PROBLEM 1

The general "axiom" relativity rests upon is that the frames of both sticks are equivalent since they are both inertial. If A sees B shorter than / longer than itself, then B also sees A longer than / shorter than itself from its frame. The contraction factor must be the same when going between frames S' and S. Now, lets assume that A sees B shortened. Therefore, B would still see A shortened, and the brushes on A would still mark B. Therefore, the brushes on A will mark B no matter how A sees B or B sees A. This is the reason why in the frame of one stick, the other one has a length of one meter, and viceversa.

PROBLEM 2

We are given that $\frac{v}{c} = 0.6 \rightarrow \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-0.6^2}} = 1.25$

Now, applying the lorentz transformations $x' = \gamma(x - vt)$ and $t' = \gamma\left(t - \frac{xv}{c^2}\right)$, we get:

(a) $x = 4$, $t = 0$ $\rightarrow$ $x' = 1.25(4 - (0.6c)0) = \boxed{5 \text{ meters}}$

$t' = 1.25\left(-\frac{4(0.6c)}{c^2}\right) = 1.25\left(-\frac{4 \cdot 0.6}{3\times10^8}\right) = \boxed{-1\times10^{8} \text{ s}}$

(b) $x = 4$, $t = 1$ $\rightarrow$ $x' = 1.25(4 - 0.6c) = \boxed{-2.25\times10^8 \text{ m}}$

$t' = 1.25\left(1 - \frac{4(0.6c)}{c^2}\right) = \boxed{1.25 \text{ seconds}}$

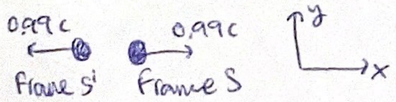
(c) $x = 1.8\times10^8$ m, $t = 1$ $\rightarrow$ $x' = 1.25(1.8\times10^8 - 0.6c) = \boxed{0 \text{ m}}$

$t' = 1.25\left(1 - \frac{1.8\times10^8 \cdot 0.6c}{c^2}\right) = \boxed{0.8 \text{ s}}$

(d) $x = 10^9$ m, $t = 2$ s $\rightarrow$ $x' = 1.25(10^9 - 0.6 \cdot 2 \cdot c) = \boxed{8\times10^8 \text{ m}}$

$t' = 1.25\left(2 - \frac{10^9 \cdot 0.6c}{c^2}\right) = \boxed{0 \text{ seconds}}$

PROBLEM 3

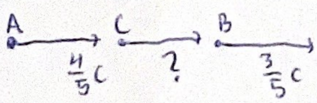


Apply formula 12.9(a) from bk.

$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$ $u_x' = \frac{-0.99c - 0.99c}{1 + 2(0.99)} = -\frac{1'98}{1'9801} c$

$\boxed{u_x' = -0'99995\,c}$ m/s, meaning that it goes at 0'99995c m/s in negative $\hat{i}$ direction.

PROBLEM 4



$V_A = \frac{4}{5}c = 0.8c$ $V_B = \frac{3}{5}c = 0.6c$ $V_A' = -V_B'$ in C frame

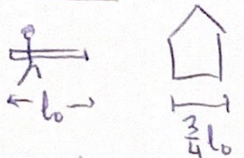
Therefore,

$V_A' = \frac{V_A - V_c}{1 - \frac{V_A V_c}{c^2}} = \frac{V_c - V_B}{1 - \frac{V_B V_c}{c^2}} = -V_B' \rightarrow (0.8c - V_c)(c - 0.6V_c) = (V_c - 0.6c)(c - 0.8V_c)$

$\rightarrow 1.4c^2 - 2'96 V_c \cdot c + 1.4V_c^2 = 0 \longrightarrow$ use Quadratic F.

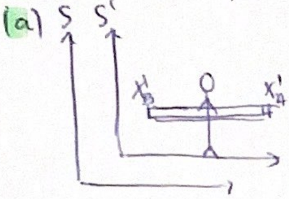
The other root is higher than c $\boxed{V_c = \frac{5}{7}c}$

PROBLEM 5



, $V_{\text{of pole}} = \frac{\sqrt{3}}{2}c \rightarrow \gamma = 2$

(a) S S'



We know that $x'_A - x'_B = l_0$. Since the Lorentz Transform says $x' = \gamma(x - vt)$, we know that

$$x'_A - x'_B = \gamma(x_A - vt_A) - \gamma(x_B - vt_B)$$

Since ~~t_0 is invariant~~, $x'_A - x'_B = l_0$ ~~too~~, Now, since $t_A = t_B = 0$, we have:

$$x'_A - x'_B = l_0 = \gamma(x_A - x_B)$$ where $x_A - x_B$ is the length of the pole in S.

Therefore, L, the length of the pole in S is:

$$L = \frac{l_0}{\gamma} = (x'_A - x'_B)\sqrt{1-\left(\frac{v}{c}\right)^2} = \boxed{\frac{(x'_A - x'_B)}{2}}$$

(b)

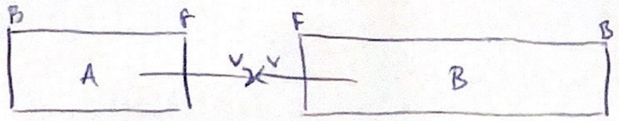
Now, we apply the length contraction to the barn.

$$\frac{1}{\gamma} l_{Barn} = l'_{Barn} \rightarrow \frac{1}{\gamma} \cdot \frac{3}{4} l_0 = l'_{Barn} = \frac{1}{2} \cdot \frac{3}{4} l_0 = \frac{3}{8} l_0$$

Therefore, $\frac{3}{8}$ of the pole is inside the barn, at t'_A. $\frac{5}{8}$ of the pole is outside the front of the barn. Therefore, the pole-vaulter sees B at $\frac{5}{8}l_0$ from the front of the barn.

(c) As shown in (b), only $\frac{3}{8}$ of the pole lie inside the barn from S' frame. Therefore, since $\frac{3}{8} < 1$, A and B don't lie inside the barn, at the same instant.

PROBLEM 6



(a) The fronts of A and B touch at E_1. From a ground frame, $L'_A = \frac{L_A}{\gamma} = \frac{L}{\gamma}$ and $L'_B = \frac{L_B}{\gamma} = \frac{2L}{\gamma}$ due to the length contraction. Therefore, at E_1, the backs of A and B are at a distance of $\frac{3L}{\gamma}$. The velocity of one relative to the other is $2v$.

Therefore, $t = \frac{d}{v} \rightarrow t = \frac{3L}{2v\gamma}$ where $\gamma = \left(\sqrt{1-\left(\frac{v}{c}\right)^2}\right)^{-1}$, so $\boxed{t_G = \frac{3L}{2v}\sqrt{1-\left(\frac{v}{c}\right)^2}}$

(b) From frame A, train A has length L and train B has length depending on γ. To find γ, we must find the relative velocity of B with respect to A.

$$V'_B = \frac{2v}{1-\frac{v^2}{c^2}} \rightarrow \gamma = 1/\sqrt{1-\left(\frac{V'_B}{c}\right)^2} \rightarrow t'_A = \frac{\left(L + \frac{2L}{\gamma}\right)}{V'_B} = \boxed{\frac{3L}{2v} - \frac{Lv}{2c^2}}$$

PROBLEM 6 (CONTINUED)

(c) Similarly to (b), from frame B, train B has length 2L and train A has length dependant on γ and therefore on the relative velocity of A with respect to B.

$$E_2' = \gamma \begin{bmatrix} 1 & v/c \\ v/c & 1 \end{bmatrix} \begin{bmatrix} \frac{3L}{2v\gamma} c \\ \frac{L}{2\gamma} \end{bmatrix} = \gamma \begin{bmatrix} \frac{3Lc}{2v\gamma} + \frac{Lv}{2c\gamma} \\ \frac{3L}{2\gamma} + \frac{L}{2\gamma} \end{bmatrix} = \begin{bmatrix} \frac{3Lc}{2v} + \frac{Lv}{2c} \\ 2L \end{bmatrix}$$

Therefore, time is: $\boxed{t_B' = \frac{3L}{2v} + \frac{Lv}{2c^2}}$

(d) Since we've set the coordinates to get that $E_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in all frames, we only look at $\begin{bmatrix} ct \\ x \end{bmatrix}$ in E_2, for each frame:

GROUND FRAME: $S^2 = (ct)^2 - x^2 = \left(c \cdot \frac{3L}{2v\gamma}\right)^2 - \left(\frac{L}{2\gamma}\right)^2 = \frac{L^2}{4}\left(\frac{9c^2}{v^2} - \frac{v^2}{c^2} - 10\right)$

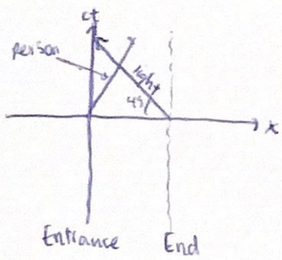
A-FRAME: $s^2 = (ct_A')^2 - x^2 = \left(\frac{3Lc}{2v} - \frac{Lv}{2c}\right)^2 - L^2 = \frac{L^2}{4}\left(\frac{9c^2}{v^2} - \frac{v^2}{c^2} - 10\right)$

B FRAME: $s^2 = (ct_B')^2 - x^2 = \left(\frac{3Lc}{2v} + \frac{Lv}{2c}\right)^2 - 4L^2 = \frac{L^2}{4}\left(\frac{9c^2}{v^2} - \frac{v^2}{c^2} - 10\right)$

Therefore, the invariant interval is the same in all three frames. QED.

PROBLEM 7

(a) Person enters tunel with $v = \frac{1}{2}c$. Since we look from tunnel's frame, the endpoints remain fixed in space through time, and the photon is emitted at $t=0$ in L.

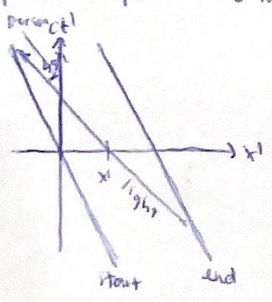


The light beam travels at 45°. The slope of the person's line is 2. Therefore we write the equations for the lines to get.

$$\begin{matrix} \text{person} \\ \text{photon} \end{matrix} \begin{cases} ct = 2x \\ ct = -x + L \end{cases} \rightarrow 2x = -x + L \rightarrow \frac{L}{3} = x.$$

Therefore, the photon and person collide at $\frac{L}{3}$, so $\boxed{f = \frac{1}{3} \text{ for } v = \frac{1}{2}c}$

(b) From the person's frame, as $t > 0$, the endpoints of the tunnel move away from her (entrance endpoint) on towards her (end endpoint). Therefore, the worldlines for the endpoints get a slope in the minkowski diagram that can be found through a Lorentz Transform.



The person's line remains in the same point in space from its frame. Taking $v = \frac{1}{2}c$, and using time dilation $t' = t\gamma$, we convert times from tunnel frame to the persons frame.

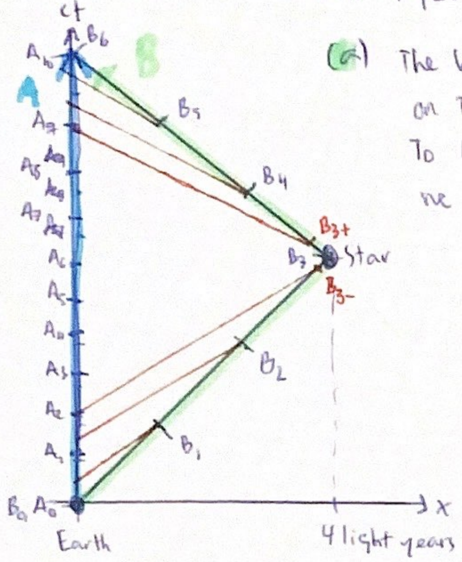
$$\gamma = \left(\sqrt{1 - \frac{(\frac{1}{2}c)^2}{c^2}}\right)^{-1} \Rightarrow \left(\frac{\sqrt{3}}{2}\right)^{-1} = \frac{2}{\sqrt{3}}$$

$$\left.\begin{matrix} \text{Time for photon to collide} = \frac{L(1-f)}{c}\gamma \\ \text{Time for person to collide} = \frac{fL}{v}\gamma \end{matrix}\right\} \text{set them equal} \rightarrow f = \frac{v}{v+c} = \boxed{\frac{1}{3}}$$

PS: due to loss of simultaneity, the light's worldline doesn't cross x axis where the tunnel's end does

PROBLEM 8

Since $v_{Rocket} = \frac{4}{5}c$, $\gamma = \frac{1}{\sqrt{1-(\frac{4}{5})^2}} = \frac{5}{3}$. B arrives back to earth after 10 years in the A frame. Using time dilation, $\frac{10}{\gamma} = 10 \cdot \frac{3}{5} = 6$ years take place from Frame B in that time. Therefore, upon arrival, B is 4 years younger than A.



(a) The birthdays for A and B, indicated by A_n and B_n are shown on the diagram.
To find what age B is in A's frame when B has a birthday, we do the Lorentz Transform:

$$ct' = \gamma\left(ct - \frac{vx}{c}\right)$$

since $t = 1, 2, 3, \ldots 6$, and $v_{Rocket} = \frac{4}{5}c$, rocket travels $\frac{4}{5}$ Lyears in 1 year. Therefore,

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \frac{5}{3}\left(1 - \frac{4}{5}\cancel{c}\frac{4}{5}\right) = 0.6$$

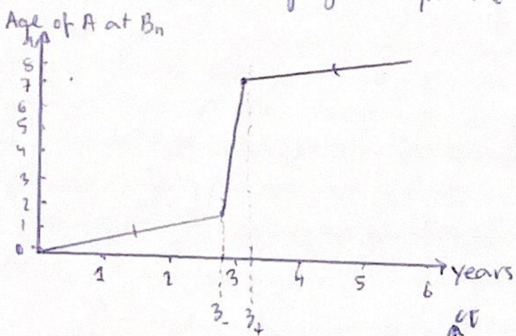
Plotting this gives us the red lines, for B_1, B_2, B_4, B_5. B_6 coincides with A_{10}. For B_3, since the system accelerates in turnaround, we must look at B_{3+} and B_{3-}

For B_{3+} and B_{3-}, we approximate values to those at B_3. X at B_3 is $\frac{12}{5}$, and $t=3$. we can write:

$$B_-: \ t' = \gamma\left(3 - \frac{4}{5} \times \frac{12}{5}\right) = 1.80$$

$$B_+: \ t' = \gamma\left(3 + \frac{4}{5} \times \frac{12}{5}\right) = 8.20$$

Now, to make a graph of the aging through the years, we do it based on the graph above;

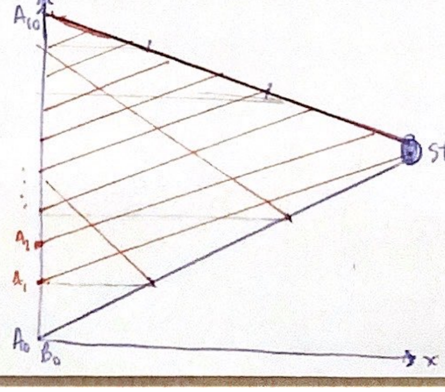


Between B_{3+} and B_{3-}, ~~there~~ B consider A to age the value we got previously to be $B_{3+} - B_{3-}$

$B_{3+} - B_{3-} = 8.20 - 1.80 \approx \boxed{6.4}$

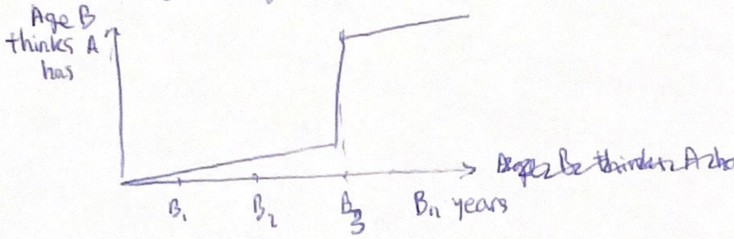
(b) Since light travels at 45° angles, we know the trajectories for light coming off events B_n, A_n. We have the diagram to the ~~left~~ right:

Note: I am bad at drawing but all the lines leaving A_n are supposed to be parallel and at an angle of 45° with the horizontal. Same for B_n.



PROBLEM 8 (CONTINUED)

(b) B sees A age when the light from A's candles reach B. Therefore, the graph looks something along these "lines". (Pun not intended)



Therefore, B sees A age at the turnaround

PROBLEM 10

(a) $\tanh(\phi) \equiv \frac{v}{c}$, therefore, $\gamma = \frac{1}{\sqrt{1-\tanh^2(\phi)}} = \cosh(\phi)$ due to trig identities.

The Lorent transformation becomes:

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \rightarrow \begin{pmatrix} ct' \\ x' \end{pmatrix} = \cosh(\phi) \begin{pmatrix} 1 & -\tanh(\phi) \\ -\tanh(\phi) & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

This can be simplified to get:

$$\boxed{\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh(\phi) & -\sinh(\phi) \\ -\sinh(\phi) & \cosh(\phi) \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \rightarrow L_\phi}$$

(b)

$$\left.\begin{aligned} S' &= L_{\phi_1} \cdot S \\ S'' &= L_{\phi_2} \cdot S' \end{aligned}\right\} \boxed{S'' = L_{\phi_2} \cdot L_{\phi_1} \cdot S}$$

Now plug in matrices:

$$\begin{pmatrix} ct'' \\ x'' \end{pmatrix} = \begin{pmatrix} \cosh(\phi_2) & -\sinh(\phi_2) \\ -\sinh(\phi_2) & \cosh(\phi_2) \end{pmatrix} \begin{pmatrix} \cosh(\phi_1) & -\sinh(\phi_1) \\ -\sinh(\phi_1) & \cosh(\phi_1) \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\begin{pmatrix} ct'' \\ x'' \end{pmatrix} = \begin{pmatrix} \cosh(\phi_2)\cosh(\phi_1) + \sinh(\phi_2)\sinh(\phi_1) & \cosh(\phi_2)\sinh(\phi_1) + \sinh(\phi_2)\cosh(\phi_1) \\ \sinh(\phi_2)\cosh(\phi_1) + \cosh(\phi_2)\sinh(\phi_1) & \sinh(\phi_2)\sinh(\phi_1) + \cosh(\phi_2)\cosh(\phi_1) \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\boxed{\begin{pmatrix} ct'' \\ x'' \end{pmatrix} = \begin{pmatrix} \cosh(\phi_2+\phi_1) & -\sinh(\phi_2+\phi_1) \\ -\sinh(\phi_2+\phi_1) & \cosh(\phi_2+\phi_1) \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}}$$

since we know the hyperbolic identities for $\cosh(a+b)$ and $\sinh(a+b)$

Therefore, the rapidity relating S and S'' is $\boxed{(\phi_1+\phi_2).}$

(c)

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh(\phi) & -\sinh(\phi) \\ -\sinh(\phi) & \cosh(\phi) \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$, substituting $ct = -i\tau$, $ct' = -i\tau'$, and $\phi = i\theta$,

$$\begin{pmatrix} -i\tau' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh(i\theta) & -\sinh(i\theta) \\ -\sinh(i\theta) & \cosh(i\theta) \end{pmatrix} \begin{pmatrix} -i\tau \\ x \end{pmatrix}$$. Since $\cosh(i\theta) = \cos\theta$ and $\sinh(i\theta) = i\sin(\theta)$

$$\begin{pmatrix} -i\tau' \\ x' \end{pmatrix} = \begin{pmatrix} \cos\theta & i\sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -i\tau \\ x \end{pmatrix}$$. Now, expand and simplify: (dividing by $-i$)

$$\boxed{\begin{pmatrix} \tau' \\ x' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tau \\ x \end{pmatrix}}$$

PROBLEM 9

Thanks to the γL displacement ← In the laser system, both cuts are simultaneous ← front cutter must advance $L\gamma$ before cutting ← to make the clocks equal. Clocks are $\frac{Lv}{c^2}$ seconds apart.

In the lab frame, the dough is contracted, so the diameter L becomes larger (it becomes γL) in the dough's frame. Therefore, when you see the cookie after it leaves the conveyor belt, it is stretched out by a factor of γ in the direction of the belts motion. Originally, the cookie was a circle, but when you buy it, it's ~~squished~~ streched in the direction of the belt. From the cookie's POV, both clocks are set at diff. times, and cookie must travel distance γL to