Experimental Derivation of the Boltzmann Constant Using a Voltage-Current Circuit and an LED

Capstone Project Lab Report

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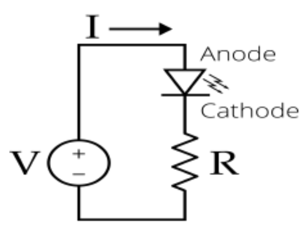
1.1 Introduction/Objectives:

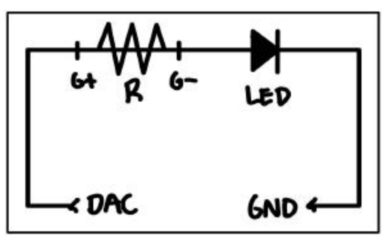
In this project, we plan to do several things. In the first place, we plan to find the optimal experimental setup that would give us a neat and clear Voltage-Intensity curve of the IOLab LED diode. We want to analyze and understand the curve we obtain, and interpret its parts. In second place, we are looking to understand the theoretical meaning of the Boltzmann Constant, and derive it mathematically. In addition, we will perform an analysis on the obtained VI curve, using linear regression methods. This should tell us the lines of best fit and an analysis of the residuals. We will try the analysis for different temperatures and voltages. Our ultimate goal is that the value of Boltzmann's Constant obtained through the theoretical prediction agrees with that obtained through the linear regression models for the different voltages and temperatures.

1.2 Theoretical Background:

Before we dive into the data obtained in this experiment, it is of high importance that we understand the theory that we are dealing with. To measure the voltage-current curve through the diode, we first need to set up a circuit that will appropriately measure the voltage through the LED, so we will set up a standard LED circuit, as shown below.



However, to fit the experiment to our needs, we decided to make an alternate circuit, which is similar to the one above, but with additional components. The precise details on how to build this circuit and setup the experiment will be provided in the Methods section. In IOLab terminology, the circuit we will be using is shown below:



As with any circuit, we can apply Kirchhoff's Laws and Ohm's Laws to it. To remind the reader of what these laws are, and since they are of great significance to this experiment, they are stated below.

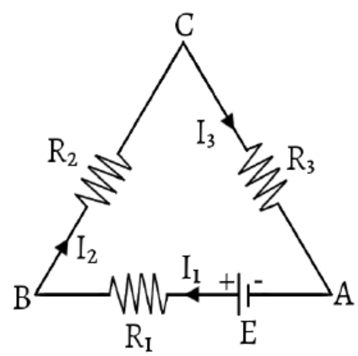
1.2.1 Laws of Circuits

Before going into the circuits, we must define three main characteristics of circuits:

- **Current:** An electric current is a stream of charged particles moving through an electrical conductor or space. It is measured as the net rate of flow of electric charge through a surface or into a control volume. The unit for current is the Ampere (A), defined by Coulumb per unit time.
- **Voltage or Potential Difference:** Voltage is the difference in electric potential between two points, which in a circuit is defined as the work needed per unit of charge to move a test charge between the two points. The units for voltage is the Volt (V), defined as Joules per unit charge.

- **Resistance:** The electrical resistance of an object is a measure of its opposition to the flow of electric current. The less more resistance, we will have a higher potential difference, since a lot of work will be done by the material to "stop" the electrons. The unit of resistance is the Ohm (Ω), which is defined as Volt/Ampere.

Now that we have a basic understanding of the main concepts, we can dive into the actual theory. It was hinted in the definition of resistance that there is some mathematical relationship between voltage and electrical current. If we were to look at a wire with some potential difference $\Delta\phi$ between its endpoints and with some current I flowing through it, we can define the resistance of the wire to be given by $R = \Delta\phi/V$. This equation is known as the simplified non-integral version of Ohm's Law ,often simplified to Ohm's Law. Now, considering these mathematical properties of Voltage, Electrical Current, and Resistance, we can start to create circuits. Circuits are collections of electrical components that all work together to allow for some current to flow through the components in the best way to the interest of the circuit's creator. Let's consider the following circuit:



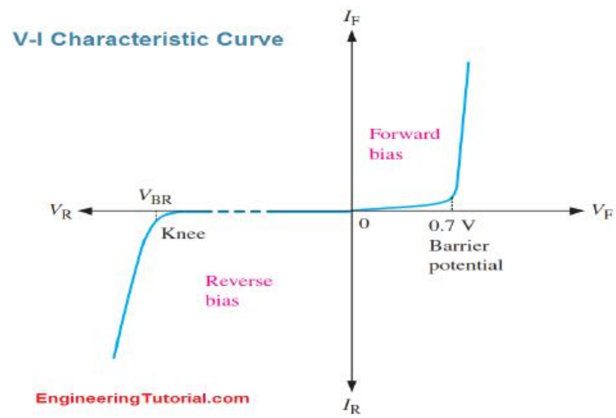
In this circuit, we have three resistors (components that have a high resistance to current flow), and we have three different currents, since the current is changed after flowing through a resistor. To solve for the different variables we have in this circuit, we can apply Kirchhoff's Law, which can be written in two statements:

1. Based on charge conservation, the current flux over a closed circuit must be 0. Closed circuit refers to the fact that the electrical current traces out a loop. Since the flux must be 0, the incoming current must equal the outgoing current, so the current is steady. This implies that the sum of all currents over the loop is 0.
2. If we go through any closed loop, the curl of the electric field is 0 for a closed loop, since this is one of the defining characteristics of the electric field. What this implies is that, since $\nabla \times E = 0$ and $E = -\nabla\phi$, we know for a fact that $\Delta\phi = 0$. This means that over a closed loop, the sum of all the potentials at the different components is 0.

Now, we have defined Ohm's Law and Kirchhoff's Law, the basic laws of circuits. With this in mind, we can look at the circuit we defined before, and how we can extract knowledge about the Boltzmann Constant from it.

1.2.2 The VI Curve for a LED and Deriving Boltzmann's Constant:

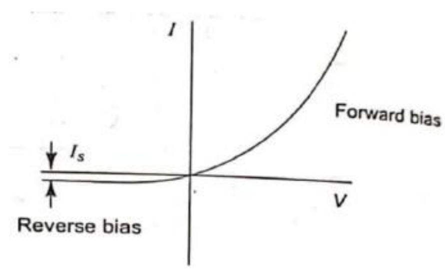
Now, we can use the IOLab and the cricuit from before to, with different voltages, measure the different currents through a Light Emitting Diode (LED). However, depending on how we flow the voltage through the LED, we get different phenomena. When the external voltage is delivered across the P-side of the diode and to the N-side of the diode, where the P-side is attached to the positive terminal and N-side is fixed to the negative side of the battery, we have a forward bias. When the conditions are the opposite to these, we say we have a reverse bias. Therefore, we must run voltage across both directions (or simply flip the diode around in the breadboard) to find different 'types' of voltages. Then, once we are done with the measurements, we expect to find this curve:



In the curve above, the barrier potential is dependant on the material the diode is made of. Also, another element of the diagram worth highlighting is the reverse saturation current. If we look at the forward bias region, we see that the height of the potential barrier at the junction with the diode will be lowered, and the diffusion current due to the electrons in the diode increases rapidly, or exponentially. In fact, the current I in this region of the graph is related to the voltage V by the equation

$$I = I_s(\exp \frac{qV}{nkT} - 1) \tag{1}$$

where q is the electronic charge, k is the Boltzmann Constant, T is the temperature of the setup, which is assumed to be stable, and n is the ideality factor of the material the diode is made of. I_s is the reverse saturation current, and it is better shown in this diagram below:



The reverse saturation constant is of extreme importance, and it is the reason for why we need to measure both the reverse and the forward bias in our experiments. For our experiment, we knew that the theoretical and experimentally confirmed values of the constants in the formula above were: $n = 1.5, q = 1.6 \times 10^{-19}, T = 300K$, and $V = 1$. Under these conditions, the Boltzmann Constant, k, takes a value of 1.3806×10^{-23}, which is the theoretical value of the Boltzmann Constant up to four decimal figures. In our analysis, we plan to perform a logarithmic linear regression of our data, to obtain a value for k that agrees with the theoretical value just derived.

1.3 Experimental Methods:

In this experiment, our ultimate goal was to find a value for the Boltzmann Constant for different voltages and temperatures. Therefore, before we show the methods to find the data, it is important to see what data we were looking for. Below is a table with the different temperatures, voltages, and resistances that we wanted to measure the current in the diode for.

Temperatures (K)	280	290	293	300	310						
Voltages (V)	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3	3.3
Resistances (Ohm)	1000	10000									

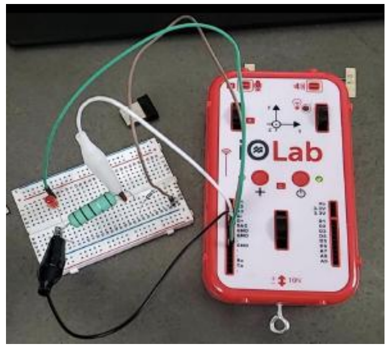
To fit for all these different conditions that we wanted to test, we had to design different experimental setups. Firstly, we wanted to measure the current with different voltages and resistances, without taking the temperature into account, just to make sure we had some good and rigorous data to present if our ideas for the temperature didn't work out. Then, we also made a setup to measure the current-voltage curve under the influence of different temperatures. In the following sections, we will describe these two setups.

1.3.1 Measuring Currents with Different Voltages and Resistances Only:

To measure current flowing through the LED at different voltages and resistors, we used the circuit we showed in the beginning of the Theory section, and used the IOLab's High Gain and DAC to have different voltages and measure them. Also, we used the different resistors that were provided to us in the Lab Kit to have different resistances. If someone with the same lab kit as ours had to carry out these measurements, they would have to follow the following steps:

1. The instrumental items needed for this part are a breadboard, the necessary circuit connective wires, the IOLab device, a diode, and three resistors.
2. Since we had no silicon or germanium diodes for which this experiment is done best, we used the lab kit's red diode, for which all the necessary details are also known, but it's just less consistent with repetitive experiments. Regarding the resistors, one must use 1K and 10K ohm resistors for the measurements and a 1 ohm resistor as a control to make sure the diode doesn't get saturated.
3. Next, one should place the resistors, wires, and diode on the breadboard as shown in the diagrams provided in the end of section 1.3.1, and plug them into the IOLab as indicated.
4. Then, turn on the High Gain sensors on the IOLab, and prepare the DAC to 0.0 V for measurement.
5. Once this is all ready, we are set to start measurements. Turn the IOLab's recording mode on, and for one resistor out of the two options we have (1K ohm and 10K ohm), switch the DAC to the voltages specified in the table in the beginning of the methods section. Then, save the resulting graph to the IOLab online repository.
6. Once this is done, change the resistor to the other option we had, and repeat step 5 for it. By now, one is expected to have two datasets saved in the IOLab online repository per group member. Each of these datasets should have 11 different voltages in it, and all with positive current flow. These datasets represent the forwards bias, and are the truly important parts of our data extraction process.
7. Now, to find the reversed bias part of the VI curve for an LED, repeat steps 5 and 6 but by changing the order of the High Gain and Low Gain wire spots or by switching around the orientation of the diode or current flow through it.

Following these steps should provide the reader with enough data to find a correlation between resistance, voltage, current, and Boltzmann Constant, assuming the entire experiment was carried out at room temperature or approximately 293 Kelvin. Then, using the agreement test which is further described in the analysis section, one should confirm that the experimentally obtained value of the Boltzmann constant agrees with that obtained in the theory section by using Equation (1). The circuit and breadboard setup for this step are shown below, taking into account that the resistor must be switched in step 6 of the method:

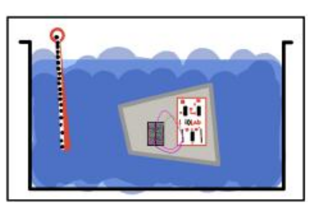


Having explained what the setup for measuring Boltzmann's Constant with regard to the resistance and voltage, we thought it would be a good idea to also explain some of the main sources of error we encountered while carrying out the measurements ourselves. The errors apply to both the measurements of the reverse and forwards bias, and they are listed below, in no particular order:

- While measuring the reverse bias, make sure the voltage does not exceed the diode's breakdown voltage, since this would lead to damages in the instrumentation and is a potential hazard for those measuring data.
- It is important to highlight the relevance of scale and units. When measuring the forwards bias, the increase in current per unit voltage is high enough to be measurable in steps of 0.3 Volts and milliamperes. However, in the reverse bias, the reverse saturation current I_s is slightly visible (and of course, measurable) if we keep this same scale. Therefore, the current flow in the reverse bias should be measured in microamperes. The top half of the y axis in the VI graph is in milliamperes and the bottom half is in microamperes, to make everything better experimentally. Not doing so could lead to systematic errors when reading or extracting results in the analysis section.
- Systematic and random errors while setting up the circuit or switching resistors are likely, especially in steps 5-7, so staying alert at these points of the experiment is highly advisable to avoid time loss.

1.3.2 Measuring Currents with Different Voltages, Resistances, and Temperatures:

Now, we will be doing something similar to that described in section 1.3.1, but we will take an additional factor into account: temperature. As shown in the table at the beginning of the methods section, we will be collecting data for 5 different temperatures. Since 293 Kelvin is room temperature, and we already covered that in section 1.3.1, we will only be focusing on 280, 290, 300, and 310 Kelvin in the upcoming section. How will we ensure that the setup remains at a given temperature for some amount of time? In short, we will be repeating the procedure from 1.3.1 but with added steps due to the complications that come with temperature. The electrical components of the setup will be the same, but we will need a tank of water to have the entire circuit remain at a specific temperature. To make sure the circuitry remains isolated from the water and to measure the temperature of the water bath, we will be using a waterproof thin plastic zip bag and a standard thermometer. Having said this, a sketch of our experimental setup is shown below, where the IOLab and the breadboard are arranged as in the figure at the top of the page.



Note that due to lockdown and Covid-19 complications, only one group member was able to extract data for this experiment, so part of the possible errors could come from here. We recommend having more datasets to average data out in future versions of this experiment. Now, if someone with the same lab kit as ours had to carry out these measurements, they would have to follow the following steps:

1. The instrumental items needed for this part are a breadboard, the necessary circuit connective wires, the IOLab device, a diode, and three resistors. Aside from this, to measure the temperatures and carry out a water bath, a standard thermometer, a waterproof bag, a straw, and a water container are required. In our experiment, we used a 15 litre container, and the same electrical setup as in section 1.3.1, to reiterate.

2. Turn the IOLab on, make sure the circuit remains stable and is correctly setup, and put it in a waterproof zip bag. Once the zip bag is almost closed, place a straw in the hole and breathe in air from inside the bag through the straw. Doing this 10-15 times will reduce the amount of air in the bag, maximising the effect of the water bath. The electrical setup is ready to go in the water tank.

3. To prepare the 15 litre tank and adjust it to the correct temperature, we heated 7.5 litres of water to 37° C, or 307 Kelvin, and then adjusted the other 7.5 litres until the temperature in the center of the tank was about the one we needed. Now, the water is ready for the temperature bath.

4. Now, one must make sure that the circuit is as surrounded by the water as possible, to maximise the effects of temperature. Therefore, we used two plastic cups (approximated to cylinders) to rest the circuit, since plastic is an insulator. In fact, we found that heat transferred from our circuit to the exterior of the water tank by the equation

$$\frac{H}{t} = \frac{k\pi r^2(T_{circuit} - T_{plastic})}{d}$$

   where k is the Boltzmann Constant derived theoretically in 1.2, $\frac{H}{t}$ is the rate of heat transfer, r is the radius of the plastic cup, and d is the height of the plastic cup. Plugging in our values, we found that for a water temperature of 300 Kelvin, the heat left the circuit at a rate of 1.38×10^{-22} Joules per second, which is negligible. We decided to neglect this effect, but still placed the circuit on plastic cups for stability.

5. Now, the circuit and the IOLab should be underwater in the plastic bag. Make sure the thermometer reads a stable temperature throughout the experiment. Now, repeat steps 5-7 from section 1.3.1, but with the twist that every time we change a resistor, we need to reset the water to its correct temperature by redoing steps 2-4 in this section.

By following these steps, we will be left with 8 datasets, two per temperature, since we have two resistors to try out per temperature. An actual underwater picture of the setup we used is shown below. Note that there have been many simplifications in the setup's mathematical description since taking into account the geometry of the water container, plastic bag, and plastic cups when analyzing the heat transfer would in itself be a valid capstone project for a thermodynamics class, such as 5CL.

Having explained what the setup for measuring Boltzmann's Constant with regard to the resistance, temperature and voltage, we thought it would be a good idea to also explain some of the main sources of error we encountered while carrying out the measurements ourselves. The errors apply to both the measurements of the reverse bias, forwards bias, and temperature stabilization, and they are listed below, in no particular order:

- To minimise the error, make sure the bag rests underwater for some time before the measurement, so that the circuit and wires really gets a stable temperature where and when we want it to.
- Also, making sure to perform several readings with the thermometers will get rid of systematic errors in the measurement. In fact, measuring the temperature near the spot where the device will go will also help.
- Turning on the IOLab's temperature "thermometer" sensor every so often in the experiment will give us a more precise idea of what the temperature is as a function of time. In fact, we will be showing some of these thermometer plots in the Appendix section.
- All of the possible sources of error from section 1.3.1 regarding circuits also apply here.

Having said this, an image that we particularly enjoyed taking throughout the experiment, since it shows a diode lighting up underwater, only protected by a layer of plastic, from water at approximately 320 Kelvin:

1.4 Data Reduction/Analysis:

In this section, we will be discussing the analysis of the data we obtained. In other words, we will perform all the calculations that we learned during the Intro Labs and discuss our results. Similarly to the Methods section in this report, we will perform the analysis and discussion part by part, since the experimental setups are different. Our goal here is to present and interpret our collected data, and to describe how we have quantitatively and critically used the data to meet the goals we laid out in the introduction section. This section will be divided into three subsections: preparation, fitting, and analysis. In each of these sections, we will be considering the data obtained from the methods in 1.3.1 and 1.3.2 separately.

1.4.1 Data Preparation:

The raw data from our experiment was not in a convenient form to be easily analyzed. To enable for the best analysis to happen, we must do some error propagation on our variables and perform any necessary filtering/processing of our data. Since most of our datasets were found separately on IOLab, in this section we will discuss how we compared and filtered them to later analyze the best data possible.

1.4.1.1 Data Preparation Corresponding to Section 1.3.1:

We needed to manipulate the data we had collected to find the current and voltage running through the diode. The raw data from these six rounds is summarized here:

A	B	C	D	E	F
VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
FIRST ROUND					
0.94874	0.00029	1000	10	2.8253	0.0041
0.84754	0.00034	1000	10	2.7128	0.0038
0.75851	0.00028	1000	10	2.6126	0.005
0.67022	0.00029	1000	10	2.5131	0.005
0.58321	0.00028	1000	10	2.4133	0.005
0.48583	0.00025	1000	10	2.3012	0.0032
0.40061	0.00036	1000	10	2.2014	0.0041
0.31629	0.0005	1000	10	2.1012	0.0046
0.23597	0.0005	1000	10	2.0014	0.0029
0.15765	0.00043	1000	10	1.901	0.004
0.08066	0.0004	1000	10	1.7884	0.0024
0.02455	0.0003	1000	10	1.6872	0.0036
-0.00431	0.00028	1000	10	1.5854	0.0027
-0.00928	0.00031	1000	10	1.4822	0.0026
-0.0051	0.00026	1000	10	1.3667	0.0021

The data above is from Round 1, where we used a 1 kΩ resistor at room temperature. The voltage across the 1 Ω resistor is listed in column A, as the voltage across resistor B.

VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
SECOND ROUND					
0.14122	0.00024	10000	100	3.2814	0.0039
0.1314	0.00025	10000	100	3.1664	0.0047
0.12217	0.00022	10000	100	3.0639	0.006
0.11237	0.00023	10000	100	2.9604	0.0047
0.10333	0.00023	10000	100	2.8577	0.0055
0.09164	0.00031	10000	100	2.742	0.0038
0.08094	0.0003	10000	100	2.6386	0.0053
0.07205	0.00028	10000	100	2.5362	0.0044
0.06348	0.00022	10000	100	2.4332	0.0047
0.05191	0.00029	10000	100	2.3172	0.0049
0.04365	0.00028	10000	100	2.2147	0.0034
0.03634	0.00049	10000	100	2.1121	0.004
0.02673	0.00021	10000	100	2.0101	0.003
0.01795	0.00024	10000	100	1.907	0.0025
0.00883	0.00023	10000	100	1.7913	0.0035
0.00166	0.00023	10000	100	1.6887	0.0026
-0.0048	0.00021	10000	100	1.5856	0.0032
-0.00799	0.00034	10000	100	1.4826	0.0032
-0.00661	0.00023	10000	100	1.3669	0.0024

Likewise, the data above is from Round 2, where we used a 10 kΩ resistor at room temperature.

VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
THIRD ROUND					
-0.01846	0.00034	1000	10	0	0
-0.00933	0.00034	1000	10	0.3	0.003
-0.00824	0.00042	1000	10	0.6	0.006
0.06601	0.00032	1000	10	1	0.01
0.29861	0.00027	1000	10	1.2	0.012
0.5616	0.00022	1000	10	1.5	0.015
0.73525	0.00024	1000	10	1.8	0.018

Likewise, the data above is from Round 3, where we used a 1 kΩ resistor at room temperature. Unfortunately, from this round onward, we had to use the default reading on the DAC as our voltage across the circuit instead of taking a reading using A7. Since the DAC output setting did not always match the voltage it actually output (we tested this experimentally and discovered a somewhat reliable pattern), we had to adjust some of the voltages manually using a chart we had compiled previously, which may have introduced some errors.

VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
FOURTH ROUND					
-0.01742	0.00021	10000	100	0	0
-0.01222	0.00022	10000	100	0.3	0.003
-0.01084	0.0002	10000	100	0.6	0.006
-0.01019	0.00022	10000	100	1	0.01
-0.00982	0.00024	10000	100	1.2	0.012
-0.00879	0.00022	10000	100	1.5	0.015
0.00545	0.00022	10000	100	1.8	0.018
0.0313	0.00023	10000	100	2.1	0.021
0.0595	0.00025	10000	100	2.4	0.024
0.08681	0.00021	10000	100	2.7	0.027
0.10681	0.00021	10000	100	3	0.03
0.13563	0.00017	10000	100	3.3	0.033

Likewise, the data above is from Round 4, where we used a 10 kΩ resistor at room temperature.

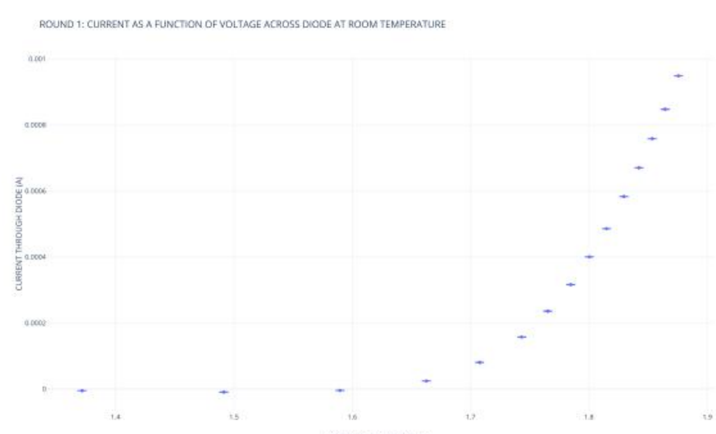
VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
FIFTH ROUND					
0.04331	0.0002	1000	10	1.7	0.017
0.24866	0.0002	1000	10	2	0.02
0.49454	0.00015	1000	10	2.3	0.023
0.67678	0.00023	1000	10	2.5	0.025
0.76378	0.00028	1000	10	2.6	0.027
0.9504	0.00026	1000	10	2.8	0.029

Likewise, the data above is from Round 5, where we used a 1 kΩ resistor at room temperature.

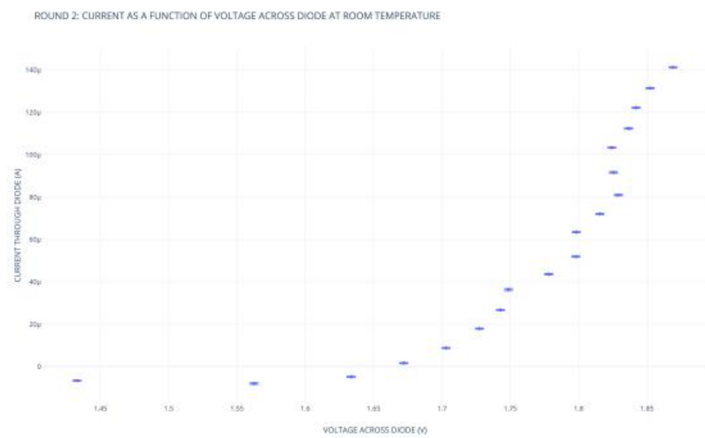
VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
SIXTH ROUND					
0.04025	0.00016	10000	100	2	0.02
0.06645	0.00018	10000	100	2.3	0.023
0.08571	0.00025	10000	100	2.5	0.025
0.10434	0.00022	10000	100	2.7	0.028
0.12412	0.00021	10000	100	2.9	0.03
0.15321	0.0002	10000	100	3.2	0.033

And finally, the data above is from Round 6, where we used a 10 kΩ resistor at room temperature.

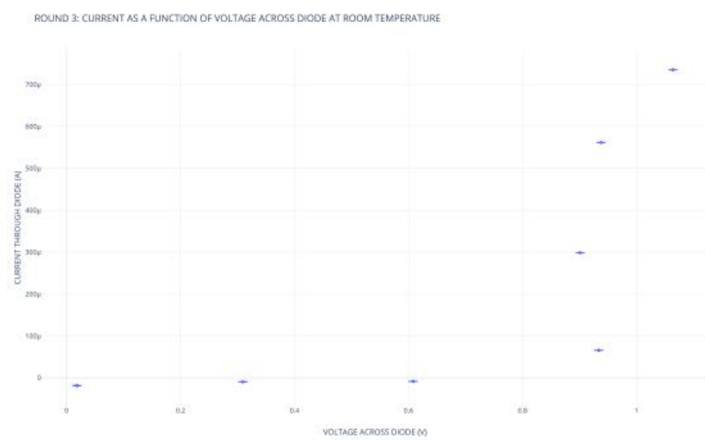
We additionally plotted this data with error bars using a simple graph maker. These graphs do not have curve fitting, we graph the data with curve fitting in a later section.



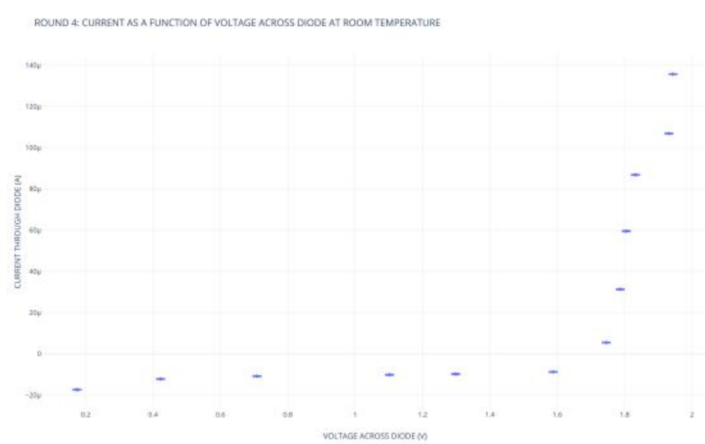
Above is the graph for Round 1.



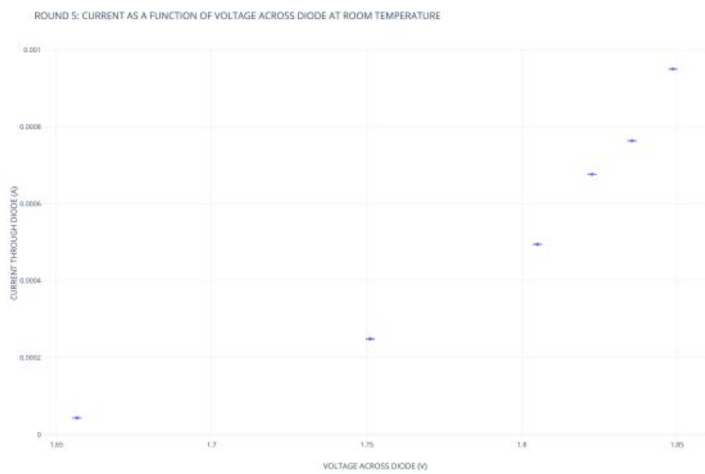
Above is the graph for Round 2.



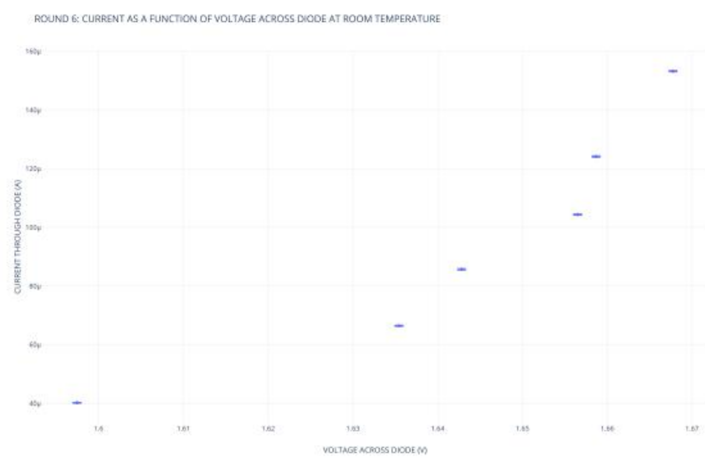
Above is the graph for Round 3.



Above is the graph for Round 4.



Above is the graph for Round 5.



Above is the graph for Round 6.

Note that the error bars are present, but are extremely small compared to the scale of the graph. As a quick preliminary analysis, we can visually confirm that using the 1 kΩ resistor gave the best results, and the data in Round 1 was the smoothest.

In order to use this data, we need to convert it to the voltage and current across the diode. The current can be found using the voltage across the 1 Ω resistor. The voltage across the diode is found using the formula $V_D = V_{\text{Total}} - I(R_A + 1)$, where the extra 1 Ω of resistance comes from the resistor we used to measure the current. We performed this calculation with a spreadsheet. The results are summarized below:

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
0.00094874	0.00000029	1.87561126	0.01033948872
0.00084754	0.00000034	1.86441246	0.009294527232
0.00075851	0.00000028	1.85333149	0.00908913081
0.00067022	0.00000029	1.84220978	0.008366824554
0.00058321	0.00000028	1.82950679	0.007687128676
0.00048583	0.00000025	1.81488417	0.005822860461
0.00040061	0.00000036	1.80038939	0.005743578722
0.00031629	0.0000005	1.78459371	0.005604858309
0.00023597	0.0000005	1.76519403	0.003772092833
0.00015765	0.00000043	1.74319235	0.004320951543
0.00008066	0.0000004	1.70765934	0.002563381306
0.00002455	0.0000003	1.66262545	0.003620835586
-0.00000431	0.00000028	1.58971431	0.00271485073
-0.00000928	0.00000031	1.49148928	0.002620096207
-0.0000051	0.00000026	1.3718051	0.002116680483

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 1.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
0.00014122	0.00000024	1.86905878	0.01484594342
0.0001314	0.00000025	1.8522686	0.01417747686
0.00012217	0.00000022	1.84207783	0.01378753267
0.00011237	0.00000023	1.83658763	0.01239561322
0.00010333	0.00000023	1.82429667	0.01192945711
0.00009164	0.00000031	1.82550836	0.01039378748
0.00008094	0.0000003	1.82911906	0.01012939466
0.00007205	0.00000028	1.81562795	0.008894582232
0.00006348	0.00000022	1.79833652	0.008199272654
0.00005191	0.00000029	1.79804809	0.007705073853
0.00004365	0.00000028	1.77815635	0.006201192875
0.00003634	0.00000049	1.74866366	0.007295255872
0.00002673	0.00000021	1.74277327	0.004533851679
0.00001795	0.00000024	1.72748205	0.003902970287
0.00000883	0.00000023	1.70299117	0.00428027418
0.00000166	0.00000023	1.67209834	0.003475430053
-0.0000048	0.00000021	1.6336048	0.003857626478
-0.00000799	0.00000034	1.56250799	0.004737162982
-0.00000661	0.00000023	1.43300661	0.003389392136

Above are the calculated voltages and currents through the diode for various voltages across the circuit in

Round 2.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
-0.00001846	0.00000034	0.01847846	0.0003871801591
-0.00000933	0.00000034	0.30933933	0.003020684725
-0.00000824	0.00000042	0.60824824	0.006015275782
0.00006601	0.00000032	0.93392399	0.01002688072
0.00029861	0.00000027	0.90109139	0.01236890614
0.0005616	0.00000022	0.9378384	0.01601836299
0.00073525	0.00000024	1.06401475	0.01944523005

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 3.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
-0.00001742	0.00000021	0.17421742	0.002728634465
-0.00001222	0.00000022	0.42221222	0.003915897349
-0.00001084	0.0000002	0.70841084	0.006416841594
-0.00001019	0.00000022	1.10191019	0.01028976817
-0.00000982	0.00000024	1.29820982	0.01227703043
-0.00000879	0.00000022	1.58790879	0.0151859675
0.00000545	0.00000022	1.74549455	0.01814216065
0.0000313	0.00000023	1.7869687	0.02135621591
0.0000595	0.00000025	1.8049405	0.02485264071
0.00008681	0.00000021	1.83181319	0.02843889314
0.00010681	0.00000021	1.93179319	0.03191386287
0.00013563	0.00000017	1.94356437	0.03571898021

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 4.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
0.00004331	0.0000002	1.65664669	0.01700669444
0.00024866	0.0000002	1.75109134	0.02015498101
0.00049454	0.00000015	1.80496546	0.02352614559
0.00067678	0.00000023	1.82254322	0.02590089038
0.00076378	0.00000028	1.83545622	0.02806090779
0.0009504	0.00000026	1.8486496	0.03051874426

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 5.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
0.00004025	0.00000016	1.59745975	0.02046365405
0.00006645	0.00000018	1.63543355	0.0240082626
0.00008571	0.00000025	1.64281429	0.0265464365
0.00010434	0.00000022	1.65649566	0.02996179774
0.00012412	0.00000021	1.65867588	0.0325341148
0.00015321	0.0000002	1.66774679	0.03643808229

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 6.

At this point, the Desmos curve fitter can fit this to our equation without any issues. However, the Desmos curve fitter cannot perform error analysis, so we need to linearize our data so that we can use our custom linear regression calculator. If we do this, we can additionally do an error-adjusted fit, which we hoped would get us a more accurate fit that the Desmos calculator. We will compare the two methods later on.

In order to linearize the diode equation, we differentiate by V on both sides then take the natural log. We also make the substitution $A = \frac{q}{nkT}$ for convenience:

$$
\begin{aligned}
I &= I_0 e^{AV} - I_0 \\
\therefore \frac{dI}{dV} &= I_0 A e^{AV} \\
\therefore \ln\left(\frac{dI}{dV}\right) &= \ln\left(I_0 A e^{AV}\right) = AV + \ln\left(I_0 A\right)
\end{aligned}
$$

Then, we can perform linear regression on $\ln\left(\frac{dI}{dV}\right)$ and V. Our calculated values for $\ln\left(\frac{dI}{dV}\right)$ and their respective errors are presented below:

LN(dI/dV)	ERROR IN LN(dI/dV)
-4.706463152	0.002561700625
-4.763200504	0.001193609148
-4.830012271	0.001067536352
-4.911979345	0.0008784390255
-4.998526557	0.0006528276376
-5.071793244	0.0006391902096
-5.185499203	0.0006980427654
-5.364908263	0.0007180072108
-5.564431525	0.0007562864528
-5.91472023	0.0006979791436
-6.405742449	0.0006152147755
-7.235675694	0.0007941669955
-8.528866852	0.001495133216
-12.52755612	0.03860850451
-10.26230047	0.00943199

Above are the linearized values for Round 1.

LN(dI/dV)	ERROR IN LN(dI/dV)
-7.444128348	0.004741713367
-7.255819239	0.002356570193
-6.714186473	0.002667555111
-6.84991248	0.002471354222
-6.281249147	0.002996210223
-5.372410504	0.004212104907
-6.223290056	0.003454718535
-7.474790375	0.002850705585
-6.77180132	0.002957886732
-6.925259776	0.002593997321
-8.062044485	0.004536004196
-7.645492665	0.002705625475
-7.049081593	0.004209238355
-7.706371629	0.002257805127
-8.131489364	0.002509518249
-8.535172484	0.002677181529
-9.337546687	0.004555539909
-11.61573229	0.01481360289
-11.44936332	0.02598002289

Above are the linearized values for Round 2.

LN(dI/dV)	ERROR IN LN(dI/dV)
-10.36903463	0.005079077183
-10.96314095	0.004822869738
-9.022830962	0.0006868467268
-6.861033392	0.0002372706517
-2.066670923	0.29575347
-5.921926128	0.0001536752267
-6.588394031	0.000287616445

Above are the linearized values for Round 3.

LN(dI/dV)	ERROR IN LN(dI/dV)
-10.77250443	0.005429387532
-11.30447852	0.00389871574
-12.72136802	0.01204780207
-13.26773442	0.0230848984
-12.75748879	0.01741985147
-10.28506053	0.002073036959
-8.510234187	0.0009328881507
-7.002913231	0.0008823779178
-6.694392821	0.0008508469508
-7.894060081	0.0008743588263
-7.735890007	0.0007157792626
-6.01233986	0.00227534925

Above are the linearized values for Round 4.

LN(dI/dV)	ERROR IN LN(dI/dV)
-6.131053387	0.0002455102987
-5.795141901	0.0001403484197
-5.117375965	0.0005554958278
-4.729575791	0.002256052694
-4.558194725	0.003536518346
-4.258396004	0.01044898211

Above are the linearized values for Round 5.

LN(dI/dV)	ERROR IN LN(dI/dV)
-7.278892316	0.001265289941
-6.905432742	0.0009515787889
-6.320543746	0.001295378072
-6.0233379	0.00176756561
-5.439060136	0.003530271057
-5.742433396	0.003472552883

Above are the linearized values for Round 6.

Throughout the rest of the analysis for this data, you may notice that we have omitted one or two values from the ends of each round. This is because the way that our calculator handled taking the natural log of a negative value was creating inaccuracies in the data. Unfortunately, there was no way to fix this, so we simply removed the points.

1.4.1.2 Data Preparation Corresponding to Section 1.3.2:

The procedure for preparing the data for the final four rounds, which were not done at room temperature, is essentially the same as for the rounds done at room temperature. The raw data is presented below:

VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
SEVENTH ROUND (280 k)					
-0.02326	0.0036	1000	10	1.8	0.018
0.29621	0.00062	1000	10	2.1	0.021
0.56074	0.0003	1000	10	2.4	0.024
0.73527	0.00032	1000	10	2.7	0.027
1.01045	0.00029	1000	10	3	0.03
1.07031	0.00016	1000	10	3.3	0.033

The data above is from Round 7, where we used a 1 kΩ resistor at 280 K.

VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
EIGHTH ROUND (280K)					
0.00527	0.00024	10000	100	1.8	0.018
0.0303	0.0002	10000	100	2.1	0.021
0.05619	0.0011	10000	100	2.4	0.024
0.07198	0.00051	10000	100	2.6	0.026
0.09942	0.00033	10000	100	2.9	0.029
0.12803	0.00025	10000	100	3.2	0.032

Likewise, the data above is from Round 8, where we used a 10 kΩ resistor at 280 K.

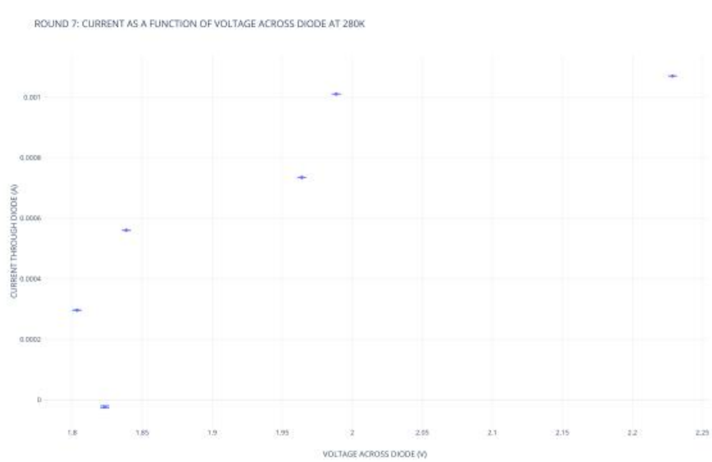
VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
NINTH ROUND (305K)					
0.07002	0.00034	1000	10	1.8	0.018
0.30426	0.00028	1000	10	2.1	0.021
0.5679	0.00022	1000	10	2.4	0.024
0.74171	0.00025	1000	10	2.6	0.026
1.01676	0.00025	1000	10	2.9	0.029
1.07014	0.000067	1000	10	3.2	0.032

Likewise, the data above is from Round 9, where we used a 1 kΩ resistor at 305 K. At this point, we decided to only use the 1 kΩ resistor, as it brought consistently better results.

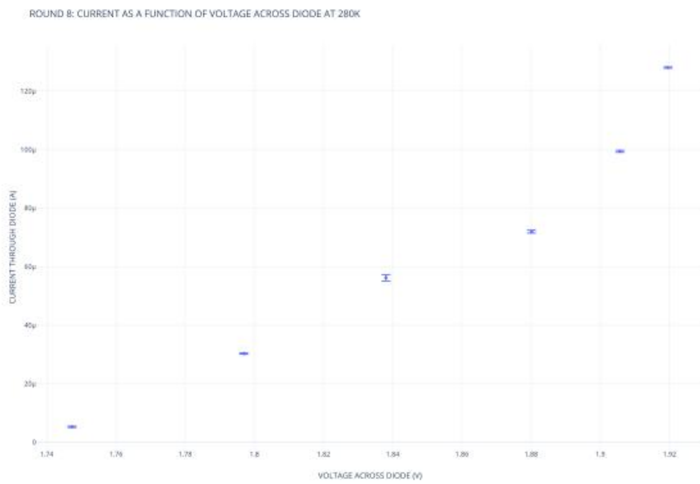
VOLTAGE ACROSS RESISTOR B (mV)	ERROR IN VOLTAGE ACROSS RESISTOR (mV)	RESISTOR A RESISTANCE (Ohms)	ERROR IN RESISTANCE (Ohms)	VOLTAGE ACROSS CIRCUIT (V)	ERROR IN VOLTAGE ACROSS CIRCUIT (V)
TENTH ROUND (320K)					
0.09294	0.00026	1000	10	1.8	0.018
0.33183	0.00026	1000	10	2.1	0.021
0.59715	0.00023	1000	10	2.4	0.024
0.7717	0.00027	1000	10	2.6	0.026
1.04781	0.00026	1000	10	2.9	0.029
1.07004	0.00012	1000	10	3.2	0.032

Likewise, the data above is from Round 10, where we used a 1 kΩ resistor at 320 K.

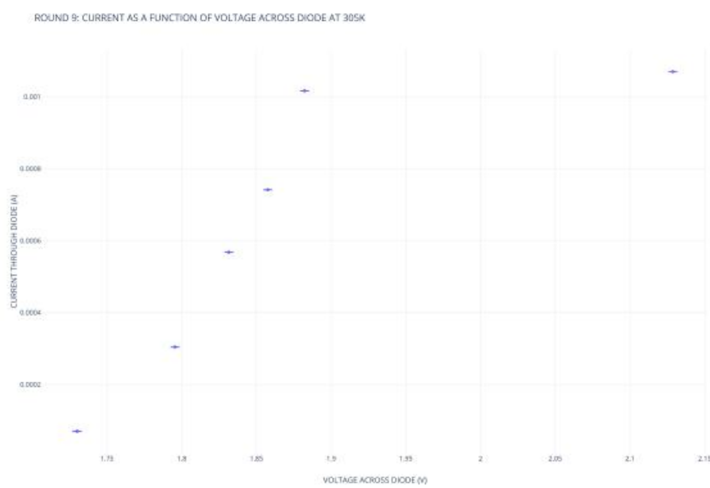
Again, we graphed this data with errors but without a fitted curve. The graphs are presented below:



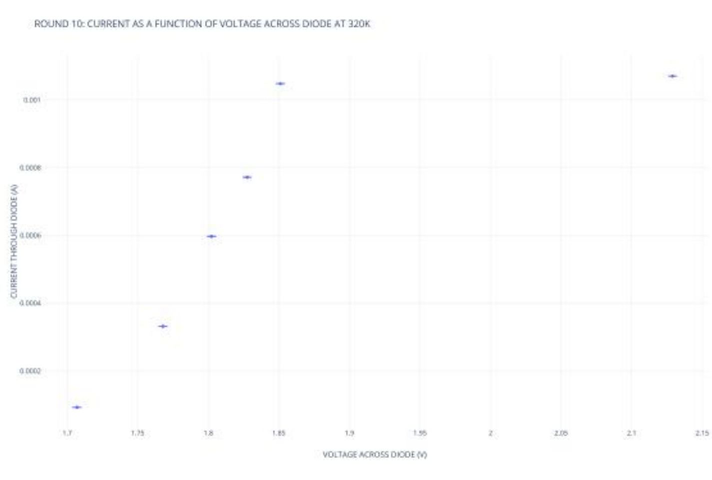
Above is the graph for Round 7.



Above is the graph for Round 8.



Above is the graph for Round 9.



Above is the graph for Round 10.

Again, we need to convert this to the current and voltage across the diode. The results of these calculations are presented below:

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
-0.00002326	0.0000036	1.82328326	0.01835865016
0.00029621	0.00000062	1.80349379	0.02121695562
0.00056074	0.0000003	1.83869926	0.02464818685
0.00073527	0.00000032	1.96399473	0.02798508178
0.00101045	0.00000029	1.98853955	0.03165730861
0.00107031	0.00000016	2.22861969	0.03469267936

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 7.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
0.00000527	0.00000024	1.74729473	0.01816697226
0.0000303	0.0000002	1.7969697	0.02131153913
0.00005619	0.0000011	1.83804381	0.02699254271
0.00007198	0.00000051	1.88012802	0.0274558993
0.00009942	0.00000033	1.90570058	0.03083399977
0.00012803	0.00000025	1.91957197	0.03455673681

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 8.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
0.00007002	0.00000034	1.72990998	0.01801682856
0.00030426	0.00000028	1.79543574	0.02122112089
0.0005679	0.00000022	1.8315321	0.0246637292
0.00074171	0.00000025	1.85754829	0.02703841707
0.00101676	0.00000025	1.88222324	0.03073178672
0.00107014	0.000000067	2.12878986	0.03374202809

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 9.

CURRENT THROUGH DIODE (A)	ERROR IN CURRENT THROUGH DIODE (A)	VOLTAGE ACROSS DIODE (V)	ERROR IN VOLTAGE ACROSS DIODE (V)
0.00009294	0.00000026	1.70696706	0.01802585697
0.00033183	0.00000026	1.76783817	0.02126214594
0.00059715	0.00000023	1.80225285	0.02473280854
0.0007717	0.00000027	1.8275283	0.0271224102
0.00104781	0.00000026	1.85114219	0.03083599058
0.00107004	0.00000012	2.12888996	0.03374185811

Above are the calculated voltages and currents through the diode for various voltages across the circuit in Round 10.

Again, we need to linearize this data. Our calculated values for $\ln\left(\frac{dI}{dV}\right)$ and their respective errors are presented below:

LN(dI/dV)	ERROR IN LN(dI/dV)
-4.126241884	0.006200597327
-3.273260227	0.02315040143
-5.901419	0.0002877428963
-5.808722342	0.0002113138094
-6.67181893	0.000163213647
-8.296719561	0.0006669251028

Above are the linearized values for Round 7.

LN(dI/dV)	ERROR IN LN(dI/dV)
-7.593181358	0.001644192743
-7.485597841	0.002953889226
-7.598480142	0.001729950634
-7.355668004	0.00361197438
-6.556391806	0.00156449174
-6.183827418	0.002591532169

Above are the linearized values for Round 8.

LN(dI/dV)	ERROR IN LN(dI/dV)
-5.633852396	0.0004290243797
-5.318657423	0.0003162926097
-4.955740927	0.0008052685316
-4.726795386	0.001464663509
-6.71644138	0.000120801115
-8.437951356	0.0005746467997

Above are the linearized values for Round 9.

LN(dI/dV)	ERROR IN LN(dI/dV)
-5.540510762	0.000427072532
-5.241643122	0.00034983878
-4.91044273	0.0008836757343
-4.686601481	0.001598782657
-6.917832575	0.0001446297223
-9.433025951	0.001365578471

Above are the linearized values for Round 10.

1.4.2 Data Fitting:

Now, our data and theoretical predictions are in a compatible form, so we will be fitting our data to optimise the analysis phase. Here, we will be discussing best-fit parameters and their uncertainties, statistical measures such as χ^2, and residuals. This section will be graph-heavy, since it is important to visualize the data.

1.4.2.1 Data Fitting Corresponding to Section 1.3.1:

In this section, we use linear regression to fit our data to the diode equation. We will be using our custom linear regression calculator. We also include the Desmos fitted curve in green on our graphs for reference. Here, we present the results of linear regression for each of the first 6 rounds:

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.87561126	0.01033948872	-4.706463152	0.002561700625
	1.86441246	0.009294527232	-4.763200504	0.001193609148
	1.85333149	0.00908913081	-4.830012271	0.001067536352
	1.84220978	0.008366824554	-4.911979345	0.0008784390255
	1.82950679	0.007687128676	-4.998526557	0.0006528276376
	1.81488417	0.005822860461	-5.071793244	0.0006391902096
	1.80038939	0.005743578722	-5.185499203	0.0006980427654
	1.78459371	0.005604858309	-5.364908263	0.0007180072108
	1.76519403	0.003772092833	-5.564431525	0.0007562864528
	1.74319235	0.004320951543	-5.91472023	0.0006979791436
	1.70765934	0.002563381306	-6.405742449	0.0006152147755
	1.66262545	0.003620835586	-7.235675694	0.0007941669955
	1.58971431	0.00271485073	-8.528866852	0.001495133216
	1.49148928	0.002620096207	-12.52755612	0.03860850451

Above is the input for our linear regression for Round 1.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	2.06E+01
	c	-4.19E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	2.00E-01
	UNCERTAINTY IN c	2.12E-02
	CHI-SQUARED	
	CHI-SQUARED	9.98E+02
	REDUCED CHI-SQUARED	8.31E+01

And above is the result of linear regression for Round 1. Here, m corresponds to A and c corresponds to $\ln(I_0 A)$.

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.86905878	0.01484594342	-7.444128348	0.004741713367
	1.8522686	0.01417747686	-7.255819239	0.002356570193
	1.84207783	0.01378753267	-6.714186473	0.002667555111
	1.83658763	0.01239561322	-6.84991248	0.002471354222
	1.82429667	0.01192945711	-6.281249147	0.002996210223
	1.82550836	0.01039378748	-5.372410504	0.004212104907
	1.82911906	0.01012939466	-6.223290056	0.003454718535
	1.81562795	0.008894582232	-7.474790375	0.002850705585
	1.79833652	0.008199272654	-6.77180132	0.002957886732
	1.79804809	0.007705073853	-6.925259776	0.002593997321
	1.77815635	0.006201192875	-8.062044485	0.004536004196
	1.74866366	0.007295255872	-7.645492665	0.002705625475
	1.74277327	0.004533851679	-7.049081593	0.004209238355
	1.72748205	0.003902970287	-7.706371629	0.002257805127
	1.70299117	0.00428027418	-8.131489364	0.002509518249
	1.67209834	0.003475430053	-8.535172484	0.002677181529
	1.6336048	0.003857626478	-9.337546687	0.004555539909
	1.56250799	0.004737162982	-11.61573229	0.01481360289

Above is the input for our linear regression for Round 2.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	1.65E+01
	c	-3.64E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	2.66E-01
	UNCERTAINTY IN c	2.00E-02
	CHI-SQUARED	
	CHI-SQUARED	7.53E+02
	REDUCED CHI-SQUARED	4.71E+01

And above is the result of linear regression for Round 2.

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	0.30933933	0.003020684725	-10.96314095	0.004822869738
	0.60824824	0.006015275782	-9.022830962	0.0006868467268
	0.93392399	0.01002688072	-6.861033392	0.0002372706517
	0.90109139	0.01236890614	-2.066670923	0.29575347
	0.9378384	0.01601836299	-5.921926128	0.0001536752267
	1.06401475	0.01944523005	-6.588394031	0.000287616445

Above is the input for our linear regression for Round 3.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	6.88E+00
	c	-1.31E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	9.35E-02
	UNCERTAINTY IN c	2.04E-02
	CHI-SQUARED	
	CHI-SQUARED	3.12E+02
	REDUCED CHI-SQUARED	7.81E+01

And above is the result of linear regression for Round 3.

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.10191019	0.01028976817	-13.26773442	0.0230848984
	1.29820982	0.01227703043	-12.75748879	0.01741985147
	1.58790879	0.0151859675	-10.28506053	0.002073036959
	1.74549455	0.01814216065	-8.510234187	0.0009328881507
	1.7869687	0.02135621591	-7.002913231	0.0008823779178
	1.8049405	0.02485264071	-6.694392821	0.0008508469508
	1.83181319	0.02843889314	-7.894060081	0.0008743588263
	1.93179319	0.03191386287	-7.735890007	0.0007157792626
	1.94356437	0.03571898021	-6.01233986	0.00227534925

Above is the input for our linear regression for Round 4.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	8.29E+00
	c	-2.28E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	1.69E-01
	UNCERTAINTY IN c	5.04E-02
	CHI-SQUARED	
	CHI-SQUARED	1.63E+02
	REDUCED CHI-SQUARED	2.33E+01

And above is the result of linear regression for Round 4.

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.65664669	0.01700669444	-6.131053387	0.0002455102987
	1.75109134	0.02015498101	-5.795141901	0.0001403484197
	1.80496546	0.02352614559	-5.117375965	0.0005554958278
	1.82254322	0.02590089038	-4.729575791	0.002256052694
	1.83545622	0.02806090779	-4.558194725	0.003536518346
	1.8486496	0.03051874426	-4.258396004	0.01044898211

Above is the input for our linear regression for Round 5.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	8.75E+00
	c	-2.08E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	1.22E+00
	UNCERTAINTY IN c	8.95E-02
	CHI-SQUARED	
	CHI-SQUARED	6.11E+00
	REDUCED CHI-SQUARED	1.53E+00

And above is the result of linear regression for Round 5.

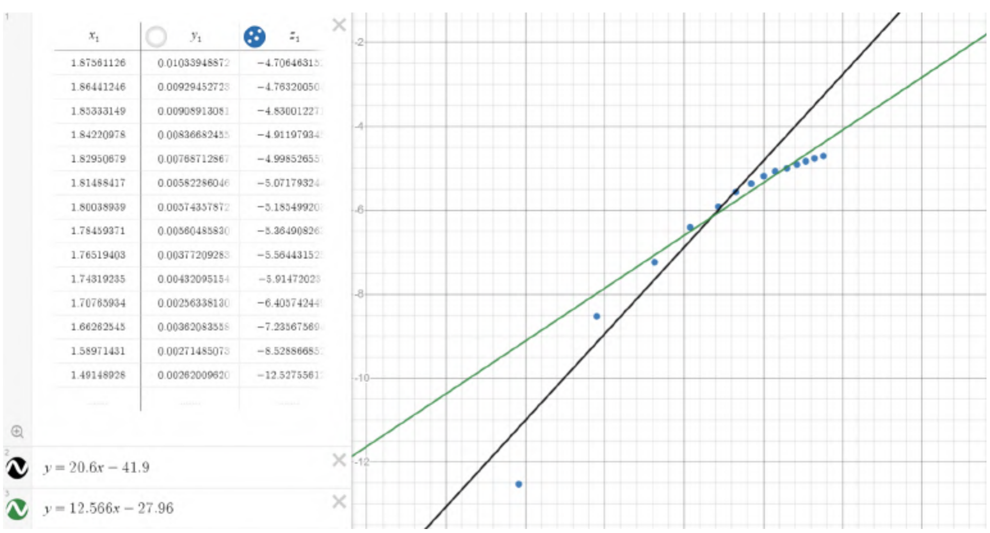
INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.59745975	0.02046365405	-7.278892316	0.001265289941
	1.63543355	0.0240082626	-6.905432742	0.0009515787889
	1.64281429	0.0265464365	-6.320543746	0.001295378072
	1.65649566	0.02996179774	-6.0233379	0.00176756561
	1.65867588	0.0325341148	-5.439060136	0.003530271057
	1.66774679	0.03643808229	-5.742433396	0.003472552883

Above is the input for our linear regression for Round 6.

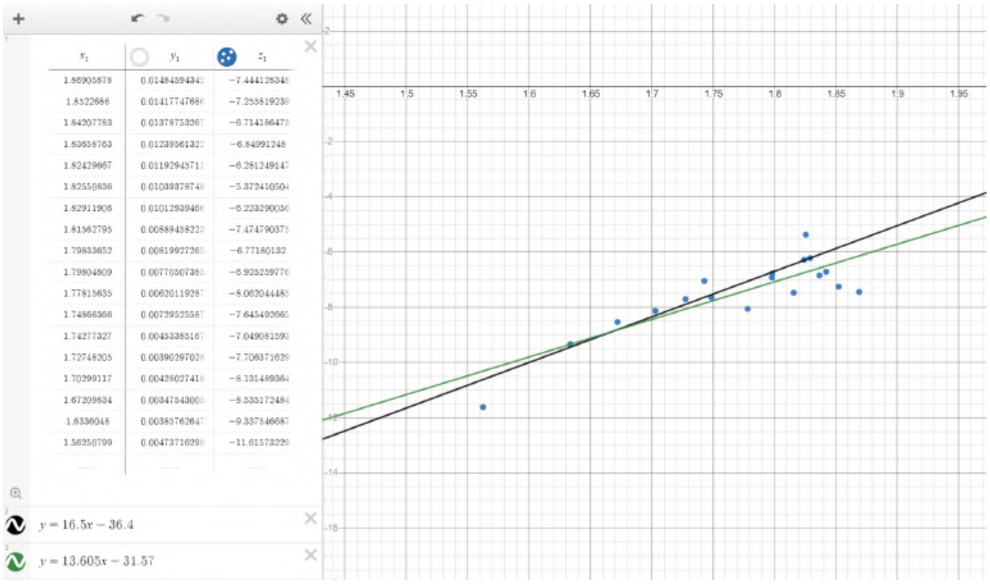
WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	2.36E+01
	c	-4.51E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	1.10E+01
	UNCERTAINTY IN c	2.77E-01
	CHI-SQUARED	
	CHI-SQUARED	8.86E-01
	REDUCED CHI-SQUARED	2.22E-01

And above is the result of linear regression for Round 6.

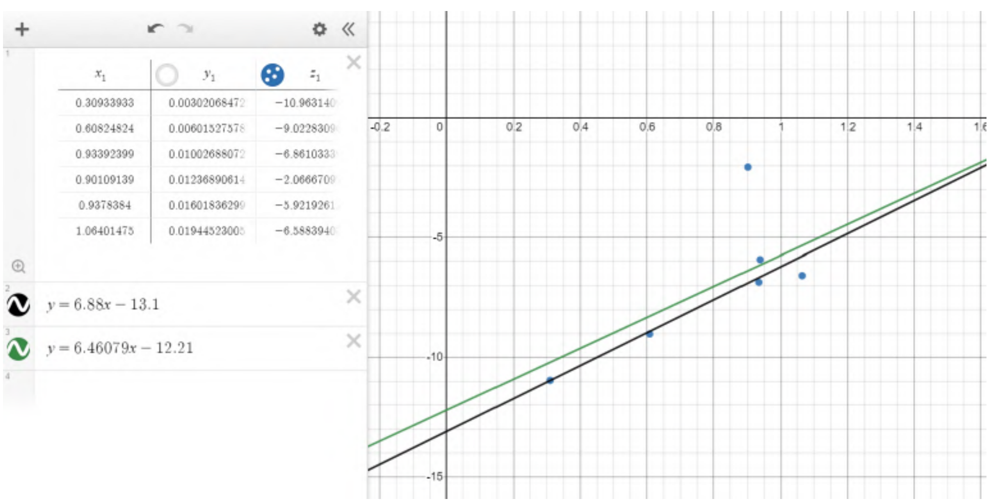
From an initial inspection, our χ^2 values are unacceptably large. This suggests that we have underestimated our errors. However, we proceed with the analysis because we do not have the luxury of rethinking our entire experiment at this point. We additionally graphed the linearized data. Again, the line calculated by our linear regression is shown in black, and the line calculated by the Desmos curve fitter is shown in green for reference:



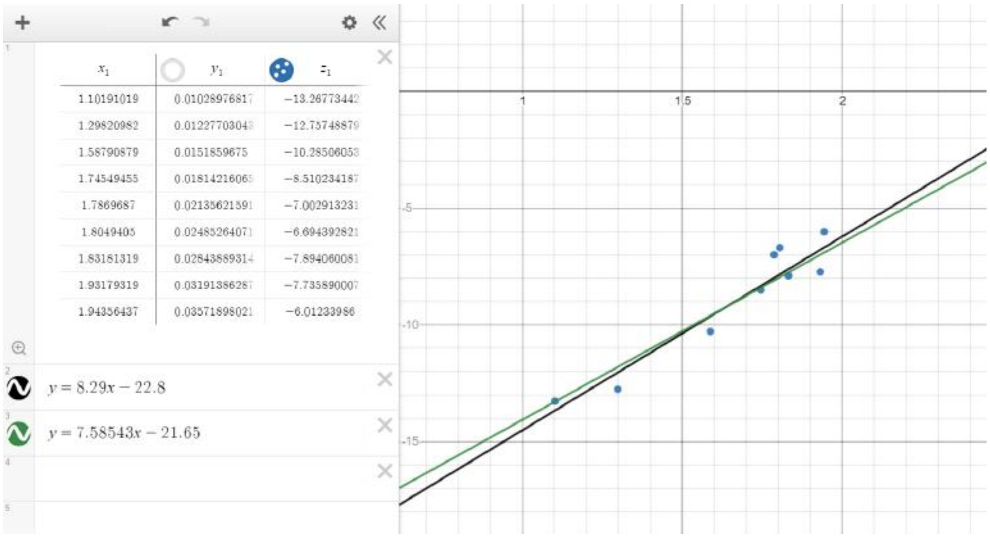
Above is the graph of the linearized data from Round 1.



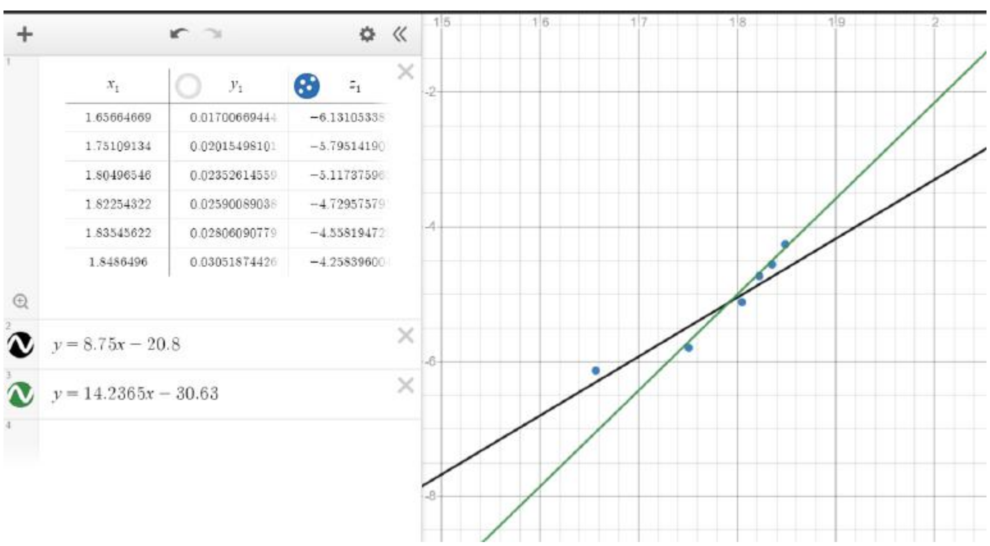
Above is the graph of the linearized data from Round 2.



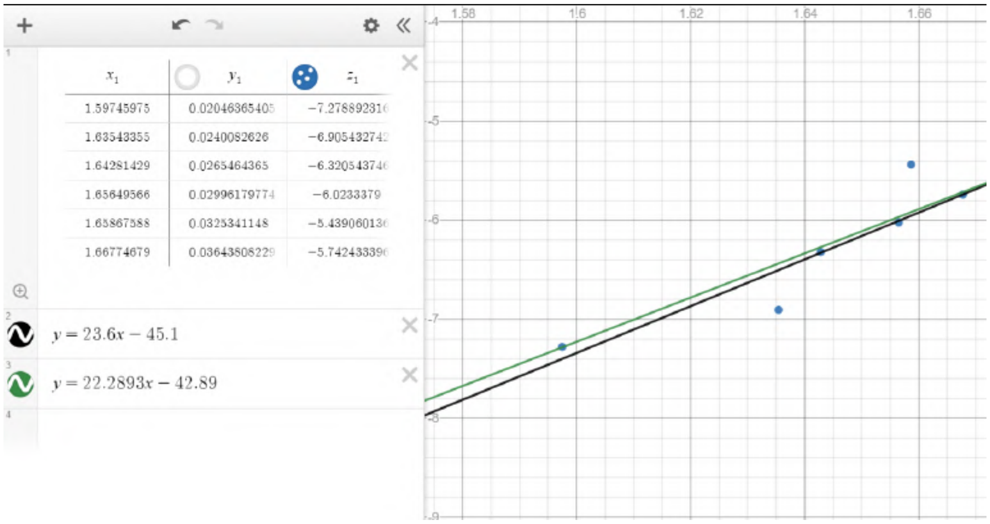
Above is the graph of the linearized data from Round 3.



Above is the graph of the linearized data from Round 4.



Above is the graph of the linearized data from Round 5.



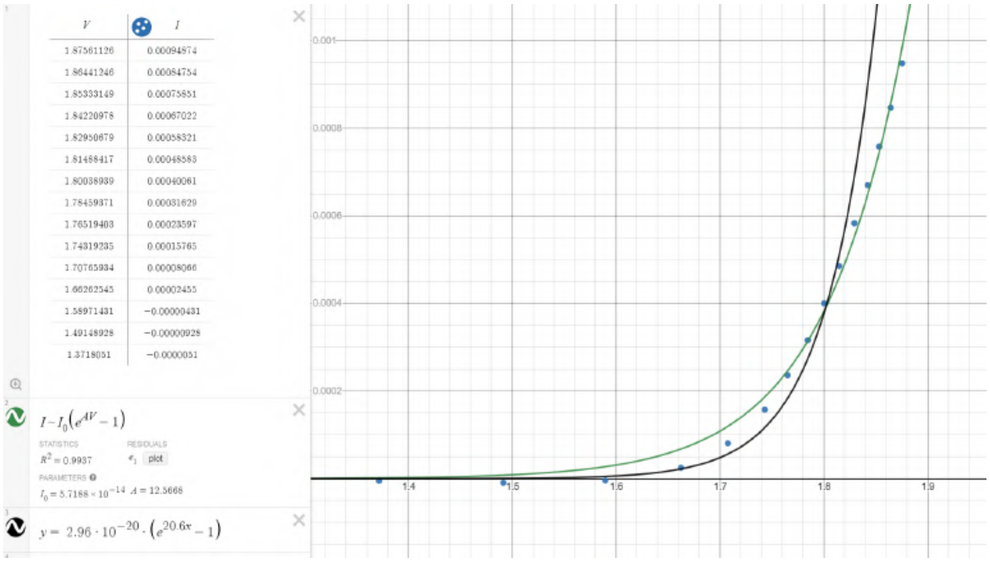
Above is the graph of the linearized data from Round 6.

Visually, most of the graphs are moderately good fits. Worryingly, however, the linearized first round appears to display some kind of pattern. We hypothesize that this is due to the quality factor of the diode changing with the current through it. Other than the first round, however, the data seems to decently fit the model. It is important to note that the graphs have all been scaled for clarity, and are not as uniform as they appear. The true scale is shown on the edges of the graphs.

We additionally used the results of our regression to calculate the coefficients of our exponential curve. We graphed this curve against our collected data of the voltage and current across the diode. Again, we used green for the Desmos fit and black for our fit:

CAPSTONE:DISPLAY	
A	2.06E+01
Error in A	2.00E-01
I_0	2.96E-20
Error in I_0	6.93E-22
Temperature (K)	293
Error in Temperature (K)	2.93
n	2
Error in n	0.2
k	1.33E-23
Error in k	1.34E-24

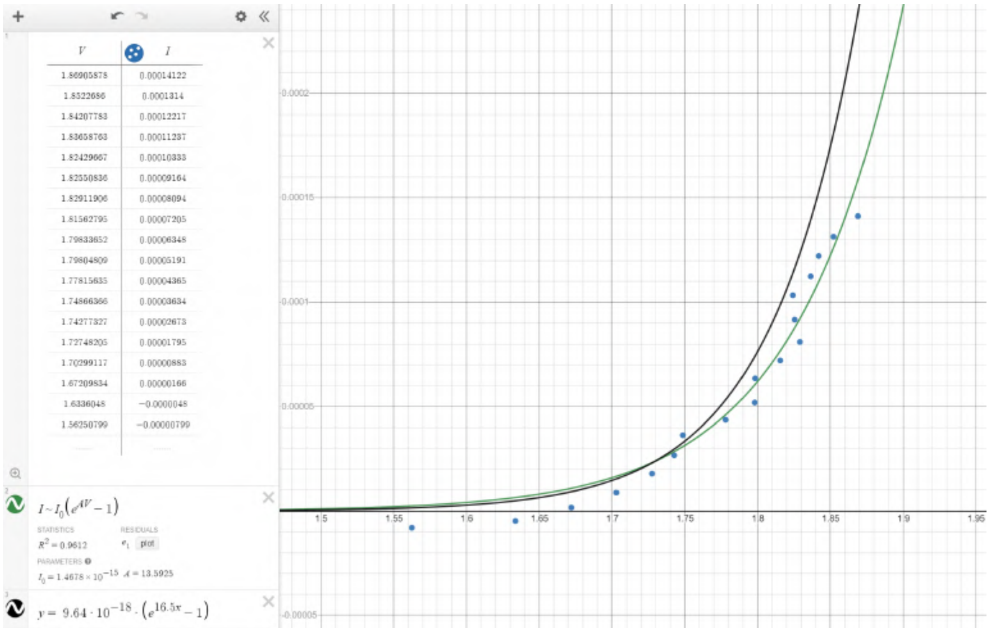
Above are the values for A and I_0 calculated for Round 1 using our regression. We additionally included some other data, including our estimate of 2 for the quality factor n of the LED. Our calculated value for Boltzmann's Constant is shown at the bottom, along with its associated error.



Above is the complete Round 1 IV graph for the LED. The values Desmos calculated for A and I_0 are shown at the side.

CAPSTONE:DISPLAY	
A	1.65E+01
Error in A	2.66E-01
I_0	9.64E-18
Error in I_0	2.48E-19
Temperature (K)	293
Error in Temperature (K)	2.93
n	2
Error in n	0.2
k	1.66E-23
Error in k	1.69E-24

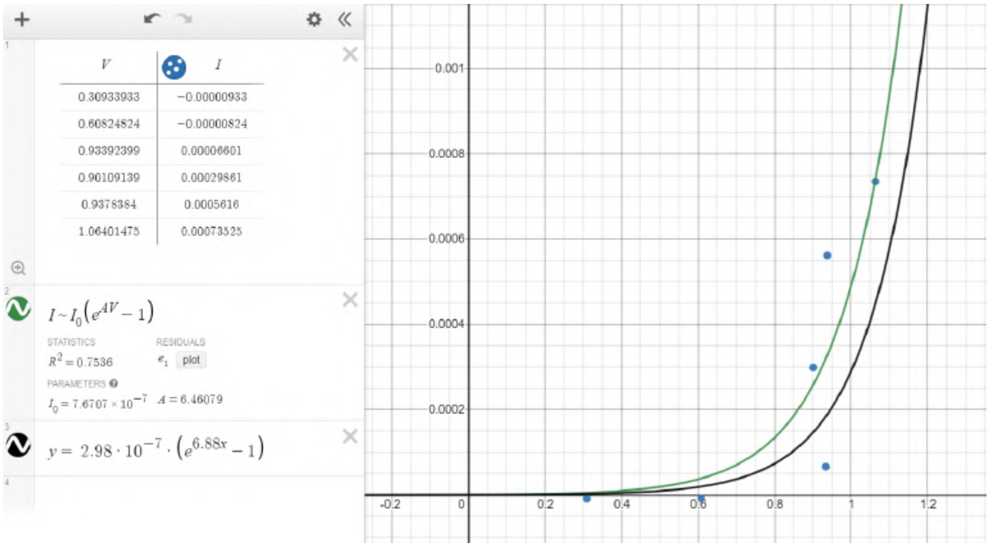
Above are the values for A and I_0 calculated for Round 2 using our regression.



Above is the complete Round 2 IV graph for the LED.

CAPSTONE:DISPLAY	
A	6.88E+00
Error in A	9.35E-02
I_0	2.98E-07
Error in I_0	7.29E-09
Temperature (K)	293
Error in Temperature (K)	2.93
n	2
Error in n	0.2
k	3.97E-23
Error in k	4.03E-24

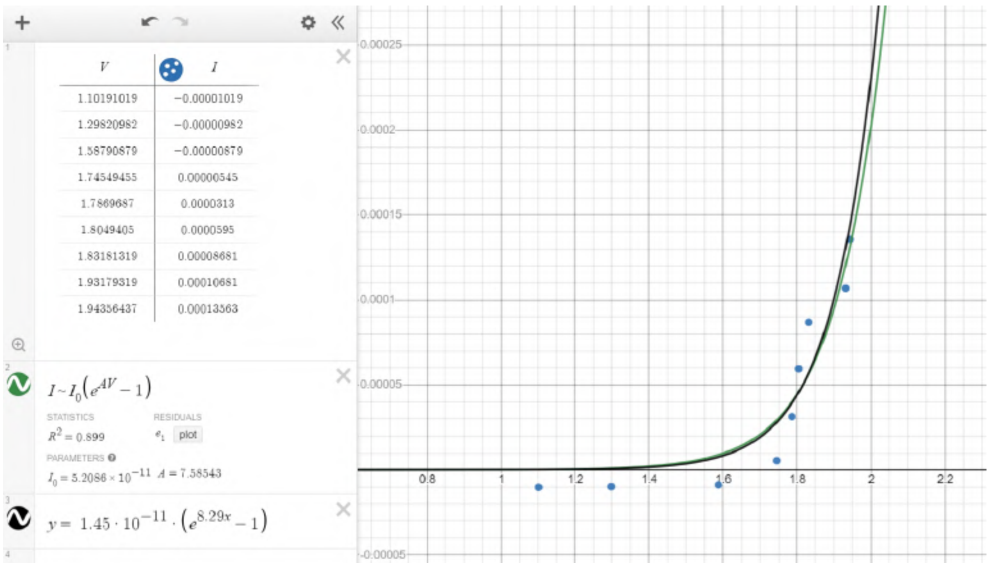
Above are the values for A and I_0 calculated for Round 3 using our regression.



Above is the complete Round 3 IV graph for the LED.

CAPSTONE:DISPLAY	
A	8.29E+00
Error in A	1.69E-01
I_0	1.45E-11
Error in I_0	7.89E-13
Temperature (K)	293
Error in Temperature (K)	2.93
n	2
Error in n	0.2
k	3.29E-23
Error in k	3.38E-24

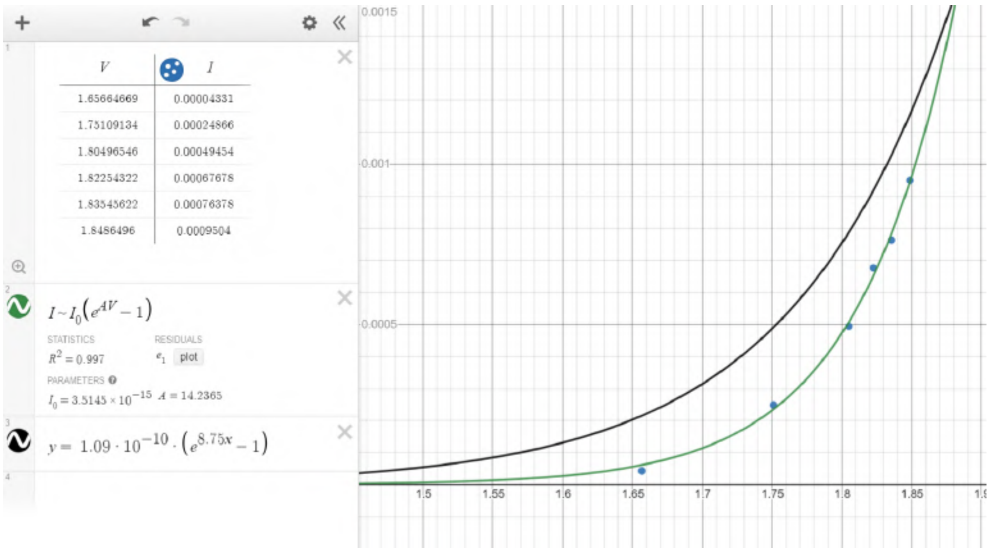
Above are the values for A and I_0 calculated for Round 4 using our regression.



Above is the complete Round 4 IV graph for the LED.

CAPSTONE:DISPLAY	
A	8.75E+00
Error in A	1.22E+00
I_0	1.09E-10
Error in I_0	1.81E-11
Temperature (K)	293
Error in Temperature (K)	2.93
n	2
Error in n	0.2
k	3.12E-23
Error in k	5.36E-24

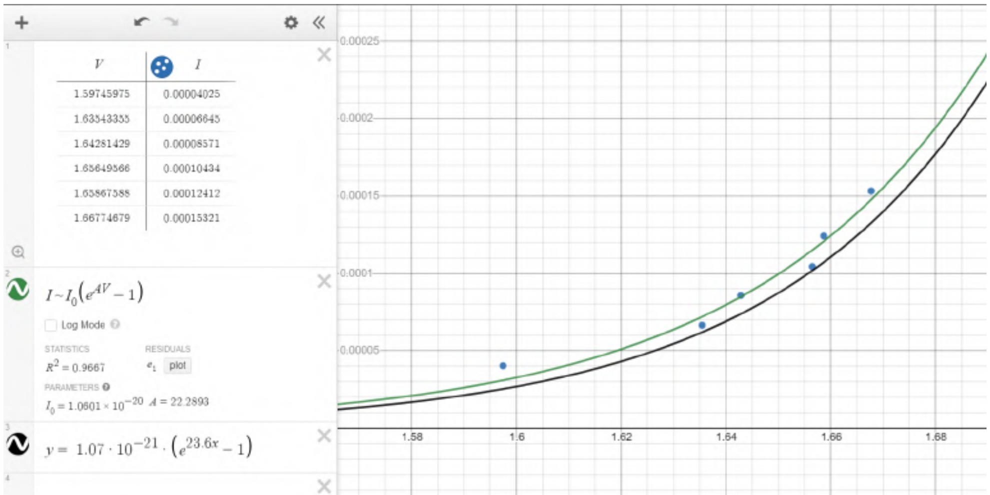
Above are the values for A and I_0 calculated for Round 5 using our regression.



Above is the complete Round 5 IV graph for the LED.

CAPSTONE:DISPLAY	
A	2.36E+01
Error in A	1.10E+01
I_0	1.07E-21
Error in I_0	5.76E-22
Temperature (K)	293
Error in Temperature (K)	2.93
n	2
Error in n	0.2
k	1.16E-23
Error in k	5.50E-24

Above are the values for A and I_0 calculated for Round 6 using our regression.



Above is the complete Round 6 IV graph for the LED.

1.4.2.2 Data Fitting Corresponding to Section 1.3.2:

In this section, we use linear regression to fit our data to the diode equation. We will be using our custom linear regression calculator. We also include the Desmos fitted curve in green on our graphs for reference. Here, we present the results of linear regression for each of the final 4 rounds:

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.82328326	0.01835865016	-4.126241884	0.006200597327
	1.80349379	0.02121695562	-3.273260227	0.02315040143
	1.83869926	0.02464818685	-5.901419	0.0002877428963
	1.96399473	0.02798508178	-5.808722342	0.0002113138094
	1.98853955	0.03165730861	-6.67181893	0.000163213647
	2.22861969	0.03469267936	-8.296719561	0.0006669251028

Above is the input for our linear regression for Round 7.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	-1.09E+01
	c	1.56E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	8.32E-01
	UNCERTAINTY IN c	1.01E-01
	CHI-SQUARED	
	CHI-SQUARED	5.33E+01
	REDUCED CHI-SQUARED	1.33E+01

And above is the result of linear regression for Round 7.

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.74729473	0.01816697226	-7.593181358	0.001644192743
	1.7969697	0.02131153913	-7.485597841	0.002953889226
	1.83804381	0.02699254271	-7.598480142	0.001729950634
	1.88012802	0.0274558993	-7.355668004	0.00361197438
	1.90570058	0.03083399977	-6.556391806	0.00156449174
	1.91957197	0.03455673681	-6.183827418	0.002591532169

Above is the input for our linear regression for Round 8.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	5.78E+00
	c	-1.78E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	1.18E+00
	UNCERTAINTY IN c	7.37E-02
	CHI-SQUARED	
	CHI-SQUARED	1.50E+01
	REDUCED CHI-SQUARED	3.76E+00

And above is the result of linear regression for Round 8.

INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.72990998	0.01801682856	-5.633852396	0.0004290243797
	1.79543574	0.02122112089	-5.318657423	0.0003162926097
	1.8315321	0.0246637292	-4.955740927	0.0008052685316
	1.85754829	0.02703841707	-4.726795386	0.001464663509
	1.88222324	0.03073178672	-6.71644138	0.000120801115
	2.12878986	0.03374202809	-8.437951356	0.0005746467997

Above is the input for our linear regression for Round 9.

WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	-6.78E+00
	c	6.71E+00
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	7.89E-01
	UNCERTAINTY IN c	8.39E-02
	CHI-SQUARED	
	CHI-SQUARED	6.78E+01
	REDUCED CHI-SQUARED	1.69E+01

And above is the result of linear regression for Round 9.

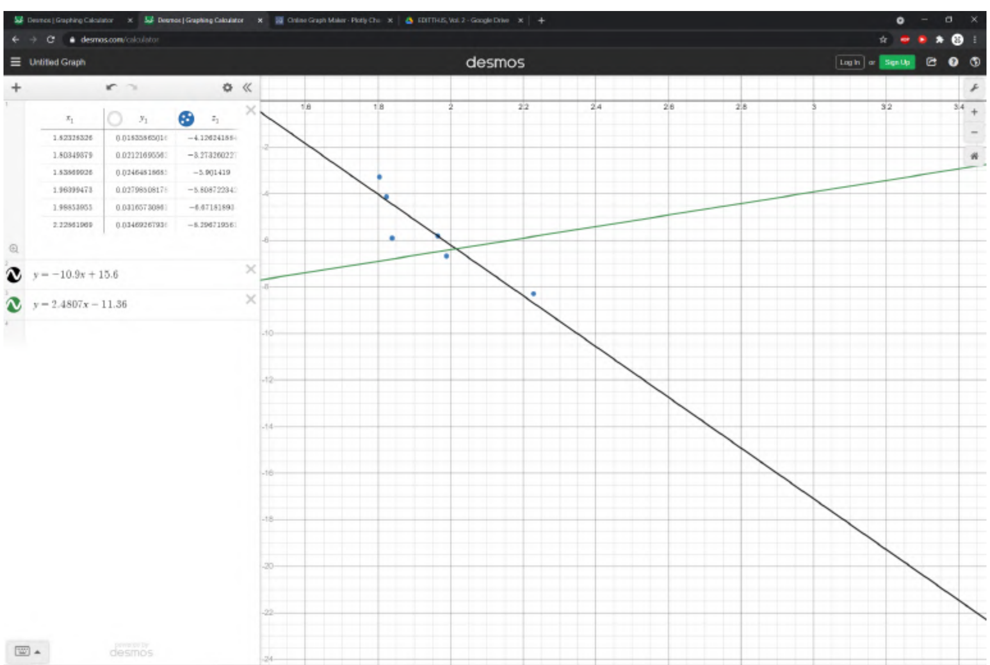
INPUT DATA	x	VARIANCE IN x	y	VARIANCE IN y
	1.70696706	0.01802585697	-5.540510762	0.000427072532
	1.76783817	0.02126214594	-5.241643122	0.00034983878
	1.80225285	0.02473280854	-4.91044273	0.0008836757343
	1.8275283	0.0271224102	-4.686601481	0.001598782657
	1.85114219	0.03083599058	-6.917832575	0.0001446297223
	2.12888996	0.03374185811	-9.433025951	0.001365578471

Above is the input for our linear regression for Round 10.

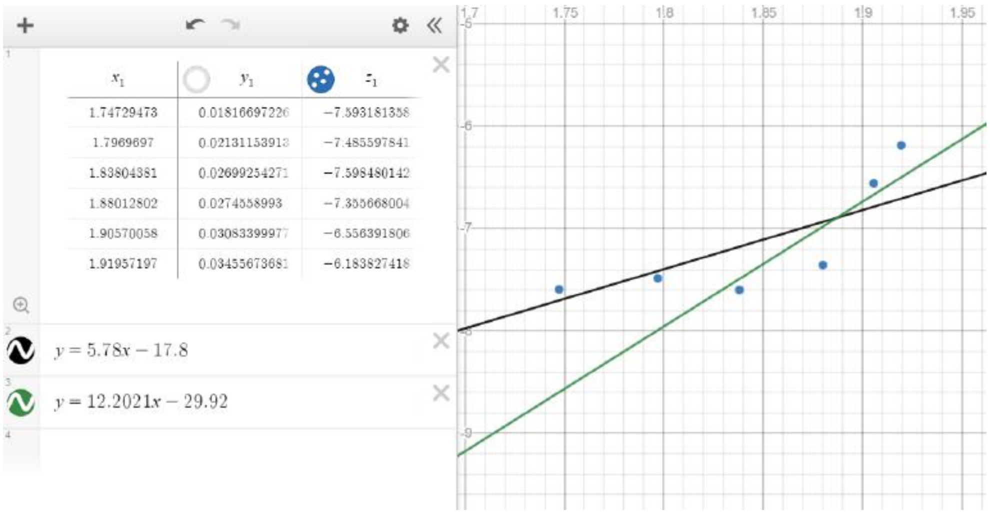
WEIGHTED LINEAR MODEL WITH ADJUSTED ERROR		
	BEST FIT PARAMETERS	
	m	-9.25E+00
	c	1.09E+01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	9.61E-01
	UNCERTAINTY IN c	1.07E-01
	CHI-SQUARED	
	CHI-SQUARED	4.94E+01
	REDUCED CHI-SQUARED	1.24E+01

And above is the result of linear regression for Round 10.

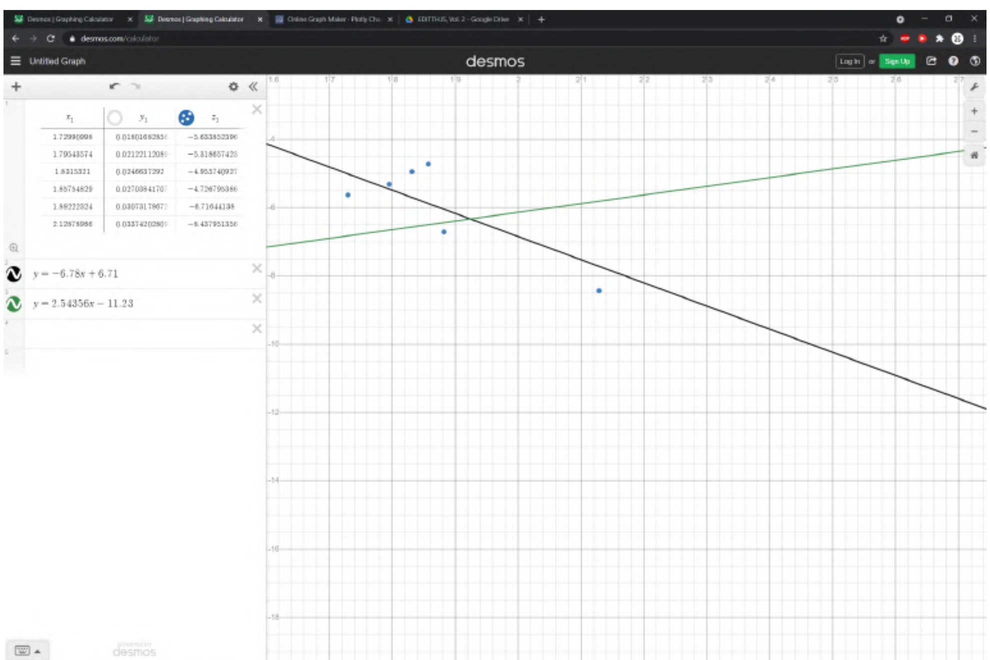
From an initial inspection, our χ^2 values are actually decent, within range of being acceptable. However, strangely, we calculated negative m values, which will contribute to making our curve for the exponential equation extremely incorrect, as we will see later on. We additionally graphed the linearized data. Again, the line calculated by our linear regression is shown in black, and the line calculated by the Desmos curve fitter is shown in green for reference:



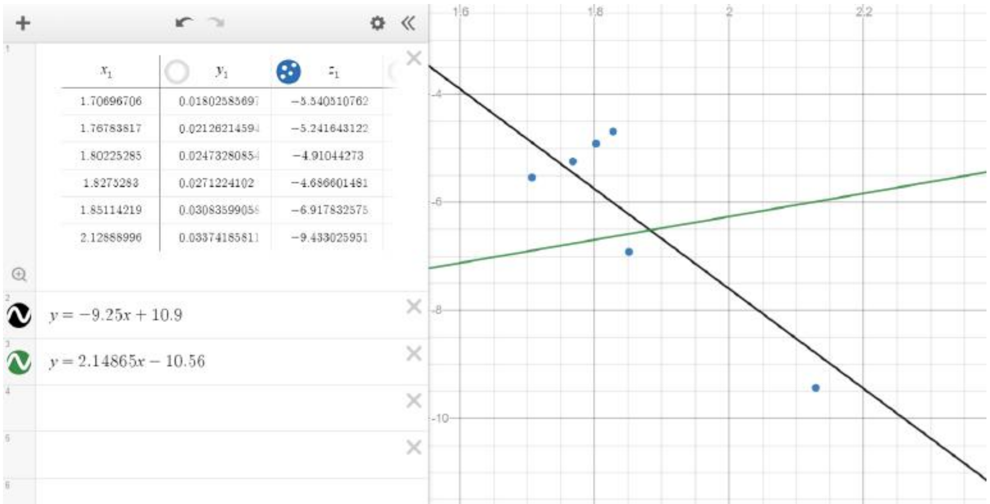
Above is the graph of the linearized data from Round 7.



Above is the graph of the linearized data from Round 8.



Above is the graph of the linearized data from Round 9.



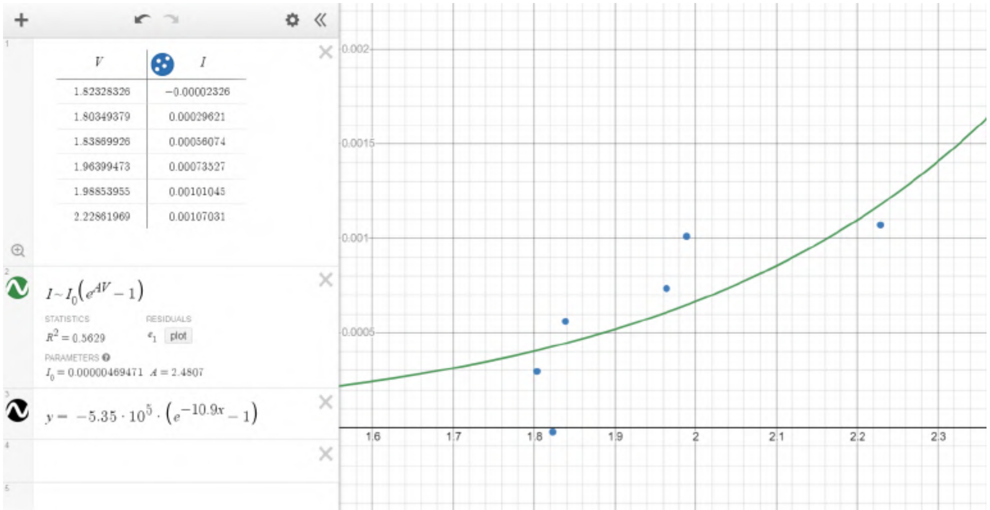
Above is the graph of the linearized data from Round 10.

Again, the lines roughly make sense given the points. However, the two fits are in extremely disagreement with each other. The Desmos fit, which was calculated by working backwards from the curve fitted to the exponential graph, consistently slopes upwards, but our line sometimes slopes downwards. This is absurd because it would imply a negative Boltzmann's Constant. We hypothesize that this is because of the way that the high gain sensor behaves at its limits. If we were to redo this experiment, we would take the measurement at many more voltages so that we could have extra data to fall back on if we were forced to remove some due to the high gain sensor maxing out. It is again important to note that the graphs have all been scaled for clarity, and are not as uniform as they appear. The true scale is shown on the edges of the graphs.

We additionally used the results of our regression to calculate the coefficients of our exponential curve. We graphed this curve against our collected data of the voltage and current across the diode. Again, we used green for the Desmos fit and black for our fit. It is important to note that the black curve, which is our fit, is not visible in some of the graphs. This is because of the negative value for A, which completely threw off the curve. There was no nice way to fix this, because removing the points at which the sensor maxed out would leave an unacceptably scant amount of data.

CAPSTONE:DISPLAY	
A	-1.09E+01
Error in A	8.32E-01
I_0	-5.35E+05
Error in I_0	-6.77E+04
Temperature (K)	280
Error in Temperature (K)	2.8
n	2
Error in n	0.2
k	-2.62E-23
Error in k	-3.30E-24

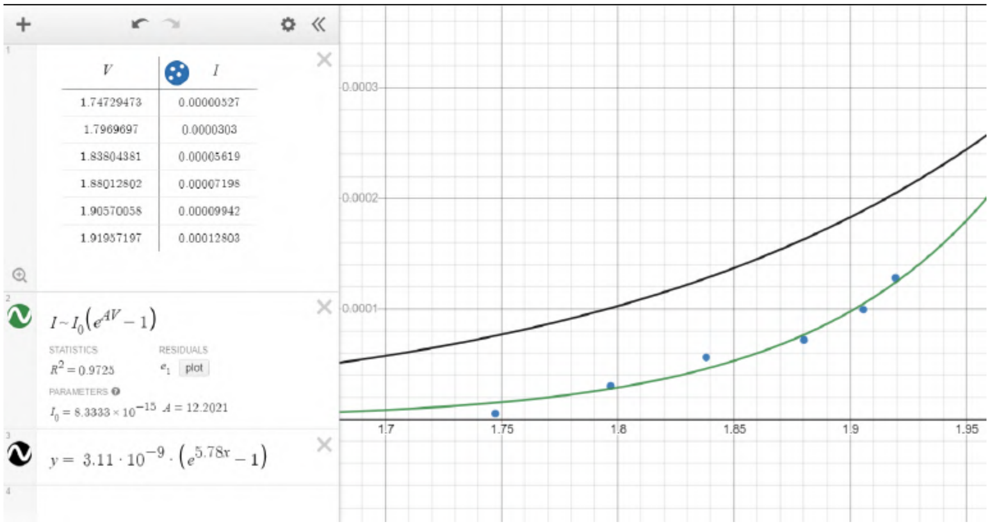
Above are the values for A and I_0 calculated for Round 7 using our regression. Note that the temperature is set to 280 instead of 293, like in the previous 6 rounds.



Above is the complete Round 7 IV graph for the LED.

CAPSTONE:DISPLAY	
A	5.78E+00
Error in A	1.18E+00
I_0	3.11E-09
Error in I_0	6.74E-10
Temperature (K)	280
Error in Temperature (K)	2.8
n	2
Error in n	0.2
k	4.94E-23
Error in k	1.12E-23

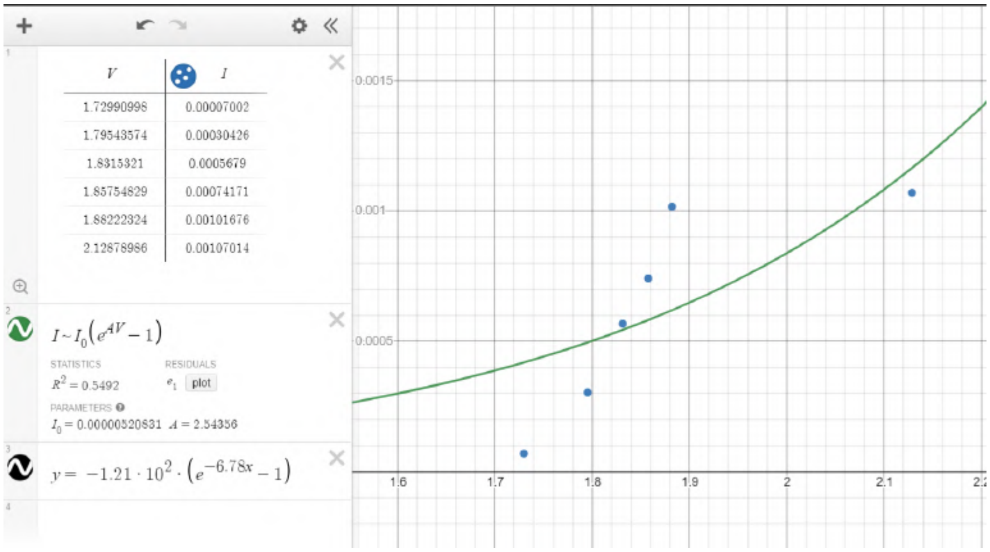
Above are the values for A and I_0 calculated for Round 8 using our regression. The temperature here is 280K.



Above is the complete Round 8 IV graph for the LED.

CAPSTONE:DISPLAY	
A	-6.78E+00
Error in A	7.89E-01
I_0	-1.21E+02
Error in I_0	-1.73E+01
Temperature (K)	305
Error in Temperature (K)	3.05
n	2
Error in n	0.2
k	-3.87E-23
Error in k	-5.94E-24

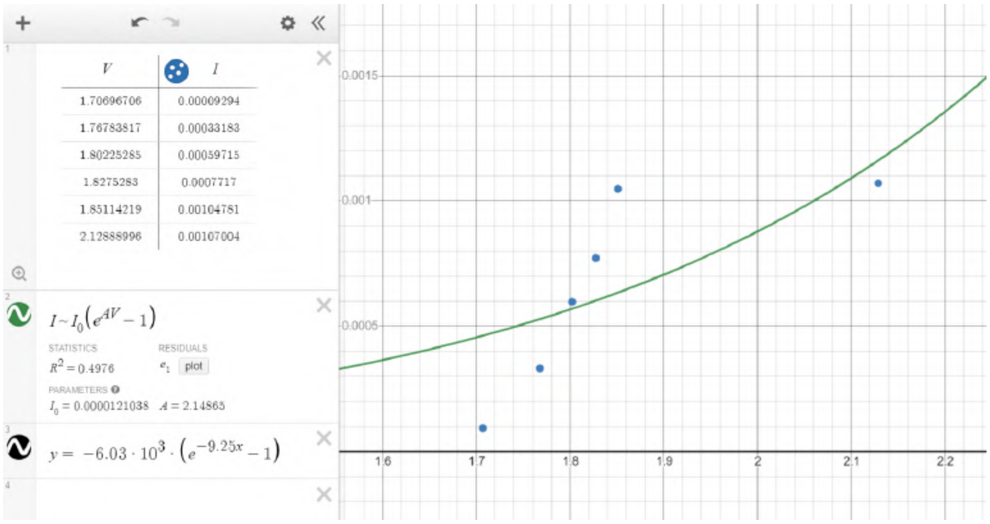
Above are the values for A and I_0 calculated for Round 9 using our regression. The temperature here is 305K.



Above is the complete Round 9 IV graph for the LED.

CAPSTONE:DISPLAY	
A	-9.25E+00
Error in A	9.61E-01
I_0	-6.03E+03
Error in I_0	-9.02E+02
Temperature (K)	320
Error in Temperature (K)	3.2
n	2
Error in n	0.2
k	-2.70E-23
Error in k	-3.91E-24

Above are the values for A and I_0 calculated for Round 10 using our regression. The temperature here is 320K.



Above is the complete Round 10 IV graph for the LED.

1.4.3 Data Analysis:

In this subsection, we will be discussing any comments, notes, or interpretations of what we found together with our final Boltzmann Constant value obtained through the experiments in 1.3.1 and 1.3.2. Our analysis will be grounded in quantitative data, qualitative observations from the experiment, and physical reasoning. In addition, we will comment the nature of our results and their validity through the agreement test.

1.4.3.1 Data Analysis Corresponding to Section 1.3.1:

In these rounds, we only varied the resistance and voltage in the circuit containing the diode. Over the six rounds, we calculated Boltzmann's constant to be, in order, $(1.33 \pm 0.134) \cdot 10^{-23}$, $(1.66 \pm 0.169) \cdot 10^{-23}$, $(3.97 \pm 0.403) \cdot 10^{-23}$, $(3.29 \pm 0.338) \cdot 10^{-23}$, $(3.12 \pm 0.536) \cdot 10^{-23}$, and $(1.16 \pm 0.550) \cdot 10^{-23}$. Boltzmann's constant is accepted to be $1.38 \cdot 10^{-23}$ with an uncertainty of 0.0000507%. Out of the six rounds, only the results for the first and last rounds agreed with this value. However, three out of the four failed results agreed with each other, perhaps suggesting that there was some uniform, systematic error in the way those rounds were performed. We hypothesize that this was due to underestimating the quality factor of the LED. This lines up with the fact that we used different LEDs between the first two rounds, the next two rounds, and the final two rounds.

We additionally want to note that the Desmos fit gave us values of $2.17 \cdot 10^{-23}$, $2.01 \cdot 10^{-23}$, $4.23 \cdot 10^{-23}$, $3.60 \cdot 10^{-23}$, $1.92 \cdot 10^{-23}$, and $1.22 \cdot 10^{-23}$. (You may recognise that the first two of these values were presented in our presentation last week.) Since Desmos is not equipped to handle regression with uncertainties, we could not perform proper agreement tests with these values. A quick inspection confirms that these values are on the same order of magnitude as Boltzmann's constant, and somewhat congregate around the correct value. However, they are off by significant percentages. We do want to note, however, that the rounds that used the same LED seem to agree with each other. Therefore, we hypothesize that the error is caused by, again, bad estimation of the quality factor n.

Overall, we feel that the Desmos curve fitter was better at handling more "chaotic" data, or data that was not very uniform. However, we feel that, given a more uniform set of data, like in Round 1, our linear regression calculator generates a better fit because it adjusts based on uncertainty in the variables. In this experiment, all the data past the first two rounds were fairly chaotic, meaning that the Desmos fit was probably more accurate than our fit. Taking this into consideration, the best result came from Round 1. We would try to recreate the conditions and procedure of Round 1 if we were to recreate this experiment.

1.4.3.2 Data Analysis Corresponding to Section 1.3.2:

In these four rounds, we varied the environmental temperature as well as the resistance and voltage in the circuit. Over the four rounds, we calculated Boltzmann's constant to be, in order, $-(2.62 \pm 0.330) \cdot 10^{-23}$, $(4.94 \pm 1.12) \cdot 10^{-23}$, $-(3.87 \pm 0.594) \cdot 10^{-23}$, and $-(2.70 \pm 0.391) \cdot 10^{-23}$. None of these results agreed with Boltzmann's constant. The negative results are likely caused by the high gain sensor reaching its maximum reading, causing the derivative $\frac{dI}{dV}$ to decrease drastically. This would make $\ln\left(\frac{dI}{dV}\right)$ decrease as V increased. Uncannily, the negative values are still on the same order of magnitude as Boltzmann's constant despite common sense telling us that they should be wildly off. We have no explanation for this behavior, and we can only hypothesize that it is due to a blind coincidence. Also note that the 8th round did not give a negative result because we used the 10 kΩ resistor instead of the 1 kΩ resistor like we did in the others.

The results Desmos gives us are similarly bad; In order, they were $1.10 \cdot 10^{-22}$, $2.23 \cdot 10^{-23}$, $1.07 \cdot 10^{22}$, and $1.27 \cdot 10^{22}$. The rounds where the 1 kΩ resistor was used strongly overestimated Boltzmann's constant. This is likely due to, again, the high gain sensor hitting its maximum reading. Indeed, looking at the graphs, these rounds consistently have points which appear to have conspicuously high voltages in comparison to the rest of the points. This causes the calculated value for A to decrease, causing the calculated value for Boltzmann's constant to increase dramatically. We recommend repeating this experiment with better values for the voltage across the entire circuit.

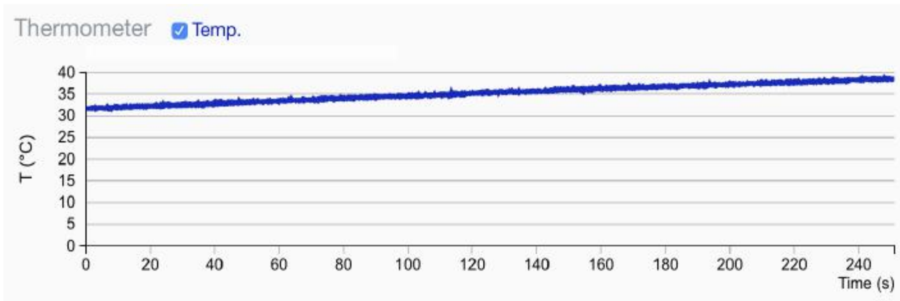
All results in these four rounds of the experiment were fairly unsuccessful. However, it was somewhat interesting to see how the temperature affected the behavior of the diode. The results, despite not agreeing with the true value, did agree roughly with each other, suggesting that the temperature affected the experiment in the way that we expected it to.

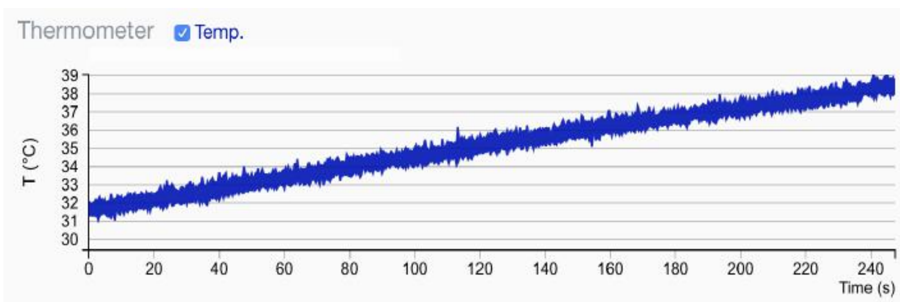
1.5 Summary/Conclusion:

Over the course of this experiment, we measured the voltage and current across a diode in an attempt to experimentally determine Boltzmann's constant. We ultimately split the experiment into 10 rounds between all 3 of us. The first 6 rounds were performed at room temperature using two different resistances and several different voltages. The final 4 rounds were performed at varying temperatures in addition to varying resistances and voltages. This allowed us to confirm that the circuit behaved as expected under temperature variation. Ultimately, out of the 10 rounds, we were only able to derive satisfactory results from the very first round of testing. If we were to repeat this experiment, we recommend recording data at more voltages, and being more careful about overloading the high gain sensor. We also recommend finding an easier method to maintain the temperature of the diode, as it was very difficult to take many measurements and retake bad data with the method we used.

Appendix: Graphs

These graphs show the IOLab heating up to some specific temperature over approximately 240 seconds. They show the relevance of waiting for the IOLab's temperature to be stable, and not measuring right after the IOLab goes in the water:





Appendix: References

https://www.shivajicollege.ac.in/sPanel/uploads/econtent/2c83b90687dc5b3149139d49a5b8c0e6.pdf

https://www.youtube.com/watch?v=DelOpL15YLA

https://en.wikipedia.org/wiki/Diode

Appendix: Presentation

Link to our presentation slides. Note that there have been some changes in our Project (especially the analysis) between the Capstone presentation and report submission:
https://docs.google.com/presentation/d/1Thq8Nr6vAaPQhkov19ro2smM2i5lrWLUqcKsLwZQT-Q/edit?usp=sharing