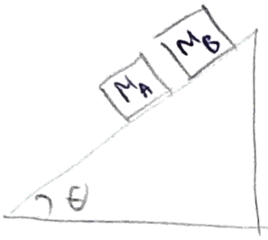
PROBLEM 1 CONTINUED.

Given $M_E = \frac{M_M R_E^2}{d^2 \theta}$, and $g = -\frac{G M_E}{R_E^2} \rightarrow g = \frac{G \frac{M_M R_E^2}{d^2\theta}}{R_E^2}$; $g = G\frac{M_m}{d^2\theta}$

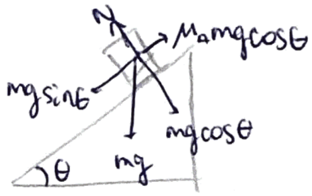
Then, $\boxed{G = \frac{g d^2 \theta}{M_m}}$

PROBLEM 10

(a)



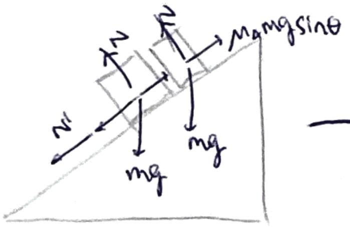
Here, since $\mu_A < \mu_B$, M_A will move first. Therefore, lets look at M_A:



Looking at this, one can see that the critical angle occurs when

$mg\sin\theta > \mu_A mg\cos\theta \rightarrow \tan\theta > \mu_A \rightarrow \boxed{\theta > \arctan(\mu_A)}$

(b) Now, since M_A is further up M_B, we are in a similar scenario. However, since $\mu_A < \mu_B$, mass A will join M_B before sliding, so we will need to take Normal forces into account.



$\rightarrow$

$$M_B g\sin\theta + N' \geq \mu_B M_B \cos\theta\, g$$
$$M_A g\sin\theta = N' + \mu_A M_A g\cos\theta$$
$$\downarrow$$
$$M_B g\sin\theta + M_A g\sin\theta - \mu_A M_A g\cos\theta \geq \mu_B M_B \cos\theta\, g$$
$$(M_B + M_A) g\sin\theta \geq (\mu_B M_B + \mu_A M_A) g\cos\theta$$
$$\tan\theta \geq \frac{\mu_B M_B + \mu_A M_A}{M_B + M_A}$$

$$\leftarrow \boxed{\theta \geq \arctan\left(\frac{\mu_B M_B + \mu_A M_A}{M_B + M_A}\right)}$$

(c) If the two blocks were to be glued together, they would behave as one mass. Therefore, my answer to (a) would remain the same, but my answer to (b) would change. Furthermore, M_{AB} would be the same as $M_A + M_B$.

PROBLEM 3.12 (PROBLEM 8) CONTINUED

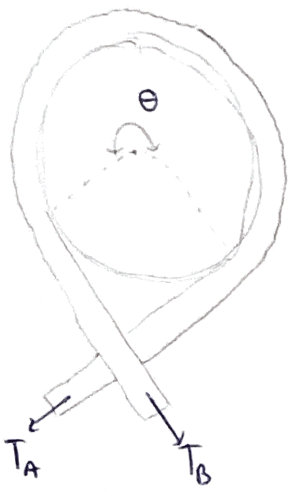
If $\mu = 0.25$, $\theta = 8\pi$ and $T_B = 100\ N$, we can plug into the equation to get:

$$100\ N = T_A e^{-0.25 \cdot 8\pi} \rightarrow T_A = 53549\ N$$

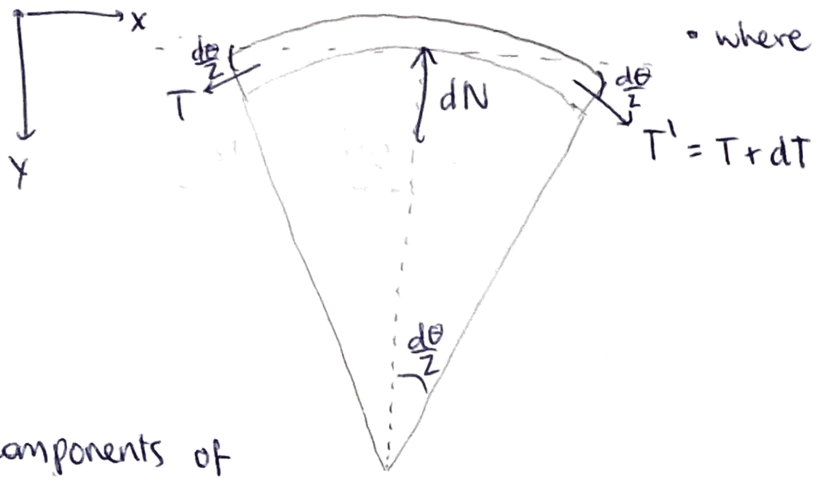
Now, since we are looking for the mass that can be hung vertically;

$$T_A = mg \rightarrow m = \frac{53549\ N}{9.8\ m/s^2} \rightarrow \boxed{m = 5464.2\ kg}$$

PROBLEM 3.12 [PROBLEM 8]

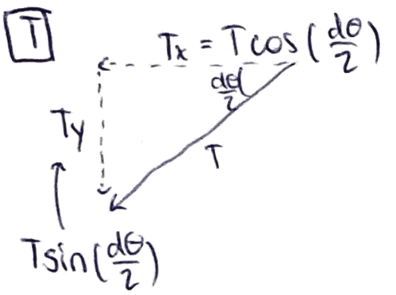


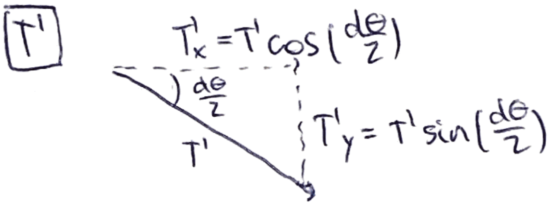
To solve this problem, we must do like in 3.11, look at an infinitesimal part of the capstan:



• where $T' > T$

Lets look at the components of T and T':





Now, lets write and solve Newton's Equations in all axees:

X-AXIS

$$T'\cos\left(\frac{d\theta}{2}\right) - \mu dN - T\cos\left(\frac{d\theta}{2}\right) = 0 \rightarrow \mu dN = T'\cos\left(\frac{d\theta}{2}\right) - T\cos\left(\frac{d\theta}{2}\right)$$

$$\mu dN = (T' - T)\cos\left(\frac{d\theta}{2}\right)$$

$$\mu dN = dT$$

Y-AXIS

small angle approx.

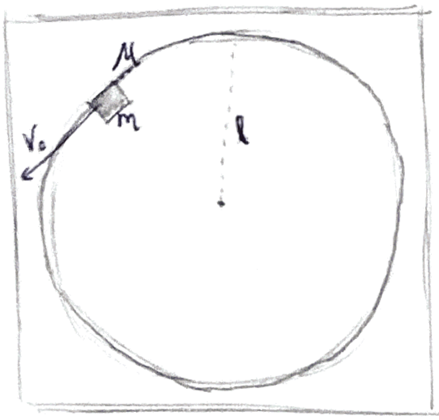
$$T'\sin\left(\frac{d\theta}{2}\right) + T\sin\left(\frac{d\theta}{2}\right) = dN \rightarrow 2T\frac{d\theta}{2} + dT\frac{d\theta}{2} = dN \rightarrow dN = Td\theta$$

cancels out due to limit product rule.

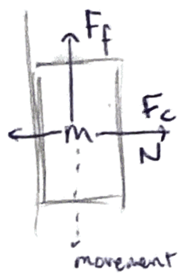
Now, solve for T_B and T_A:

$$\begin{cases} dN = Td\theta \\ dN = \frac{dT}{\mu} \end{cases} \quad \int_{T_A}^{T_B} \frac{dT}{T} = \mu\int_0^{\theta} d\theta \rightarrow \ln(T_B) - \ln(T_A) = \mu\theta \rightarrow \boxed{T_B = T_A e^{-\mu\theta}}$$

PROBLEM 3.23 (PROBLEM 7)



(a)



- Mass m is moving in circles, so there must be a centripetal force.

$F_f = \mu N$. However, the Normal force is the same as the centripetal force!

$$F_f = m\frac{v^2}{r}\mu = m\frac{v^2}{\ell}\mu$$

Now, since F_f is negative respect the object's movement, lets write newtons Law.

$$F = ma \rightarrow F_f = ma \rightarrow -m\frac{v^2}{\ell}\mu = ma \rightarrow a = -\frac{v^2}{\ell}\mu$$

$$\frac{dv}{dt} = -\frac{v^2}{\ell}\mu \rightarrow \int_{V_0}^{V}\frac{1}{v^2}dv = -\int_0^t \frac{\mu}{\ell}dt' \rightarrow \cancel{-\frac{1}{V} = -\frac{\mu}{\ell}(t-t_0)}$$

$$-\frac{1}{V} + \frac{1}{V_0} = -\frac{\mu t}{\ell} \rightarrow \boxed{V = \frac{V_0}{1+\frac{\mu V_0 t}{\ell}}}$$

(b) Now, to find the position, we must think about what defines the position in a circle: the radial position(ℓ) and the angle with respect to the starting angle (θ_0). Since the integral of θ is w and w and r are deeply connected, lets look at the following integral.

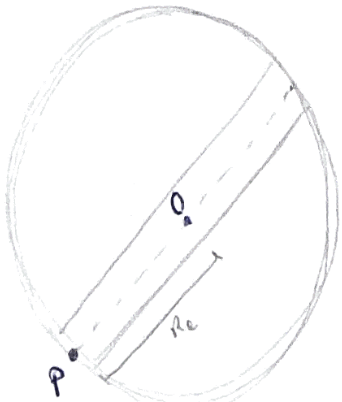
$$\int_0^t \frac{d\theta}{dt}dt = \int \frac{V_0}{\left(1+\frac{\mu V_0 t'}{\ell}\right)\ell}dt' \rightarrow u = \frac{\mu V_0 t}{\ell},\quad du = \frac{\mu V_0}{\ell} \rightarrow \int_0^t d\theta = \frac{1}{\mu}\int \frac{du}{1+u}$$

Since $\frac{d\theta}{dt} = W = \frac{V}{\ell}$

$$\rightarrow \theta(t) - \theta_0 = \frac{1}{\mu}\ln\left(1+\frac{\mu V_0 t}{\ell}\right) \rightarrow \boxed{\theta(t) = \theta_0 + \frac{1}{\mu}\ln\left(1+\frac{\mu V_0 t}{\ell}\right)}$$

This expression is enough for the position since the radius ℓ has already been taken into account in the integrals.

PROBLEM 3.15 (PROBLEM 5)



We must use mechanics to obtain an expression of Simple Harmonic Motion. When the ball is on the surface, at point P, the only force it feels is gravity.

$$\underbrace{\vec{F}}_{\text{gravity}} = m \underbrace{\vec{a}}_{\text{change in } \dot{r} \text{ through time}}$$

Therefore, the Newton Equation for the mass is.

$$-G\frac{Mm}{r^2} = m\ddot{r} \rightarrow \ddot{r} = -\underbrace{\frac{GM}{r^2}}_{\text{Gravitational Field}} \rightarrow \ddot{r} + g = 0.$$

Nevertheless, g isn't a constant here! This is because the mass is moving. Depending on the position of the mass with respect to point O (center of Earth), the Mass under the object and radius changes! We must therefore find an expression for M as a function of the radius. Since density is constant, we can do this:

$M(r) = \frac{4}{3}\pi r^3 \cdot \rho$ ⟶ This is because $V(r) \cdot \rho = M(r)$ and the Volume is a sphere. ⟶ if we let M be $M(r)$ and substitute, we are left with:

$$\ddot{r} + \frac{GM(r)}{r^2} = 0 \longrightarrow \ddot{r} + \frac{4}{3}\pi\rho G r = 0.$$ Now, let w^2 be $\frac{4}{3}\pi\rho G$. Therefore,

$$\boxed{\ddot{r} + w^2 r = 0}$$ This equation represents simple harmonic motion, so our point is proven!

Now, we must find the Period of the SHM. $T = \frac{2\pi}{w}$.

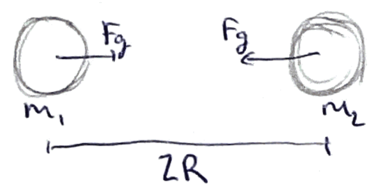
$w^2 = \frac{4}{3}\pi G\rho$. However, $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$. Therefore, $w^2 = +\frac{g}{R_E} \rightarrow w = \cancel{\sqrt{\frac{GM_E}{R_E^3}}}$ ~~or~~ $\boxed{\sqrt{\frac{g}{R_E}}}$

$\cancel{T = \frac{2\pi}{\sqrt{\frac{GM_E}{R_E^3}}}}$ Therefore, $T = 2\pi\sqrt{\frac{R_E}{g}} \rightarrow \boxed{9080 \text{ seconds}}$

To solve the last part of the problem, we can state that $F_g = F_{centripetal}$, since motion is circular!

$$m\frac{v^2}{r} = G\frac{Mm}{r^2} \rightarrow v = \sqrt{\frac{GM_E}{R_E}}.$$ Also, $mR_E w^2 = mg \rightarrow \boxed{w = \sqrt{\frac{g}{R_E}}}$ which is the same in orbit and in tunnel

PROBLEM 3.14 (PROBLEM 4)



- The distance between the spheres is the diameter of a circle.
- The spheres are made of platinum, where $\rho = 21.5\ g/cm^3$ or $\rho = 21.5 \times 10^3\ \frac{kg}{m^3}$

$F_g = G\frac{M_1 M_2}{4R^2}$. Since this is circular motion, $F_g = F_{centripetal}$ such that

$G\frac{M_1 M_2}{4R^2} = m_1 r w^2$. Also, since $M_1 = M_2$, $G\frac{M^2}{4R^2} = MRw^2 \rightarrow 4w^2 = \frac{GM}{R^3}$

Now, since both are spheres of $\rho = 21.5 \cdot 10^3\ \frac{kg}{m^3}$, we can calculate M.

$\rho = \frac{m}{V} \rightarrow m = \frac{4}{3}\pi r^3 \cdot 21.5 \cdot 10^3$

However, we are asked to calculate the SHORTEST period of orbit. This happens when $2R_{orbit} = 2r_{sphere}$, so that the spheres orbit in contact with eachother.

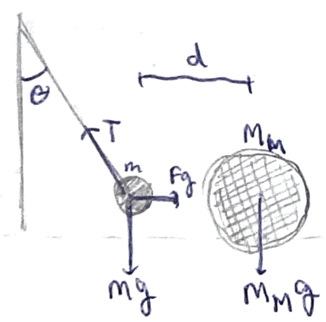
$M = \frac{4}{3}\pi R^3 \cdot 21.5 \cdot 10^3 \rightarrow 4w^2 = \frac{GM}{\frac{3M}{4\pi \cdot 21.5 \cdot 10^3}} \rightarrow \frac{4\pi \cdot 21.5 \cdot 10^3 G}{3} = 4w^2$

$R^3 = \frac{3M}{4\pi \cdot 21.5 \cdot 10^3}$

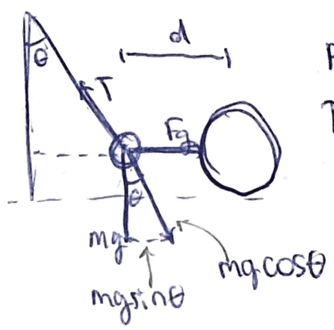
Now, we can find w. Since $T = \frac{2\pi}{w}$, we can find T_{min}:

$w = \sqrt{\frac{21.5 \cdot 10^3 \cdot \pi G}{3}}$ .... $T = \frac{2\pi}{\sqrt{\frac{21.5 \cdot 10^3 \pi G}{3}}} = \boxed{5127.99 \text{ seconds}}$

PROBLEM 11



Lets zoom into the pendulus to find the forces acting on it.



For small values of θ, $\sin\theta \approx \theta$. Therefore, since T and $mg\cos\theta$ cancel eachother out, we are left with

$mg\theta = F_g'$ (between M_M and m)

where

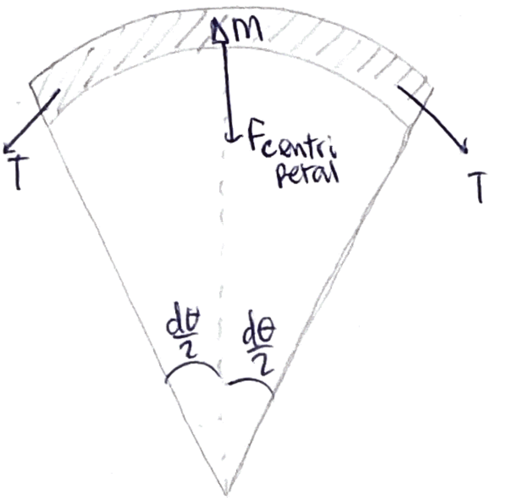
$F_g' = -G\frac{M_M m}{d^2}$

The downward pull in the pendulum is:

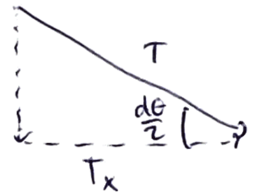
$F_g = -G\frac{M_E m}{R_E^2}\theta$ $\longrightarrow$ $\boxed{M_E = \frac{M_M R_E^2}{d^2 \theta}}$

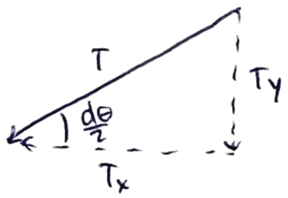
PROBLEM 3.11 (PROBLEM 3)

For this problem, lets look at the following sketch of an infinitessimally small part of the string.



To solve this, lets decompose T first.





Where $T_x = T\cos\left(\frac{d\theta}{2}\right)$ and $T_y = T\sin\left(\frac{d\theta}{2}\right)$. Here we can see that when both are combined, T_x's cancel out and T_y's add up.

$$F_{centripetal} = 2T_y = 2T\sin\left(\frac{d\theta}{2}\right) \xrightarrow[\text{small angle approx.}]{} F_{centripetal} = T\,d\theta$$

Now, the mass of this infinitely small part of the string, ΔM, is also:

$\Delta M = M\frac{\Delta\theta}{2\pi}$, since its a portion of a circle (that's what $\frac{\Delta\theta}{2\pi}$ represents).

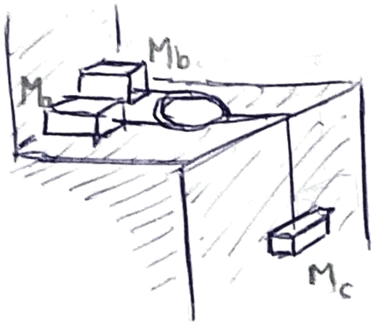
Also, since the circumference is $2\pi r$ and has length ℓ, $r = \frac{\ell}{2\pi}$.

Now we can substitute into the formula of centripetal acceleration.

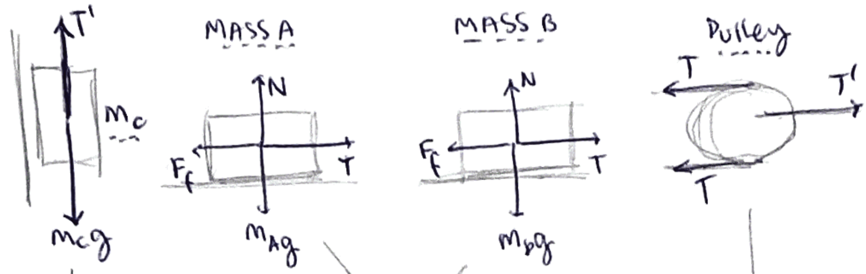
$$\begin{cases} F_{centripetal} = T\,d\theta \\ F_{centripetal} = (\Delta M) r w^2 = M\dfrac{\Delta\theta \cdot \ell w^2}{4\pi^2} \end{cases} \quad T\,d\theta = \frac{M\Delta\theta \ell w^2}{4\pi^2} \rightarrow \Delta\theta = d\theta$$

Therefore, $\boxed{T = \dfrac{M\ell w^2}{4\pi^2}}$

PROBLEM 3.7: (PROBLEM 2)

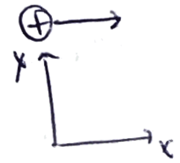


(a) Sketches:



Where the coordinates are

Where positive is in the x direction such that

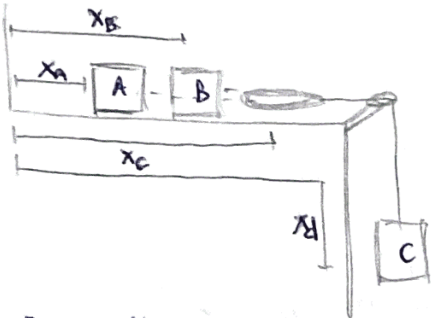


This is useful for setting up a constraint equation

$T' = 2T$

This is because the pulley is massless.

(b) How are the accelerations related?



From this drawing we can infer that $x_d - x_c$ is a constant. Also, the length of the string joining A, B, and the pulley must be:

$$l = (x_c - x_A) + (x_c - x_B)$$

If we do the 2nd Derivative, we obtain a relationship between the accelerations:

- $2\ddot{x}_c = \ddot{x}_A + \ddot{x}_B$
- $\ddot{x}_D = \ddot{x}_c$

(c) Tension on the rope:

Lets write newton's law for each mass:

MASS A: $M_A \ddot{X}_A = T - F_{fA}$

MASS B: $M_B \ddot{X}_B = T - F_{fB}$

where $F_{fA} = \mu M_A g$ and $f_{fB} = \mu M_B g$

MASS C: $M_c \ddot{X}_c = M_c g - T'$

Therefore, if we plug in the values for $\ddot{X}_A$, $\ddot{X}_B$ and $\ddot{X}_c$ into the acceleration equation:

$$2\left(\frac{M_c g - T'}{M_c}\right) = \frac{T - \mu M_A g}{M_A} + \frac{T - \mu M_B g}{M_B} \rightarrow T'\left(-2M_A M_B + \frac{M_A M_c + M_c M_B}{2}\right) = -2g M_A M_B M_c (1+\mu)$$

Now we can solve for T' and therefore, for T too.

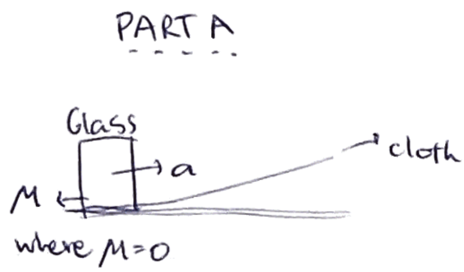
$$\boxed{T' = \frac{4g M_A M_B M_c (1+\mu)}{M_A M_c + M_B M_c + 4 M_A M_B}}$$

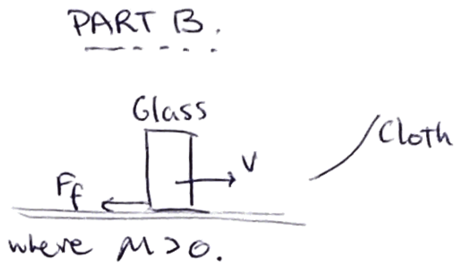
→ Now, $T' = 2T$. Therefore,

$$T = \frac{T'}{2} = \boxed{\frac{2g(1+\mu) M_A M_B M_c}{M_B M_c + 4 M_A M_B + M_A M_c} = T}$$

PROBLEM 3.6 (PROBLEM 1)

For finding the time it takes for the glass to come to a stop at the edge, we must find the maximum time the cloth can act on the glass first. Therefore, lets divide the problem in two.





Now, we must find the time that happens in part A, since this is the time the cloth acts on the glass:

$V_0 = 0$

$V_f = V_0 + a_x t \;\downarrow\; V_f = \mu g t$

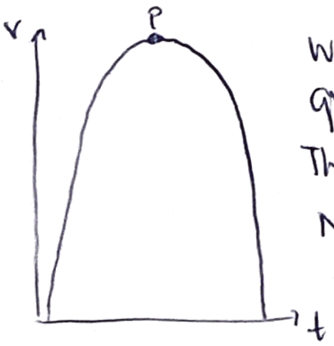
$f = ma_x$

$\mu mg = ma_x \;\downarrow\; a_x = \mu g$

$x = x_0 + V_0 t + \frac{1}{2} a_x t^2$

$6\text{ inches} = 0 + 0 + \frac{1}{2} a_x t^2 \;\downarrow\; a_x t^2 = 2 \cdot 0.1524\,m$

Now, since the distance the glass moves under the influence of the cloth is the same as the distance it moves under the influence of the table, the time that happens is the same for both part a and b. This is only because motion can be graphed like this:



Where P is the point in which the cloth separates from the glass.

Therefore, $2a_x t^2 = 2 \cdot 0.1524\,m \rightarrow a_x t^2 = 0.1524$.

Now we can solve for t using $a_x = \mu g$.

$a_x t^2 = 0.1524 \rightarrow t = \sqrt{\frac{0.1524}{\mu g}} \rightarrow \boxed{t = 0.17 \text{ seconds}}$

Since after 0.17 seconds the block is left under the influence of the table, If $t > 0.17$, the glass would fall off. Therefore, 0.17 seconds is the maximum amount of time the cloth can act upon the glass.