<u>Problem 1</u>: 500 nm lightwave in vacuum enters a glass plate of index 1.6 and propagates perpendicularly across it. How many waves span the glass if it's 1 cm thick?

The wavelength of the wave in space is $500 \cdot 10^{-9}$ m. Inside the glass, it becomes $\frac{500}{1.6} \cdot 10^{-9} = 312.5 \cdot 10^{-9}$ m

Using the formula for how many waves span 1 cm with wavelength $312.5 \cdot 10^{-9}$ m, we find that $N = \frac{0.01 \text{ m}}{312.5 \cdot 10^{-9} \text{ m}}$.

Therefore, $\boxed{N = 32000}$

## Problem 2:

$$n_i \sin\theta_i = n_t \sin\theta_t \longrightarrow \sin\theta_t = \frac{n_i}{n_t}\sin\theta_i \quad \text{and} \quad \cos\theta_t = \sqrt{1-\sin^2\theta_t}.$$

$$r_\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{n_i\cos\theta_i - n_t\sqrt{1-\sin^2\theta_t}}{n_i\cos\theta_i + n_t\sqrt{1-\sin^2\theta_t}} \approx -0.2334$$

$$r_{\parallel} = -\frac{\frac{n_i}{\cos\theta_i} - \frac{n_t}{\cos\theta_t}}{\frac{n_i}{\cos\theta_i} + \frac{n_t}{\cos\theta_t}} \approx 0.1979$$

Now, we can find the amplitude of the reflected field:

$$E_{r,\perp} = r_\perp \cdot E_{incoming,\perp} = -0.2334 \cdot 20 = -4.668\ V/m$$

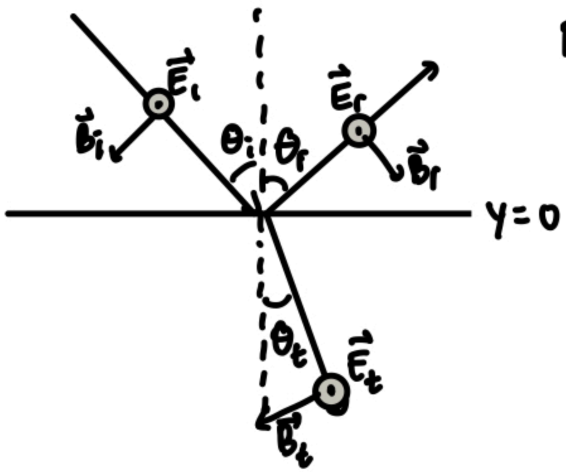
$$E_{r,\parallel} = r_\parallel \cdot E_{incoming,\parallel} = 0.1979 \cdot 10 = 1.9786\ V/m$$

## Problem 3:

Here, we just use the equation of the critical angle between two surfaces:

$$\sin\theta_c = \frac{n_r}{n_i} \longrightarrow \theta_c = \sin^{-1}\left(\frac{n_{water}}{n_{glass}}\right) = \sin^{-1}(0.858) = \boxed{1.03 \text{ rad.}}$$

# Problem 4




Boundary condition:

- The tangential electric field is continuous. Therefore, on the interface $y=0$, we have:

  $$\vec{E}_i(y=0) + \vec{E}_r(y=0) = \vec{E}_t(y=0)$$

- The tangential magnetic fields is continuous. Therefore,

  $$-B_i\cos\theta_i + B_r\cos\theta_r = -B_t\cos\theta_t$$

We now have:

$$\begin{cases} E_{0i} + E_{0r} = E_{0t} \\ -B_{0i}\cos\theta_i + B_{0r}\cos\theta_r = -B_{0t}\cos\theta_t \end{cases}$$

However, from Maxwell's and other laws, we know that $\theta_i = \theta_r$ and $B = \frac{En}{c}$. Therefore, we have:

$$-B_{0i}\cos\theta_i + B_{0r}\cos\theta_r = -B_{0t}\cos\theta_t \rightarrow n_i(E_{0r}-E_{0i})\cos\theta_i = -n_t E_{0t}\cos\theta_t$$

$$\rightarrow n_i(E_{0r}-E_{0i})\cos\theta_i = -n_t(E_{0r}+E_{0i})\cos\theta_t$$

Now, we can solve for $E_{0r}$ and $E_{0i}$. Since the reflection coefficient $r_\perp = \frac{E_{0r}}{E_{0i}}$ and the transmission coefficient $t_\perp = \frac{E_{0t}}{E_{0i}}$, we have:

$$\boxed{r_\perp = \frac{(n_i\cos\theta_i - n_t\cos\theta_t)}{(n_i\cos\theta_i + n_t\cos\theta_t)} \quad \text{and} \quad t_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}}$$