Lab Report 4

Tier 1: Tier 1-RC and RLC circuits

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1.1 Introduction/Objectives:

Throughout this lab, there were two main parts. In the first one, we had to figure out how to measure how to measure the time constants for different circuits excited by a DC source which is abruptly being turned on and off. In the second one, we studied the behavior of RLC circuits as a pose to the RC circuits in the first part. However, even though both these parts focused on different concepts and theory, they did have in common the measurement of time constants together with coming up with the slowest or fastest series circuits achievable with the Lab Kit.

1.2 Theory:

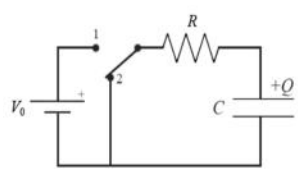
Before we go on to showcase our work, it is important that we understand the theory behind the experiments we carried out. This theory section will mostly be divided in two parts: understanding RC and DC circuits and understanding transient behaviors in RLC circuits.

1.2.1 Capacitance and DC-RC Circuits

First of all, capacitance is the ratio of the amount of electric charge stored on a conductor to a difference in electric potential. This means that if we have two planes sheets of conductor, for example, and they have charges $+Q$ and $-Q$, respectively, there will be a potential difference between them. This potential difference V will be proportional to the strength of the $\mathbf{E}$ field and the separation between the planes, d. The capacitance is defined as the relationship between V and the total charge of the system, Q. The capacitance is therefore

$$C \equiv \frac{Q}{V}$$

The capacitance is therefore a geometrical property of an object. Since C depends on V and V depends on $\mathbf{E}$, changing $\mathbf{E}$ in dramatic or special ways can drastically change the capacitance. In fact, for some particular value of $\mathbf{E}$, the system suffers a dielectric breakdown, and the space between the two planes also becomes a conductor! This has a lot of importance, and failing to take this into account in real-life scenarios can lead to broken circuits. Now that we understand what capacitance and a capacitor (the system of two conducting planes) are, we can start creating circuits with them. An example is shown below:



The RC Circuit in this lab consists of a capacitor C and a resistor R connected in series to a DC source V_0, hence the name, RC-DC circuit. Initially, we have that the switch has been in Point 2 for a long time so that the capacitor is completely discharged when we start the experiment. However, when the switch is switched to Point 1, the current starts to flows out of the battery and begins to charge the capacitor. If the switch keeps toggling, the capacitor starts charging and discharging. To mathematically explain this behavior, we use Kirchhoff's Laws and Ohm's Law to find that the behavior is exponential, and it looks like this:

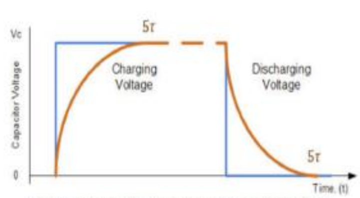


Figure 2 Charging and discharging behavior of a capacitor

1.2.2 Transient Behavior in RLC Circuits

In this part of the theory section, we will be combining the RC circuits in section 1.2.1 with the RL circuits we talked about in previous labs. We will do so by making use of the RLC circuit, which consists of a resistor, an inductor, and a capacitor all in series. This circuit type is particularly interesting because the energy transfers from the **E** field in the capacitor to the **B** field in the inductor, and does so for every loop the current completes around the circuit. Therefore, the energy will 'oscillate' between the two fields, so the RLC will behave as the electromagnetic counterpart of the harmonic oscillator. Our goal now is to describe this harmonic motion of the energy using a differential equation. We can use the definitions of V, R, I, and L to come up with the differential equation

$$L\frac{d^2I(t)}{dt^2} + R\frac{dI(t)}{dt} + \frac{I(t)}{C} = 0$$

We can see that the differential equation relates R, I, and L, and is of similar form to the harmonic motion differential equation in classical mechanics. Therefore, its solutions are equivalent, and we can just substitute in the symbols to find that the resulting harmonic motion can be underdamped, overdamped, or critically damped, depending on the damping constant, R. Our three scenarios are given by

Underdamped ($\alpha < \omega_0$)	$I(t) = A\omega e^{-\alpha t}\left(\cos\omega t + \frac{\alpha}{\omega}\sin\omega t\right),$
Critically ($\alpha = \omega_0$)	$I(t) = e^{-\alpha t}(A + Bt),$
Overdamped ($\alpha > \omega_0$)	$I(t) = e^{-\alpha t}(Ae^{bt} + Be^{-bt}),$

Note that above the amplitude A depends on the initial conditions of our differential equation and the frequency ω of the oscillations depends on various conditions, and is mathematically expressed by

$$\omega = \sqrt{\omega_0^2 - \alpha^2}$$

With this in mind, we can now start with the experiments, and this is covered in section 1.3.

1.3 Methods:

This Lab is separated into two separate experiments, each with sub-experiments, for which we will outline the procedure to perform them. For this lab, the equipment needed includes:

- IOLab
- Breadboard
- Wires
- Resistors
- Capacitors (we couldn't find the 56 μF ones so we used the 47 μF ones)
- Inductor
- Aluminum Foil
- Paper and scissors
- Glue and tape

1.3.1 Method for Experiment 1a

In this experiment we want to measure the time constant τ. to do so we set up a circuit with a 10 kΩ resistor in series with a 47 μF capacitor (we couldn't find the 56 μF ones), as shown in the image. We then use the A1 and A2 sensors on the IOLab to measure Voltage before and after the resistor. For the measurement:

1. Set the DAC to 2.0 V
2. Start recording and turn on the DAC
3. Once enough time has passed, turn off the DAC
4. Repeat twice then stop recording.

1.3.2 Method for Experiment 1b

In this part, we are expected to find the combination of RC circuit elements that yields the slowest and the fastest time constant, τ. To do this, each person needs their own creativity, but we thought of the following two setups. To make the time constant be the smallest possible, we are interested in making the resistance as small as possible while reducing the net capacitance of the system. To do this, we thought that having many resistors in parallel is the only way, since parallel resistors are follow a $\frac{1}{R}$ law. Therefore, what we have to set up is a chain of as many resistors as possible with high resistance. A picture of our setup is shown in the appendix. Therefore, with resistors in parallel the time will be the fastest it can be. On the other hand, to ensure that our time constant is the largest possible, we can have a system of capacitors in parallel. Again, our setup is shown in the Appendix. The reason why many capacitors in parallel work is that when placed like this, the capacitance adds together, so the time constant will have larger values. Therefore, with capacitors in parallel the time will be the slowest it can be, since a larger time constant means more time.

1.3.3 Method for Experiment 1c

In this part of the experiment we build our own capacitor using paper and aluminum foil. To build the capacitor, we cut pieces of aluminum foil that are slightly smaller than the paper, and then glue the aluminum foil to the both sides of the paper. Since capacitance is inversely proportional to separation distance between the two plates, it is very important to make sure the foils are flat and stick very well to the paper. It is also important that we make sure the layers of tin foil are occupying the same parts of the page, just on opposite sides. We then replace the capacitor in the circuit from experiment 1a with this homemade capacitor, and repeat the experiment. We do so by just placing the metal cables in contact with the foil sheets and connecting them to the breadboard. In the event where the Capacitor is not being charged, this is likely due to the capacitor losing charge to the surface it is placed on, so adjust the capacitor so as to prevent this. Now, the goal is to measure the capacitance of your home-built capacitor using the charging and discharging curves, and we will do so in the analysis section.

1.3.4 Method for Experiment 2a

For the entirety of part 2, we will be shifting our attention to RLC circuits and how they oscillate. A brief description of this is provided too in the Theory section. In this experiment, we observe the curves made by measuring voltage in an overdamped RLC circuit. As described in the Theory section, the overdamped oscillations of energy in the circuit should follow some equation. In this experiment, we wanted to prove this. To do so:

1. Construct the circuit with a 104 mH inductor, a 47 μF capacitor, a 10k Ω resistor, and a 1 Ω resistor in series
2. Use wires to connect A1 sensor before the inductor and and the A2 sensor after the capacitor
3. Set the DAC to 3.0 V
4. Start recording and turn the DAC on, then after enough time has passed turn it off.
5. Continue turning DAC on and off until enough data has been collected.

1.3.5 Method for Experiment 2b

Here we again observe the transient behavior from the curves formed from an overdamped RLC circuit, this time using the high gain sensor across the 1 Ω. As such for this experiment we remove the A1 and A2 sensors and place the high gain before the 1 Ω resistor and the low gain after the resistor, and repeat the steps from experiment 2a, but this time measuring the high gain by activating the corresponding sensor on the IOLab.

1.3.6 Method for Experiment 2c

Here we observe the transient behavior for a circuit that is slightly underdamped. to create the circuit, we place the inductor, two 47 μF capacitors, two 22 μF capacitors, a 10 kΩ resistor, and a 1 Ω resistor, and measure the high gain across the 1 Ω resistor. To do so, we connect the high gain and low gain sensors before and after the resistor respectively, and repeat the procedure of experiments 2a and 2b.

1.3.7 Method for Experiment 2d

For this part, we decided to observe the behavior of the overdamped RLC circuit under the smallest R we could produce and the largest R we could produce. to do so, we once again build an RLC circuit using the 104 mH indcutor and a 47 μF capacitor in series with three 10k Ω resistors in parallel for the smallest R (as many as would fit), and four in series for the largest R (could have added even more resistors, but kept at 4 for the sake of simplicity) followed by a 1 Ω resistor, through which we measure the high gain with 3.0 V using the DAC, repeating the same measurement steps as in the parts a-c until sufficient data has been collected.

1.4 Analysis and Data Interpretation:

In this section, we will be discussing the analysis of the data we obtained. In other words, we will perform all the calculations that we learned during the Intro Labs and discuss our results. Similarly to the Methods section in this report, we will perform the analysis and discussion part by part, since the experimental setups are different. For each part, we will be considering and answering questions such as why we chose a specific dataset, how our results match the accepted values, whether the behaviour of our mass fits the expected behavior given by the formulae developed in the Theory section, and why all the possible discrepancies could happen.

1.4.1 Analysis for Experiments 1a and 1b:

In this analysis, we had two main goals: 1) finding the RC time constant for Experiment 1a, and 2) comparing the experimental results with the expected results for Experiment 1b. We are hoping to see that our experimental results in part a match with our theoretical results. To find the theoretical results, we will just have to plug numbers into formulae provided in the Theory section and in the Lab Manual. However, to get to that point, we first need to analyze and understand our data from 1a and 1b. To do so, we answer the questions in the lab manual: **1) Determine the discharge time constant for each Experiment's circuit(s):**, **2) Solve for the fit amplitude A and the fit time constant B, and the associated errors:**, **3) Compare the fitted B to the theoretical time constant τ using the agreement test:**, **4) If we don't find agreement, what are the possible reasons?:**

Our answer to all these four questions is below, we indicate with a number the specific spot where we start our answer:

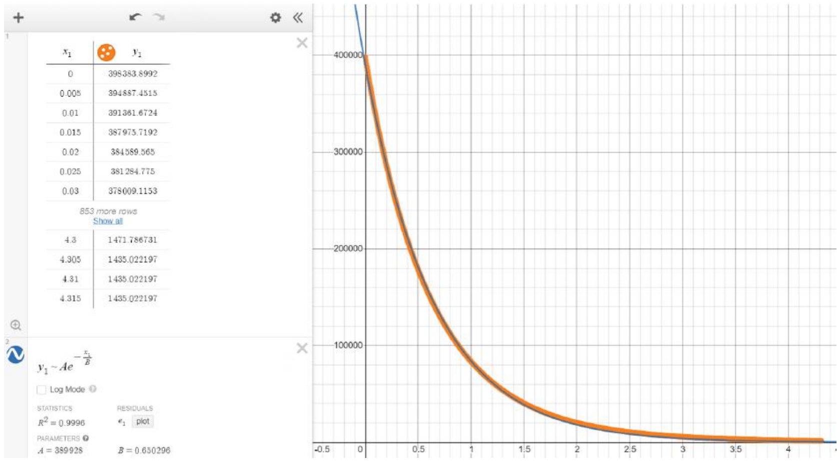
Over the course of experiment 1a, we switched the DAC voltage on and off 3 times for a total of 6 curves. We fitted the data that we obtained in 1a to the charging and discharging curves that we know from theory. The charging and discharging formulas are, respectively

$$\frac{V(t)}{V_0} = A - Ae^{-\frac{t}{B}} \quad \text{and} \quad \frac{V(t)}{V_0} = Ae^{-\frac{t}{B}}$$

Now, using these equations and the data we had, we created some fits. In fact, we fitted our three datasets (one for each group member). Below are shown one for charging and one for discharging, but the final results we show will be based on the average of our three fitting results. The fitting plots are:

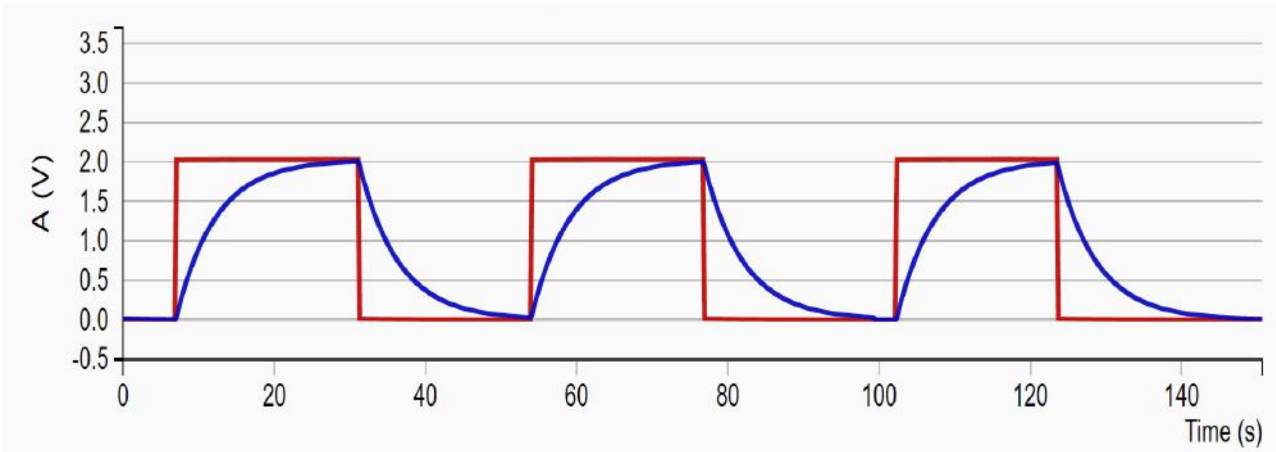
When charging:

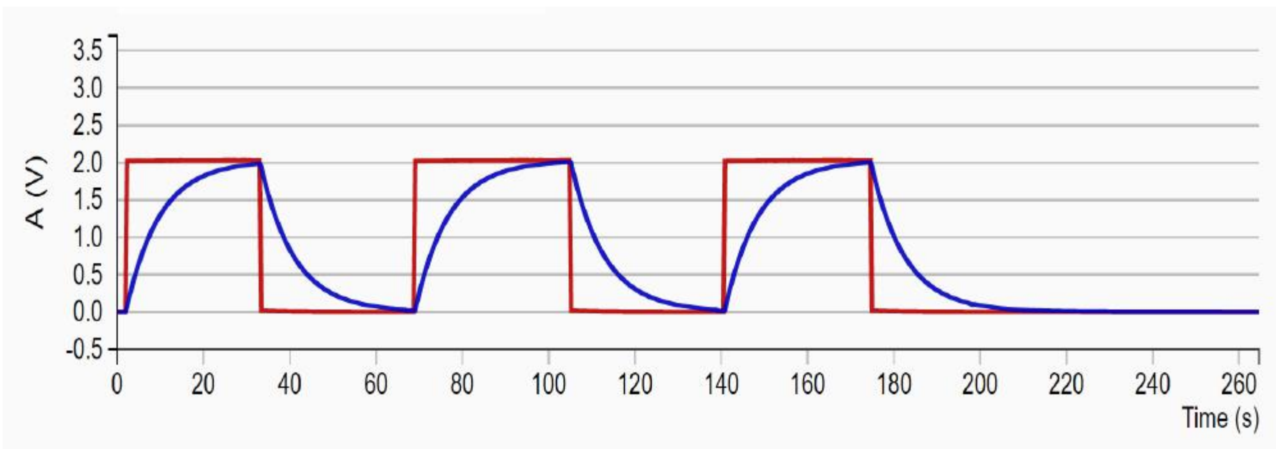
When discharging:

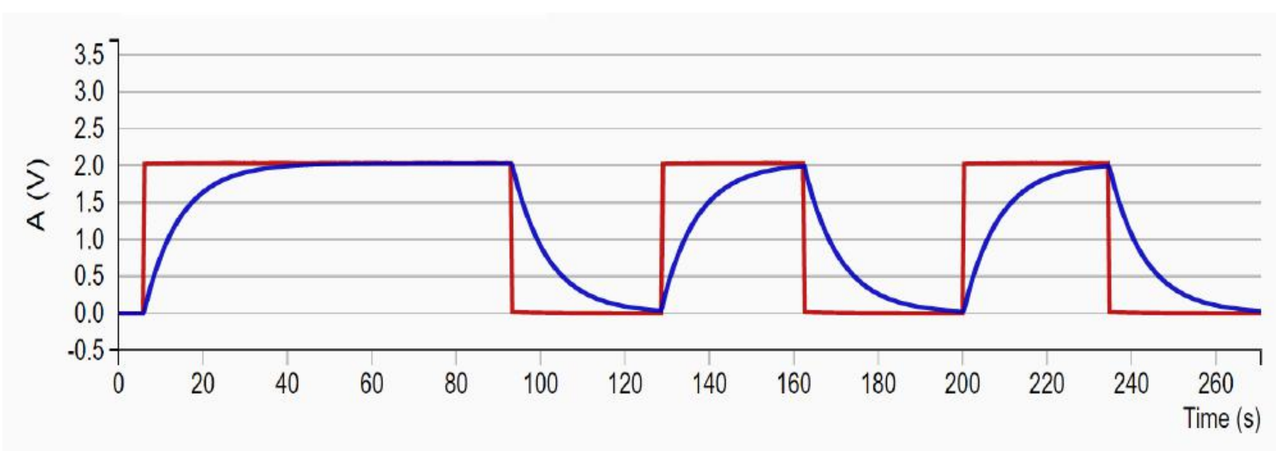


In our graphs, you can see the fitting values for A and the fitting time constant B (also known as τ) in the bottom left corner. **(2)** We averaged the different values we got for A, and found the fitted value to be .... **(1)** By taking the average of the three fittings we did (the 6 entire graphs can be found in the second appendix), we found the value for τ that the charging curves give us to be 0.610 ± 0.0020, and the value for τ that the discharging curves give us to be 0.651 ± 0.0005. **(3)** Theoretically, we expected τ to be $47 \cdot 10^{-6} \cdot 10^4 = 0.47$. Therefore, the values we calculated from the charging and discharging curves using the agreement test were off by 29.8% and 38.5%, respectively. The values we calculated deviate significantly from the expected values. Furthermore, the time constant calculated from the charging curve does not agree with the time constant calculated from the discharging curve. **(4)** We make hypotheses about both the difference between the charging and discharging values and about the difference between the theoretical and experimental values. We hypothesize that the difference between the time constants calculated from the charging and discharging curves is caused by the fact that, while charging, real capacitors experiences a parasitic resistance due to charge leakage, slowing down the charging. Discharging, meanwhile, is unaffected and even accelerated by the parasitic resistance. Similarly, the theoretical value for the time constant does not account for additional resistances introduced by the capacitor being unideal. In addition to this, there could've been measurement errors or logical flaws in our breadboard, but we have revised them and didn't find anything.

In this section, we also attempted to maximize the decay time by increasing the capacitance. We did this by adding increasing amounts of capacitors in parallel. In the graphs below, the first is from a circuit with effective capacitance 440 μF, the second with 660 μF, and the third with 707 μF.
As expected, the decay time is maximized when the capacitance is the highest.

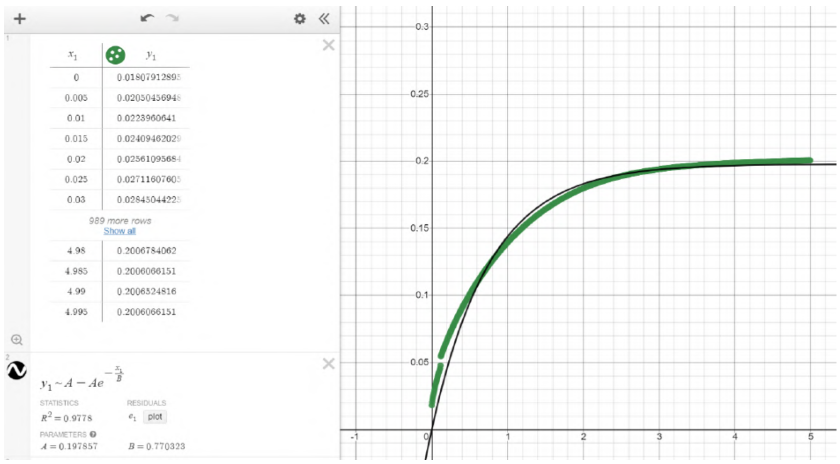

A (V)
Time (s)


A (V)
Time (s)

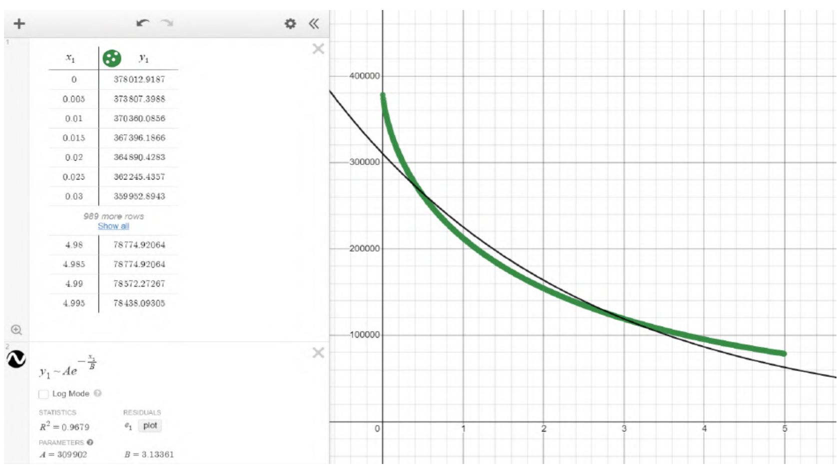

A (V)
Time (s)

1.4.2 Analysis for Experiment 1c:
In this analysis, our goal was to derive the capacitance of a homemade capacitor, and to determine how well this aligned with an estimate made based on the materials used and the dimensions of the capacitor. We were also tasked with determining an effective dielectric thickness of our homemade capacitor, and finding out how commercial capacitors achieve extremely high capacitance while remaining very small.

When charging:



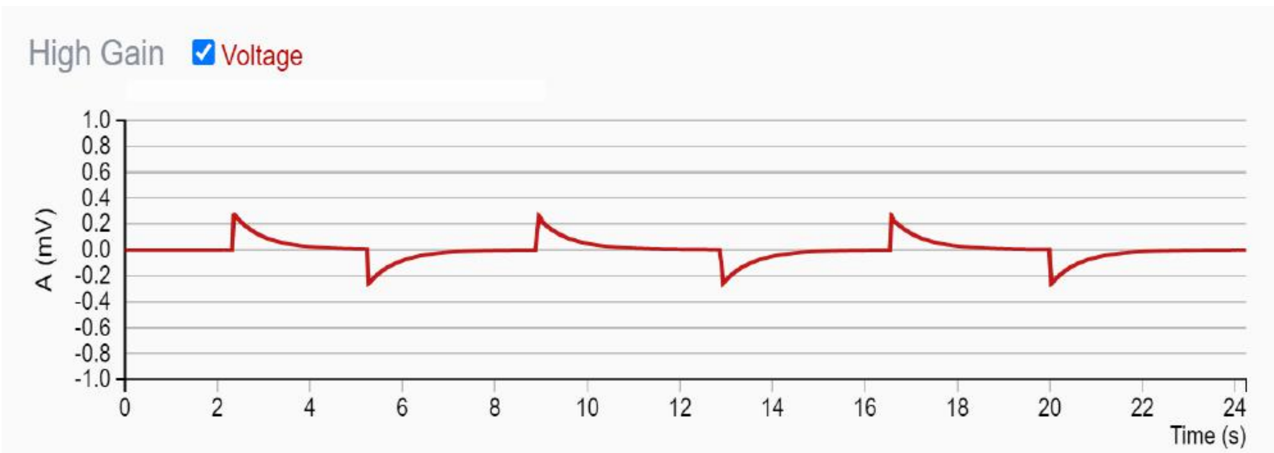
When discharging:



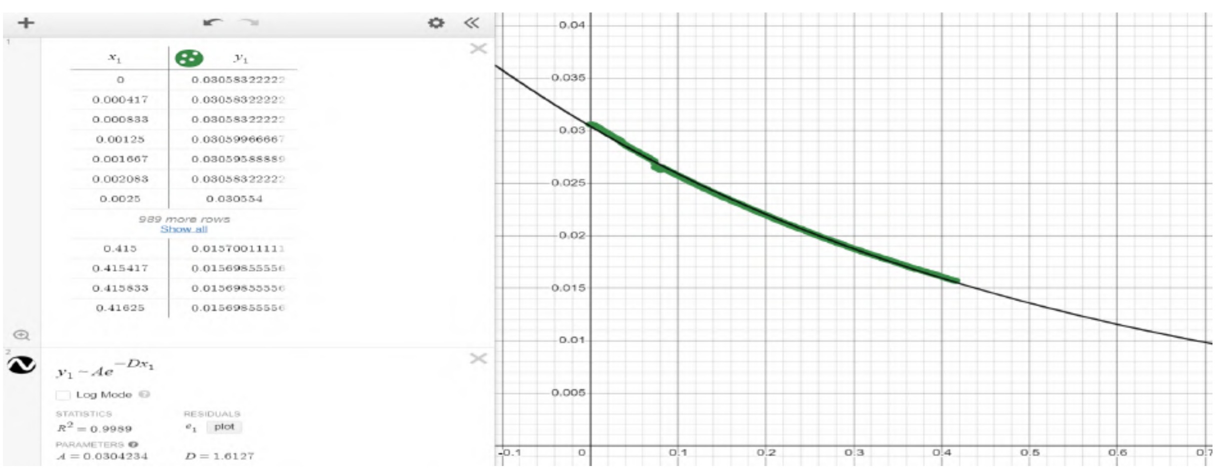
In our graphs, you can see the fitting values for A and the fitting time constant B (also known as τ) in the bottom left corner. We averaged the different values we got for A, and found the fitted value to be .... By taking the average of the three fittings we did (the 6 entire graphs can be found in the second appendix), we found the value for τ that the charging curves give us to be 0.793 ± 0.0095, and the value for τ that the discharging curves give us to be 3.15 ± 0.012. These results are extremely unusual; the difference between charging and discharging speed is huge. We also noticed that the homemade capacitor was unable to fully charge, regardless of changes we made to the setup. We hypothesize that the capacitor leaked enough charge to greatly affect the time constant we derived from the experiment. This is the same as the parasitic resistance we discussed in 1a, albeit in a more extreme form. We decided to use the discharging value to calculate the capacitance, as the discharging would be less affected by the parasitic resistance. Our calculated capacitance is $3.15/10^4 = 0.000315$ F. The theoretical capacitance we calculated using the properties of the homemade capacitor is $1.51628823 \cdot 10^{10}$ F. These values do not agree, and indeed are off by several orders of magnitude. This is probably due to the huge amount of uncertainties and approximations we ignored in the making and use of our homemade capacitor. For example, leaked charge, the properties of the glue used, the thickness of the paper, and the properties of the foil and paper used were all either ignored or essentially guessed. We estimate the effective dielectrical thickness, or the thickness of the paper, to be .1 mm. Commercial capacitors can be very small while still having very high capacitance. According to our research, this can be achieved through rolling up the capacitor and using very strong dielectrics.

1.4.4 Analysis for Experiment 2b:

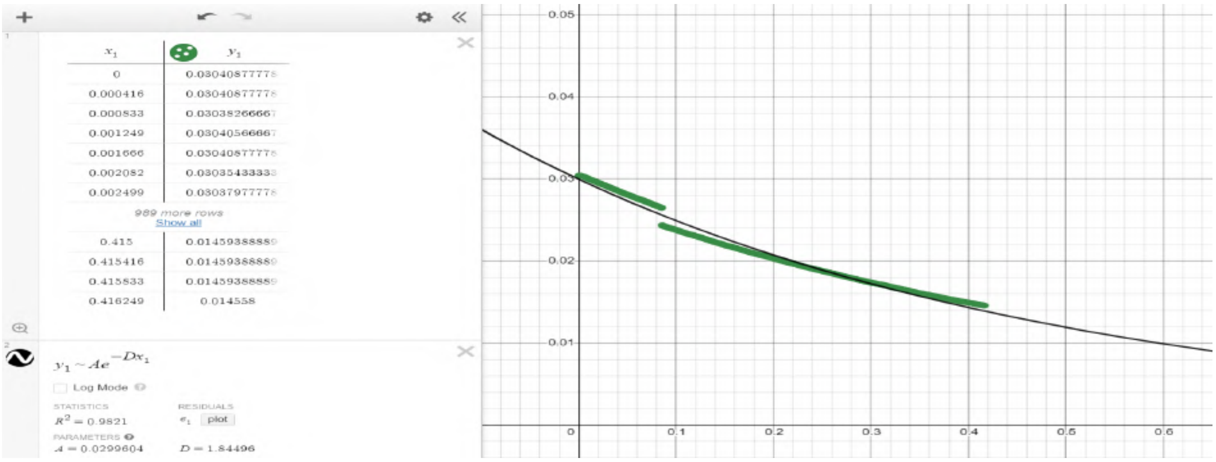
In this section, we are asked to consider the qualitative properties of the graph we recorded and to calculate the decay time constant of the circuit. The graph of our recorded voltages is shown here:

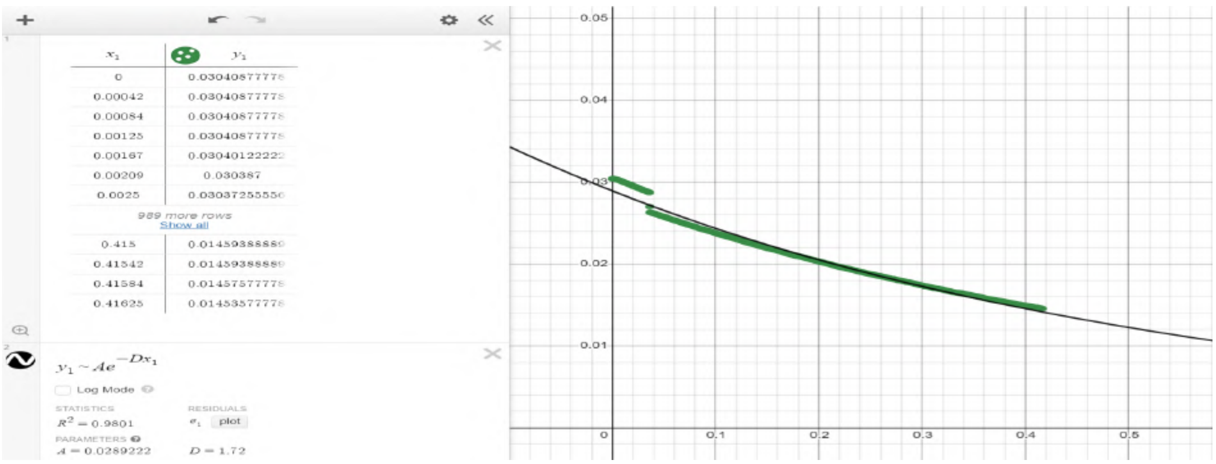


This graph does indeed qualitatively resemble the leftmost graph in figure 9. This is reassuring. We also calculate the decay time constant of the circuit. We performed several fits computationally with a graphing program. These are included below:



The average value for the decay time constant was 1.72 ± 0.054. Our expected theoretical value for this constant was $10^4/2 \cdot 10^{-1} = 5 \cdot 10^4 \pm 49998$. The variance of our constant is huge due to the large difference between our resistor and inductor strength, and our value manages to agree with it.





1.4.6 Analysis for Experiment 2c:

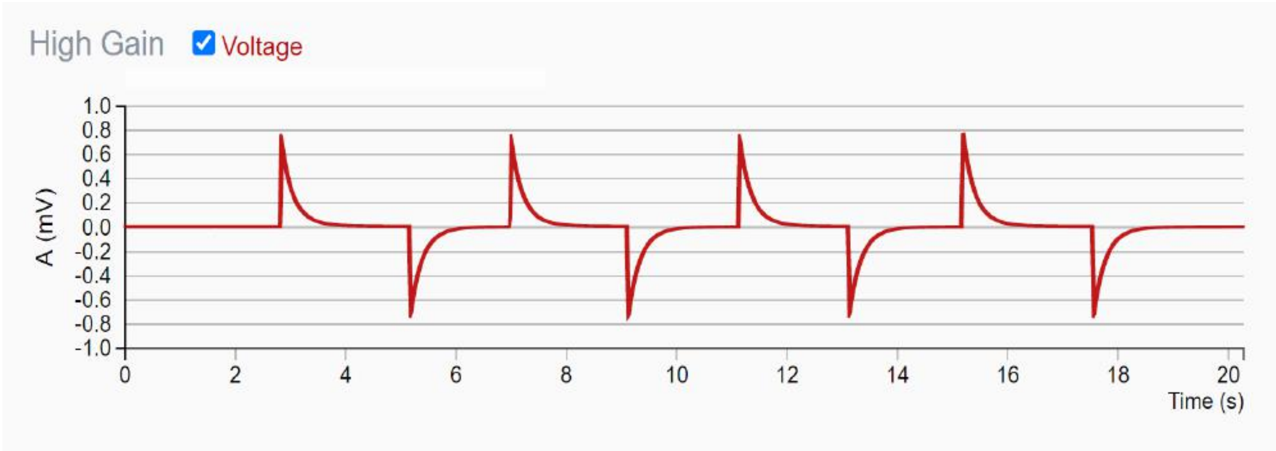
For this experiment we were asked the question:

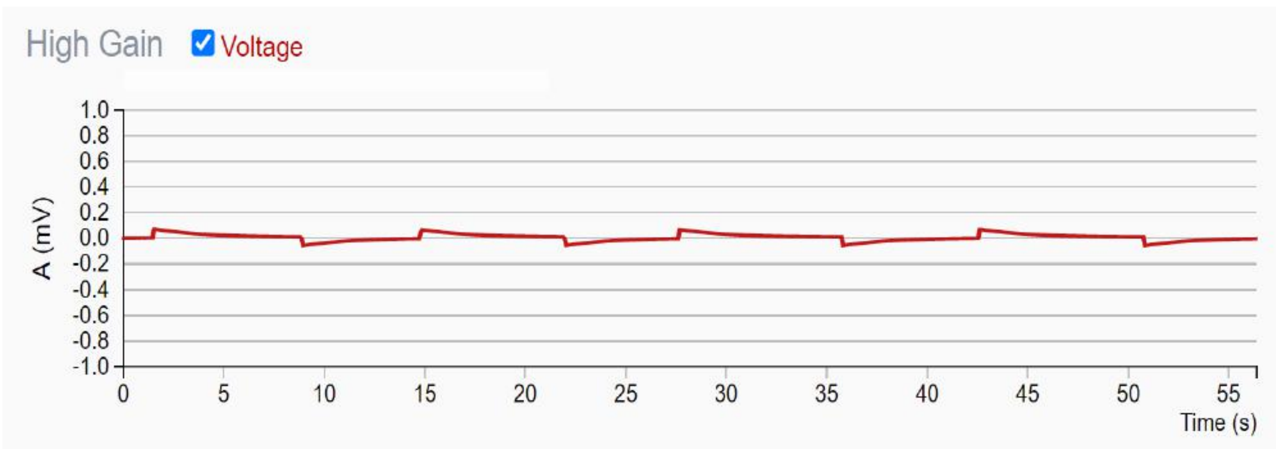
- Why do you think the instructors chose this circuit with four capacitors in series? Does the positive transient recorded resemble the middle curve in Figure 9?

We believe this circuit was chosen to demonstrate the slightly underdamped behavior of an RLC circuit, when both the value of R is large compared to L. The resulting shape of the curve does in fact match that of thee middle curve in figure 9, however, due to the large value of R, it is hard to observe, as the decay factor is very fast, faster than that observed in overdamped circuits, despite still exhibiting underdamped behavior.

1.4.6 Analysis for Experiment 2d:

In this experiment, we attempted to minimize the decay time by adding more and more resistors in parallel to the circuit, reducing the effective resistance and making the decay time constant smaller. In the top graph, the effective resistance was 3333 Ω, while in the bottom graph, the effective resistance was 40000 Ω.

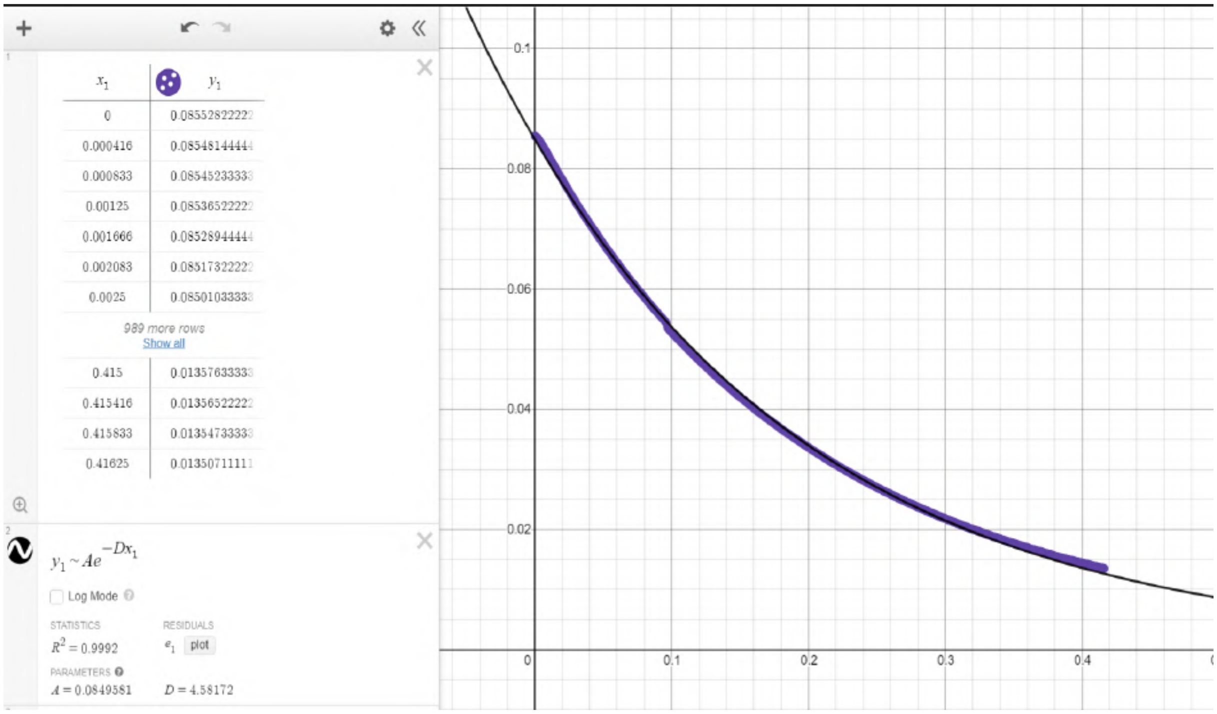


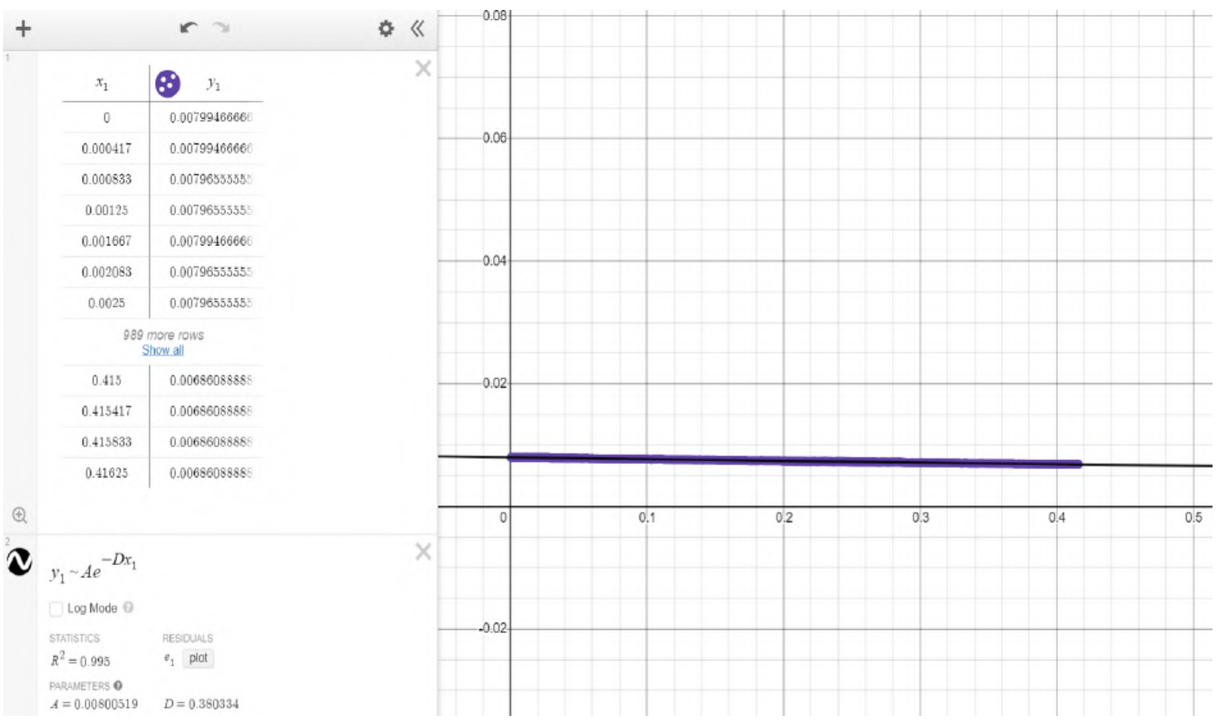


We fitted these to the formula

$$\frac{V_R(t)}{V_0} = Ae^{-Dt}$$

The results are presented in order below:



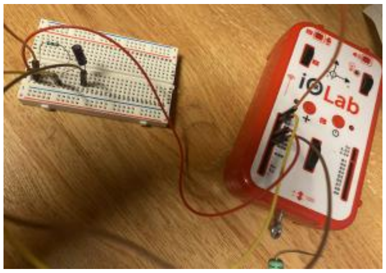


The decay time constant calculated for the 3333 Ω circuit was 4.58, while the decay time constant calculated for the 40000 Ω circuit was 0.380. As expected, the circuit with high effective resistance has a much lower time decay constant.

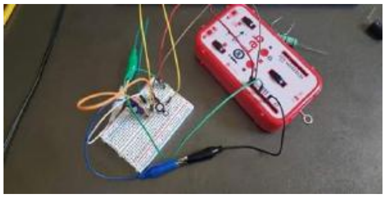
1.5 Summary and Conclusion: In this lab we first attempt to determine the time constant for an RC circuit. In building our own capacitors for the RC circuit, we quickly realized that, despite our best efforts, the handmade capacitors would never charge completely, since there was too much charge leaking out to the surroundings, and that this resulted in large differences between the measured time constants for charging and discharging. From this intuition we were able to observe that the small difference in the time constant for the Rc circuit when it is charging versus when it is discharging is also due to this leakage, although it occurs to a much smaller degree in the store bough capacitors. We also observed the transient behavior of RLC circuits. The experimental behavior conformed well with our theoretical expectations for the overdamped cases in 2b and 2d, and the underdamped case in 2c was a good demonstration of the factg that the underdamped case does not necessarily decay slower than the overdamped case, which is to be expected since whether or not the circuit behaves underdamped or overdamped depends on how α compares to Ω_0, which in the case of a constant L, depends on the ratio of R to C, whereas the value of α does not actually depend on C. Overall the Experiments appear to align reasonably well with our theoretical predictions.

Appendix: Photos of our electric circuit/setup for the different experiments:

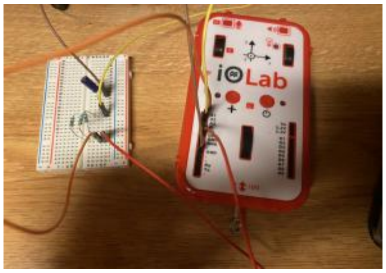
Experiment 1a



Experiment 1b, slowest time

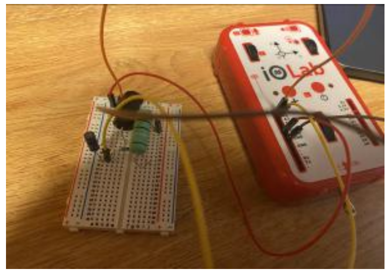


Experiment 1b, fastest time

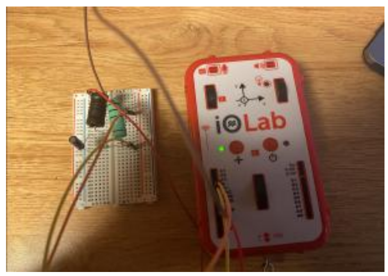


Experiment 1c

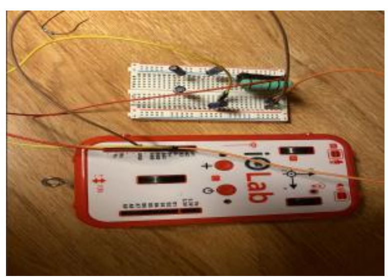
Experiment 2a



Experiment 2b



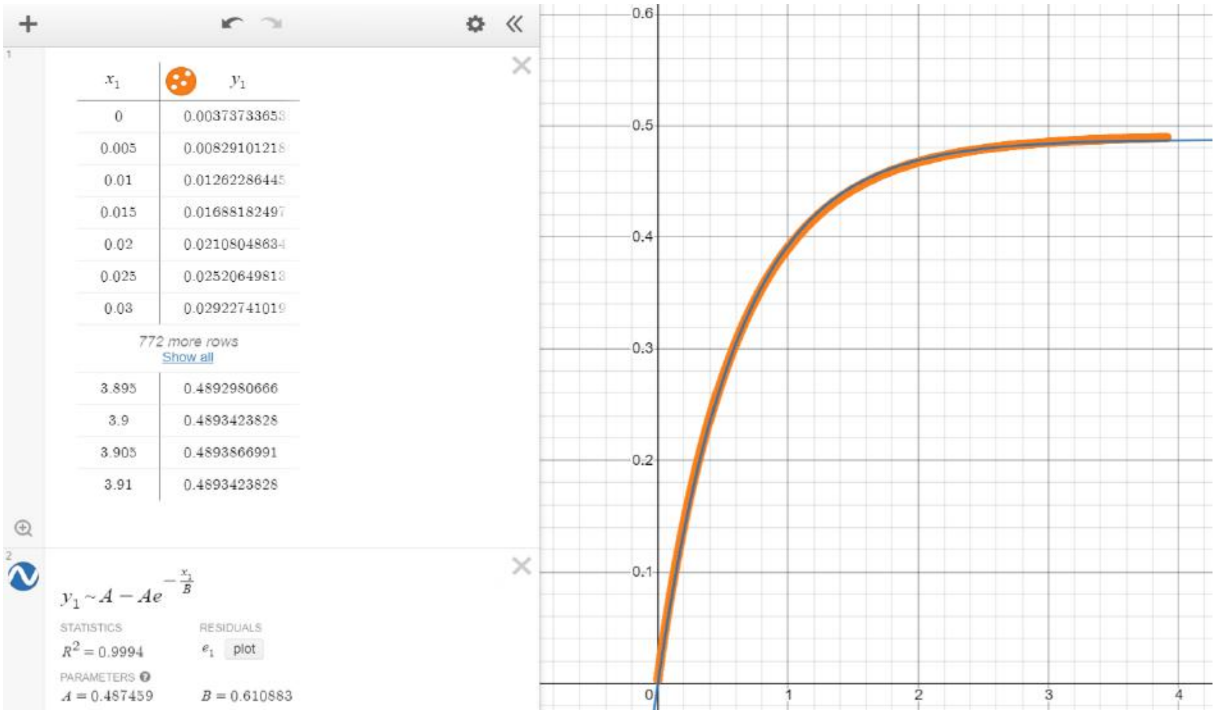
Experiment 2c



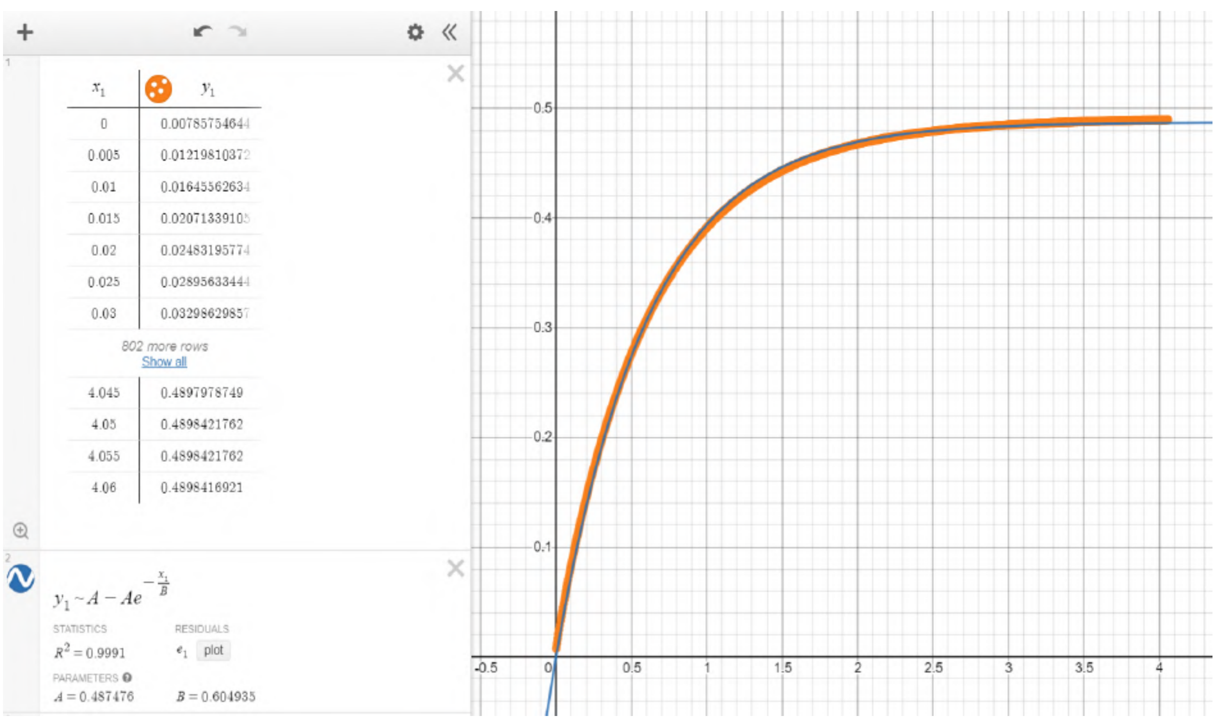
Experiment 2d

ANALYSIS 1A and 1B

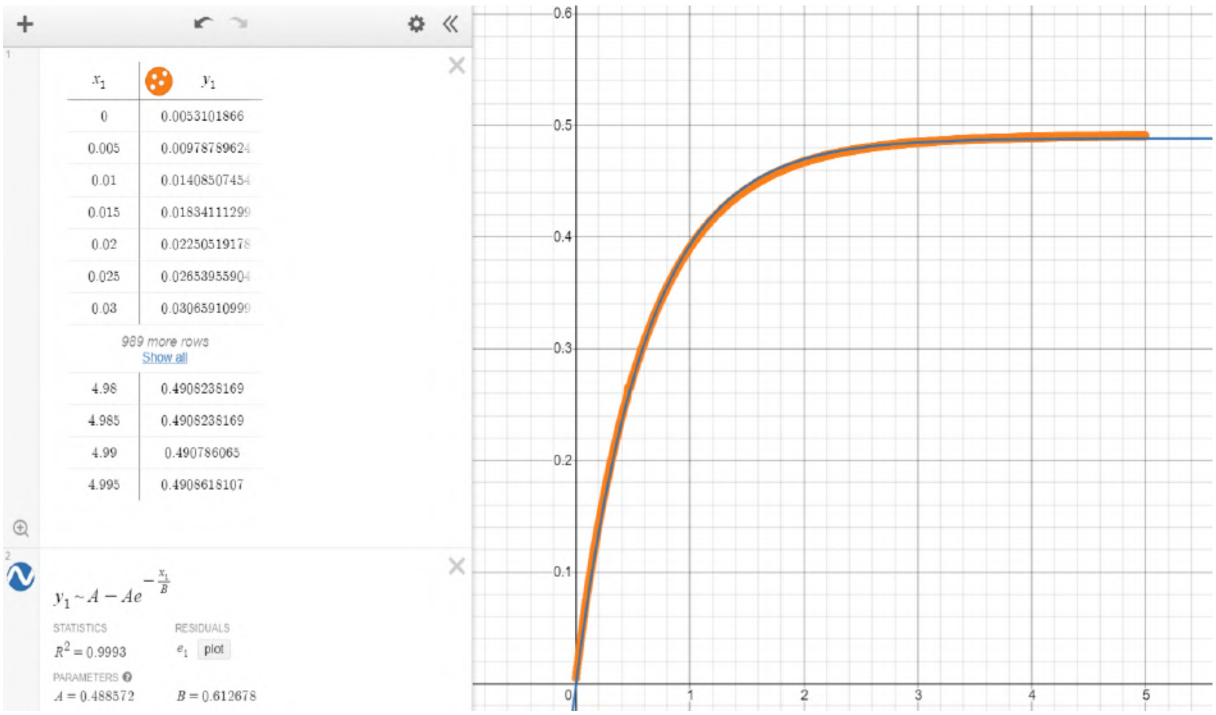
Charging:



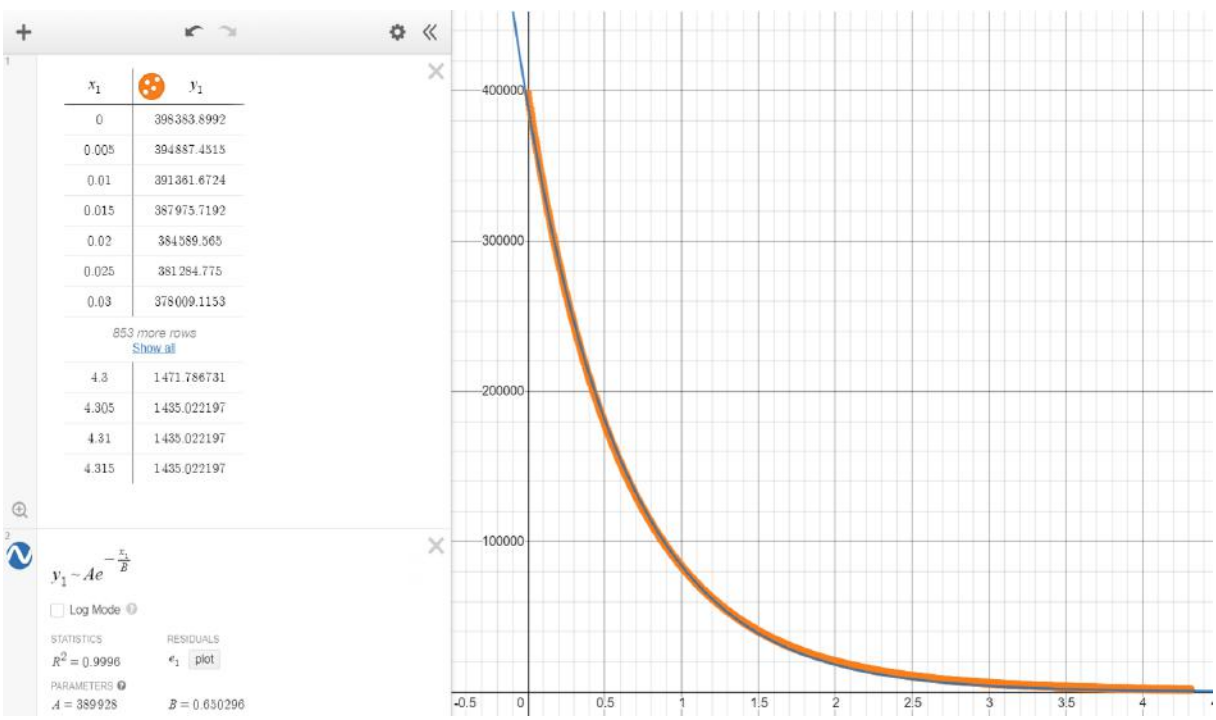
x_1
y_1
0 0.00373733653
0.005 0.00829101218
0.01 0.01262286445
0.015 0.01688182497
0.02 0.02108048634
0.025 0.02520649813
0.03 0.02922741019
772 more rows
Show all
3.895 0.4892980666
3.9 0.4893423828
3.905 0.4893866991
3.91 0.4893423828
$y_1 \sim A - Ae^{-\frac{x_1}{B}}$
STATISTICS
RESIDUALS
$R^2 = 0.9994$
e_1 plot
PARAMETERS
$A = 0.487459$
$B = 0.610883$

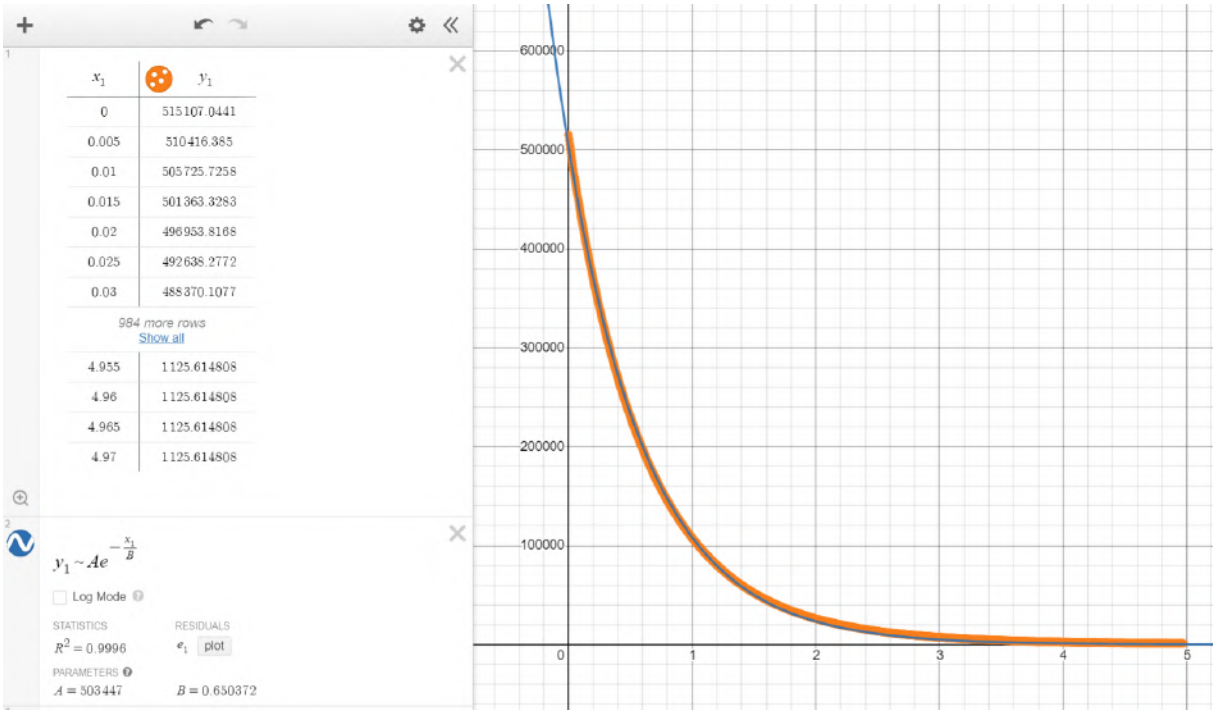


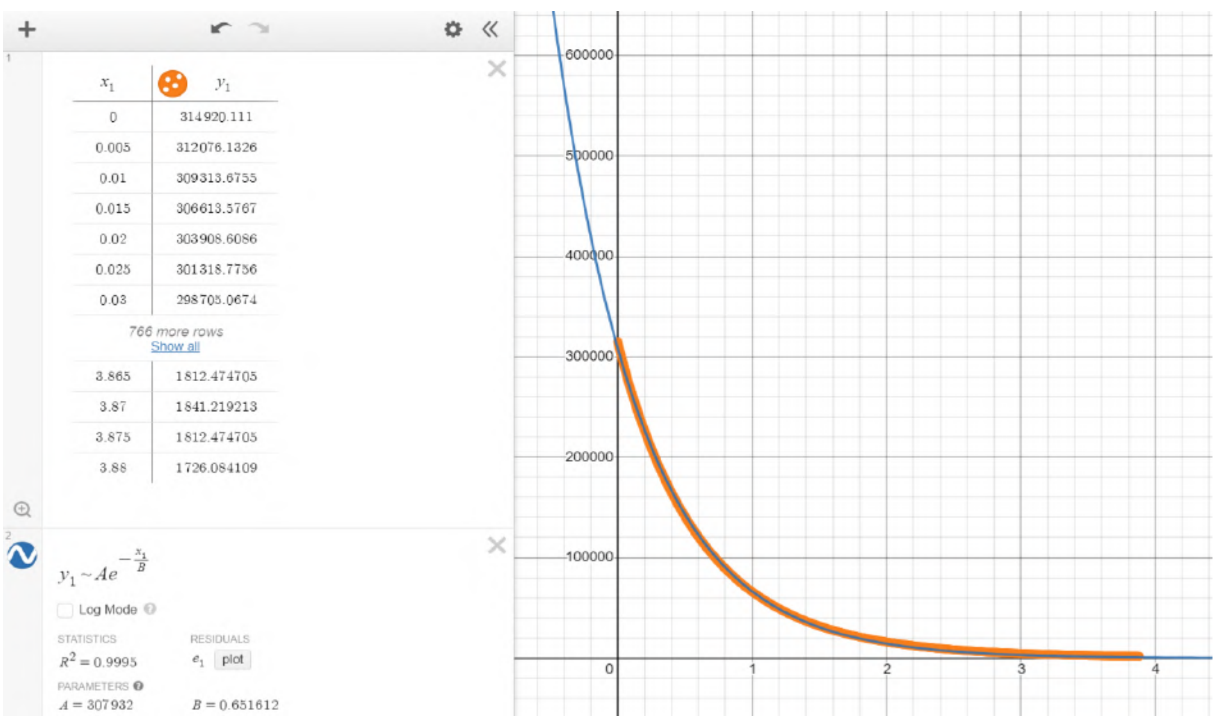
x_1
y_1
0 0.00785754644
0.005 0.01219810372
0.01 0.01645562634
0.015 0.02071339105
0.02 0.02483195774
0.025 0.02895633444
0.03 0.03298629857
802 more rows
Show all
4.045 0.4897978749
4.05 0.4898421762
4.055 0.4898421762
4.06 0.4898416921
$y_1 \sim A - Ae^{-\frac{x_1}{B}}$
STATISTICS
RESIDUALS
$R^2 = 0.9991$
e_1 plot
PARAMETERS
$A = 0.487476$
$B = 0.604935$



Discharging:

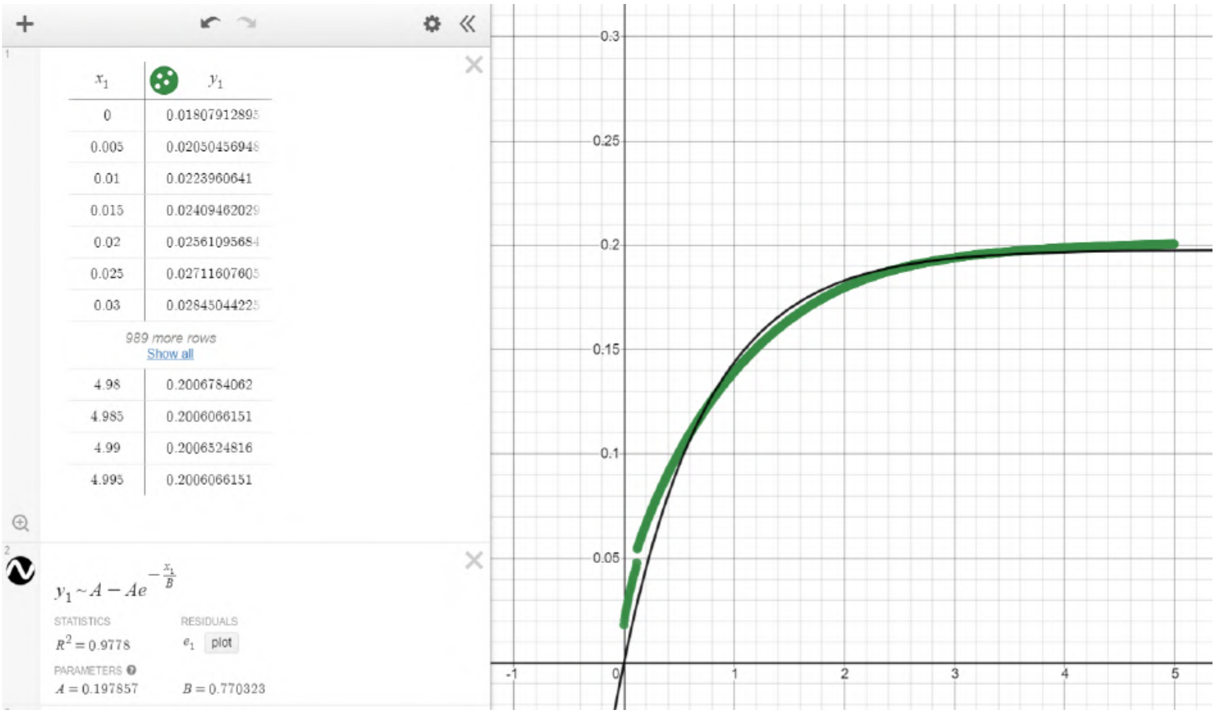


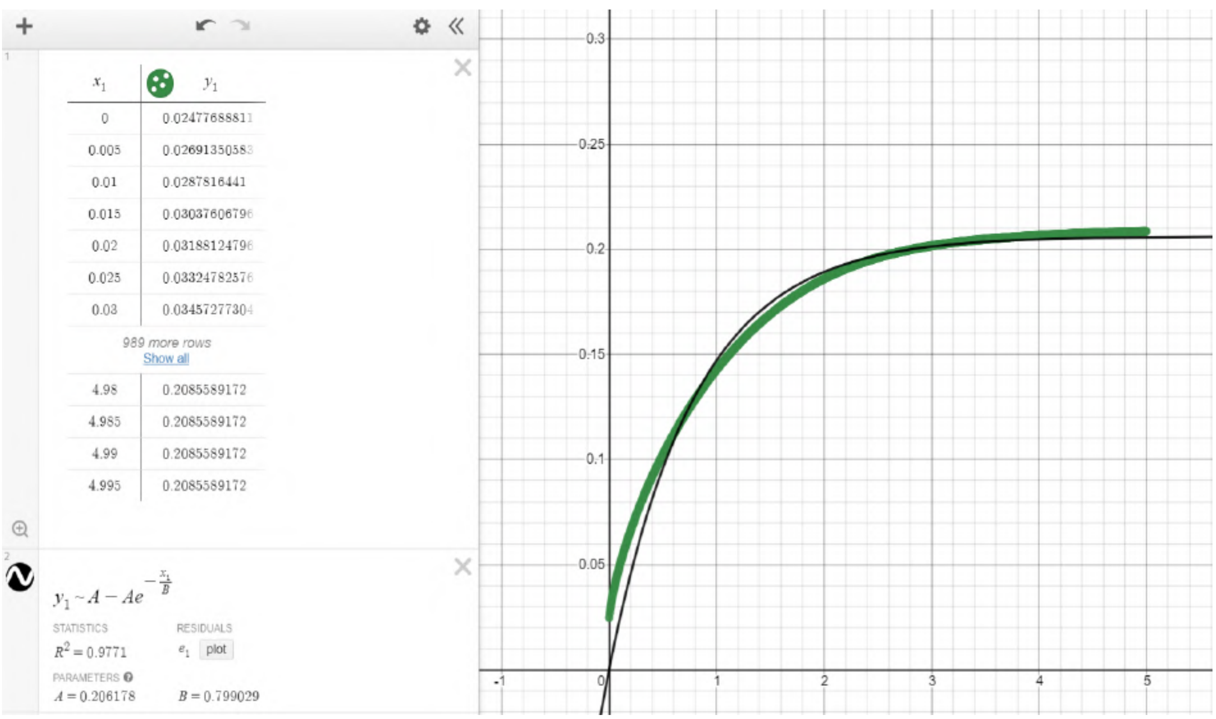

x_1 y_1
0 515107.0441
0.005 510416.385
0.01 505725.7258
0.015 501363.3283
0.02 496953.8168
0.025 492638.2772
0.03 488370.1077
984 more rows
Show all
4.955 1125.614808
4.96 1125.614808
4.965 1125.614808
4.97 1125.614808
$y_1 \sim Ae^{-\frac{x_1}{B}}$
Log Mode
STATISTICS
RESIDUALS
$R^2 = 0.9996$
e_1 plot
PARAMETERS
$A = 503447$
$B = 0.650372$
600000
500000
400000
300000
200000
100000
0
1
2
3
4
5

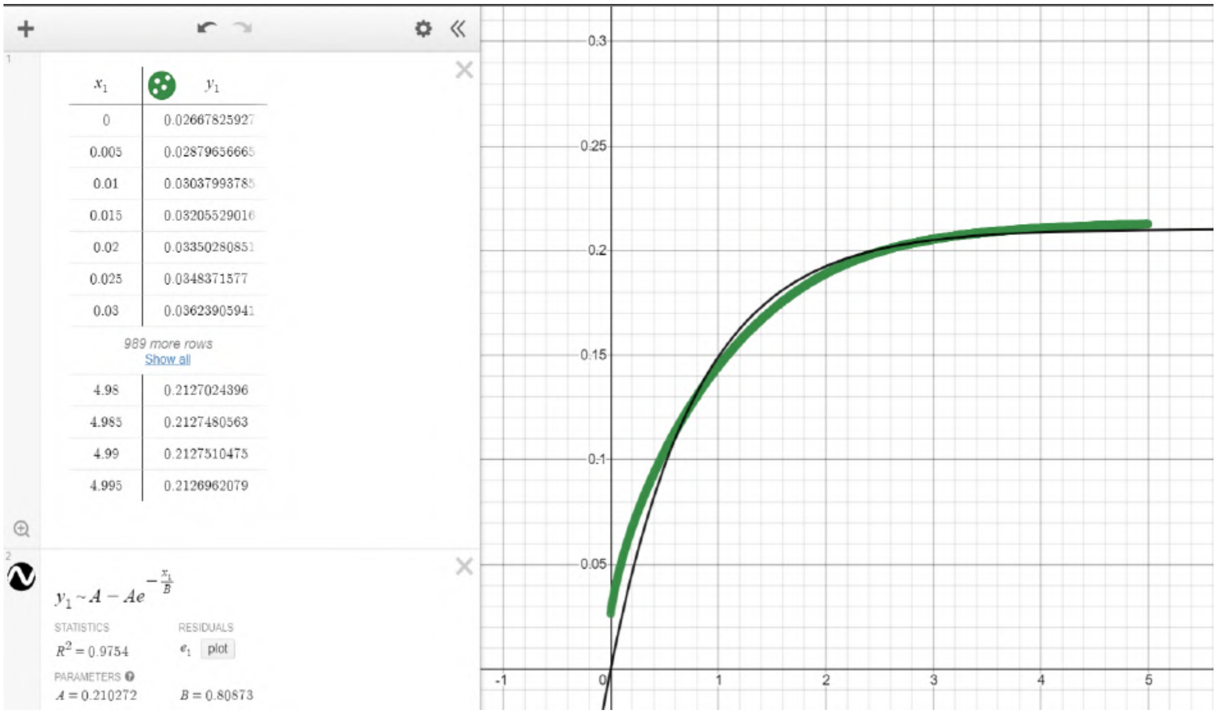

x_1 y_1
0 314920.111
0.005 312076.1326
0.01 309313.6755
0.015 306613.5767
0.02 303908.6086
0.025 301318.7756
0.03 298705.0674
766 more rows
Show all
3.865 1812.474705
3.87 1841.219213
3.875 1812.474705
3.88 1726.084109
$y_1 \sim Ae^{-\frac{x_1}{B}}$
Log Mode
STATISTICS
RESIDUALS
$R^2 = 0.9995$
e_1 plot
PARAMETERS
$A = 307932$
$B = 0.651612$
600000
500000
400000
300000
200000
100000
0
1
2
3
4

ANALYSIS 1C

Charging:

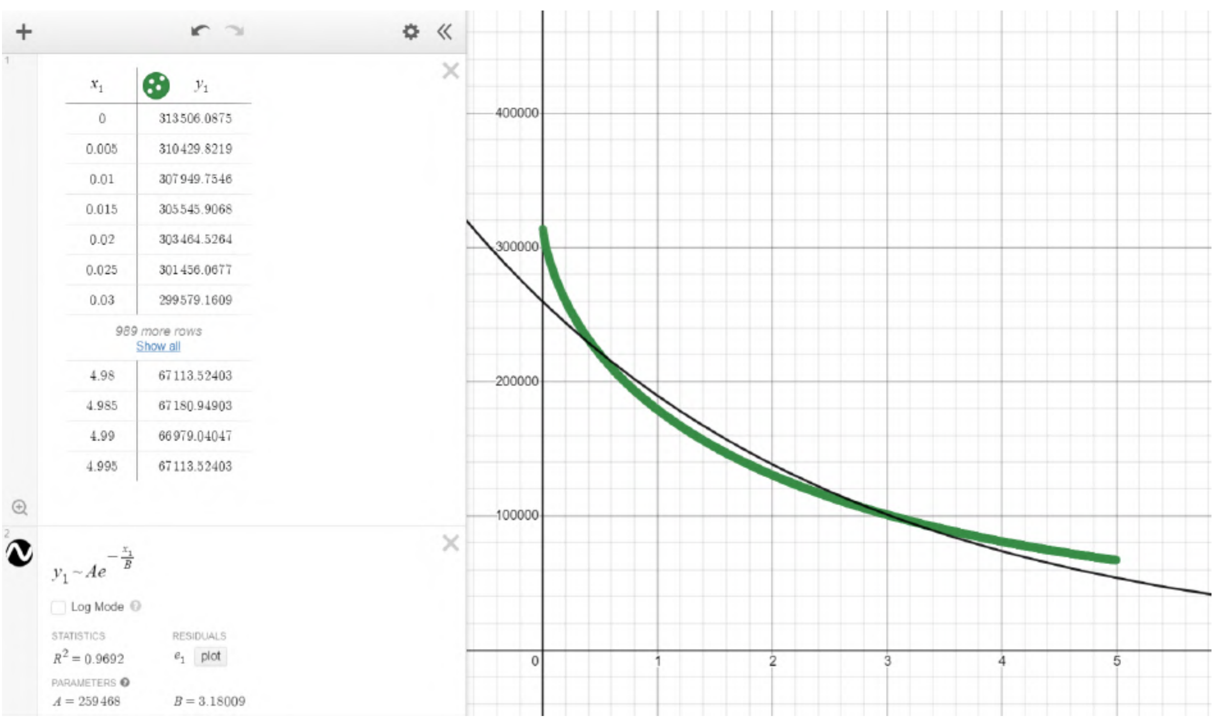






Discharging:


x1
y1
0
351289.3818
0.005
347608.5898
0.01
344731.7261
0.015
342123.6772
0.02
339705.602
0.025
337480.436
0.03
335388.2096
989 more rows
Show all
4.98
74086.78903
4.985
73888.84739
4.99
73812.10307
4.995
73812.10307
y1 ~ Ae^(−x1/B)
Log Mode
STATISTICS
R^2 = 0.9687
RESIDUALS
e1 plot
PARAMETERS
A = 289711
B = 3.14533
400000
300000
200000
100000
0
1
2
3
4
5


x1
y1
0
313506.0875
0.005
310429.8219
0.01
307949.7546
0.015
305545.9068
0.02
303464.5264
0.025
301456.0677
0.03
299579.1609
989 more rows
Show all
4.98
67113.52403
4.985
67180.94903
4.99
66979.04047
4.995
67113.52403
y1 ~ Ae^(−x1/B)
Log Mode
STATISTICS
R^2 = 0.9692
RESIDUALS
e1 plot
PARAMETERS
A = 259468
B = 3.18009
400000
300000
200000
100000
0
1
2
3
4
5