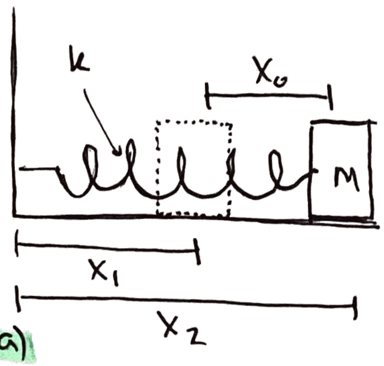
PROBLEM 4 (kk 5.8)



$X_0 = X_2 - X_1$

(a)

Energy of m at X_2: $\frac{1}{2}m\cancel{v_0^2}^{\,0} + \frac{1}{2}kx_0^2 = \frac{1}{2}k(x_2-x_1)^2$

Energy of m after one cycle returning to $x_2 - \Delta x$ due to friction: $\frac{1}{2}m\cancel{v^2}^{\,0} + \frac{1}{2}kx^2 = \frac{1}{2}k(x_2-x_1-\Delta x)^2$

$$\Delta U = \frac{1}{2}k\left[(x_2-x_1-\Delta x)^2 - (x_2-x_1)^2\right]$$

Now, due to the definition of work, the work done by friction is $4f(x_2-x_1)$ for one full cycle. Since $W = -\Delta U$, we can write that:

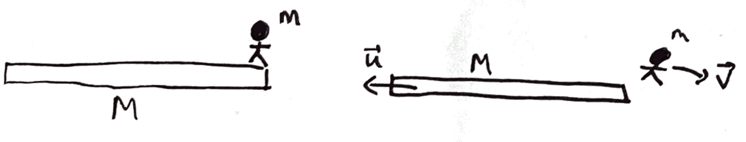
$$4f\cancel{(x_2-x_1)} = -\frac{1}{2}k\left[(x_2-x_1-\Delta x)^2-(x_2-x_1)^2\right] = k\Delta x\,\cancel{(x_2-x_1)}$$

$\therefore\ 4f = k\Delta x \rightarrow \boxed{\Delta x = \frac{4f}{k}}$ $\therefore$ Δx is constant, and therefore the decrease of amplitude is the same for all cycles.

(b)

$n\cdot\Delta x = (x_2-x_1) \rightarrow n\frac{4f}{k} = (x_2-x_1) \rightarrow \boxed{n = \frac{k(x_2-x_1)}{4f}}$

PROBLEM 1

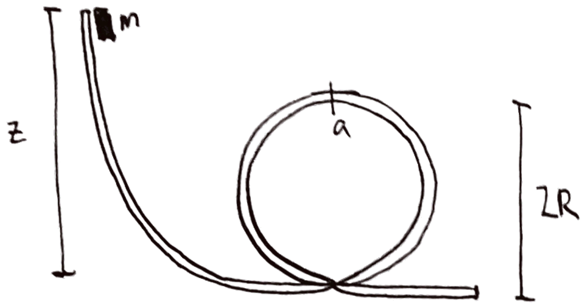


$$\left.\begin{array}{l}\vec{P}_i = 0 \\ \vec{P}_f = m\vec{v} + M\vec{u}\end{array}\right\} \Delta\vec{P} = 0 = m\vec{v} + M\vec{u} \quad \therefore -M\vec{u} = m\vec{v} \rightarrow \vec{u} = \frac{-m\vec{v}}{M}$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}Mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{-mv}{M}\right)^2 = \frac{1}{2}mv^2\left(1+\frac{m}{M}\right)$$

Now, since $E_{man} = \frac{1}{2}mv^2$, $E = E_{man}\left(1+\frac{M}{M}\right)$, so the fraction of the energy carried by the man is $\boxed{\frac{E_m}{E} = \frac{1}{1+\frac{m}{M}}} = \frac{M}{m+M}$. If $m \ggg M$, this tends to 0. This means that the man would "carry" a vast part of the total energy. If $M \ggg m$, this tends to 1, so nearly 100% of the energy is "carried" by M.

PROBLEM 2. (KK 5.1)



Energy when $t=0$: $\frac{1}{2}mv_0^2 + mgz \rightarrow mgz$ since $V_0=0$.

Energy when m is at point a: $\frac{1}{2}mv^2 + mg(2R)$

Since energy is conserved in this system, we can say that: $\Delta E = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(2R) - \cancel{m}gz = 0$

$$\therefore \quad v^2 = 2gz - 4gR$$

At point a, $N + mg = \frac{mv^2}{r}$, and $N = mg$, so $2mg = \frac{mv^2}{r}$. By substituting v^2 we get:

$$2mg = \frac{m}{R}(2gz - 4gR) = \frac{2gzm}{R} - 4gm \rightarrow 6\cancel{mg} = \frac{2\cancel{g}\cancel{m}z}{R} \rightarrow 3 = \frac{z}{R} \rightarrow \boxed{z = 3R}$$

PROBLEM 3 (KK 5.5)

Since all the motion is due to a radial force, we know that tangential acceleration is zero: $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$ since $\dot{\theta} = \frac{d\theta}{dt} = w$, we can write

$$\int_{w_1}^{w_2} \frac{dw}{w} = -2\int_{r_1}^{r_2} \frac{dr}{r}, \text{ so } w_2 = \frac{r_1^2 w_1}{r_2^2}$$

, as we can see, the numerator depends on the initial conditions, and it corresponds to $mr^2\dot{\theta}$, so therefore it remains constant.

$$w_2 = \frac{C}{r_2^2}$$

Now, apply conservation of energy to show that $W_{21} = \Delta KE$.

$$\Delta k^{\text{Tangential}} = \frac{1}{2}\frac{mC^2}{r_2^2} - \frac{1}{2}\frac{mC^2}{r_1^2} \qquad \Delta k^{\text{radial}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\Delta E = \frac{1}{2}mv_f^2 + \frac{1}{2}\frac{mC^2}{r_2^2} - \frac{1}{2}\frac{mC^2}{r_1^2} - \frac{1}{2}mv_i^2 \qquad (1)$$

Now, since Work $= \int F_{\text{radial}}\, dr$, find the work. After some integration, we get:

$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, and comparing to (1) we see that Work corresponds to the change in KE.