Lab 4

Hooke's Law: Fitting curves to derive parameters

Lab Objectives

- Produce good graphical representation of data
- Introduce linear regression to analyze data
- Using graphs to estimate random and systematic errors
- Introduce the χ^2 statistic

Group members,
and their group roles:

Overview

You will perform an experiment that require you to practice all the skills introduced in the course so far and extend them further. You'll determine the spring constant k of a single spring by measuring the spring's equilibrium length with different suspended masses and measuring the resultant equilibrium length. You will analyze your data with methods presented in this course so far, which require you to assume that k is constant, and then using Hooke's Law to fit the dataset with linear regression.

Be especially careful to …

- Look for ways to reduce errors as you notice them!
- Make sure your data and analyses are clear, so you can refer to them again later!
- Generate good scientific graphs

What to Turn In

This lab submission differs from past ones because only one experiment will be undertaken for Intro Lab 4. Each group member must submit their own lab worksheet pdf individually.

Each student is responsible for acquiring a full dataset for the experiment. The group, as a whole, can decide which of the three generated datasets each person will analyze and provide a *justification* for the decision. Options: each student uses the best group dataset or the individual uses their own dataset.

Learning regression analysis is crucial for success in the Physics 5 and 111 series. Therefore, **each student in a group** will be responsible for analyzing *one* dataset and answering the related questions. Group members can discuss the analysis and questions, but each student's written submission including the graphs must be their own work. **Each Gradescope pdf submission should include the filled-in worksheet with the specified 4 plots. A separate bCourses submission is required for the Python or Excel used in the analysis.**

If the group decides using the best group dataset for each member's analysis is appropriate, **that dataset must be included** in the other group member's individual lab reports either as an Appendix, or highlighted to indicate the dataset analyzed when combining in a single table. The data sets should be properly attributed in all cases.

Developing a Measurement Procedure Overview

Throughout the 5 series labs, you will be more and more responsible for devising, carrying out, analyzing, and adjusting experimental procedures.

You should first identify the goal of the experiment, the primary data you will need to collect, and the equipment you will need. Note that there may be multiple strategies that you have to choose between. Can you come up with another approach that works better?

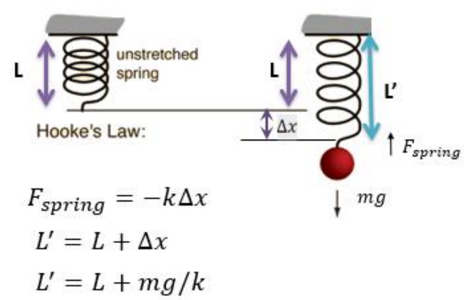
You should also carefully consider your experimental setup. This includes your work surface, positioning and orientation of the equipment, strategies to reduce "noise", stability, and reproducibility.

For each measurement you will need a measurement strategy. This includes: minimizing random errors in your measurement; identifying and eliminating of systematic errors; performing multiple independent measurements; etc.

You should have a clearly laid out procedure before you begin the experiment. However, during the experiment you will often find unanticipated roadblocks, flaws, or sources of error in your design. You will need to adapt! Always be on the lookout for these things and adjust your procedure to account for them. In your report, you should note when and where you saw the need for adjustment and how you and your group adapted your procedure.

Theory

A Hookean spring of spring constant k exerts a force $F = -k\Delta x$ when displaced a distance Δx from equilibrium. For a Hookean spring, k is a constant. When hung from a ceiling, a massive spring will stretch somewhat under its own weight, but the effect of this is simply to establish an equilibrium length L, and Hooke's Law ($F = -k\Delta x$) still applies for this new equilibrium.



If a mass m is suspended from the spring, the spring will stretch a distance $\Delta x = mg/k$ to balance the gravitational force, resulting in a new equilibrium length $L' = L + \Delta x$, or

$$\Delta L = mg/k \qquad (1)$$

where $\Delta L \equiv L' - L$ is the change in equilibrium length, L is the equilibrium length with no suspended mass, L' is the equilibrium length with a mass m suspended, and g is the acceleration due to gravity.

Equipment

- Spring from IOLab kit, longest one
- 250 gm weight set
- Tape measure
- Homemade measurement setup
- Ruler
- Wooden dowel
- Gloves

Weight set. Unpack the weight set. The kit gloves were provided for handling the brass. Finger tighten the hooked hanger into the base of the hanger. The weights are slotted, and must be removed from the top. The values stamped on each weight is accurate to 0.1 gm.

Spring from the IOLab kit. Use the longest spring, which is also the most tightly wound one.

Measurement instrument. Choose either the ruler or tape measure, as appropriate for your setup.

Homemade measurement setup. You will be hanging different weights on the spring and measuring the spring extension. The spring fits perfectly on the dowel and can be used as the spring support on top. You will need some type of dowel support that's tall enough to suspend the spring with the heaviest mass and simultaneously measure the stretched spring length.

Figure 1 Example of Intro Lab 4 setup

Prelab

This prelab should go quickly.

> **Task 1: Construct the measurement setup and submit a picture.** Set up the dowel support, hang the spring from it with and without the weight set attached, and figure out how to measure the spring length. *Be sure your apparatus functions properly for all masses!* Do not leave the weight set hanging for when not actively taking measurements, to avoid stretching out the spring.

Procedure and Data Collection

You will measure the equilibrium lengths of the spring with various masses suspended from it. The following steps will guide you through this measurement process and data analysis.

1. ***Propagate error symbolically.*** Solve for k in Equation (1) and propagate error to find the fractional error α_k/k in terms of $\alpha_{\Delta L}/\Delta L$, and α_m/m. You can neglect the error in g. Solve for the error $\alpha_{\Delta L}$ in terms of the quantities you'll be measuring, L' and L, to arrive at a final α_k/k.

$\Delta L = \frac{mg}{K} \rightarrow K = \frac{mg}{\Delta L}$. Since k depends on ΔL and m, $\alpha_k = \sqrt{\left(\frac{\partial k}{\partial m}\alpha_m\right)^2 + \left(\frac{\partial k}{\partial \Delta L}\alpha_{\Delta L}\right)^2}$. To simplify, we must first find α_m and $\alpha_{\Delta L}$.

Since $\Delta L \equiv L' - L$, $\alpha_{\Delta L} = \sqrt{\left(\frac{\partial \Delta L}{\partial L'}\alpha_{L'}\right)^2 + \left(\frac{\partial \Delta L}{\partial L}\alpha_L\right)^2}$. Also, we can rearrange eq.(1) to be: $m = \frac{k\Delta L}{g}$. Therefore, $\alpha_m = \sqrt{\left(\frac{\partial m}{\partial \Delta L}\alpha_{\Delta L}\right)^2 + \left(\frac{\partial m}{\partial k}\alpha_k\right)^2}$.

Now, we have:

$$\alpha_{\Delta L} = \sqrt{\left(\frac{\partial \Delta L}{\partial L'}\alpha_{L'}\right)^2 + \left(\frac{\partial \Delta L}{\partial L}\alpha_L\right)^2}$$

$$\alpha_m = \sqrt{\left(\frac{\partial m}{\partial \Delta L}\alpha_{\Delta L}\right)^2 + \left(\frac{\partial m}{\partial k}\alpha_k\right)^2}$$

$$\alpha_k = \sqrt{\left(\frac{\partial k}{\partial m}\alpha_m\right)^2 + \left(\frac{\partial k}{\partial \Delta L}\alpha_{\Delta L}\right)^2}$$

$\rightarrow$ Simplifying by solving the partial derivatives. $\rightarrow$

$$\alpha_{\Delta L} = \sqrt{(\alpha_{L'})^2 + (-\alpha_L)^2}$$

$$\alpha_m = \sqrt{\left((k\alpha_{\Delta L})^2 + (\Delta L\alpha_k)^2\right)\frac{1}{g^2}}$$

$$\alpha_k = \sqrt{\left((g\alpha_m)^2 + \left(-\frac{mg}{\Delta L}\alpha_{\Delta L}\right)^2\right)\frac{1}{\Delta L^2}}$$

$\rightarrow$ Now, solving for $\frac{\alpha_k}{k}$, $\frac{\alpha_m}{m}$, and $\frac{\alpha_{\Delta L}}{\Delta L}$. $\rightarrow$

$$\frac{\alpha_{\Delta L}}{\Delta L} = \frac{1}{(L'-L)}\sqrt{(\alpha_{L'})^2 + (-\alpha_L)^2}$$

$$\frac{\alpha_m}{m} = \frac{1}{k\Delta L}\sqrt{(k\alpha_{\Delta L})^2 + (\Delta L\alpha_k)^2}$$

$$\frac{\alpha_k}{k} = \frac{1}{mg}\sqrt{(g\alpha_m)^2 + \left(-\frac{mg}{\Delta L}\alpha_{\Delta L}\right)^2}$$

Now, we can plug $\frac{\alpha_{\Delta L}}{\Delta L}$ into $\frac{\alpha_k}{k}$:

$$\frac{\alpha_k}{k} = \frac{1}{mg}\sqrt{(g\alpha_m)^2 + \left(\frac{-mg}{L'-L}\sqrt{(\alpha_{L'})^2 + (-\alpha_L)^2}\right)^2} \rightarrow \frac{\alpha_k}{k} = \frac{1}{mg}\sqrt{(g\alpha_m)^2 + \left(\frac{m^2g^2}{(L'-L)^2}\left((\alpha_{L'})^2 + (\alpha_L)^2\right)\right)} = \boxed{\sqrt{\frac{\alpha_m^2}{m^2} + \frac{1}{\Delta L^2}(\alpha_{L'}^2 + \alpha_L^2)}}$$

2. ***Record instruments.*** Before taking any data, record the precision errors of your instrument(s):

INSTRUMENT	PRECISION ERROR
RULER	±0.05 cm
TAPE MEASURE	±0.05 cm

3. **Experimental design**. You will assuming more responsible for devising, carrying out, analyzing, and adjusting experimental procedures during the semester. This section highlights the design elements which are needed for the group's measurement procedure.
   a. Choose a measuring instrument.
   b. Reading the spring length consistently and accurately with a range of weights is crucial for this experiment. One suggestion: Choose a point at the top and one at the bottom of the spring that you will measure between throughout the lab. The point at the top should be fully above all the coils, and the point at the bottom should be fully below all the coils. Otherwise, exactly which points you choose is not significant because your calculations will only include differences in length ($L' - L$). However*, <u>it is very important that you measure length between these same two points for all measurements in this experiment</u>.* This is so that the length differences you calculate are only due to physical stretching, and not due to mistakenly shifting the reference point of measurement. Will need to measure a range of masses.
   c. Make sure each setup works for the range of weights, especially the fully loaded spring.
   d. Decide how you'll minimize parallax error when reading the scale. The setup may need to be modified at this step.

4. **Describe your measurement procedure.** It should address topics such as: what was your goal, what challenges did you encounter and how did you design the experimental protocol to account for them? This section could include: how the length was determined, the measuring instrument(s) used and why chosen, how random errors were minimized, what systematic errors were identified, etc.

The goal of this experiment was to measure the relationship between three variables: k (the spring constant), deltaL (the length the spring has shifted by after a mass hangs from it), and m (the mass hanging from it). We were going to do this by measuring the equilibrium length of a spring (the length when no mass is hanging from it), and then linearly adding masses and measuring the respective lengths of the accordingly stretched springs. To do this, as in all experiments, we thought it would be a good idea to zero out and unstretch the spring and measuring apparatus before each measurement. This would reduce systematic errors. We also decided to measure the length of the spring without including the hooks of the spring, since this would increase our accuracy. Our procedure for the experiment was given by:
Step 1: Measure the length several times when the spring is in equilibrium without a weight.
Step 2: Hang a mass and measure the new length. Measure all lengths using a ruler or the calipers, since they aren't malleable instruments and they are as precise as we can get.
Step 3: Do this for several masses that have a linear increase if possible, to see if results are proportional as the experiment goes on.
Step 4: Find k using physics and then plot results, analyzing our experiments.

5. ***Measure unloaded equilibrium length L.*** Measure the equilibrium length L of the spring without any masses or the mass holder hanging from it.

Unloaded Equilibrium Length Data and Results		
Trial #	**Unloaded Equilibrium Length (units: cm)**	**Notes**
1	5.78 ± 0.05	
2	5.83 ± 0.05	
3	5.84 ± 0.05	
4	5.86 ± 0.05	
5	5.83 ± 0.05	

Mean of Length, $\bar{L}$ = 5.828 ± 0.05 cm **Reported result with error**

Standard Deviation = 0.029 ± 0.05 cm $L =$ 5.828 ± 0.013 cm

Standard Error $\alpha_L =$ 0.013 ± 0.05 cm

6. ***Measure loaded equilibrium lengths.*** Measure the equilibrium length L' of the spring, loaded with different masses ranging from 50 gm to 250 gm hanging from it.
   a. Record the total mass $\boldsymbol{m_i}$ (hanger + various masses) in the table heading.
   b. Now measure the new spring equilibrium length $\boldsymbol{L_i'}$ for the mass $\boldsymbol{m_i}$. Repeat this for each of six masses chosen by your group. You need only take five data points for each mass, for the sake of time. Be sure to note the total mass for each run.
   c. Use the mass values stamped on the brass disks. We will take the mass error α_m in each case to simply be the precision error of 0.1gm.

Loaded Equilibrium Length Data and Results, for Various Masses (Mass error $\alpha_m =$ 0.1 g)					
Mass $m_1 =$ 50 g			**Mass $m_2 =$ 70 g**		
Trial #	**Eq. Length L_1' (units: cm)**	**Notes**	**Trial #**	**Eq. Length L_2' (units: cm)**	**Notes**
1	5.86 ± 0.05		**1**	5.91 ± 0.05	
2	5.88 ± 0.05		**2**	5.92 ± 0.05	
3	5.87 ± 0.05		**3**	5.91 ± 0.05	
4	5.86 ± 0.05		**4**	5.94 ± 0.05	
5	5.89 ± 0.05		**5**	5.93 ± 0.05	
Mean of Length = 5.872 ± 0.05			**Mean of Length =** 5.93 ± 0.05 cm		
Standard Error = 0.006 ± 0.05			**Standard Error =** 0.01 ± 0.05 cm		

Result with Error L' = 5.872 ± 0.006 cm	Result with Error L_2' = 5.93 ± 0.01 cm

Mass m_3 = 90 g

Trial #	Eq. Length L_3' (units: cm)	Notes
1	5.98 ± 0.05	
2	5.97 ± 0.05	
3	5.99 ± 0.05	
4	5.97 ± 0.05	
5	5.98 ± 0.05	

Mean of Length = 5.982 ± 0.05 cm

Standard Error = 0.004 ± 0.05 cm

Result with Error L_3' = 5.982 ± 0.004 cm

Mass m_4 = 110 g

Trial #	Eq. Length L_4' (units: cm)	Notes
1	6.04 ± 0.05	
2	6.05 ± 0.05	
3	6.03 ± 0.05	
4	6.04 ± 0.05	
5	6.05 ± 0.05	

Mean of Length = 6.043 ± 0.05 cm

Standard Error = 0.004 ± 0.05 cm

Result with Error L_4' = 6.043 ± 0.004 cm

Mass m_5 = 130 g

Trial #	Eq. Length L_5' (units: cm)	Notes
1	6.97 ± 0.05	
2	6.95 ± 0.05	
3	6.91 ± 0.05	
4	6.99 ± 0.05	
5	6.94 ± 0.05	

Mean of Length = 6.096 ± 0.05 cm

Standard Error = 0.005 ± 0.05 cm

Result with Error L_5' = 6.096 ± 0.005 cm

Mass m_6 = 150 g

Trial #	Eq. Length L_6' (units: cm)	Notes
1	6.15 ± 0.05	
2	6.15 ± 0.05	
3	6.16 ± 0.05	
4	6.15 ± 0.05	
5	6.16 ± 0.05	

Mean of Length = 6.1533 ± 0.05 cm

Standard Error = 0.0044 ± 0.05 cm

Result with Error L_6' = 6.1533 ± 0.0044 cm

7. ***Summarize + Compute the derived quantity for quick analysis***. Summarize your measured results in the first two columns below, then use these measured quantities and their errors to compute the spring constant and its error using the theory and your error propagation from earlier. Compute a value of the spring constant for each mass using

$$k_i = m_i g / \Delta L_i$$

where $\Delta L_i \equiv L_i' - L$, L is the equilibrium length with no suspended mass, L_i' is the equilibrium length with a mass m_i suspended from the spring, and $g = 9.800$ m/s^2 is the acceleration of gravity in Berkeley.

Mass: m_i (error $\alpha_m =$ 0.1 g)	**Change in Eq. Length ΔL:** (units: cm)	**Spring Constant: k_i** (units: N/m)
$m_1 =$ 50 g	0.044	11.136
$m_6 =$ 150 g	0.1253	11.732

Assuming the spring constant k is indeed constant (i.e., independent of the spring's stretching), each derived measurement of k_i above should be the same value. Let's check that assumption!

Compute an average k value

$$\bar{k} = \left(\sum_{i=1}^{N} k_i\right)/N \quad (2)$$

Since we are asked k for m_1 and m_6, $\bar{k} = \frac{k_1+k_6}{2} = 11.434$ N/m

Result:

$\bar{k} = 11.434$ N/m

As part of the analysis in the next section, you'll fit a line to the data and check how well it matches.

8. ***Compare results to accepted values.*** The accepted spring constant of your spring is $\mathbf{12} \pm \mathbf{10}\%$ N/m. Does your experimental result "agree" with the accepted value? (Circle one) **Yes** or **No**

9. ***Discussion.*** Use the space below to briefly answer the following questions:

Did you need to refine your measurement procedure? If so, explain.

Yes, as I did more and more experiments, I tried to make sure that my measurements for the length of the stretched spring were reliable. I thought about measuring the length of the spring including the hooks, but I thought that it wouldn't really matter. Perhaps changing that would've yielded better results. I also tried to minimise errors by letting the springs rest and relax for a while between measurements.

If your results do not agree with the accepted value, what might be the reasons? Consider experimental design, random errors, systematic errors, modeling errors etc.

Perhaps the measurement of the length was wrong. Also, something that could've changed results significantly was considering that k might not be the same for all parts of the spring, but I didn't really know how this can be described or even shown experimentally, so I left it to myself as food for thought. There might be modelling errors in our setup, but as I said before, I consider the main source of errors to be the measurement of the length, which can cause systematic errors.

What could be done to improve your measurement?

Perhaps being able to measure more precisely the lengths, using either more advanced instrumentation or having a different setupo that allows us fto measure the length without rotating the apparatus, for example.

Justification for dataset used in analysis: Each student is responsible for acquiring a full dataset for the experiment. The group, as a whole, can decide which of the three generated datasets to analyze and provide a *justification* for the decision.

We decided to use Allen's data set, since even though mine is closer to the actual valid result for k, Allen's had more consistency in its results, and less standard error. Also, his experiment had a wider range in masses, which gave him more values for which to find k. Even though his k isn't the accepted value, his results are more reliable and broader , so they would give us a better understanding behind the experimental procedure to find k. Also, the value for k wasn't too far off, so we concluded that Allen's dataset would give us a better value.

Analysis

Fitting curves to test models and extract parameters with errors

In the data taking portion of this Experiment, you measured the spring constant k of a spring by hanging various masses from it and measuring the equilibrium length for each mass. This gave you the spring constant according to the model

$$mg = k\Delta L \tag{3}$$

where $\Delta L_i \equiv L_i{}' - L$. As a preliminary analysis of the data, you used equation (3) to derive several measurements of $k_i = m_i g/\Delta L_i$. You then combined these in an average to find your result $\bar{k} = (\sum_{i=1}^{N} k_i)/N$, for a preliminary analysis.

Notice that this analysis inherently makes several assumptions. One is that the model in equation 3 accurately describes the data. Another is that all the k_i represent measurements of the same value k—that is, the analysis assumes that k is constant.

We can find a better value of k by graphing our data, then using curve-fitting to rigorously determine whether or not the linear model in Equation (3) is a good model for our data.

Our aim is developing the best model to fit the observed data. The method of least squares will be used to deduce the best-fit parameters for a proposed direct proportionality model, $y = mx$. Once the best fit parameter has been obtained, a residual plot will be generated as a preliminary test of the quality of the fit. Based on the information contained in the residual plot, an improved model, $y = mx + c$, is evaluated for a better fit.

- You'll also calculate a "goodness of fit" parameter χ^2, for use during the Intro Lab 5 analysis. A discussion of χ^2 will be provided next time.

1. Overview of simple linear least-squares fit

We use a least-squares approach, assuming all the uncertainties $\{\alpha_i\}$ are the same (See Hughes and Hase, Chapter 5 and *5BL Statistics Review* Section 5.2)

For the **direct proportionality model** $y(x) = mx$, which is a linear model constrained to pass through the origin, the best fit parameters are:

Best-fit parameters: $$m = \sqrt{\frac{\sum x_i y_i}{\sum x_i{}^2}}$$

Common uncertainty: $$\alpha_{CU} = \sqrt{\frac{1}{N-1}\sum (y_i - mx_i)^2}$$

Uncertainty in best-fit parameter:

$$\alpha_m = \alpha_{CU}\sqrt{\frac{1}{\sum x_i^2}}$$

For the **linear model**, $y(x) = mx + c$, the best fit parameters are:

Best-fit parameters:

$$m = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{\Delta}$$

$$c = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{\Delta}$$

Common uncertainty:

$$\alpha_{CU} = \sqrt{\frac{1}{N-2}\sum (y_i - mx_i - c)^2}$$

Uncertainties in best-fit parameters:

$$\alpha_m = \alpha_{CU}\sqrt{\frac{N}{\Delta}}$$

$$\alpha_c = \alpha_{CU}\sqrt{\frac{\sum x_i^2}{\Delta}}$$

$$\Delta = N\sum x_i^2 - \left(\sum x_i\right)^2$$

2. Preparation for curve-fitting: Considering the direct proportionality model

Our spring model in equation 3 has the form $y = mx$. You will plot $(x_i, y_i) = (m_i g, \Delta L_i)$ for each of your data points. A line going through them should have slope $1/k$—assuming that a linear model describes them well.

3. Preparation for curve-fitting: Recasting the data for the direct proportionality model

In light of our first model, then, let's start by summarizing the data in a way that will be most useful for plotting a linear fit., In the table below, use the six mass values and the associated change in spring equilibrium lengths ΔL_i to write down the *gravitational force* and *change in equilibrium length* for each of the six masses. You can neglect any error in g.

Mass: m_i (error α_{m_i} = 0.1 g) (units: grams)	Gravitational Force: $m_i g$ (error $\alpha_{m_i g}$ = 0.00098 N) (units: N)	Change in Eq. Length: $\Delta L_i = L_i' - L$ (units: CM)	Change in Eq. Length Error: $\alpha_{\Delta L_i}$ (units: cm)
50	0.491	0.17	0.047
90	0.882	3.03	0.045
130	1.274	6.08	0.045
170	1.666	9.41	0.050
210	2.058	12.50	0.046
250	2.453	15.74	0.053

4. Perform the linear regression and plot the result

Now that you have your data points $(x_i, y_i) = (m_i g, \Delta L_i)$, use a simple linear least-squares approach to fit the data to a direct proportionality model, $y = mx$.

Next, plot your data with the vertical error bars $\alpha_{\Delta L_i}$ from the table above and a line $y = mx$ using the model "best fit" parameter m you just calculated. Make sure you label your plot fully! It should have a descriptive title (like, "Spring stretch length vs gravitational force"), axis labels with units, the equation of the line-of-best-fit, the values of the slope m with its error α_m, and the value of χ^2 (which we'll learn how to interpret next time). If the error bars are not visible, include the information on the plot. **Include your plot with the submission.** **I ADDED ALL THE LABELLED GRAPHS AT THE BOTTOM OF THIS PDF**

You may neglect the error in the x-coordinates $m_i g$, since the fractional error in the x-coordinates is much smaller than the fractional error in the y-coordinates: $m_i g / \alpha_{m_i g} \ll \Delta L_i / \alpha_{\Delta L_i}$. We will discuss this during Intro Lab 6.

5. Examine the plot and residuals

- We need to answer the question: Does the initial fit function choice $y = mx$ adequately fit the data?" Visually inspect the plot and answer: **yes or no**

Next, prepare a residual data plot, graphing $(x_i, y_i) = (m_i g, R_i)$, where the residual is defined as:

$$R_i = y_i - y(x_i) = y_i - mx_i$$

where R_i is the residual, y_i, is the ith measurement and $y(x_i)$ is the value of the model fit for the ith measurement. If the model adequately fits the data, the residuals should have randomly distributed y values centered on 0, reflecting the random uncertainty in our experiment.

- Are the residuals randomly distributed around 0 i.e. show no structure?
  yes or no

Include your residuals plot with the submission. **I ADDED ALL THE LABELLED GRAPHS AT THE BOTTOM OF THIS PDF**

6. Perform the regression with the linear model, plot the result

Clearly, the theoretical equation $y = mx$ does not adequately fit our data. But wait, we can do better! Use the revised theoretical equation $y = mx + c$, where an offset c has been added to the direct proportionality model, and fit your data points $(x_i, y_i) = (m_i g, \Delta L_i)$ using linear regression.

Plot your data with vertical error bars and a line $y = mx + c$ using the revised linear model "best fit" parameters m and c you just calculated.

Make sure you label your plot fully! It should have a descriptive title, axis labels with units, the equation of the line-of-best-fit, the values of the fitted slope m and intercept c with their errors, and the value of χ^2 (which we'll learn how to interpret next time). **Print out your plot and attach it to this worksheet.**

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7. Examine the plot and residuals for the linear model

- Does our improved function choice $y = mx + c$ adequately fit the data? Visually inspect the plot and answer: **yes or no**

Next, prepare a residual data plot, for $(x_i, y_i) = (m_i g, R_i')$, where

$$R_i' = y_i - y(x_i) = y_i - (mx_i + c).$$

where R_i' is the residual, y_i, is the ith measurement and $y(x_i) = mx_i + c$ is the value of the model fit for the ith measurement. If the model adequately fits the data, the residuals should now have random y values centered on 0, reflecting the random uncertainty in our data.

- Are the residuals now randomly distributed around 0 i.e. show no structure?
  yes or no

Include your residuals plot with your submission. I ADDED ALL THE LABELLED GRAPHS AT THE BOTTOM OF THIS PDF

8. Compare k to accepted value and to the previous analysis

In the table below, write four values: The accepted value of your spring constant k, the measured value of k from the original analysis (by averaging measurements of k derived from single data points), the measured value of k from curve fitting to $y = mx$, and the measured value of k from the curve-fitting to $y = mx + c$. Include error values for your curve fitting measured k values.

Accepted value of k	**Measured value of k , by averaging**	**Measured value of k, by curve-fitting $y = mx$**	**Measured value of k, by curve-fitting, $y = mx + c$**
$12 \pm 1.2\ N/m$	64.7 ± 13.32 N/m	16.8 ± 1.31 N/m	12.5 ± 0.14 N/m

Does your value of k from curve-fitting match the accepted value of k, using the agreement test? Show your work.

Measured value of k , by averaging	**Measured value of k, by curve-fitting $y = mx$**	**Measured value of k, by curve-fitting, $y = mx + c$**
NO	YES	YES

Values agree if $|x-y| \leq 2\sqrt{\alpha_m^2 + \alpha_k^2}$. Using eqs found above, find α_k and α_m and verify the table above

Briefly, how does the value of k from curve-fitting analysis compare to the value of k from your average value analysis? Which is more accurate?

The value for k obtained using direct proportionality was closer to the accepted value than the curve fitted linear model was. Therefore, curve-fitting always gives us a closer value to the accepted one. Curve-fitting the linear or the direct proportionality models gives us a more accurate value for k than the average value analysis, since both the models gave better and closer values for k than the acerage value analysis.

Why did we include the non-zero intercept for the improved fit equation? What does it mean physically?

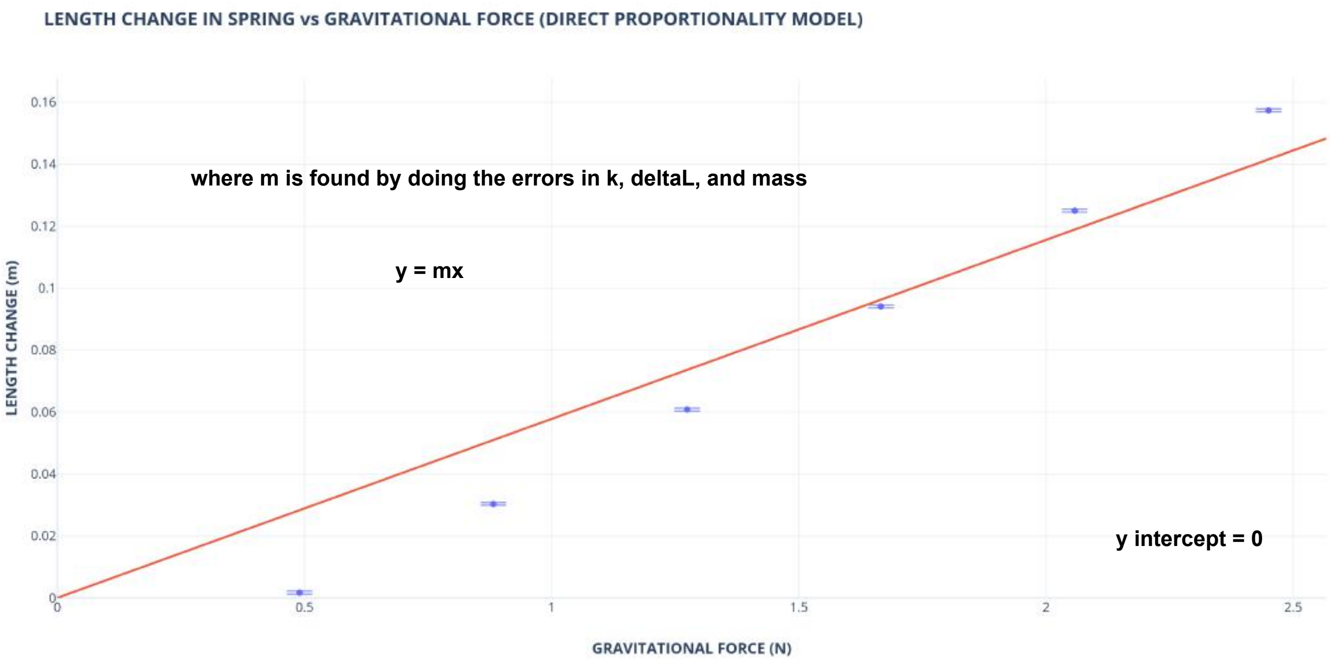
Our intercept is a positive value, and it fits the data better, so it improved our fit, as the name indicated. Physically, this question is more interesting. When we have the line cross through the origin, we are implicitly saying that the relaxed spring with no mass hanging from it has k=0.Nevertheless, this is wrong! by adding the positive x-intercept, we find that a relaxed spring also has a positive k. This matches what physics predicts, and therefore fits our data better.

Does your residual plot from curve-fitting to $y = mx + c$ show the data has only random errors? Explain.

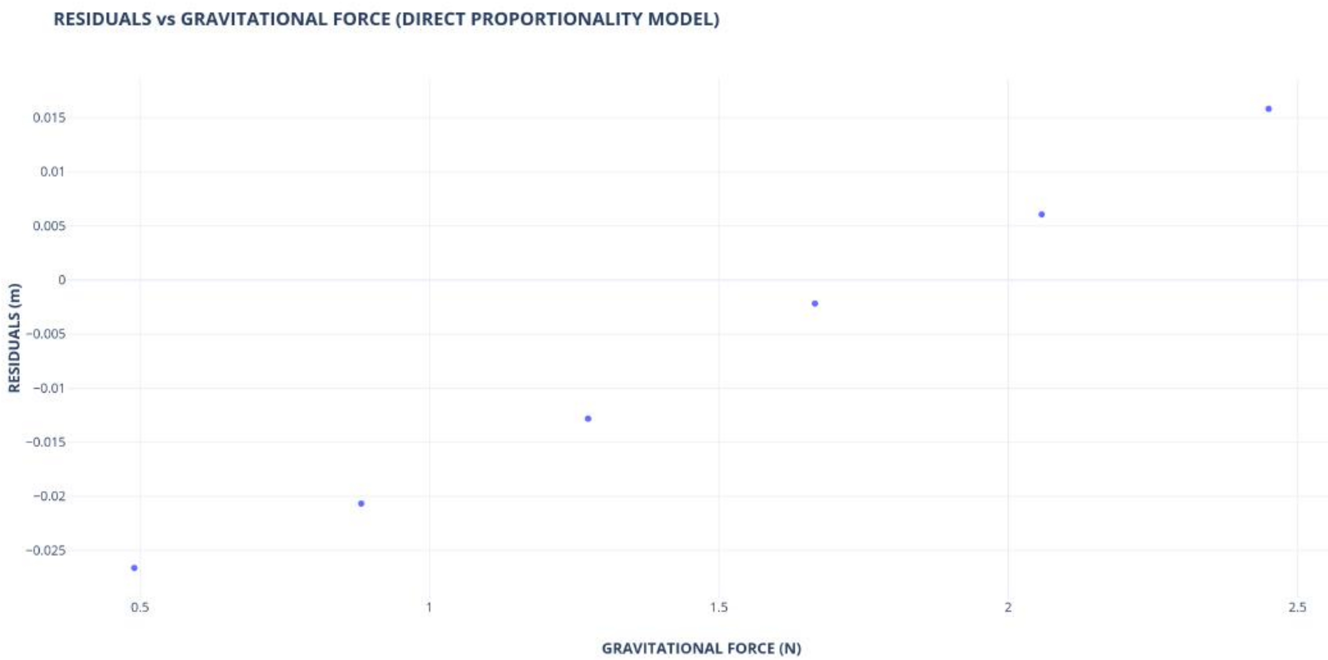
Since the linear model doesn't have a systemic error in account, the residuals are not structured. On the other hand, when we look at the rirect proportionality model, we see that a model with systemic errors produces structured residuals.

REMINDER: KEEP A COPY OF YOUR χ^2 CALCULATIONS TO ANALYZE DURING THE NEXT LAB!!

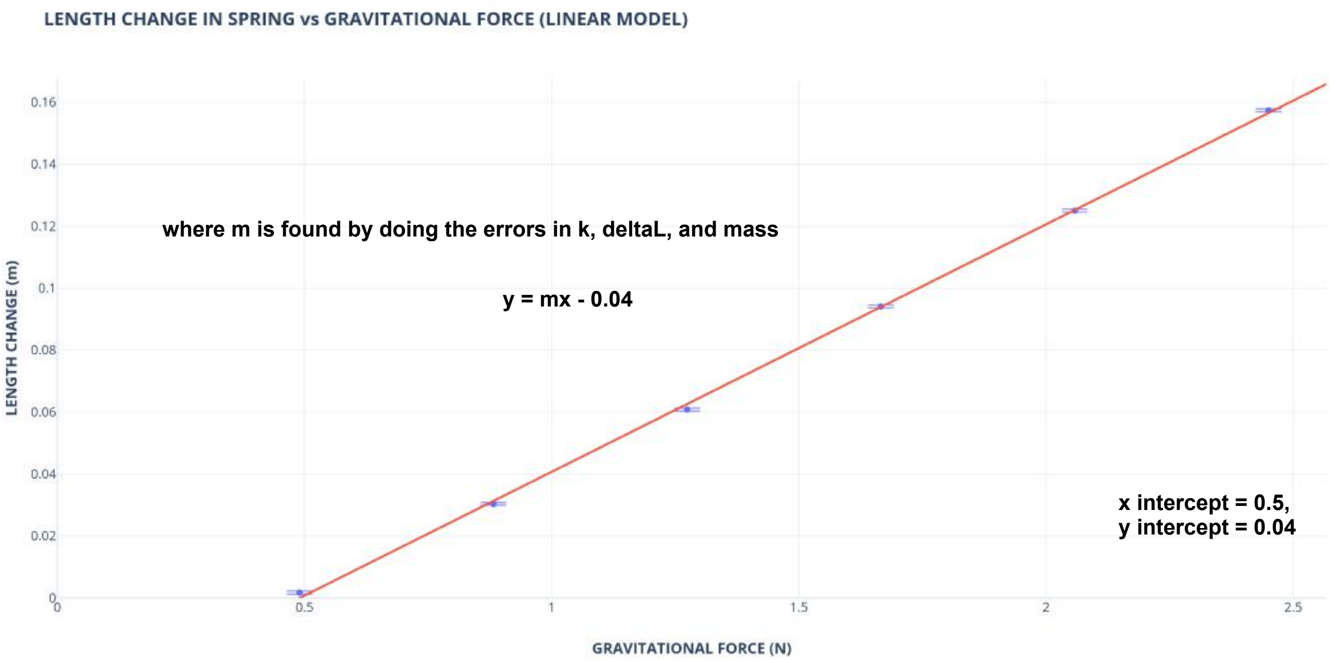
Q4


LENGTH CHANGE IN SPRING vs GRAVITATIONAL FORCE (DIRECT PROPORTIONALITY MODEL)
where m is found by doing the errors in k, deltaL, and mass
y = mx
y intercept = 0
LENGTH CHANGE (m)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.5
1
1.5
2
2.5
GRAVITATIONAL FORCE (N)

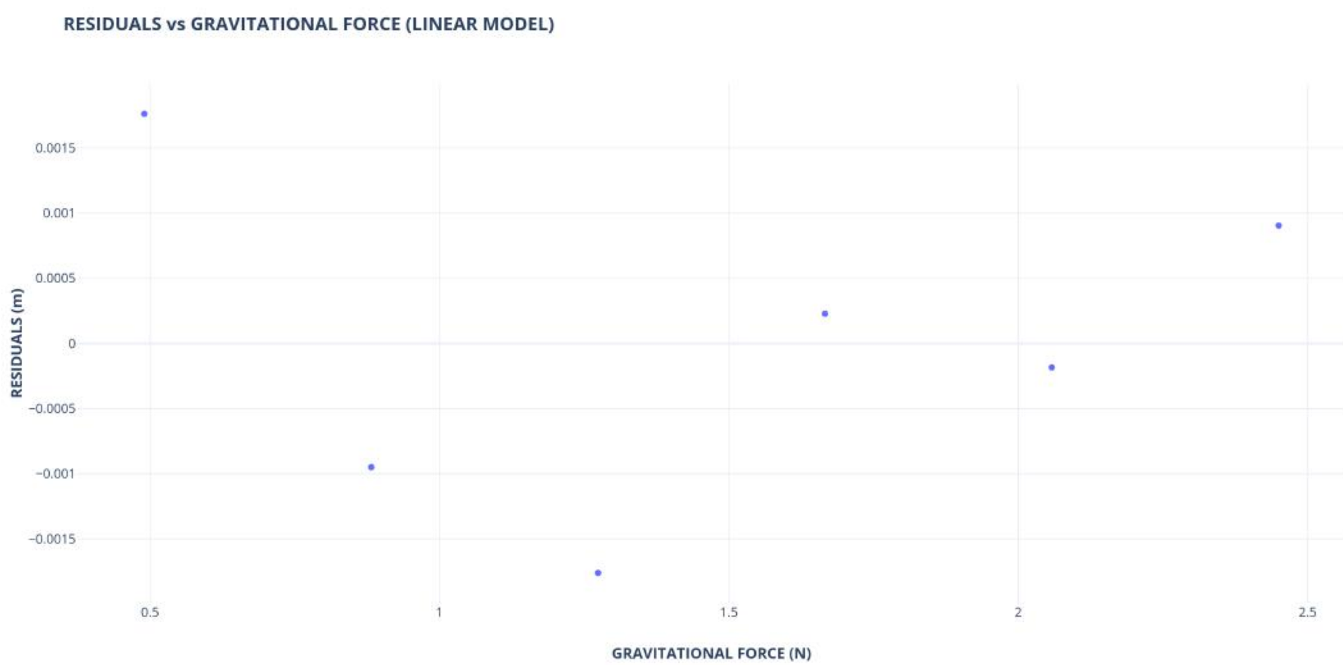
Q5


RESIDUALS vs GRAVITATIONAL FORCE (DIRECT PROPORTIONALITY MODEL)
RESIDUALS (m)
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
−0.025
0.5
1
1.5
2
2.5
GRAVITATIONAL FORCE (N)

Q6


LENGTH CHANGE IN SPRING vs GRAVITATIONAL FORCE (LINEAR MODEL)
where m is found by doing the errors in k, deltaL, and mass
y = mx - 0.04
x intercept = 0.5,
y intercept = 0.04
LENGTH CHANGE (m)
GRAVITATIONAL FORCE (N)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.5
1
1.5
2
2.5

Q7


RESIDUALS vs GRAVITATIONAL FORCE (LINEAR MODEL)
RESIDUALS (m)
0.0015
0.001
0.0005
0
−0.0005
−0.001
−0.0015
0.5
1
1.5
2
2.5
GRAVITATIONAL FORCE (N)