Lab Report 2

Tier 1- EM1: Introduction to DC Electronics

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Introduction/Objectives:

In this lab, our goal was to measure and explore Ohmic and non-Ohmic behaviors in resistors and LEDs. To do this, we would learn about circuit construction on a breadboard, construction and analysis of DC circuits and electric current prediction in a complex circuit with multiple voltage sources. Furthermore, one we had accomplished our experimental goals in learning the basics of circuit components and electrical measurement, we would apply the error analysis skills from previous labs. It could be said that the ultimate goal of this lab was to introduce us to the experimental side of electromagnetism while teaching us some basic intuition on the inner workings of circuits.

Theory:

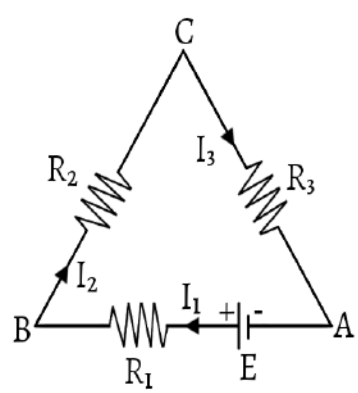
In order to understand all measurements we will be performing in this experiment regarding circuits and electromagnetism, it is necessary to know what the theoretical framework is. Before going into the maths, we must define three concepts:

- **Current:** An electric current is a stream of charged particles moving through an electrical conductor or space. It is measured as the net rate of flow of electric charge through a surface or into a control volume. The unit for current is the Ampere (A), defined by Coulumb per unit time.

- **Voltage or Potential Difference:** Voltage is the difference in electric potential between two points, which in a circuit is defined as the work needed per unit of charge to move a test charge between the two points. The units for voltage is the Volt (V), defined as Joules per unit charge.

- **Resistance:** The electrical resistance of an object is a measure of its opposition to the flow of electric current. The less more resistance, we will have a higher potential difference, since a lot of work will be done by the material to "stop" the electrons. The unit of resistance is the Ohm (Ω), which is defined as Volt/Ampere.

Now that we have a basic understanding of the main concepts, we can dive into the actual theory. It was hinted in the definition of resistance that there is some mathematical relationship between voltage and electrical current. If we were to look at a wire with some potential difference $\Delta\phi$ between its endpoints and with some current I flowing through it, we can define the resistance of the wire to be given by $R = \Delta\phi/V$. This equation is known as the simplified non-integral version of Ohm's Law ,often simplified to Ohm's Law. Now, considering these mathematical properties of Voltage, Electrical Current, and Resistance, we can start to create circuits. Circuits are collections of electrical components that all work together to allow for some current to flow through the components in the best way to the interest of the circuit's creator. Let's consider the following circuit:



In this circuit, we have three resistors (components that have a high resistance to current flow), and we have three different currents, since the current is changed after flowing through a resistor. To solve for the different variables we have in this circuit, we can apply Kirchhoff's Law, which can be written in two statements:

1. Based on charge conservation, the current flux over a closed circuit must be 0. Closed circuit refers to the fact that the electrical current traces out a loop. Since the flux must be 0, the incoming current must equal the outgoing current, so the current is steady. This implies that the sum of all currents over the loop is 0.

2. If we go through any closed loop, the curl of the electric field is 0 for a closed loop, since this is one of the defining characteristics of the electric field. What this implies is that, since $\nabla \times E = 0$ and $E = -\nabla\phi$, we know for a fact that $\Delta\phi = 0$. This means that over a closed loop, the sum of all the potentials at the different components is 0.

By using these characteristics of a closed circuit, we will be solving for the different currents, resistances, and voltages in the experiments throughout this lab report. It is also important that we look at some simpler circuit arrangements to remind ourselves of the quantitative descriptions of the currents, voltages, and resistances. As we mentioned before, circuits can be in series or parallel. When in series, all the elements in a circuit are connected in a straight line. Since $V = IR$, we find that the total voltage is the sum of all voltages through all components, and this implies that $V_{Tot} = IR_1 + IR_2 + ... + IR_n$, where I is a steady current and R1 ... Rn represent the resistances of the first n components. Another way to arrange components in a circuit is in parallel, when the circuit splits in two different paths at some point. In this case, the voltage remains constant and the current divides accordingly between the two paths. We now have that $I_{Tot} = \frac{V}{R_1} + \frac{V}{R_2} + ... + \frac{V}{R_n}$. Now, we know different ways to describe resistance in circuits, which we will also use in the Analysis section.

Methods:

For the methods section, we thought we would give an overview of all the necessary steps and items to carry out each of the experiments we had to carry out here. Therefore, we will do a methods subsection for each of the 3 experiments and their respective parts. All equipment necessary for these experiments, aside from the batteries, were found in the lab kit:

- Wires
- Breadboard
- IOLab device
- 1, 1k, 4.7k, and 10k Ω resistors
- 1.5 V (AAA) batteries
- Battery pack
- Green and red LEDs

Method for Experiment 1a

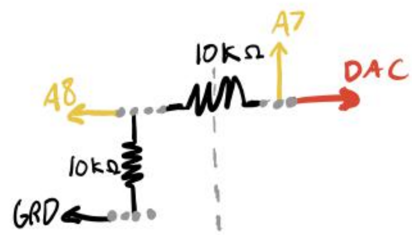
To carry out this experiment, we will require the breadboard, 3 wires, 2 10k Ω resistors, and the iOLab, which can all be found in the lab kit. As a general procedure, we will be using certain steps:

1. Set the breadboard to look like the one in the image provided to us in the Lab Manual. Make sure all the connections are well embedded in the breadboard and the circuit makes sense in terms of current flow.

2. Record sensors A7 and A8 by activating them on the IOLab website.

3. On the IOLab app/website, set the DAC to 2.0 V. Do this by clicking on Settings ¿ Expert Mode ¿ Output Configuration, and the selecting the appropriate voltage.

4. Start the IOLab recording, making sure the correct variables or outputs are being recorded.
5. Turn the DAC on and off, and make sure the final result makes sense.

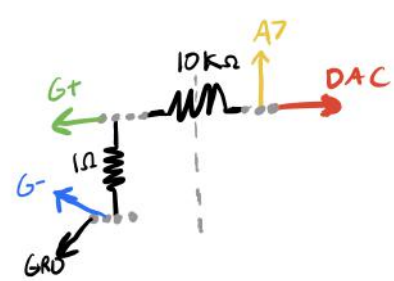


We measure the voltage before and after the first resistor using the IOLab's A7 and A8 outputs so as to determine the transfer ratio of the voltage. In this case where there are two of the same type of resistor wired in a series, we expect the transfer ratio to be 0.5. A more in-depth analysis of this will be provided in the Analysis section.

Method for Experiment 1b

To carry out this experiment, we will require the breadboard, 5 wires, a 10k Ω resistor, a 1 Ω resistor, and the iOLab, which can all be found in the lab kit. As a general procedure, we will be using certain steps:

1. Replace the 2nd 10k Ω resistor with a 1 Ω resistor. This is just to make sure that the correct resistor is being used, in order to avoid errors.
2. Build the electric circuit like the one on the image below.
3. Record sensors A7 and High Gain by activating them on the IOLab website.
4. Like before, on the IOLab app/website, set the DAC to 2.0 V.
5. Start the IOLab recording, making sure the correct variables or outputs are being recorded.
6. Turn the DAC on and off, and make sure the final result makes sense.

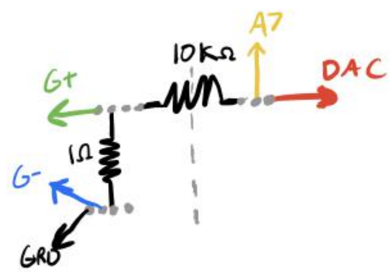


We measure Voltage through A7 and the high gain so as to measure the current using the voltage measured through the 1 Ω resistor and Ω's law. It is important to understand that by measuring the High Gain and by changing the resistor's resistance, we are inherently doing an experiment different to that in 1a, so we can expect different values than those from 1a.

Method for Experiment 2a

To carry out this experiment, we will require the breadboard, 5 wires, a 10 kΩ resistor, a 1 Ω resistor, and the iOLab, which can all be found in the lab kit. As a general procedure, we will be using certain steps:

1. Build the same circuit as the one used in experiment 1b.
2. Select 10 points that are evenly distributed in the range of 0 V and 3.3 V. We will use these points to select some voltage in the DAC and then have 10 data points from the A7 and High Gain sensors.
3. Record A7 and High Gain sensor on the IOLab website.
4. Turn DAC on and set to different selected values. We are expecting our data from the A7 to look alike a staircase with 10 steps, corresponding to our 10 data points.

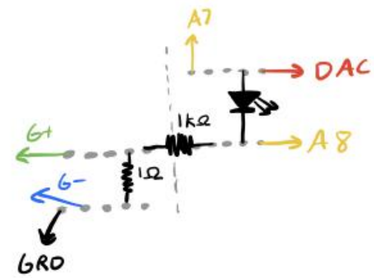


We measure voltage through A7 and High Gain so as to confirm that they obey the linear relation described by V = IR, since the voltage measured in the High Gain is equal to the Current. Note that the setup here is the same as the one in 1b, with the exception that we measure the High Gain through one particular resistor, and that's why we place the G+ and G- in the endpoints of one of the resistors in Step 1.

Method for Experiment 2b

To carry out this experiment, we will require the breadboard, 6 wires, a 1 kΩ resistor, a 1 Ω resistor, an LED, and the iOLab, which can all be found in the lab kit. As a general procedure, we will be using certain steps:

1. Build the circuit as shown in the image below.
2. In the final circuit we have, make sure that the longer end of the LED is at the higher potential. If we have it the other way around, the circuit won't work.
3. Like before, select 10 points in the range of 0 V and 3.3 V. We will use these points to select some voltage in the DAC and then have 10 data points from the A7, A8, and High Gain sensors.
4. Record A7, A8, and High Gain sensors on the IOLab website.
5. Turn DAC on and set to different selected values, to obtain the different data points.
6. Repeat until data has been gathered with both the green and red LEDs.

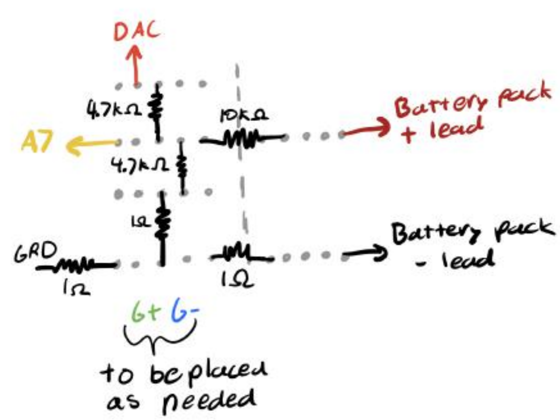


We measure the voltage, again with the high gain voltage through the 1 Ω resistor acting as the equivalent measurement to current, in order to confirm that in this case, the relationship between voltage and current is not linear.

Method for Experiment 3

To carry out this experiment, we will require the breadboard, 4 wires, the battery pack, 3 1.5 V (AAA) batteries, 3 1 Ω resistors, 2 4.7 kΩ resistors, 1 10 kΩ resistor, and the iOLab. As a general procedure, we will be using certain steps:

1. Place batteries into the battery pack. Make sure they all have the correct orientation, and each is a 1.5 V battery.
2. Build the circuit as shown in the image below.
3. Set the DAC to 3.3 V.
4. Place High Gain leads so as to measure the voltage across one of the 1 Ω resistors. In other words, place the G+ connection on the higher potential end of the resistor and do the opposite with the G- connection.
5. Record A7 and High Gain sensors on the IOLab website.
6. Turn on the Battery Pack, and then the DAC. The order is important.
7. Repeat experiment until voltage across all three 1 Ω resistors have been measured. Change what resistor is being measured by changing the position of the G+ and G- connections.



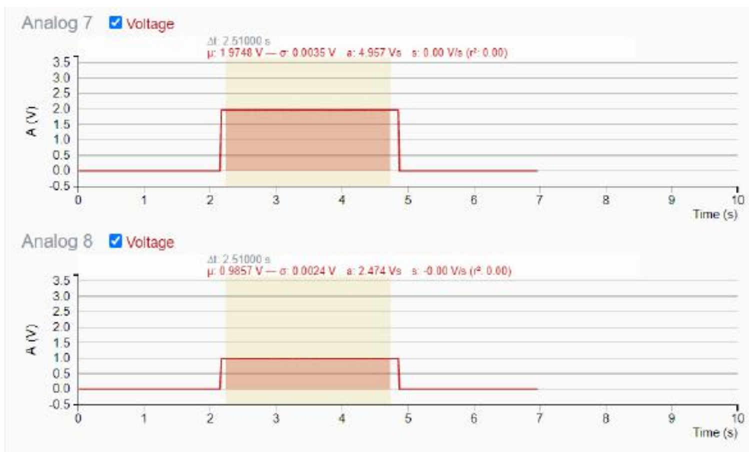
The High Gain voltage measurements again act as our equivalent measurements for current. We use this experiment to measure the experimental current and compare our results to the theoretical values derived using Kirchhoff's law and the rules for resistors in parallel and in series.

Calculations and Data Reduction/Analysis:

The analysis for these experiments was carried out mostly in Excel, and uses the formulas and laws found in the theory section. To reiterate, the most important formulas were Ohm's Law, $V = IR$, the equivalent resistance of resistors in series, $R_{\text{total}} = R_1 + R_2 + ... + R_n$, and Kirchhoff's Laws $\sum \Delta V_{\text{loop}} = 0$ and $\sum I_{\text{node}} = 0$. The data sets were all similar enough to the point that the one we chose would not matter. Therefore, we used Allen's data for all the experiments, except for the green LED in experiment 2b, where we used Javier's data because he was the only one who used green.

Analysis for Experiment 1

In part a, A7 read an average voltage of 1.974 V and A8 read an average voltage of 0.986 V over the period of time that the voltage was on.

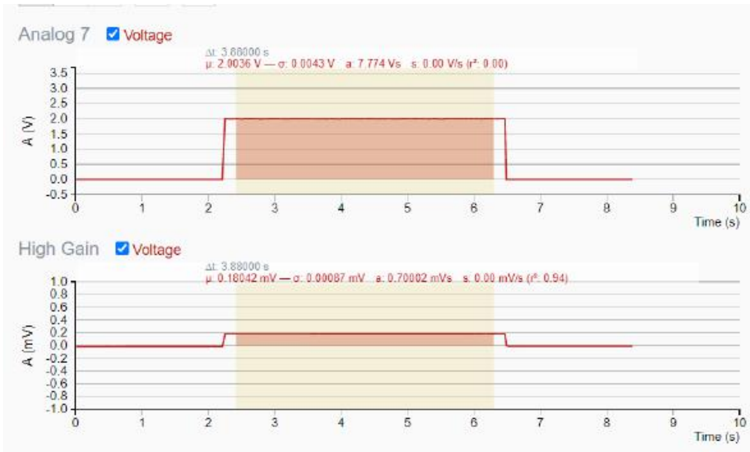


Then, $V_{\text{in}} = 1.974$, $V_{\text{out}} = 0.986$ V, and $H = \dfrac{V_{mathrmout}}{V_{mathrmin}} = 0.499$. We can use theory to determine what we expect H to be:

$$H = \frac{\dfrac{V}{R_1 + R_2} \cdot R_1}{V} = \frac{\dfrac{2.0}{10^4 + 10^4} \cdot 10^4}{2.0} = 0.5$$

The experimental value we determined for H matches the theoretical value well. Then, we confirmed that the two resistors have the same resistance and the voltage divider circuit functions as it is supposed to.

In part b, the high gain sensor read an average voltage of 0.180 mV over the period of time that the voltage was on.

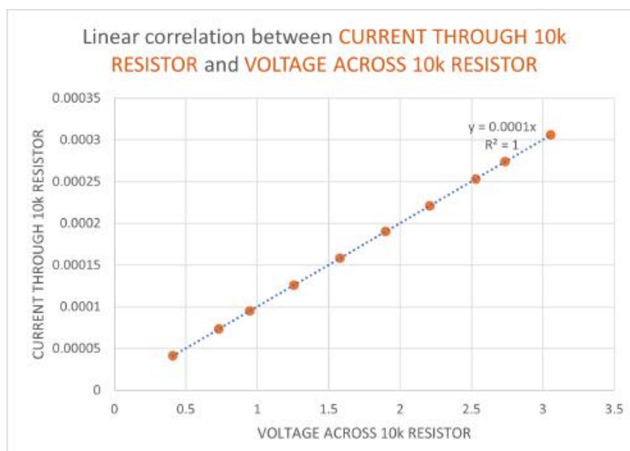


Then, the current over the 1 Ω resistor is $1.80 \cdot 10^{-4}$ A. We expect a current of 2.0/10001 A = $2.0 \cdot 10^{-4}$ A. Our experimental result for the current through the circuit was off by 10%.

Analysis for Experiment 2

In part a, we found that the current and voltage across a 10 kΩ resistor conforms extremely well to a linear relationship.

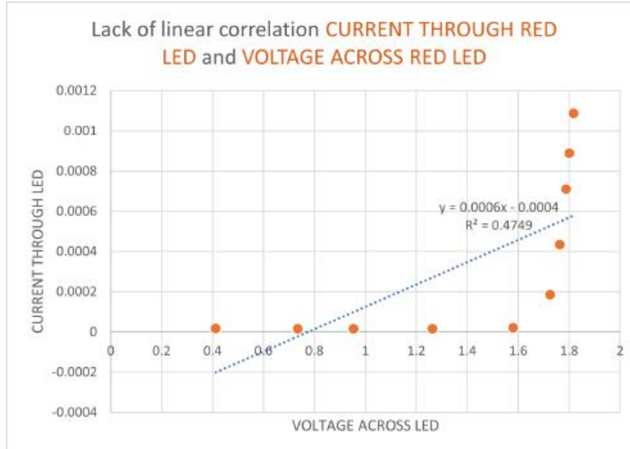
CURRENT THROUGH 10k RESISTOR	VOLTAGE ACROSS 10k RESISTOR
4.10642E-05	0.41064239
7.31712E-05	0.7317118
9.50091E-05	0.95009104
0.000125866	1.25866134
0.000157993	1.57993077
0.00019007	1.90070015
0.000220897	2.20897009
0.000252994	2.52993959
0.000273552	2.73552022
0.000305599	3.0559894



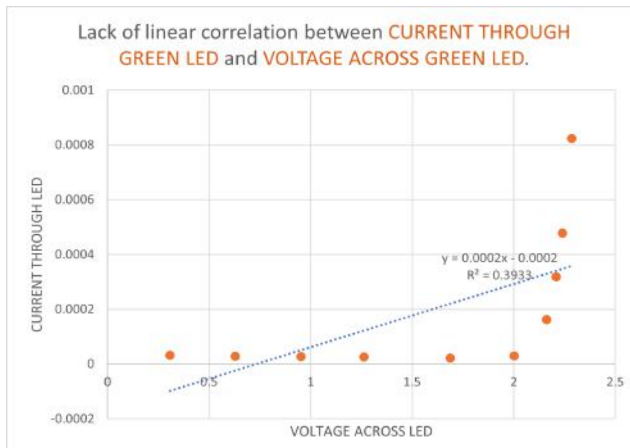
The resistance we found fit well with our expectations. The regression yielded $R = 10^4\,\Omega$ as we expected. This shows that the resistor conforms well to Ohm's Law. The residuals were all 0 once they were reduced down to significant figures, so we have not included their graph here.

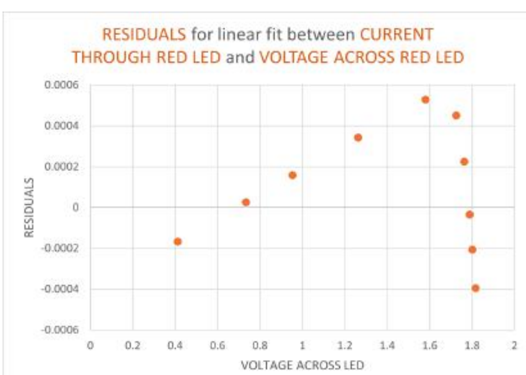
However, we found that the colored LEDs did not fit well with a linear model.

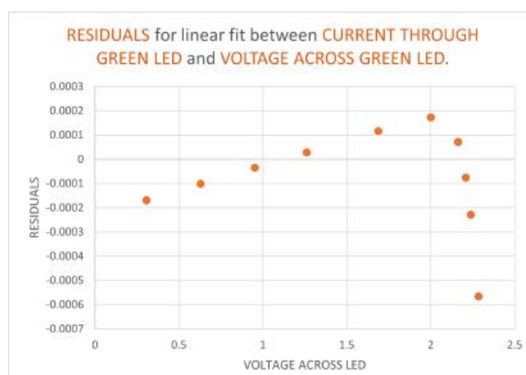
CURRENT THROUGH RED LED	VOLTAGE ACROSS RED LED
0.00001591	0.413
0.00001522	0.7346
0.00001532	0.9539
0.00001539	1.2635
0.00002094	1.5805
0.00018492	1.7257
0.00043417	1.764
0.00070908	1.7888
0.00088821	1.8012
0.00108596	1.8174



CURRENT THROUGH GREEN LED	VOLTAGE ACROSS GREEN LED
0.00003134	0.3081
0.00002817	0.6304
0.00002593	0.9525
0.00002465	1.2626
0.00002154	1.6881
0.00002831	2.0024
0.00016203	2.1632
0.00031779	2.2106
0.00047747	2.2405
0.0008232	2.2857



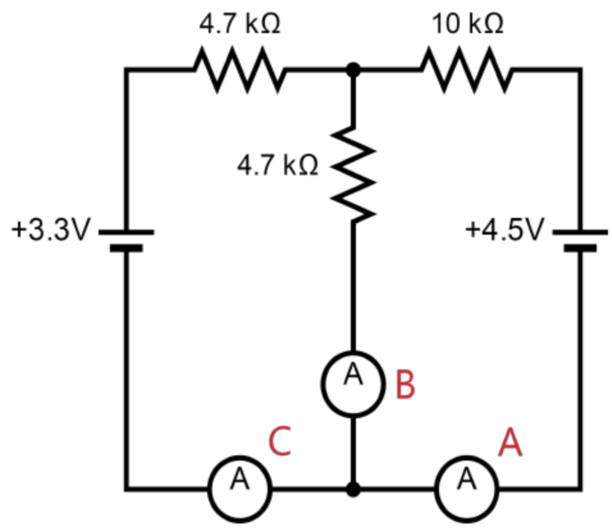




The residuals display a strong pattern, suggesting that the linear model is incorrect. It is reasonable to conclude that the colored LEDs do not conform to Ohm's Law. This is probably because LEDs are semiconductors, and their resistance is not constant. The green and red LEDs probably have different curves because of differences in the properties of the semiconductors inside them.

Analysis for Experiment 3

We assigned A, B, and C to the "ammeters" as they are labelled in the following image:



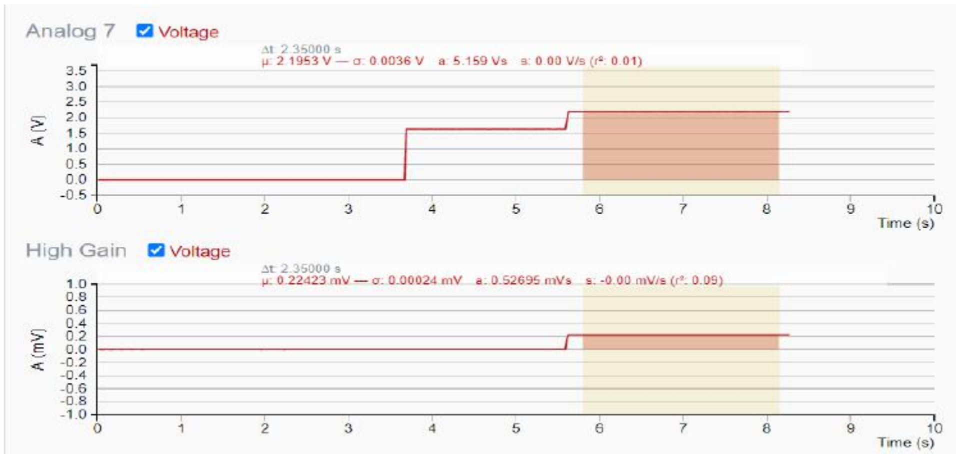
We can find our expected values for the current drops across A, B, and C using Kirchhoff's Laws:

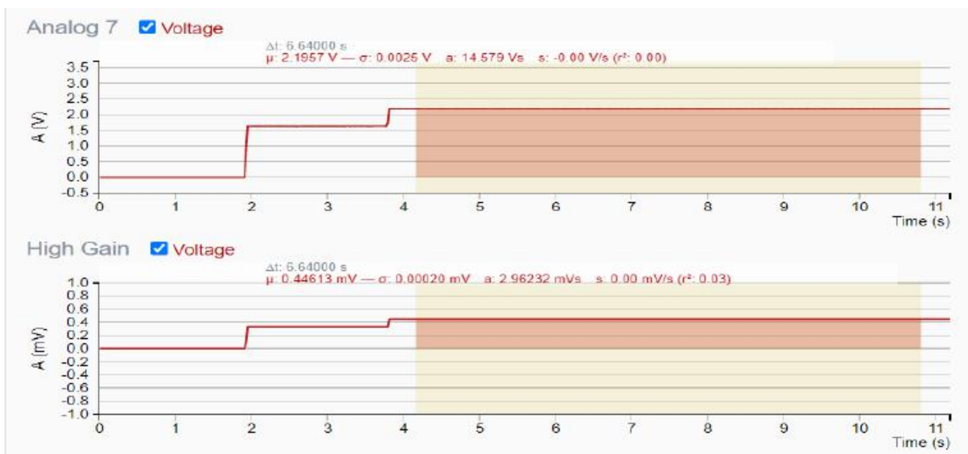
$$\begin{cases} I'_B + I'_C - 3.3 + 4700I'_C + 4700I'_B = 0 \\ I'_B + I'_A - 4.5 + 10000I'_A + 4700I'_B = 0 \\ I'_A + I'_C = I'_B \end{cases}$$

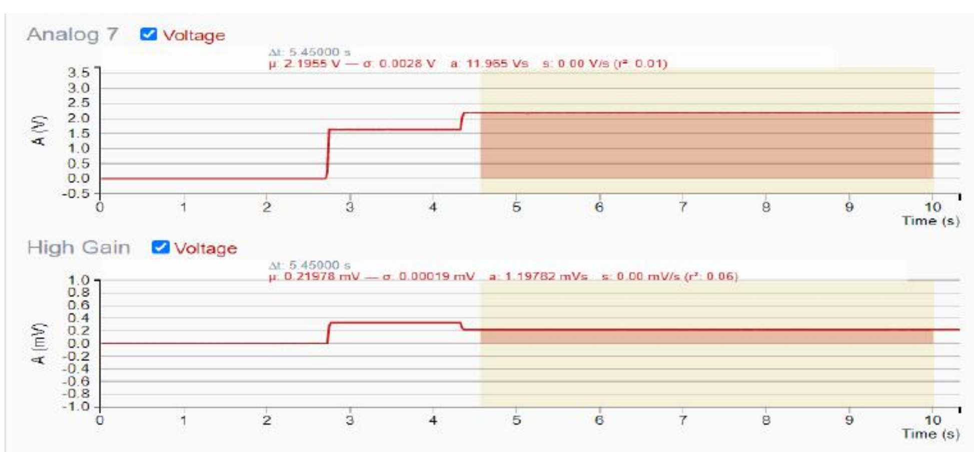
$$\implies \begin{cases} 4701I'_B + 4701I'_C - 3.3 = 0 \\ 4701I'_B + 10001I'_A - 4.5 = 0 \\ I'_A + I'_C = I'_B \end{cases}$$

$$\implies \begin{cases} I'_A = 2.3 \cdot 10^{-4}\,\mathrm{A} \\ I'_B = 4.6 \cdot 10^{-4}\,\mathrm{A} \\ I'_C = 2.3 \cdot 10^{-4}\,\mathrm{A} \end{cases}$$

We measured the voltages across these three "ammeters" in the order A, B, then C:

A: 

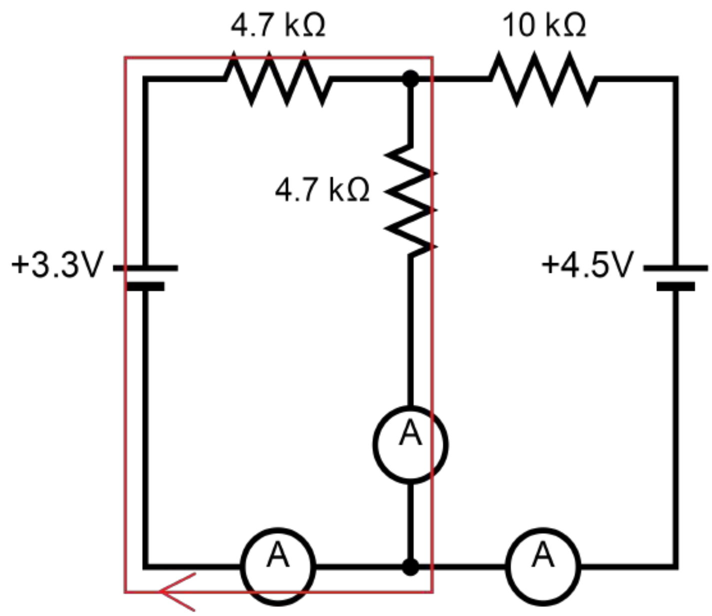
B: 

C: 

We measured $V_A = 0.224$ mV, $V_B = 0.446$ mV, and $V_C = 0.220$ mV. Then, the currents across the three "ammeters", in the direction running towards the junction closest to them, are $I_A = -2.24 \cdot 10^{-4}$ A, $I_B = 4.46 \cdot 10^{-4}$ A, and $I_C = -2.20 \cdot 10^{-4}$ A. These conform relatively well with our expected values. We can perform a sanity check with Kirchhoff's Current Law:

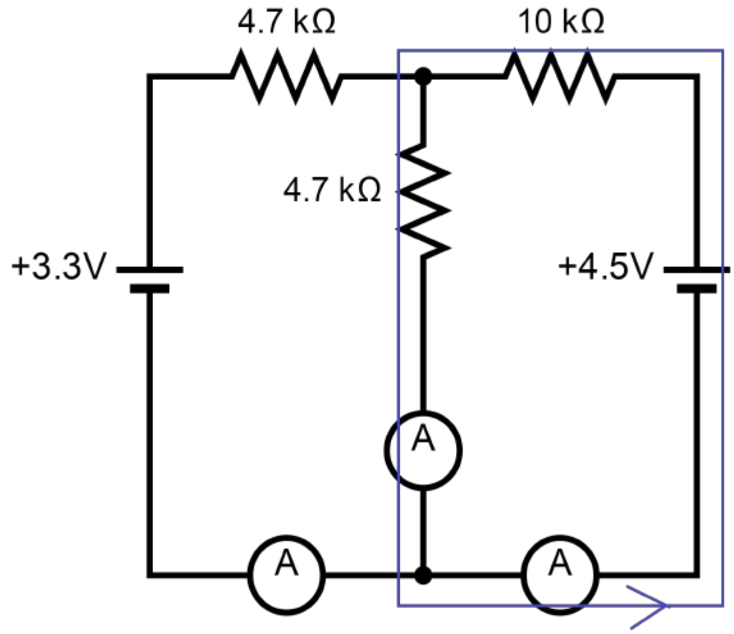
$$\sum I_{\text{node}} = -2.24 \cdot 10^{-4} + 4.46 \cdot 10^{-4} - 2.20 \cdot 10^{-4} = 2.0 \cdot 10^{-6}$$

This number is small enough for us to reasonably assume that Kirchhoff's Current Law is satisfied here. Next, we consider Kirchhoff's Loop Rule over the three loops in the circuit. We will include the voltage drop caused by the ammeters for the sake of rigor.



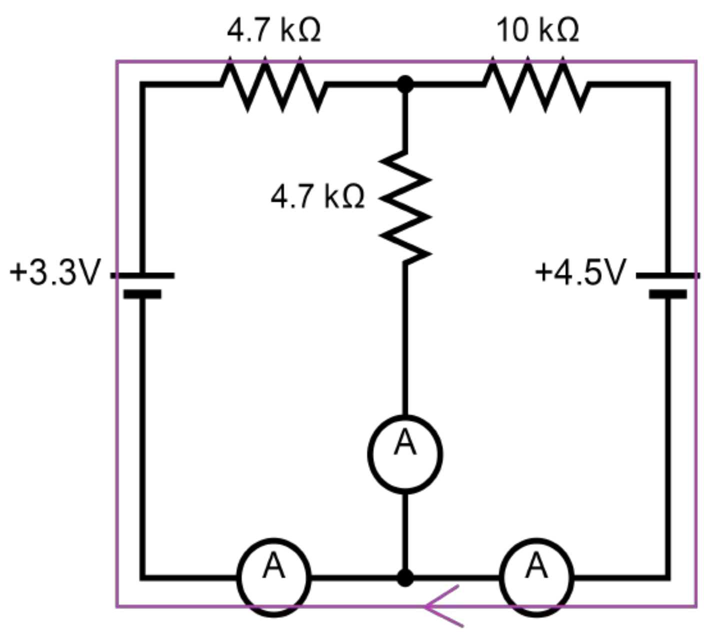
Moving in the direction of the arrow,

$$\begin{aligned}\sum V_{\text{Redloop}} &= 4.46 \cdot 10^{-4} + 2.20 \cdot 10^{-4} - 3.3 + 4.7 \cdot 2.20 \cdot 10^{-1} + 4.7 \cdot 4.46 \cdot 10^{-1} \\ &= -0.169\text{V}\end{aligned}$$



Moving in the direction of the arrow,

$$\begin{aligned}\sum V_{\text{Blueloop}} &= 4.46 \cdot 10^{-4} + 2.24 \cdot 10^{-4} - 4.5 + 2.24 + 4.7 \cdot 4.46 \cdot 10^{-1} \\ &= -0.163\text{V}\end{aligned}$$



Moving in the direction of the arrow,

$$\begin{aligned}\sum V_{\text{Purpleloop}} &= 2.20 \cdot 10^{-4} - 3.3 + 4.7 \cdot 2.20 \cdot 10^{-1} - 2.24 + 4.5 - 2.24 \\ &= -0.006\text{V}\end{aligned}$$

The data fits reasonably well with our expectations. In general, the voltages we measured were slightly less than the expected voltages. This could be because, at some point in the complicated connections made in the circuit, extra resistance was introduced somehow, dissipating some of the voltage before it could reach our "ammeters".

Summary and Conclusion:

In this section, we will be discussing our overall perspective on this Lab Report and the experiments carried out within it. As originally mentioned, our goal was to understand and measure the Ohmic, Non-Ohmic, and Kirchhoff-like behavior of an electric circuit in several different arrangements with several different components. To do this, we had three different experimental setups that were described in the Methods section. With these setups, we looked at the data and analyzed it for the three different setups, and we were looking to prove different physical laws in each part. For each experiment, we looked at the behavior of the classical concepts in a circuit (resistance, current, voltage) under different conditions, and analyzed the data that we had previously obtained in the Analysis section.

Conclusion: Experiment 1

In this experiment, we were looking to find the transfer ratio H. We wanted to see what the output voltage of the experiment was. Theoretically, as shown in the Analysis section, the expected value was 0.5, and we got the exact same result. This proves not only that our setup was correct and that we minimised as many sources of error as possible, but it also shows that voltage is effectively being outputted from this circuit and in the right amount. In part b of this same experiment, our goal was to, using the setup of a DMM, measure the voltage drop, and since the resistance is known, we will be able to determine the current. However, we found our current to be off by ten percent, which shows that we had some errors. Even though our understanding of the experiment was reasonable, we had some errors, probably in the breadboard (that gave a small percent error in themselves). Other errors we didn't include were the internal resistance of the wires, the resistors, and other components, as well as a loss of electrical current in the breadboard pins. Aside from that, we attributed small errors to errors in the setup, since the procedure was tested repeatedly and the errors were fairly consistent.

Conclusion: Experiment 2

In this experiment, we found that the 10 kΩ resistor conformed well with Ohm's Law. We calculated the resistance of the resistor to be 10 kΩ. This matches our expectations, a factory-made resistor is very unlikely to deviate very far from how it is labeled. However, we found that the LED lights did not conform very well to Ohm's Law at all. We found that a linear model was probably inadequate to describe the behavior of the LED lights. This makes sense, given that LED lights are semiconductors. Other errors we didn't include were the internal resistance of the wires, the resistors, and other components, as well as a loss of electrical current in the breadboard pins. Aside from that, we attributed small errors to errors in the setup, since the procedure was tested repeatedly and the errors were fairly consistent.

Conclusion: Experiment 3

In this experiment, we confirmed that Kirchhoff's Laws held in a simple circuit. We used three "ammeters" to find the currents through three branches of a circuit. We performed the experiment multiple times, as for some reason we could not determine, the first few times we performed the experiment our data was extremely flawed. We ultimately adjusted the layout of the circuit to avoid touching between wires where they were not meant to be. We found that the current entering a junction was 0, confirming Kirchhoff's Junction Rule. We also compared our data against Kirchhoff's Loop Rule. Our voltages summed around loops generally conformed to Kirchhoff's Loop Rule, but were off enough that we had to consider sources of error. Our currents were lower by a small amount compared to the expected currents which we calculated theoretically, which we attributed to random resistance introduced by connections between wires and the wires themselves. Furthermore, we concluded that the total voltage sum in the loop of circuit was very close to 0, just of by a small part in a thousand, further confirming Kirchhoff's Law.