Lab 6

Weighted Curve Fitting: Torsional Pendulum

Objectives

- Give students practice applying chi-square analysis
- Illustrate using linear regression for a nonlinear model
- Illustrate simple methods to analyze data with both vertical and horizontal error bars
- Use weighted least squares fit

Group members,
and their group roles:

Activity Overview

This experiment focusses on a torsional pendulum, the rotational analogue of a "swinging" pendulum. You will measure the torsional spring constant of a wire by measuring the periods of oscillation for several disks. You will need to measure the disks to find moments of inertia, measure the period of oscillation for each disk, and finally plot and fit this data appropriately to the theory. Your conclusions will use your new knowledge of chi-square to assess your agreement or disagreement with the theory and known values.

Be especially careful to …

- Design your experimental technique to minimize your error sources when noticed!
- Make sure your data and analyses are clear, so you can refer to them again later!
- Generate clear scientific plots!

What to Turn In

Intro Lab 6 contains only one experiment and each student should learn the analysis techniques. For this lab, each group can choose to their submission method; individual submissions or combining the individual worksheets into a single submission.

Each student is responsible for acquiring a full dataset for the experiment. The group, as a whole, can decide which of the three generated datasets each person will analyze and provide a *justification* for the decision. Options: each student uses the best group dataset or the individual uses their own dataset.

E**ach student in a group** will be responsible for analyzing *one* dataset and answering the related questions. Group members can discuss the analysis and questions, but each student's written submission including the graphs must be their own work. **All code authors must be properly cited**. Each Gradescope pdf submission should include the filled-in worksheet with the specified plots. **A separate bCourses submission is required for the Python code or Excel worksheet used in the analysis.**

If the group decides using the best group dataset for each member's analysis is appropriate, **that dataset must be included** in the other group member's individual lab reports either as an Appendix, or highlighted to indicate the dataset analyzed when combining in a single table. The data sets should be properly attributed in all cases.

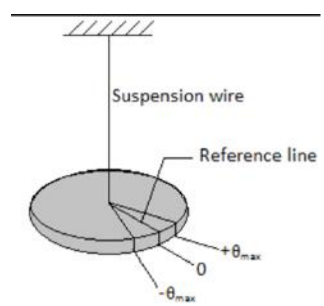
Procedure Overview

Throughout each experiment, be sure to identify the goal, primary data you need to collect, and equipment you will use. Carefully consider each experimental setup, including the environment around your setup. Try to identify strategies to reduce experimental noise, ensure stability, and allow your experiment to be reproducible.

For each measurement, be sure you have outlined a clear measurement strategy that includes identifying and minimizing any random and systematic errors.

During the experiment you may need to adjust your procedure to account for errors or flaws in the design. In your report, you should note when and where you saw the need for adjustment and how you and your group adapted your procedure.

Theory



A torsional pendulum, or torsion pendulum, is the rotational analogue of the more familiar gravitational ("swinging") pendulum. In a torsional pendulum, an object such as a disk is suspended parallel to the ground from a wire attached to its center. When the disk is rotated by a sufficiently small angle (small enough not to plastically deform the wire), the twisted wire exerts a restoring force proportional to the angle of rotation away from equilibrium: $F = -\kappa\Delta\theta$. Notice this is just Hooke's Law. (For a normal spring, $F = -k\Delta x$.) The disk then rotates back and forth with a period of oscillation given by

$$T = 2\pi\sqrt{I/\kappa} \tag{1}$$

where I is the moment of inertia of the disk and κ is the "torsional spring constant". The moment of inertia of a disk with mass M and radius R rotated about its center is given by

$$I = \frac{1}{2}MR^2 = \frac{1}{8}MD^2 \tag{2}$$

Equipment

- Ruler
- Calipers, kit
- Manganin wire, kit
- Stopwatch
- Homemade torsional pendulum setup
- 250 gm weight set with gloves, kit
- Protractor, kit
- Wooden dowel or a wire support
- Scotch tape
- Scale (optional)

Manganin wire: originally located on the back of the protractor, the manganin should be on the sandpaper's paper covering. The wire can be bent, cut or twisted as needed. Aim for 20 cm between the pivot and the weight set attachment.

Measurement instrument. Choose either the ruler or calipers, as appropriate.

Weight set: use only the weight holder and the 20 gm disks.

Homemade measurement setup. You will be hanging different weights to the wire and measuring the torsional pendulum period. A fixed suspension point for the wire and a low-friction pivot works best. The wire can be attached to the dowel or other support: the wires should hang down but not move lengthwise along the dowel. Several design iterations might be necessary to keep the wire from unraveling on the dowel.

You will need some type of dowel support that's tall enough to allow a **20 cm length** of wire between the pivot point and the top of the weight set holder.

The wire can be attached to the weight set holder by repeated wraps. You might need to alternate sides to keep the weight set from slipping off.

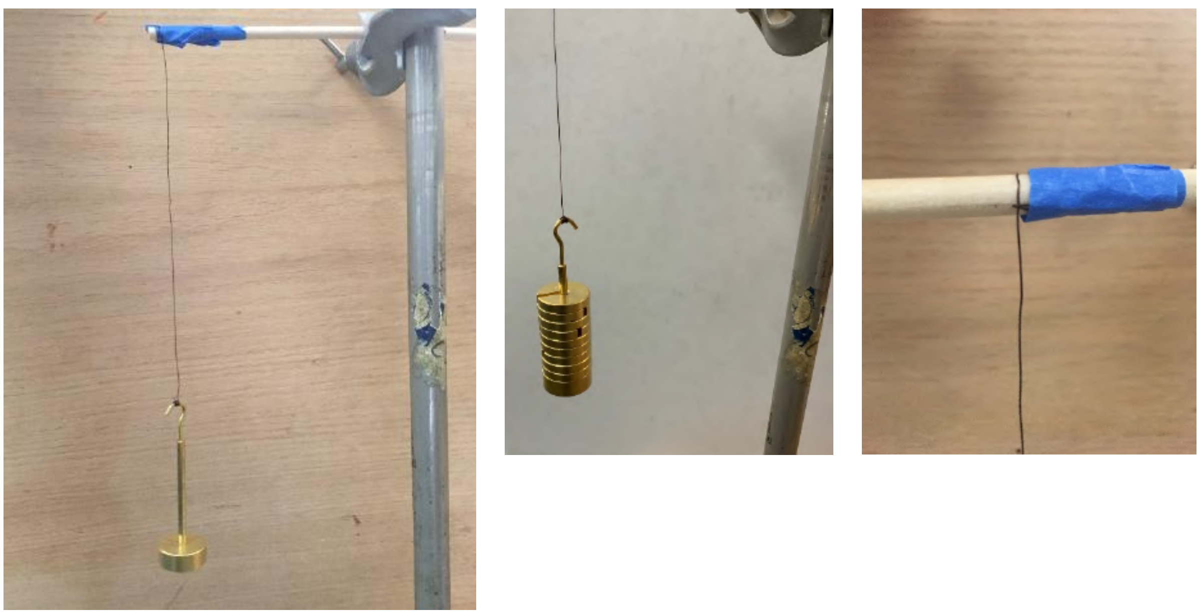


Figure 1: Example of Experimental Setup and Details

Prelab

Task 1. Propagate error symbolically for I. You will be calculating and plotting the moment of inertia I. You will be measuring the *diameters* D, with the mass provided. Write I (Equation 2) in terms of M and D and propagate the error for I.

Task 2: Construct the measurement setup and submit a picture. Set up the dowel or other support, attach the manganin wire to it with a low-friction pivot, hang the wire from it, measure 20 cm and attach the weight holder to the wire. *Be sure your apparatus functions properly!*

Procedure and Data Collection

You will measure the moments of inertia and periods of oscillation of the various disks in the torsion pendulum. You will use the moment of inertia and period data to fit your data to the theory and find the torsion spring constant κ. The following steps will guide you through this measurement process and data analysis.

1. ***Error propagation.*** In the prelab, you already wrote the moment of inertia I, and calculated its error α_I. We've provided space to copy those results below, if you choose. **Before beginning the linear regression analysis, additional error propagation is necessary and a classroom check required in section 9.**

From Eq 2, $I = \frac{1}{8}MD^2 \rightarrow \alpha_I = \sqrt{\left(\frac{\partial I}{\partial M}\alpha_m\right)^2 + \left(\frac{\partial I}{\partial D}\alpha_D\right)^2} = \boxed{\frac{D}{4}\sqrt{\frac{1}{4}D^2\alpha_m^2 + M^2\alpha_D^2}}$

2. ***Record instruments.*** Before taking any data, record the precision errors of your measurement instrument(s). In addition to the usual importance of this step for others trying to reproduce your results, you will need to use these same instruments next time.

INSTRUMENT	PRECISION ERROR
RULER	± 0.05 cm
CALIPERS	± 0.05 cm
STOPWATCH	± 0.01 s

3. ***Measurements for moments of inertia.*** Measure the diameters of the disks and holder, in preparation for computing their moments of inertia. In this experiment, let us treat the disks the same, so you only need to measure one of the nine 20 gm disks you have.

A) Disk dimensions: Measure its diameter, using whichever instrument is appropriate.

Disk mass: $\mathbf{20 \pm 0.1\ gm.}$

Diameter of Disk: Choose the appropriate measurement instrument and provide a justification:

I chose to use the calipers since they are more comfortable to measure widths than the rules. This is because the long parts of the calipers can be used to support the calipers snd keep them in place, minimizing random errors. The ruler would be more uncomfortable, and some numbers would have to be eyeballed.

Trial #	Diameter of disk (units: cm)
1	3.16 ± 0.05
2	3.14 ± 0.05
3	3.15 ± 0.05
4	3.16 ± 0.05
5	3.16 ± 0.05

Mean of diameter $\bar{d}$: 3.154 ± 0.05 cm

Standard error α_{disk}: 0.004 ± 0.05 cm

Diameter of disk including error: $\boxed{3.154 \pm 0.004 \text{ cm}}$

B) Mass holder dimensions:

Holder mass: $50 \pm 0.1\ gm$. Note: the hooked center rod mass is included in this value.

Diameter of Holder Base: Choose the appropriate measurement instrument. If you chose a different instrument to measure the holder, include a justification:

Trial #	Diameter of disk holder (units: cm)
1	3.14 ± 0.05
2	3.15 ± 0.05
3	3.16 ± 0.05
4	3.16 ± 0.05
5	3.15 ± 0.05

Mean of diameter $\bar{d}$: 3.152 ± 0.05 cm

Standard error α_{holder}: 0.004 ± 0.05 cm

Diameter of disk holder including error: $\boxed{3.152 \pm 0.004 \text{ cm}}$

4. ***Compute moments of inertia***. Calculate the numeric value of the moment of inertia I for the holder and one disk using Equation 2 and your error propagation from the prelab:

Moment of Inertia of the disk holder I_{holder}:

$$I = \frac{1}{8}MD^2 = \frac{1}{8}(50\text{ grams})(3.152\text{ cm})^2 = \frac{1}{8}(0.05\text{ kg})(0.03152\text{ m})^2 = 0.0000062\text{ kgm}^2 = 6.2\times10^{-6}\text{ kgm}^2$$

Moment of Inertia error of the disk holder α_{holder}:

$$\alpha_I = \frac{D}{4}\sqrt{\frac{1}{4}D^2\alpha_m^2 + M^2\alpha_D^2} = \frac{0.03152}{4}\sqrt{\frac{1}{4}(0.03152)^2(0.0001)^2 + (0.05)^2(0.00004)^2} = 2.0\times10^{-8}\ kgm^2$$

- **Reported value $I_{holder} \pm \alpha_{holder}$:** $\boxed{6.2\times10^{-6} \pm 0.020\times10^{-6}\ kgm^2}$

Moment of Inertia of a single disk I_{disk}:

$$I = \frac{1}{8}MD^2 = \frac{1}{8}(0.02)(0.03154)^2 = 2.5\times10^{-6}\ kgm^2$$

Moment of Inertia error of a single disk α_{disk}:

$$\alpha_I = \frac{D}{4}\sqrt{\frac{1}{4}D^2\alpha_m^2 + M^2\alpha_D^2} = \frac{0.03154}{4}\sqrt{\frac{1}{4}(0.03154)^2(0.0001)^2 + (0.02)^2(0.00004)^2} = 1.4\times10^{-8}\ kgm^2$$

- **Reported value $I_{disk} \pm \delta I_{disk}$:** $\boxed{2.5\times10^{-6} \pm 0.014\times10^{-6}\ kgm^2}$

5. **Experimental design**. Now you are ready to measure pendulum oscillation period as a function of mass by attaching disks to the pendulum holder and measuring the period. This section highlights the design elements needed for the group's measurement procedure.
   - Ensure the pendulum is balanced when adding disks (slot orientation).
   - Establish an equilibrium position with the pendulum at rest.
   - Rotate the disks at a small angle from equilibrium, say 10° or 20° or so.
   - After any wobbles settle out from rotating the assembly, pick one of the extremal positions of the disk and start timing with your stopwatch.
   - You already learned the advantages of timing multiple pendulum periods to derive the single period time in Intro Lab 2. Decide how many periods are appropriate – you'll justify the choice later.
   - Repeat as indicated.

6. **Describe your experimental methodology.** Take a minute to develop your experimental procedure and ensure data reproducibility. Topics addressed should include: what was your goal, what challenges did you encounter and how did you design the experimental protocol to account for them? This section could include: how did you ensure the measurements were as reproducible as possible, how random errors were minimized, what systematic errors were identified, etc.

Throughout this experiment, our goal was to measure the time it took for the weight to perform 10 periods pf oscillation about it's vertical axis of rotation. To find this period, we decided to start the weight at an angle, and then measure the time it took for the entire 10 periods to happen. Then, we would divide our result by 10 and find a single period. However, we encountered some sources of error that we had to fix, or at least minimise. These included measuring the angle that we initially rotated the weights by so that every part of the experiment was as identical as possible, making sure the weight was in equilibrium and at rest before starting the rotation, making sure our counting was correct, and minimising the reaction time errors, which are random but relevant. To reduce all these errors, we came up with this procedure:
Step 1: zero out the setup. Make sure the weight is at equilibrium and everything is stable before measuring
Step 2: measure the angle we rotate the weight by, to ensure reproducible experiments
Step 3: Let the weight rotate a few times, and 2 or 3 periods after, start the stopwatch at one of the endpoints.
Step 4: Stop the timer after the weight has completed 10 periods.
Step 5: Change the mass on the system and repeat steps 1-4 until the experiment is complete.

7. **Measure periods of oscillation**

10-Period Times

10-Period Time for 0 disk

Trial #	10-Period Time (units: s)
1	10.47 ± 0.01
2	10.41 ± 0.01
3	10.43 ± 0.01
4	10.46 ± 0.01
5	10.47 ± 0.01

Result with total error:
$\mathbf{10 \cdot T_{0}} =$ 10.452 ± 0.011 s

10-Period Time for 1 disk

Trial #	10-Period Time (units: s)
1	12.44 ± 0.01
2	12.46 ± 0.01
3	12.52 ± 0.01
4	12.47 ± 0.01
5	12.43 ± 0.01

Result with total error:
$\mathbf{10 \cdot T_{1}} =$ 12.464 ± 0.016 s

10-Period Time for 3 disks

Trial #	10-Period Time (units: s)
1	16.69 ± 0.01
2	16.60 ± 0.01
3	16.62 ± 0.01
4	16.59 ± 0.01
5	16.64 ± 0.01

Result with total error:
$\mathbf{10 \cdot T_{3}} =$ 16.628 ± 0.012 s

10-Period Time for 5 disks

Trial #	10-Period Time (units: s)
1	19.71 ± 0.01
2	19.69 ± 0.01
3	19.74 ± 0.01
4	19.72 ± 0.01
5	19.69 ± 0.01

Result with error:
$\mathbf{10 \cdot T_{5}} =$ 19.711 ± 0.009 s

10-Period Time for 7 disks

Trial #	10-Period Time (units: s)
1	22.41 ± 0.01
2	22.45 ± 0.01
3	22.46 ± 0.01
4	22.44 ± 0.01
5	22.41 ± 0.01

Result with error:
$\mathbf{10 \cdot T_{7}} =$ 22.434 ± 0.010 s

10-Period Time for 9 disks

Trial #	10-Period Time (units: s)
1	24.41 ± 0.01
2	24.42 ± 0.01
3	24.42 ± 0.01
4	24.45 ± 0.01
5	24.43 ± 0.01

Result with error:
$\mathbf{10 \cdot T_{9}} =$ 24.426 ± 0.007 s

DATA TAKING NOTES:

8. ***Calculate periods of oscillation from data.*** You have now measured the time it takes for each set of disks to complete 10 periods of oscillation (or whatever number your group chose). Use this data to calculate the time it takes for a single period of oscillation for each disk set. <u>Be careful to calculate the proper error on these single periods</u>.

# of Disks	**10-Period Time with error:** $10 \cdot T \pm \alpha_{10 \cdot T}$ (units: s)	**Period with error:** $T \pm \alpha_T$ (units: s)
0	10.452 ± 0.011 s	1.0452 ± 0.0011
1	12.464 ± 0.016 s	1.2464 ± 0.0016
3	16.628 ± 0.012 s	1.6628 ± 0.0012
5	19.711 ± 0.009 s	1.9711 ± 0.0009
7	22.434 ± 0.010 s	2.2434 ± 0.0010
9	24.426 ± 0.007 s	2.4426 ± 0.0007
Extra space		

9. ***Propagate error symbolically (classroom check).***

Before starting the linear regression analysis, let's evaluate the errors.

a) Write the equation for the **<u>total</u>** moment of inertia I for the disks plus the holder, then symbolically calculate its error, δI. Use n for the number of disks in your formula. You can assume the error for each disk is independent and therefore disk errors add in quadrature.

$$I_{TOTAL} = I_{HOLDER} + n \cdot I_{DISKS} = \frac{1}{8} M_{holder} D^2 + n \cdot \frac{1}{8} M_{Disk} D^2$$

$$\alpha_{I_{tot}} = \sqrt{\left(\frac{\partial I_H}{\partial M_H}\alpha_{M_H}\right)^2 + \left(\frac{\partial I_H}{\partial D}\alpha_D\right)^2 + n\left[\left(\frac{\partial I_D}{\partial M_D}\alpha_{M_D}\right)^2 + \left(\frac{\partial I_D}{\partial D}\alpha_D\right)^2\right]}$$

$$\boxed{\alpha_{I_{tot}} = \frac{D}{8}\sqrt{\left(\alpha_{M_H}^2 + 4M_H^2\alpha_D^2 + n^2\alpha_{M_D}^2 + 4n^2 M_D^2 \alpha_D^2\right)}}$$

b) In the Analysis section, you will be graphing $(x, y) = (T^2, I)$ Calculate the error α_{T^2} in terms of measured quantity T and the associated α_T.

$$\boxed{\alpha_{T^2} = \sqrt{\left(\frac{\partial T^2}{\partial T}\alpha_T\right)^2}}$$

10. ***Discussion.*** Use the space below to briefly answer the following questions:

Did you need to refine your measurement procedure? If so, explain.

As we went on with the experiment, we didn't really need to refine our experimental procedure, since our results seemed to be fairly similar and had some type of pattern.

What could be done to improve your measurement?

To refine our experiment, we thought that there were some things that could be standardised, such as the number of periods we waited before starting the stopwatch. Waiting for longer might reduce the timed result due to the effects of air resistance. Also, if there was some way to reduce reaction time, we would lose some random errors. Aside from this, we concluded that our measurements were fairly standard and good given the instruments available.

Justification for dataset used in analysis: Each student is responsible for acquiring a full dataset for the experiment. The group, as a whole, can decide which of the three generated datasets to analyze and provide a *justification* for the decision.

As a group, we decided to have each person analyze their own dataset, since it would avoid complications. Also, all datasets were fairly similar, so we couldn't really find an advantage in using any of the others.The standard error was similar in all of them, hence our decision to perform individual analysis on each individual's dataset.

Analysis

1. Overview of weighted least squares fit

We use a ***weighted*** **least-squares approach** when we have unequal errors and in our data points. In general, for weighted least squares fit we want data points with low uncertainty to "matter more" than data points with high uncertainty so we attach a ***weight*** to each data point,

$$w_i = \frac{1}{\alpha_i^2}. \tag{3}$$

This weight gets attached to our *Q* from earlier, which now becomes the same as χ^2

$$\chi^2 = \sum w_i\big(y_i - y(x_i)\big)^2 = \sum w_i(y_i - mx_i - c)^2.$$

For a linear model, $y(x) = mx + c$, the best fit parameters are:

Best-fit parameters:

$$m = \frac{\sum w_i \sum w_i x_i y_i - \sum w_i x_i \sum w_i y_i}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2},$$

$$c = \frac{\sum w_i x_i^2 \sum w_i y_i - \sum w_i x_i \sum w_i x_i y_i}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2} = \frac{\sum w_i y_i - m \sum w_i x_i}{\sum w_i}.$$

Uncertainties in best-fit parameters:

$$\alpha_m = \sqrt{\frac{\sum w_i}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2}},$$

$$\alpha_c = \sqrt{\frac{\sum w_i x_i^2}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2}}.$$

If you have uncertainties α_x and α_y in our independent and dependent variables which are approximately equal, you proceed slightly differently. The first thing to do is *eliminate* the uncertainty in x.

To do this, you perform a simple least-squares linear regression to find a best-fit slope m_{simple}, then exchange the uncertainty in *x* for additional uncertainty in *y:*

$$\alpha_{equiv,i} = \sqrt{\alpha_{y,i}^2 + m_{simple}^2 \cdot \alpha_{x,i}^2}. \tag{4}$$

replacing α_i in Equation 3, and in χ^2, with $\alpha_{equiv,i}$. Then you proceed with normally with the weighted fit. This procedure is spelled out in the next sections.

2. Preparation for plotting: linearize

You will be using the non-linear equation:

$$I = (\kappa/4\pi^2)\,T^2 \tag{5}$$

to describe the pendulum motion. To use our linear regression techniques, we need to convert Equation 5 to a linear model. Therefore, we "linearize" using $(x, y) = (T^2, I)$. The data should fall on a line with slope $\kappa/4\pi^2$.

Note! When we derived our linear regression formulae, we assumed throughout that our T and I data were normally distributed. In this analysis, we further assume that T^n are normally distributed, so we can proceed with our linear regressions as usual.

In preparation for plotting, then, let's now summarize our data as the pairs $(x, y) = (T^2, I)$. Note: you already recalculated the errors α_{T^2} from the errors α_T in section 9, and should use the result here. **To calculate total moment of inertia, remember to include both disks and disk holder, as you did in section 9.**

# of Disks	**Period squared with error:** $\boldsymbol{T^2} \pm \alpha_{T^2}$ (units: s)	**Total moment of Inertia with error:** $\boldsymbol{I} \pm \alpha_I$ (units: Kgm^2)
0	1.092 ± 0.002	$6.22\times10^{-6} \pm 2.01\times10^{-6}$
1	1.554 ± 0.004	$8.71\times10^{-6} \pm 5.57\times10^{-6}$
3	2.765 ± 0.004	$1.37\times10^{-5} \pm 1.25\times10^{-6}$
5	3.885 ± 0.004	$1.87\times10^{-5} \pm 2.01\times10^{-6}$
7	5.033 ± 0.004	$2.37\times10^{-5} \pm 2.79\times10^{-6}$
9	5.966 ± 0.003	$2.87\times10^{-5} \pm 3.77\times10^{-6}$

3. Implementing weighted least squares fit: horizontal error bars

Looking at the table above, you notice that both the x- and y-values have error bars. Since the errors are comparable in your data and the relative magnitudes for α_x or α_y are not constant, you cannot neglect this issue in the experiment. But the linear regression model we've used so far can only handle approximately equal y error bars!

For this experiment, you will exchange the uncertainty in x for additional uncertainty in y using Equation (4). So, for $(x, y) = (T^2, I)$ so you will add a contribution $\alpha_{I_0} = (\kappa_0/4\pi^2)\alpha_{T^2}$ to the experimentally derived error of I, where κ_0 is your estimate of κ from simple linear regression. This error gets added in quadrature to the existing error α_I in I, so the new total error in I is

$$\alpha_{I'} = \sqrt{(\alpha_I)^2 + \left(\alpha_{I_0}\right)^2}$$

You then use this new error to fit the data and obtain a better estimate of κ, using weighted linear regression.

D. York, N. Evensen, M. Lo, M. Nez, J. De, B. Delgado, "Unified equations for the slope, intercept, and standard errors of the best straight line", Am. J. of Physics 72, 367 (2004).

For a fully rigorous method, a regression technique such as Deming regression would be used. Physics 5BL will not require this level of analysis.

Recalculate your y axis error bars and fill in the chart:

# of Disks	Period squared: T^2 (units: s)	Total moment of Inertia with error: $I \pm \alpha_{I'}$ (units: Kg m²)
0	1.092	$6.22 \times 10^{-6} \pm 2.01 \times 10^{-6}$
1	1.554	$8.71 \times 10^{-6} \pm 5.57 \times 10^{-6}$
3	2.765	$1.37 \times 10^{-5} \pm 1.25 \times 10^{-6}$
5	3.885	$1.87 \times 10^{-5} \pm 2.01 \times 10^{-6}$
7	5.033	$2.37 \times 10^{-5} \pm 2.79 \times 10^{-6}$
9	5.966	$2.87 \times 10^{-5} \pm 3.77 \times 10^{-6}$

- **What is your κ_0?**

$\boxed{K = 1.78 \times 10^{-4}}$

4. Evaluate and perform the weighted linear regression

As we've just discussed, simple linear regression is appropriate when the y data point errors are essentially the same. For this data set, weighted linear regression will be required.

Now plot your $(x, y) = (T^2, I)$ data from Section 4 and fit with the theoretical curve $y = mx + c$ using **weighted linear regression**.

- **BE SURE TO PRINT OUT YOUR PLOT AND ATTACH IT TO THIS WORKSHEET.**

Make sure to label your plot fully! It should have a descriptive title, axis labels with units, the equation of the line-of-best-fit, data with error bars, the values of the slope and intercept with errors, and the value of $\tilde{\chi}^2$.

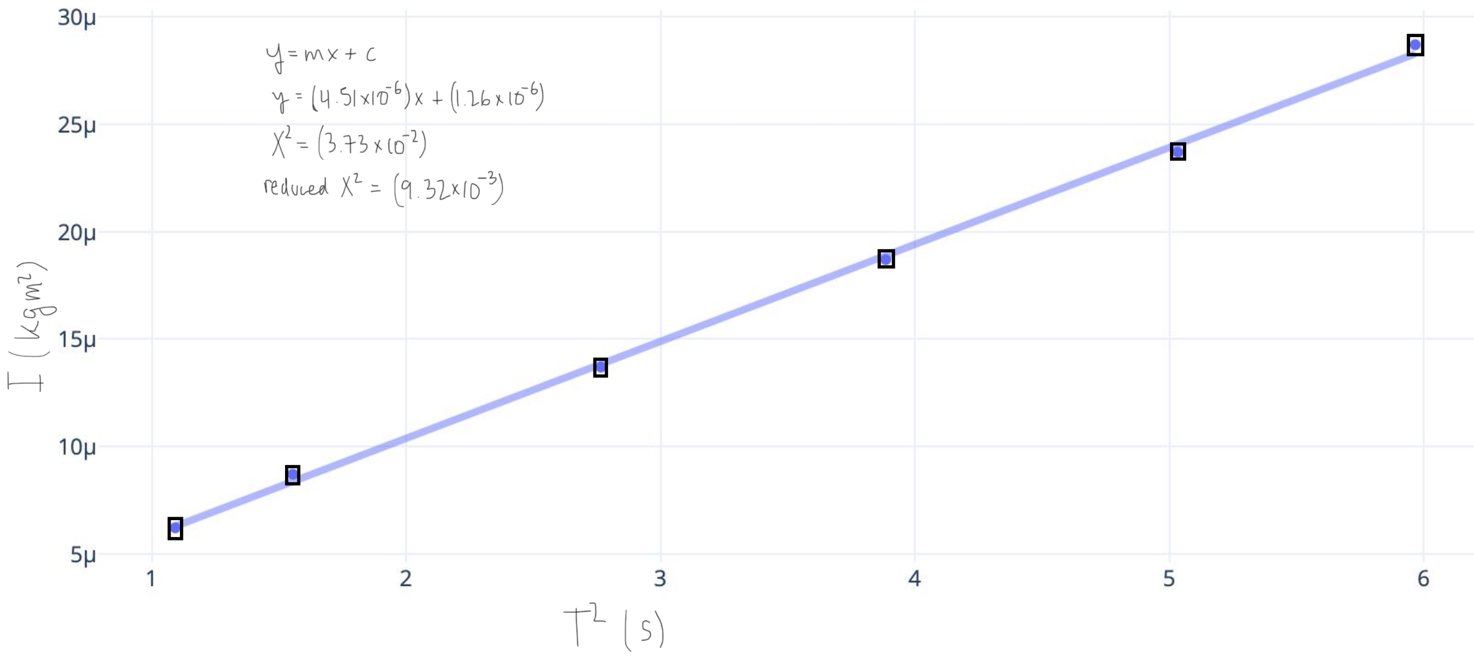
5. Plot the residuals

For Intro Labs 3-4, you have been calculating the residuals associated with your fit, $R_i = y_i - y(x_i)$, creating scatter plots of the residuals, checking for randomness in the scatter plot and that at least 2/3 of the error bars intersect the line of best fit.

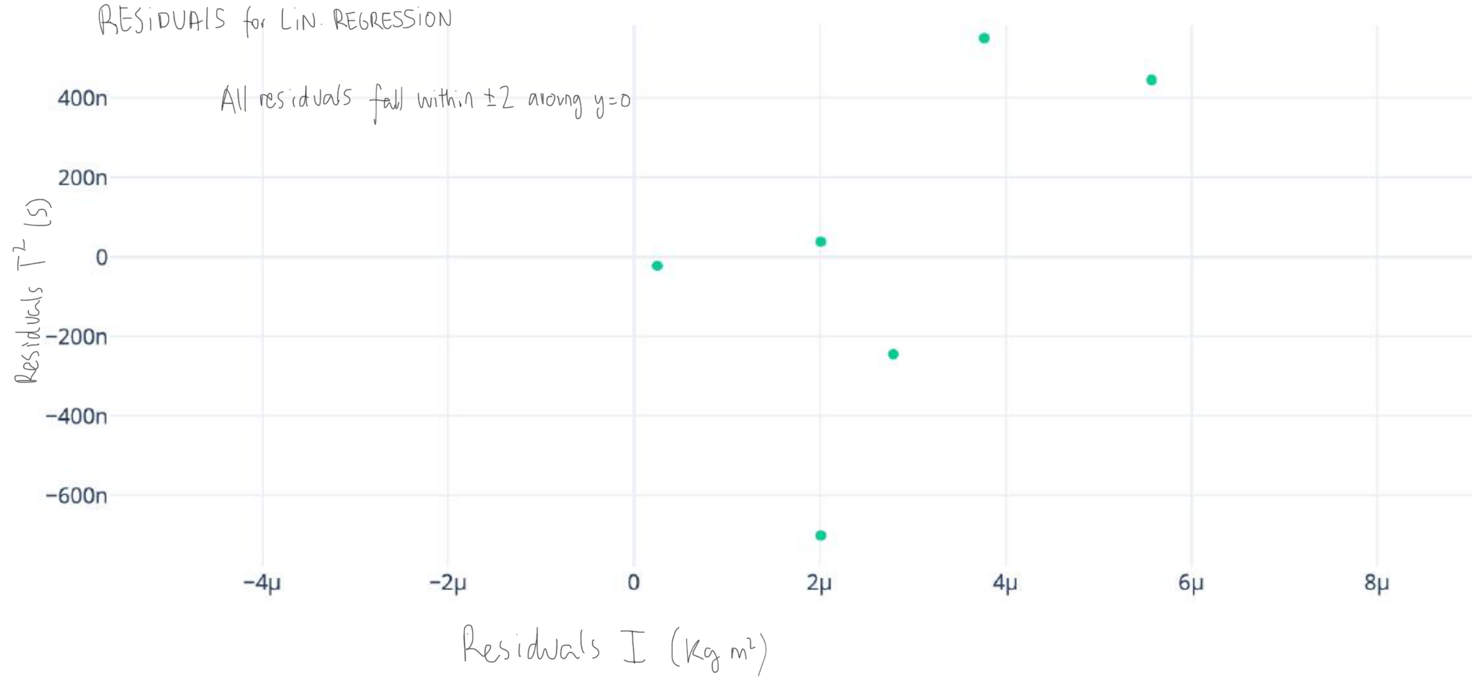
For data with non-uniform error bars, the raw residuals can be more difficult to use. We define instead the normalized residual, R_i

$$R_i = \frac{y_i - y(x_i)}{\alpha_i}$$

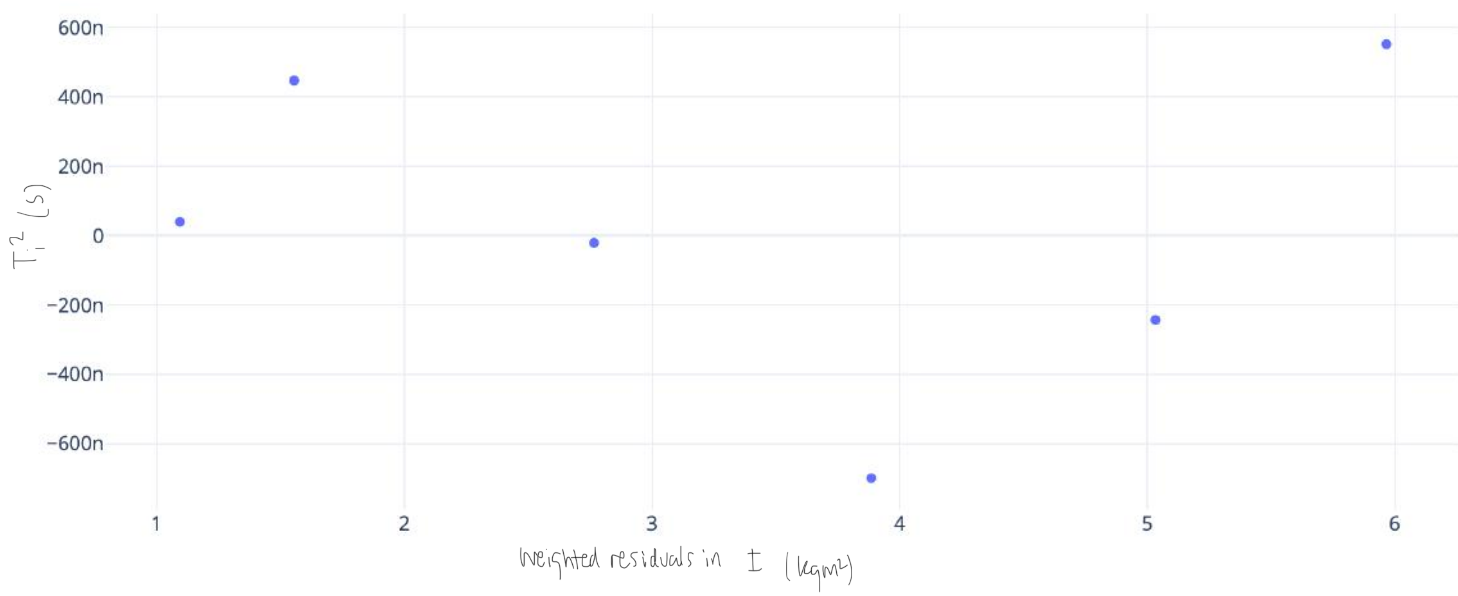
WEIGHTED LINEAR REGRESSION. T^2 vs I.



RESIDUALS for LIN. REGRESSION



NORMALIZED RESIDUALS FROM GRAPH ABOVE



where y_i, α_i and $y(x_i)$ are the i^th measurement, its uncertainty and the value of the fit, respectively. The normalized residuals should be randomly scattered, with 95% falling within ± 2 around 0 on the y -axis for a good fit.

Plot the normalized residuals

$$(x_i, y_i) = \left(T_i{}^2, \frac{I_i - I(x_i)}{\alpha_{I'}}\right)$$

Are the residual y values including the error bars randomly distributed?

Yes **No**

PRINT OUT YOUR NORMALIZED RESIDUALS PLOT AND ATTACH IT TO THIS WORKSHEET.

6. Calculate $\boldsymbol{\kappa}$ from the line-of-best fit

Now that you have a line-of-best fit, with its slope and error $m \pm \alpha_m$ you are ready to calculate the torsional spring constant $\kappa \pm \alpha_\kappa$. Write the equations you use to relate the slope m and its error to κ and its error on the next page.

Equation relating $\boldsymbol{m}$ and $\boldsymbol{\kappa}$	Equation relating $\boldsymbol{\alpha_m}$ and $\boldsymbol{\alpha_\kappa}$	$\boldsymbol{m \pm \alpha_m}$ from line-of-best-fit	$\boldsymbol{\kappa \pm \alpha_\kappa}$
$K = m \cdot 4\pi^2$	$\alpha_\kappa = 4\pi^2 \alpha_m$	$\frac{K}{4\pi^2} \pm \frac{\alpha_\kappa}{4\pi^2}$	$m4\pi^2 \pm \alpha_m 4\pi^2$

7. χ^2 analysis (revised: $0.5 \leq \tilde{\chi}^2 \leq 3$)

At last, you have all your results. Now it's time to determine whether or not your model and data are "acceptable". Then we will determine whether or not your measured value of the torsional spring constant κ can meaningfully be said to agree with the accepted value.

First, write down your chi-square, count the number of degrees of freedom, and calculate your reduced chi-square.

Chi-square: χ^2	Number of degrees of freedom: ν	Reduced chi-square: $\tilde{\chi}^2 = \chi^2/\nu$
3.73×10^{-2}	4	9.32×10^{-3}

Now, determine whether or not your reduced chi-square is too big, too small, or acceptable, with acceptable: $\mathbf{0.5} \leq \tilde{\chi}^2 \leq \mathbf{3}$, too big: $\tilde{\chi}^2 \geq \mathbf{3}$, too small $\tilde{\chi}^2 \leq \mathbf{0.5}$

My reduced chi-square $\tilde{\chi}^2 =$ 3.73×10^{-2} is …

Too big **Too small** **Acceptable**

Finally, interpret what this means. Based on your value of $\tilde{\chi}^2$, what can you conclude about your experimental results? If your value of $\tilde{\chi}^2$ is too big or too small, why do you think this might this have happened?

SInce our chi squared is too small, this can be interpreted as saying that our errors were overestimates. this means that we took the errors to be larger than what they actually were. Our experimental results were therefore not only incorrect, but pessimistic.

What would be your next steps to resolve any possible issues?

Quite probably, even though we could do a re-assessment of our values for the errors ,the quickest thing to do would be to redo the experiment, being more careful about how correct our results and errors are.

8. Discussion

If your results do not agree with the accepted value, what might be the reasons? Consider experimental design, random errors, systematic errors, modeling errors etc.

Aside from all the errors mentioned previously in the experimental procedure section, our results might've been incorrect due to errors in our modelling or anaylisis of the data fitting. A mistake in the data analysis can lead to a different set of normalized results.

What could be done to improve your measurement?

Being more careful in the data analysis could reduce random errors we could've made. Furthermore, we could've measured the angle better, which wouldve changed our value for k_0, changing all our results.

Do you suspect any appreciable error from your fitting procedure? Explain.

As mentioned above, we suspect we confused the normalized weighted residuals with the weighted residuals, which would've changed our results. I did the calculations after this was finished and found that this would've yielded a chi squared of 0.372, which is considerably better. Any other errors that make chi squared be 0.372 and not 0.5 are random errors during the measurement.