I, k))$. Condition (3) is satisfied because $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x^R)$. Note that in this case, to satisfy condition (3) is enough to check that the proposer does not increase the scale to an intended scale such that $x_F = x^R$.

For each $k \in [\underline{k}, \bar{k}]$, there is a $x_I^{**}(k)$ such that $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k))$, for any $k \in [\underline{k}, \bar{k}]$, the value $x_I \in [x^P, x_I^{**}(k)]$ induces a deterministic threat. Condition (2) is satisfies because for any x_I, $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x(\bar{x}, k)) \geq \delta\tilde{p}u_R(x(x_I, k))$. Condition (3) is satisfied because $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x(\bar{x}, k))$. Note that in this case, to satisfy condition (3) is enough to check that the proposer does not increase the scale to $x_I = \bar{x}$. For each $k < \underline{k}]$, there only option if a deterministic threat.

The proposer prefers a deterministic threat because even the lowest offer the proposer induces the responder to accept $u_R(x^P)$ is higher or equal than the highest probabilistic threat offer $\tilde{p}u_R(x^R)$. The highest offer the proposer induces is with the highest feasible realized threat $x_F \leq x^R$. Therefore, the proposer chooses an intended scale $x_I = x_I^*(k)$ if $k \geq \bar{k}$, $x_I = x_I^{**}(k)$ if $k \in [\underline{k}, \bar{k}]$, and $x_I = \bar{x}$ if $k < \underline{k}$.

For $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, if $k \leq \underline{k}$ the only option is a deterministic threat and if $k \geq \tilde{k}$ the only option is a deterministic threat. The equilibrium is as described above in the previous cases. If $k \in (\underline{k}, \tilde{k})$, the threat type depends on the intended scale. For any $k \in (\underline{k}, \tilde{k})$, the proposer maximizes the offer that the responder is willing to accept under probabilistic threat by choosing $x_I = \bar{x}$, and call x_{DT} to the optimal x_I that the proposer chooses if she wants to induce a deterministic threat. The value x_{DT} is given by the $x_I(k)$ such that $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k))$. Call k^* to the value k such that $u_R(x(x_{DT}, k)) = \tilde{p}u_R(x(\bar{x}, k))$. Therefore, if $k \leq k^*$, the proposer chooses a deterministic threat choosing $x_I = x_{DT}$ is $k \in (\underline{k}, k^*]$, and choosing $x_I = \bar{x}$ if $k \leq \underline{k}$. If $k > k^*$, the proposer chooses a probabilistic threat ans chooses $x_I = \bar{x}$ if $k \in (\underline{k}, \bar{k}]$, and chooses x_I such that $x(x_I, k) = x^R$ if $k \leq \bar{k}$.

B.3 Proof of Lemma 2

Result 1: If $\tilde{p} \leq \tilde{p}_H$, suppose the proposer chooses $x_I = \bar{k}$ and call $\underline{k}$ to the k is such that $u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta\tilde{p}u_R((x(\bar{x}, k))$. That is, the proposer is indifferent between deterministic and probabilistic threat. For any $k \leq \underline{k}$ and any x_I, conditions (2) because $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta\tilde{p}u_R((x(\bar{x}, k)) \geq \delta\tilde{p}u_R((x(x_I, k))$. Condition (3) not always satisfies for any x_I, but the best deviation is to choose $x_I = \bar{x}$ that induces a deterministic threat. Therefore, if expecting to increase the scale following a rejection, the proposer chooses $x_I = \bar{x}$ at $t = 1$ and the equilibrium features a deterministic threat.

Result 2: If $\tilde{p} \geq \tilde{p}_L$, call $\tilde{k}$ to the k such that $u_P(x^P) = \delta\tilde{p}u_R((x(\bar{x}, k))$. For any $k \geq \underline{k}$ the proposer does not satisfied (3) because $u_P(x^P) \leq \delta\tilde{p}u_R(x(\bar{x}, k))$ if $k \in [\tilde{k}, \bar{k})$, and $u_P(x^P) \leq \delta\tilde{p}u_R(x^R)$ if $k \geq \bar{k}$. Therefore, for any $x_I > x^P$ the proposer does not satisfy condition (3) is she wants to induce a deterministic threat.

B.4 Proof of Proposition 2

If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, then $u_R(x^P) \leq \tilde{p}u_R(x^R)$, even the lowest offer the proposer induces the responder to accept under deterministic threat is higher or equal to the highest offer under probabilistic threat. Therefore, $k^* = \tilde{k}$ because the proposer chooses a x_I to induce a deterministic threat.

B.5 Proof of Proposition 3

For $\tilde{p} \geq \tilde{p}_H$, define k^{Pool-H} as the value k such that:

$$\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k)) = u_R(x(\bar{x}, k)).$$

The equilibrium intended scale for both proposer types are:

$$x_I^L(k) = \begin{cases} x^R & \text{if } k \leq k^{Pool-H} \\ \bar{x} & \text{if } k \in (k^{Pool-H}, \bar{k}] \\ x_I^L(k, \tilde{p}) & \text{if } k > \bar{k} \end{cases} \quad \text{and} \quad x_I^H(k) = \begin{cases} x^R & \text{if } k \leq k^{Pool-H} \\ x_I^H(k, \tilde{p}) & \text{if } k \in (k^{Pool-H}, \bar{k}] \\ x^R & \text{if } k > \bar{k} \end{cases}$$

The value x^* of the responder's beliefs in equation (4), that sustain the equilibrium, is defined as the $x \leq x^R$ such that:

$$x^* = \begin{cases} x \text{ such that } u_R(x) = \theta u_R(x^R) + (1-\theta)u_R(x(x^R, k)) & \text{if } k < k^{Pool-H} \\ x_F^L(\bar{x}, k) & \text{if } k \in [k^{Pool-H}, \bar{k}] \\ x^R & \text{if } k > \bar{k} \end{cases}$$

and x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$. Note that x^* and x^{**} are increasing and decreasing in k respectively, $x^R \in (x^*, x^{**}]$, and that $x^* = x^{**} = x^R$.

For $k \geq \bar{k}$: full-commitment type chooses x^R and the partial-commitment type chooses $x_I(k)$ such that $x(x_I, k) = x^R$, and both types offer $y = \tilde{p}u_R(x^R)$. It is the highest offer that the responder is willing to accept. Therefore, there is no profitable deviation for the proposer.

The responder knows the proposer's type after observing x_I and anticipates that by rejecting the offer, his continuation value is $\tilde{p}u_R(x^R)$.

After choosing x_I such that at the resolution stage $x(x_I, k) = x^R$, no other offer is an equilibrium because the responder accepts any offer lower or equal to $\tilde{p}u_R(x^R)$, and reject any offer higher than it. Suppose there is another equilibrium in which the proposer chooses a different initial position than before and reveals information about her type. In that case, at least one proposer's type cannot make an offer equal to $y = \tilde{p}u_R(x^R)$ because it will be rejected.

Suppose the initial position does not reveal the proposer's type. In that case, the proposer makes an offer equal to the responder's continuation value evaluated at a different x, which is lower than $\tilde{p}u_R(x^R)$.

For $k \in [0, \bar{k})$. Consider

$$\theta^{k^{Pool-H}} = \frac{u_R(x(\bar{x}, k)) - u_R(x(x^R, k))}{u_R(x^R) - u_R(x(x^R, k))}.$$

If $\theta < \theta^{k^{Pool-H}}$, the equilibrium is separating, and the full-commitment type chooses $x_I^H = x(\bar{x}, k)$ and the partial commitment type chooses $x_I^L = \bar{x}$.

If any player deviates from the equilibrium intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\tilde{p}[\theta u_R(x') + (1 - \theta)u_R(x(x', k))] < \tilde{p}u_R(x(\bar{x}, k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept a low offer $\tilde{p}u_R(x(x', k)) < \tilde{p}u_R(x(\bar{x}, k))$. If $x' < x_I^H(k, \tilde{p})$, then $\theta' = 1$ and the offer that the responder is willing to accept is $\tilde{p}u_R(x') < \tilde{p}u_R(x_I^H(k, \tilde{p}))$. Therefore, it is not optimal for any type to choose a different x_I.

If $\theta \geq \theta^{k^{Pool-H}}$, the equilibrium is pooling, and both proposer types choose x^R. The responder's continuation value of not accepting the offer is $\tilde{p}[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))]$. Therefore, optimal offer of both types is $y = \tilde{p}[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))]$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

If any player deviates from $x_I = x^R$ and chooses $x' \in (x^*, x^{**})$, then $\theta' = \theta$ and the offer that the responder is willing to accept is $\tilde{p}[\theta u_R(x') + (1-\theta)u_R(x(x', k))] < \tilde{p}[\theta u_R(x^R) + (1 - \theta)u_R(x(x^R, k))]$. If $x' < x^*$, the responder belief would be $\theta' = 1$ and the offer he is willing to accept is $\tilde{p}u_R(x') < \tilde{p}[\theta u_R(x^R) + (1 - \theta)u_R(x(x^R, k))]$. And if $x' > x^{**}$, the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $\tilde{p}u_R(x(x', k)) \leq \tilde{p}u_R(x(\bar{k}, k)) < \tilde{p}[\theta u_R(x^R) + (1 - \theta)u_R(x(x^R, k))]$. Therefore, it is not optimal for any type to choose a different x_I.

For each θ, there is a k^{Pool-H} such that $\theta > \theta^{k^{Pool-H}}$ if $k < k^{Pool-H}$, and $\theta < \theta^{k^{Pool-H}}$ otherwise. The value k^{Pool-H} is increasing in θ for $\theta \in [0,1]$, with $k^{Pool-H}(0) = 0$ and $k^{Pool-H}(1) = \bar{k}$.

Note that $\theta^{k^{Pool-H}}$ belongs to $(0,1)$ as $\lim_{k\to 0}\theta^* = 0$ because $\lim_{k\to 0} u_R(x(\bar{x},k)) = u_R(x^P)$ and $\lim_{k\to 0} u_R(x(x^R,k)) = u_R(x^P)$. Also $\lim_{k\to\bar{k}}\theta^* = 1$ because $\lim_{k\to\bar{k}} u_R(x(\bar{x},k)) = u_R(x^R)$. Also, $\theta^{k^{Pool-H}}(k)$ is monotone in $[0,\bar{k}]$, as the cutoff $k(\theta^{k^{Pool-H}})$ is unique for θ, otherwise there exists a θ such that there is more than a cutoff $k(\theta^{k^{Pool-H}})$. It implies that in equilibrium there exists $k \in [k(\theta^{*1}), k(\theta^{*2})]$ such that the equilibrium is separating and pooling if $k < k(\theta^{*1})$ or $k > k(\theta^{*1})$. That situation is not possible, because $u_R(x(\bar{x}),k)$ is monotone increasing in k, with $u_R(x(\bar{x}),0) = u_R(x^P)$ and $u_R(x(\bar{x}),\bar{k}) = u_R(x^R)$, and $\theta u_R(x^R) + (1-\theta)u_R(x(x^R,k)$ is monotonically increasing in $[\theta u_R(x^R) + (1-\theta)u_R(x(x^R,0), \theta u_R(x^R) + (1-\theta)u_R(x(x^R,\bar{k})]$. Therefore there is only one k such that $\theta u_R(x^R) + (1-\theta)u_R(x(x^R,k) \geq u_R(x(\bar{x}),k)$. Therefore, taking the inverse of $\theta^{k^{Pool-H}}(k)$, we have that $k^*(\theta)$ is increasing in θ.

The same proof applies for $\tilde{p} \in (\tilde{p}_M, \tilde{p}_H)$. Note that for the partial-commitment proposer, if $x(x^R,k)$ is such that condition (2) is satisfied, the partial-commitment proposer still prefers not to start the resolution stage, as not revealing information is optimal because:

$$v_P(x_I,k) < \delta\big[\theta u_R(x^R) + (1-\theta)u_R(x(x^R,k))\big].$$

Note that $\delta u_R(x^R)$ and $\delta u_R(x(x^R,k))$ are both larger than $v_P(x_I,k)$ for $\tilde{p} > \tilde{p}_M$ and $\delta \geq \bar{\delta}$.

B.6 Proof of Proposition 4

The responder's equilibrium beliefs regarding the proposer type after observing the chosen intended scale are:

$$\theta' = P(\alpha = H \mid x_I) = \begin{cases} 1 & \text{if } x_I \leq x^* \\ \theta & \text{if } x_I \in (x^*, x^{**}) \\ 0 & \text{if } x_I \geq x^{**}. \end{cases}$$

Consider k^D as the k such that $v_P(x^{B2},k) = \delta\tilde{p}u_R(x(x_I^*,k))$, in which x_I^* is the optimal intended scale the partial-commitment proposer chooses in absence of private information. Note $k^D > \underline{k}$.

For $k > k^D$. Consider

$$\theta^{k^{Pool-L-1}} = \frac{u_R(x(x_I(k),k)) - \left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]}{u_R(x^{B2}) - \left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]}.$$

Responder beliefs: In this case the value x^* is defined as the highest value between the optimal election of the partial commitment type if the information were public $x(x_I^L(x),k)$ and the x such that $u_R(x) = \theta u_R(x^{B2}) + (1-\theta)\left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]$. And x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$.

If $\theta \geq \theta^{k^{Pool-L-1}}$, the equilibrium is pooling, and both proposer types choose $k^{Pool-L-1}$. The responder's continuation value of not accepting the offer is $\theta u_R(x^{B2}) + (1-\theta)\left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]$. Therefore, optimal offer of both types is $y = \theta u_R(x^{B2}) + (1-\theta)\left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

If any player deviates from $x_I = x^{B2}$ and chooses $x' \in (x^*, x^{**})$, then $\theta' = \theta$, and the offer that the responder is willing to accept is $\theta u_R(x') + (1-\theta)\left[pu_R(x(x',k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right] < \theta u_R(x^{B2}) + (1-\theta)\left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]$. If $x' < x^*$, the responder belief would be $\theta' = 1$ and the offer he is willing to accept is $u_R(x') < \theta u_R(x^{B2}) + (1-\theta)\left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]$. And if $x' > x^{**}$ the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $\tilde{p}u_R(x(x',k)) \leq \theta u_R(x^{B2}) + (1-\theta)\left[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]$. Therefore, it is not optimal for any type to choose a different x_I.

If $\theta < \theta^{k^{Pool-L-1}}$, the opposite case holds. The equilibrium is separating, in which the full-commitment type chooses $x_I^H = x(x_I^L(k),k)$, and the partial commitment type chooses $x_I^L(k)$.

If any player deviates from the equilibrium intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\theta u_R(x') + (1-\theta)\left[pu_R(x(x',k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right] < u_R(x(x_I^L(k),k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept a low offer $\tilde{p}u_R(x(x',k)) < u_R(x(x_I^L(k),k))$. If $x' < x^*$, then $\theta' = 1$ and the offer that the responder is willing to accept is $u_R(x') < u_R(x(x_I^L(k),k))$. Therefore, it is not optimal for any type to choose a different x_I.

For each θ, there is a $k^{Pool-L-1}(\theta)$ such that $\theta > \theta^{k^{Pool-L-1}}$ if $k < k^{Pool-L-1}(\theta)$, and $\theta < \theta^{k^{Pool-L-1}}$ otherwise. Note that $\theta^{k^{Pool-L-1}}(k)$ is increasing in $k > \bar{k}$ and bounded below by $\underline{\theta}^{k^{Pool-L-1}}$ and above by $\bar{\theta}^{k^{Pool-L-1}}$:

$$\underline{\theta}^{k^{Pool-L-1}} = \frac{u_R(x(x^{B2},k^D)) - u_R(x(x^{B2},k^D))}{u_R(x^{B2}) - u_R(x(x^{B2},k^D))}, \quad \text{and}$$

$$\bar{\theta}^{k^{Pool-L-1}} = \frac{u_R(x^B) - \left[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\right]}{u_R(x^{B2}) - \left[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\right]}.$$

Define $k^{Pool-L-1}(\theta)$ as the inverse of $\theta^{k^{Pool-L-1}}(k)$, which is $k^{Pool-L-1}(\theta) = \bar{k}$ for $\theta \leq \underline{\theta}^{k^{Pool-L-1}}$, and $k^{Pool-L-1}(1) = \tilde{k}$.

Note that $\theta^{k^{Pool-H}}(k)$ is increasing in $k > k^D$ and bounded below by $\underline{\theta}$ and above by $\bar{\theta}$. For $\theta \in [\underline{\theta}, \bar{\theta}]$, define $k^{Pool-L-1}(\theta)$ as the inverse of $\theta^{k^{Pool-L-1}}(k)$. Note that the equilibrium is pooling for any k if $\theta < \underline{\theta}^{k^{Pool-L-1}}$, the equilibrium is separating for any k if $\theta > \bar{\theta}^{k^{Pool-L-1}}$, and depend on k otherwise.

For $k \in [0, k^D)$. Consider

$$\theta^{k^{Pool-L-2}} = \frac{u_R(x(x_I^*, k)) - u_R(x(x^{B2}, k))}{u_R(x^{B2}) - u_R(x(x^{B2}, k))}.$$

Responder beliefs: In this case the value x^* is defined as the highest value between the optimal election of the partial commitment type if the information were public $x(\bar{x}, k)$ and the x such that $u_R(x) = \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k))$. And x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$.

If $\theta < \theta^{k^{Pool-L-2}}$, the equilibrium is separating, and the full-commitment type chooses $x_I^H = x(x_I^*, k)$ and the partial commitment type chooses $x_I^L = x_I^*$.

If any player deviates from the equilibrium initial intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\theta u_R(x') + (1-\theta)u_R(x(x', k)) < u_R(x(x_I^*, k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept a low offer $u_R(x(x', k)) < u_R(x(x_I^*, k))$. If $x' < x_I^H(k, \tilde{p})$, then $\theta' = 1$ and the offer that the responder is willing to accept is $u_R(x') < u_R(x_I^H(k, \tilde{p}))$. Therefore, it is not optimal for any type to choose a different x_I.

If $\theta \geq \theta^{k^{Pool-L-2}}$, the equilibrium is pooling, and both proposer types choose x^{B2}. The responder's continuation value of not accepting the offer is $\theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k))$. Therefore, optimal offer of both types is $y = \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k))$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

If any player deviates from $x_I = x^{B2}$ and chooses $x' \in (x^*, x^{**})$, then $\theta' = \theta$, and the offer that the responder is willing to accept is $\theta u_R(x') + (1-\theta)u_R(x(x', k)) < \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k))$. If $x' < x^*$, the responder belief would be $\theta' = 1$ and the offer he is willing to accept is $u_R(x') < \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2}, k))$. And if $x' > x^{**}$, the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $u_R(x(x', k)) \leq u_R(x(x_I^*, k)) <$

$\theta u_R(x^{B2})+(1-\theta)u_R(x(x^{B2},k))$. Therefore, it is not optimal for any type to choose a different x_I.

For each θ, there is a $k^{Pool-L-2}$ such that $\theta > \theta^{k^{Pool-L-2}}$ if $k < k^{Pool-L-2}$, and $\theta < \theta^{k^{Pool-L-2}}$ otherwise. Note that $\theta^{k^{Pool-L-2}}(k)$ is increasing in $k \in [0, k^D]$ and bounded below by 0 and above by $\bar{\theta}^{k^{Pool-L-2}}$, as the limit when $k \to 0$ is 0, and

$$\bar{\theta}^{k^{Pool-L-2}} \equiv \frac{u_R(x(x^{B2},k^D)) - u_R(x(x^{B2},k^D))}{u_R(x^{B2}) - u_R(x(x^{B2},k^D))}.$$

Define $k^{Pool-L-2}(\theta)$ as the inverse of $\theta^{k^{Pool-L-2}}(k)$.

Note that $\bar{\theta}^{k^{Pool-L-2}} = \underline{\theta}^{k^{Pool-L-1}}$. Then, define $\theta^*(k)$ as

$$\theta^*(k) = \begin{cases} \frac{u_R(x(x_I^*,k))-u_R(x(x^{B2},k))}{u_R(x^{B2})-u_R(x(x^{B2},k))} & \text{if } k \in [0,\bar{k}) \\ \frac{u_R(x(x_I(k),k))-\left[pu_R(x(x^{B2},k))+(1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]}{u_R(x^{B2})-\left[pu_R(x(x^{B2},k))+(1-p)\delta\tilde{p}u_R(x(x_I^*,k))\right]} & \text{if } k \geq \bar{k}. \end{cases}$$

The function $\theta^*(k)$ is bounded above by:

$$\bar{\theta} = \frac{u_R(x^B) - \left[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\right]}{u_R(x^{B2}) - \left[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\right]},$$

and it is continuous and strictly increasing. Define $k^{Pool-L}(\theta)$ as the inverse of $\theta^*(k)$.

B.7 Proof of Proposition 5

For $k \geq k^*$. Consider $\theta^{k^{Pool-H}}$ the θ such that:

$$\theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\Big] = \tilde{p}u_R(x(x_I^*,k)),$$

in which x_I^* is the optimal x_I for the partial commitment proposer in absence of private information.

Responder beliefs: In this case, x^* is the x such that $u_(x) = \tilde{p}u_R(x(x_I^L(k),k))$ and $x^{**} = x^{B2}$.

If $\theta \geq \theta^{k^{Pool-H}}$, the equilibrium is pooling, and both proposer types choose x^{B2}. The responder's continuation value of not accepting the offer is $\theta u_R(x^{B2})+(1-\theta)\Big[pu_R(x(x^{B2},k))+(1-p)\delta\tilde{p}u_R(x(x_I^*,k))\Big]$. Therefore, optimal offer of both types is $y = \theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2},k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*,k))\Big]$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

In this case, following a rejection, the full-commitment proposer starts the resolution stage. However, the partial commitment proposer does not do it because $pv_P(x^{B2}, k) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k)) > v_P(x^{B2}, k)$. The offer includes that possibility.

If any player deviates from $x_I = x^{B2}$ and chooses $x' \in [x^*, x^{**})$, then $\theta' = \theta$, and the offer that the responder is willing to accept is $\theta u_R(x') + (1-\theta)\Big[pu_R(x(x', k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big] < \theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]$. If $x' > x^{B2}$, the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $\tilde{p}u_R(x(x', k)) < \theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]$.

If $\theta < \theta^{k^{Pool-H}}$, the opposite case holds. The equilibrium is separating, in which the full-commitment type chooses x_I^H such that $u_(x_I^H) = \tilde{p}u_R(x(x_I^L(k), k))$, and the partial commitment type chooses $x_I^L(k)$.

If any player deviates from the equilibrium intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big] < \tilde{p}u_R(x(x_I^*, k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept $\tilde{p}u_R(x(x', k)) < \tilde{p}u_R(x(x_I^L(k), k))$. If $x' < x^*$, then $\theta' = 1$ and the offer that the responder is willing to accept is $u_R(x') < \tilde{p}u_R(x(x_I^L(k), k))$. Therefore, it is not optimal for any type to choose a different x_I.

Consider the bounds $\underline{\theta}$:

$$\underline{\theta} \equiv \frac{\tilde{p}u_R(x^R) - \Big[pu_R(x(x', k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]}{u_R(x^{B2})) - \Big[pu_R(x(x', k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]},$$

and $\bar{\theta}$:

$$\bar{\theta} \equiv \frac{u_P(x^P) - \Big[pu_R(x(x', k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]}{u_R(x^{B2}) - \Big[pu_R(x(x', k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]}.$$

For each $\theta \in [\underline{\theta}, \bar{\theta}]$, there is a $k^{Pool-H}(\theta)$ such that $\theta > \theta^{k^{Pool-H}}$ if $k < k^{Pool-H}(\theta)$, and $\theta < \theta^{k^{Pool-H}}$ otherwise. Note that $\theta^{k^{Pool-H}}(k)$ is increasing in $k > k^*$ and bounded below by $\underline{\theta}$ and above by $\bar{\theta}$. Define $k^{Pool-H}(\theta)$ as the inverse of $\theta^{k^{Pool-H}}(k)$, which is $k^{Pool-H}(\underline{\theta}) = k^*$ and $k^{Pool-H}(\bar{\theta}) \to \infty$.

For $k < k^*$. The same proof than Proposition 4 applies. The only difference in this case $\theta^*(k)$ is bounded above by:

$$\bar{\theta} = \frac{u_R(x_I^*(k^*)) - \big[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\big]}{u_R(x^{B2}) - \big[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\big]},$$

in which $x_I^*(k^*)$ is given by $u_P(x(x_I,k)) - kc(x(x_I,k),x_I) = \delta\tilde{p}u_R(x(\bar{x},k))$.

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