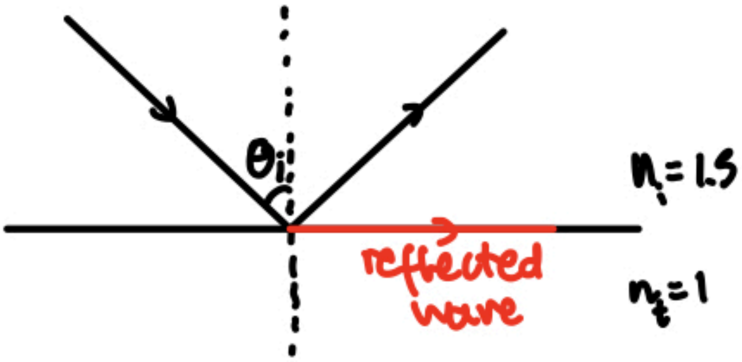
Problem 1

We are given:
- Vacuum wavelength: 600 nm
- Travelling in glass: $n = 1.5$
- Angle of incidence = 45°, total internal reflection.



We have to:
- Find the distance into air at which the the amplitude of the evanescent light has dropped by $1/e$ of max.

We have $\hat{E}_t = \hat{E}_{t_0} \cdot e^{i(k\sin\theta_t x - \omega t)} e^{-\beta k z}$ where $k = \frac{2\pi}{\lambda}$. Also, $n_i \sin\theta_i = n_t \sin\theta_t$, so $\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i$. Also, $\beta = \sqrt{\sin^2\theta_t - 1}$.

Furthermore, if we want the amplitude to drop by $1/e$, we want the exponent in $e^{-\beta k_t z}$ to be -1, so $\beta k_t z = 1$.

Therefore:

$$z = \frac{1}{\beta k} = \frac{\lambda}{2\pi\sqrt{\sin^2\theta_t - 1}} = \frac{\lambda}{2\pi\sqrt{\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1}}$$

Plugging in values,

$$z = \frac{600}{2\pi\sqrt{(1.5^2 \sin^2 45) - 1}} = 270.1 \text{ nm}.$$

Problem 2

We have to: Show that the phase shift for total internal reflection of $\vec{E}_{\parallel}$ polarization (p-polarization) is given by:

$$\phi = 2\arctan\left(\frac{n_i}{n_t}\alpha\right), \text{ where } \alpha = \frac{1}{\cos\theta_i}\sqrt{\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1}$$

Using terminology from problem (1), $\alpha = \frac{\beta}{\cos\theta_i}$.

We know that $r_{\parallel} = \frac{-\bar{z}}{z}$, which is a complex ratio. By multiplying by the complex conjugate, we have $r_{\parallel} = -\frac{\bar{z}}{z}\frac{\bar{z}}{\bar{z}} = \frac{-\bar{z}^2}{|z|^2} = -\left(\frac{E_{or}}{E_{oi}}\right)_{\parallel}$, where $E_{or} = E_{oi}e^{\phi}$. Also, $\frac{-\bar{z}^2}{|z|^2} = \frac{-|z|^2 e^{2i\phi}}{|z|^2} = e^{2i\phi}$. $\therefore$ Therefore, the phase angle is $2\phi_{\parallel}$ of z, where we have $\phi_{\parallel z} = \arctan\left(\frac{\text{Im}|z|}{\text{Re}|z|}\right)$ and $z = \frac{n_t}{n_i}\cos\theta_i - i\sqrt{\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1}$.

Therefore:

$$\frac{\text{Im}|z|}{\text{Re}|z|} = \frac{-1}{\frac{n_t}{n_i}\cos\theta_i}\sqrt{\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1}$$, which can be written as $-\frac{n_i}{n_t}\alpha$.

This implies that

$$\phi_{\parallel} = 2\arctan\left(-\frac{n_i}{n_t}\alpha\right).$$

Problem 3

We know that $\lambda = 530\,nm$, $d = 5\,m$, and $a = 0.1\,mm$.

Therefore, $\Delta y = \lambda \frac{d}{a} = \frac{530\,nm \cdot 5\,m}{0.1\,mm} = 0.0265\,m.$