Lab 5

Chi-squared analysis: Static Friction between Various Surfaces

Lab Objectives

- Calculate χ^2 and $\tilde{\chi}^2$ and their interpretation
- Give students practice fitting lines to analyze data
- Give students additional practice on all skills introduced in the course so far

Group members, and their group roles:

Allen Chen: Analyst
Javier Santillán:Manager
Pablo Castaño: Skeptic

Overview

You will measure the coefficients of static friction for several surface material pairings by using a spring scale to measure the force required to move a wooden block on various surfaces. Changing the mass loaded into the block will vary the applied normal force. A model will be fitted to your data using linear regression and the coefficient of static friction extracted for each material pairing. The "goodness of fit" will be evaluated using the dimensionless parameters χ^2 and $\tilde{\chi}^2$.

Be especially careful to …

- Design your experimental method to minimize your error sources!
- NOT CHERRY PICK DATA! The surfaces are non-uniform and the spring scale operation challenging
- Make sure your data and analyses are clear, so you can refer to them again later!
- Generate clear scientific plots!

What to Turn In

The **group as a whole** will submit a single copy of the lab worksheet pdf, with all measurements, calculations and question answers included. ***Clearly list the group members*** on the first page and their ***roles***: manager/communicator (who submits the group lab report), skeptic or analyst (if these roles are assigned within the group).

Each student is responsible for acquiring a full dataset for two of the three materials pairings, similar to the Intro Lab 3 procedure. *All six* individual datasets must be included in the single group lab report, and properly attributed.

Each student in a group will be responsible for analyzing *one* dataset from *one* material pairing and answering related questions. The group, as a whole, can decide which of the six datasets each person will analyze and provide a *justification* for the decision.

Group members can discuss the analysis and questions but each student's analysis, including the graphs, must be their own work. **Each Gradescope pdf submission should include a worksheet pdf with the specified plots attached. A separate bCourses submission is required for the three Python or Excel files used in the analysis.**

Procedure Overview

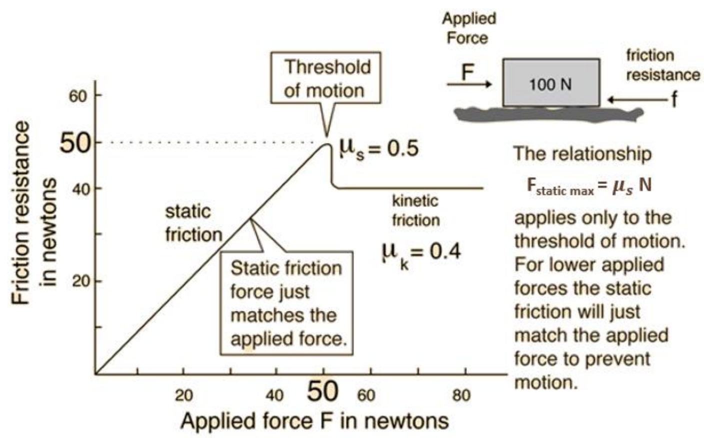
Throughout each experiment, be sure to identify the goal, primary data you need to collect, and equipment you will use. Carefully consider each experimental setup, including the environment around your setup. Try to identify strategies to reduce experimental noise, ensure stability, and allow your experiment to be reproducible.

For each measurement, be sure you have outlined a clear measurement strategy that includes identifying and minimizing any random and systematic errors.

During the experiment you may need to adjust your procedure to account for errors or flaws in the design. In your report, you should note when and where you saw the need for adjustment and how you and your group adapted your procedure.

Theory

The friction forces that arise between any two contacting surfaces are divided into two broad categories based on the relative motion of the surfaces (see Figure 1). Starting with a mass at rest, as you increase the applied force F_{applied}, an equal and opposite static friction force arises to match and keep the mass motionless. When F_{applied} equals the maximum value of static friction, the mass starts to move. The threshold of motion is characterized by:



$$F_{\text{s,max}} = \mu_s N, \qquad (1)$$

where $F_{\text{s,max}}$ is the applied force when motion occurs, N is the normal force and μ_s is the "coefficient of static friction." If the contact area between the object and the surface is parallel to the ground (e.g., a block on a horizontal table), then the normal force is $N = mg$, where m is the mass of the object and g is the acceleration due to gravity. If the applied force exceeds $F_{\text{s,max}}$, static friction is no longer present and the box now slides in the direction of the applied force. A kinetic friction force now acts to opposite the motion but unlike its static counterpart, does not match the applied force up to a maximum value.

Equipment

- Cork with adhesive backing, kit
- Metal ball, kit
- 2.5 N spring scale
- Wooden block, kit
- Table top or another surface
- Sandpaper, kit
- IOLab or a 200 gm weight, kit
- 250 gm weight set with gloves, kit
- IOLab cardboard box, kit
- Scotch tape

Table top of another firm surface: Most important is not damaging the surface. The cork adhesive sticks well to plastic or shiny surfaces and can be easily detached when completing the lab. Finished plywood and table surfaces also work but when in doubt, opt for another surface.

Cork sheet: to adhere, peel off the backing and press down firmly. Cork oriented along the direction the backing writing might be less uniform than perpendicular.

Sandpaper: remove the sheet from the paper wrapping. The sandpaper should be firmly secured to the surface with non-damaging tape. The sandpaper is used throughout 5BL, and must be saved.

Paper wrapping, sandpaper: the attached manganin wire and 330 Ω resistors will be used shortly. Please keep the wrapping and return the sandpaper sheet to it after the lab is finished.

Cardboard: all students should have the IOLab cardboard box. Either the top or bottom is suitable, whichever one isn't damaged and is strong enough to not bend under the weights.

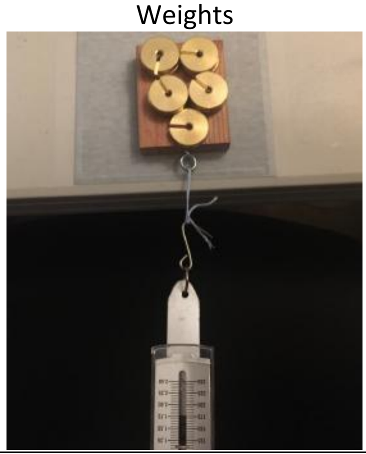
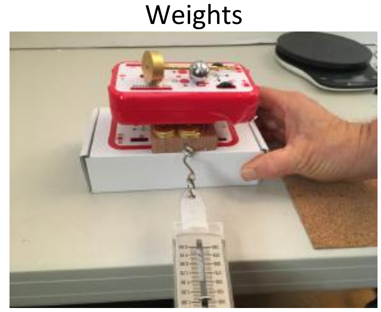
Weights: use the weight set and ball, primarily. Those weights are known accurately and recorded later in the lab manual. The IOLab can be used as a mass, as needed, but please try not to drop it!

Wood block: both sides will be used. One side may be slightly smoother than the other and should be used for the cork and cardboard measurements, while the rougher side should be used for the sandpaper measurements. To determine the slightly rougher side, lightly run your finger over the surface and mark.

Spring scale: attach one end to the block eye screw. Make sure the spring scale is as horizontal as possible so that all your force is exerted parallel to the surface. Check the zero frequently.

Experimental Setup

Three different Setups used in this lab, one for each materials pairing. Examples are shown below:

Setup A: **Cork/Wood**	**Setup B:** **Sandpaper/Wood**	**Setup C:** **Cardboard/Wood**
Cork Wood block Weights 	Sandpaper Wood block Weights 	Cardboard, IOLab box top Wood block Weights 

As you'll discover, the static friction coefficient, μ_s, substantially changes for different materials pairing.

Prelab

This prelab should help you prepare for the class activities.

Task 1: Construct the Setups A, B, and C. Submit a picture(s) showing each Setup.

Task 2: Determine the largest mass for each of Setups A, B, and C that can be reasonably measured with the spring scale (2.25-2.5N reading). Submit the mass value for each Setup.

Task 2 considerations:

- Attach one end of the spring scale to the block, and hold the other end in your hand. **Make sure the spring scale is as horizontal as possible**, so that all of your force is exerted parallel to the surface.
- Gently and slowly pull, paying close attention to the reading on the spring scale. Record the reading on the scale when the object first starts to move, the threshold of motion. This is one reason why it's important to move slowly and pay close attention, as the reading will change suddenly when the object starts to move, and you will need to remember and record what it was just before the jump.
- To check the largest mass your spring scale can handle:
    - Load the block with a single 20g mass, and pull it as if you were making a measurement, just to practice getting a reading for this relatively small force.
    - Repeat this with a full 250g load, to make sure the scale won't max out before the object starts to move.
    - Adjust the load as needed to determine the largest mass.

You may use the nominal weights listed in the following section, "Procedure and Data Collection, 2. Record the masses".

Procedure and Data Collection

You will measure the force it takes to begin the wood block across different materials and loaded with various masses across the table. The following steps will guide you through this measurement process and data analysis.

1. ***Record instruments.*** Before taking any data, record the precision errors of your measurement instrument(s):

INSTRUMENT	PRECISION ERROR	SCALE RANGE (UNITS: N)
MASS SCALE (IF OWNED)	I don't have one with me	N/A
SPRING SCALE	0.01 N	2.5

2. ***Record the masses* (only if you own a scale).** Measure the four masses, and record your results in the table below in grams. You don't need to take multiple mass measurements in this lab, as your error will be dominated by your spring scale measurements.

Wood block: m_{block}	Kit weight set: m_{weight}	IOLab mass: m_{IOLab}	Metal ball mass: m_{ball}
$\mathbf{32.5\ \pm 0.1\ gm}$	$\mathbf{250.0\ \pm 0.1\ gm}$	$\mathbf{202\ \pm 1\ gm}$	$\mathbf{25.5\ \pm 0.1\ gm}$

3. **Experimental design**. Now you are ready to measure the minimum forces $F_{\text{s,max}}$ required to start an object sliding across the surface. This section highlights the design elements needed for the group's measurement procedure.

   To measure the threshold forces $F_{\text{s,max}}$, place the block with its weights onto the surface.
   - For each Setup, you will measure $F_{\text{s,max}}$ with **six** different mass loads that you choose, starting with the block+20gm and continuing to the (block + the maximum load determined in the prelab). Pick the loads judiciously to ensure a good spread of (x_i, y_i) values for the curve fitting analysis.
   - The cork, sandpaper, and cardboard surfaces may not be uniform. You'll want to position the wood block as reproducibly as possible.
   - Attach one end of the spring scale to the block, and hold the other end in your hand. **Make sure the spring scale is as horizontal as possible**, as discussed in the Prelab Task 2. Gently and slowly pull, paying close attention to the reading on the spring scale. Record the reading on the scale when the object first starts to move, the threshold of motion, $F_{\text{s,max}}$.

4. **Describe your experimental methodology.** It should address topics such as: what was your goal, what challenges did you encounter and how did you design the experimental protocol to account for them? This section could include: how did you ensure the measurements were as reproducible as possible, how random errors were minimized, what systematic errors were identified, etc.

   Note: since each group member is doing two of the three setups, this should be a combination of notes from all three group members, properly attributed i.e. "Student A notes: ; Student B Notes: ", etc.

   Setup A:

   Student A and B Notes combined and sumarized: We wanted to measure the coefficient of friction for a wooden block on cork, so we first made sure the surface was uniform, and the spring was horizontal to the plane. This would simplify and reduce the errors in our experiment. Our procedure would virtually consist of repeating the same experiment over and over again, making sure to zero the instruments between measurements. Aside from this, we just followed the steps described here.

   Setup B:

   Student A and B Notes combined and sumarized: We wanted to measure the coefficient of friction for a wooden block on sandpaper, so we first made sure the surface was uniform, and the spring was horizontal to the plane. This would simplify and reduce the errors in our experiment. Our procedure would virtually consist of repeating the same experiment over and over again, making sure to zero the instruments between measurements. Aside from this, we just followed the steps described here. Furthermore, we had to make sure our experiments were fairly accurate, since the reaction time was low.

   Setup C:

   Student A and B Notes combined and sumarized: We wanted to measure the coefficient of friction for a wooden block on carboard, so we first made sure the surface was uniform, and the spring was horizontal to the plane. This would simplify and reduce the errors in our experiment. Our procedure would virtually consist of repeating the same experiment over and over again, making sure to zero the instruments between measurements. Aside from this, we just followed the steps described here. Furthermore, we had to make sure our experiments were fairly accurate, since the reaction time was low.
   Our reaction time isn't that high, so we also made sure to be careful with it, especially since the cardboard was very slippery. Perhaps, if we had been more careful, we would've gotten better results.

   The next three pages have tables for your data. Each table is labeled by Setup materials. **In the tables, the mass m_i is the TOTAL mass moved each time, including the block mass, and $F_{\text{s,max}}$ is the minimum required force to start the loaded block moving.**

The data on this table was found experimentally by Javier, my groupmate.

Setup A: Minimum force, $F_{s,max}$ required to move wooden block from rest, for various masses

Surface Material: Cork; Block Mass = 30.8 g

Min. Force for Total Mass m_1 = 30.8 g

Trial #	Min. Force (units: N)	Notes
1	0.06 ± 0.01	
2	0.08 ± 0.01	
3	0.06 ± 0.01	
4	0.08 ± 0.01	
5	0.06 ± 0.01	

Result with Error $F_{s,max,cork,1}$ = 0.07 ± 0.01 N

Min. Force for Total Mass m_2 = 80.8 g

Trial #	Min. Force (units: N)	Notes
1	0.28 ± 0.01	
2	0.26 ± 0.01	
3	0.24 ± 0.01	
4	0.30 ± 0.01	
5	0.24 ± 0.01	

Result with Error $F_{s,max,cork,2}$ = 0.26 ± 0.01 N

Min. Force for Total Mass m_3 = 130.8 g

Trial #	Min. Force (units: N)	Notes
1	0.42 ± 0.01	
2	0.40 ± 0.01	
3	0.38 ± 0.01	
4	0.38 ± 0.01	
5	0.36 ± 0.01	

Result with Error $F_{s,max,cork,3}$ = 0.39 ± 0.01 N

Min. Force for Total Mass m_4 = 180.8 g

Trial #	Min. Force (units: N)	Notes
1	0.52 ± 0.01	
2	0.52 ± 0.01	
3	0.50 ± 0.01	
4	0.52 ± 0.01	
5	0.54 ± 0.01	

Result with Error $F_{s,max,cork,4}$ = 0.521 ± 0.006 N

Min. Force for Total Mass m_5 = 230.8 g

Trial #	Min. Force (units: N)	Notes
1	0.66 ± 0.01	
2	0.67 ± 0.01	
3	0.63 ± 0.01	
4	0.64 ± 0.01	
5	0.65 ± 0.01	

Result with Error $F_{s,max,cork,5}$ = 0.651 ± 0.007 N

Min. Force for Total Mass m_6 = 280.8 g

Trial #	Min. Force (units: N)	Notes
1	0.72 ± 0.01	
2	0.78 ± 0.01	
3	0.78 ± 0.01	
4	0.74 ± 0.01	
5	0.76 ± 0.01	

Result with Error $F_{s,max,cork,6}$ = 0.76 ± 0.01 N

Note: Total masses $m_1, m_2, \ldots, m_6$ include the mass of the weights AND the wooden block.

Setup B: Minimum force, $F_{s,max}$ required to move wooden block from rest, for various masses

Surface Material: Sandpaper; Block Mass = 32.5 g ± 0.1 g

Min. Force for Total Mass m_1 = 52.5 g

Trial #	Min. Force (units: N)	Notes
1	0.28 ± 0.01	
2	0.29 ± 0.01	
3	0.27 ± 0.01	
4	0.25 ± 0.01	
5	0.32 ± 0.01	

Result with Error $F_{s,max,sp,1}$ = $\boxed{0.282 \pm 0.012 \text{ N}}$

Min. Force for Total Mass m_2 = 72.5 g

Trial #	Min. Force (units: N)	Notes
1	0.44 ± 0.01	
2	0.43 ± 0.01	
3	0.39 ± 0.01	
4	0.46 ± 0.01	
5	0.45 ± 0.01	

Result with Error $F_{s,max,sp,2}$ = $\boxed{0.434 \pm 0.012 \text{ N}}$

Min. Force for Total Mass m_3 = 92.5 g

Trial #	Min. Force (units: N)	Notes
1	0.60 ± 0.01	
2	0.63 ± 0.01	
3	0.57 ± 0.01	
4	0.62 ± 0.01	
5	0.59 ± 0.01	

Result with Error $F_{s,max,sp,3}$ = $\boxed{0.602 \pm 0.011 \text{ N}}$

Min. Force for Total Mass m_4 = 112.5 g

Trial #	Min. Force (units: N)	Notes
1	0.86 ± 0.01	
2	0.77 ± 0.01	
3	0.88 ± 0.01	
4	0.85 ± 0.01	
5	0.79 ± 0.01	

Result with Error $F_{s,max,sp,4}$ = $\boxed{0.832 \pm 0.021 \text{ N}}$

Min. Force for Total Mass m_5 = 132.5 g

Trial #	Min. Force (units: N)	Notes
1	0.93 ± 0.01	
2	0.95 ± 0.01	
3	0.89 ± 0.01	
4	0.96 ± 0.01	
5	0.99 ± 0.01	

Result with Error $F_{s,max,sp,5}$ = $\boxed{0.944 \pm 0.017 \text{ N}}$

Min. Force for Total Mass m_6 = 152.5 g

Trial #	Min. Force (units: N)	Notes
1	0.108 ± 0.01	
2	0.106 ± 0.01	
3	0.103 ± 0.01	
4	0.104 ± 0.01	
5	0.106 ± 0.01	

Result with Error $F_{s,max,sp,6}$ = $\boxed{0.1054 \pm 0.0009 \text{ N}}$

Setup C: Minimum force, $F_{s,max}$ required to move wooden block from rest, for various masses

Surface Material: Cardboard; Block Mass = 32.5 g

Min. Force for Total Mass m_1 = 92.5 g			Min. Force for Total Mass m_2 = 112.5 g		
Trial #	**Min. Force (units: N)**	**Notes**	**Trial #**	**Min. Force (units: N)**	**Notes**
1	0.15 ±0.01		1	0.32 ±0.01	
2	0.13 ±0.01		2	0.31 ±0.01	
3	0.17 ±0.01		3	0.29 ±0.01	
4	0.15 ±0.01		4	0.25 ±0.01	
5	0.19 ±0.01		5	0.26 ±0.01	
Result with Error $F_{s,max,cb,1}$ =	0.158 ± 0.010 N		Result with Error $F_{s,max,cb,2}$ =	0.286 ± 0.014 N	

Min. Force for Total Mass m_3 = 132.5 g			Min. Force for Total Mass m_4 = 152.5 g		
Trial #	**Min. Force (units: N)**	**Notes**	**Trial #**	**Min. Force (units: N)**	**Notes**
1	0.36 ±0.01		1	0.45 ±0.01	
2	0.39 ±0.01		2	0.46 ±0.01	
3	0.35 ±0.01		3	0.44 ±0.01	
4	0.42 ±0.01		4	0.45 ±0.01	
5	0.40 ±0.01		5	0.48 ±0.01	
Result with Error $F_{s,max,cb,3}$ =	0.384 ± 0.013 N		Result with Error $F_{s,max,cb,4}$ =	0.456 ± 0.007 N	

Min. Force for Total Mass m_5 = 172.5 g			Min. Force for Total Mass m_6 = 192.5 g		
Trial #	**Min. Force (units: N)**	**Notes**	**Trial #**	**Min. Force (units: N)**	**Notes**
1	0.55 ±0.01		1	0.57 ±0.01	
2	0.53 ±0.01		2	0.59 ±0.01	
3	0.54 ±0.01		3	0.56 ±0.01	
4	0.57 ±0.01		4	0.60 ±0.01	
5	0.51 ±0.01		5	0.61 ±0.01	
Result with Error $F_{s,max,cb,5}$ =	0.54 ± 0.01 N		Result with Error $F_{s,max,cb,6}$ =	0.586 ± 0.009 N	

5. ***Discussion.*** Use the space below to briefly answer the following questions ***for each Setup***:

   Did you need to refine your measurement procedure? If so, explain.

   Throughout the experiment for all the setups, we didn't modify the experiiment intentionally. We thought that the measurements we were taking were fairly consistent, so we decided to just repeat the same procedure.

   If your results do not agree with the accepted value, what might be the reasons? Consider experimental design, random errors, systematic errors, modeling errors etc.

   Our results up to here agree with what was expected and with what we compared with other groups

   What could be done to improve your measurement?

   We are fairly convinced that the main source of error in our measurements was the reaction time. When the block started sliding, the value of F we recorded was not really the true one, since a few milliseconds had passed. Improving this would've solved many problems.

Analysis

Fitting curves to test the model and extract parameters

1. Propagate error symbolically.

You will be plotting $F_{s,max}$ vs N. The applied force $F_{s,max}$ you measured directly, but the normal force you will be calculating from mass measurements. Propagate error for the normal force $N = mg$ here:

N= mg . The variables that can yield us errors are m, which we measured, and the errors in g, which are given by where/how precisely g is measured.

$$\alpha_N = \sqrt{\left(\frac{\partial N}{\partial m}\alpha_m\right)^2 + \left(\frac{\partial N}{\partial g}\alpha_g\right)^2} = \boxed{\sqrt{(g\alpha_m)^2 + (m\alpha_g)^2}}$$

2. Preparation for curve-fitting: Recasting the data

Prepare for plotting. To find the coefficient of static friction, you will be plotting $F_{s,max}$ vs N and fitting a line to extract the slope μ_s, according to the model: $F_{s,max} = \mu_s N$. Before we can plot the points $(x_i, y_i) = (N_i, F_{s,max,i})$, we need to calculate the normal forces N_i using

$$N_i = m_i g.$$

Let $g = 9.800$ m/s². Record the "best" data pairs in the table below, which are then ready to be plotted.

Setup A: Cork/Wood		Setup B: Sandpaper/Wood		Setup C: Cardboard/Wood	
Normal Force: N_i (units: N)	Min. Force: $F_{s,max,cork,i}$ (units: N)	Normal Force: N_i (units: N)	Min. Force: $F_{s,max,sp,i}$ (units: N)	Normal Force: N_i (units: N)	Min. Force: $F_{s,max,cb,i}$ (units: N)
0.302	0.07	0.525	0.282	0.907	0.158
0.792	0.26	0.711	0.434	1.103	0.286
1.282	0.39	0.907	0.602	1.299	0.384
1.772	0.521	1.103	0.832	1.495	0.456
2.262	0.651	1.299	0.944	1.691	0.540
2.752	0.76	1.495	1.054	1.887	0.586

3. Perform the linear regression and plot the result

Plot data and fit curve from theory. Now each group members should plot the data for one data set $(x_i, y_i) = (N_i, F_{s,max,i})$ above. Fit the simple linear regression function $y = mx + c$ to the data in each plot. The slopes of each line-of-best-fit should be the respective coefficients of static friction μ_s for each material pairing.

- Assessing the fit: Perform a visual assessment of the two plots with fits
- Check the residual plots show no structure.
- Is the y-axis intercept term for a material pairing (cork-wood, sandpaper-wood or cardboard-wood) needed to properly fit? If so, explain the source of the offset (random, systematic, model, equipment or some other type of error)?

Setup A

Setup B

CHECK APPENDIX

Setup C

Make sure you label your plots fully! They should have descriptive titles (which include mention of the material pairings), axis labels with units, the equation of the line-of-best-fit, the values of the slope and intercept with errors, and the value of χ^2. The residuals plots were defined in the Intro Lab 4.

- **BE SURE TO PRINT OUT YOUR PLOTS (ONE FOR EACH MATERIAL PAIRING AND THE ASSOCIATED RESIDUAL PLOT) AND ATTACH THEM TO THIS WORKSHEET.**

4. Compare to accepted values

Compare results to accepted values. In the table below, write your measured value of the coefficient of friction (with errors!), as derived from your plots.

Material Pairing	Accepted value of μ_s	Measured value of μ_s
Cork-Wood*	$0.76 \pm 10\%$	1.77 ± 0.04
Sandpaper-Wood	**Nothing appeared here**	
Cardboard-Wood		

*depends on room conditions

Using an agreement test, does your measured value of μ_s match the accepted value of μ_s for cork-wood? **(Yes/No)**

Note that any apparent match is only naïve—it's not meaningful without a "chi-square" analysis to say whether your data is well-described by the theory.

5. Using chi-squared

So far in Intro Lab 5, you measured coefficients of static friction for various material pairings using curve-fitting to ultimately extract values and errors of μ_s for each case, examined the residuals to preliminarily assess the fit of the curve, and compared your μ_s result to the "accepted" value. You used the same procedure in Intro Lab 4 to evaluate the spring constant *k* with two different models. In both Labs, a theoretical model(s) was proposed and optimal values of the fit parameters along with their uncertainties obtained using the minimization procedure discussed in lecture.

We can do better when testing the quality of the fit by considering the $\chi^2$and $\tilde{\chi}^2$ statistics:

Chi-squared:
$$\chi^2 = \sum \frac{\left(y_i - y(x_i)\right)^2}{\alpha_i^2}.$$

Reduced chi-squared
$$\tilde{\chi}^2 = \frac{\chi^2}{\nu}, \qquad \nu = N - q$$

where $y(x_i)$ is the proposed theoretical model, and the number of degrees of freedom, ν, is defined as the number of data points N minus the number of parameters in the fit q. For example, $\nu = N - 1$ for the equation $y = mx$ because there is only one fit parameter ($q = 1$), whereas $\nu = N - 2$ for $y = mx + c$ because two parameters ($q = 2$) are used for the fit function.

Given good data with well-understood and constrained errors and an appropriate model, $\tilde{\chi}^2$ should be close to one. There are many reasons why $\tilde{\chi}^2$ may be significantly greater (or less than!) 1, however.

- If $\tilde{\chi}^2 > 1$ then the data and/or model is falling outside the expected uncertainty range. The larger $\tilde{\chi}^2$ is, the less likely the discrepancy is due to random statistical variations. Therefore, if $\tilde{\chi}^2 \gg 1$ then our ***results are suspect***. There are a few possibilities for why this occurs.
    - Your "best fit" model is incorrect.
    - Your *uncertainties* are incorrect. You may have incorrectly evaluated the uncertainties or made invalid assumptions about them.
- If $\tilde{\chi}^2 < 1$ then the model is falling within the expected uncertainty range, so you cannot determined whether it is incorrect. If $\tilde{\chi}^2 \ll 1$ then our results are ***also suspect*** since it indicates that the actual variation of the data is not as large as a normal distribution based on your uncertainty calculations have suggested! This suggests that you have underestimated the errors.

We may ask what "close to 1" means for evaluating $\tilde{\chi}^2$. The answer depends on a number of factors including the numbder of degrees of freedom *v*; the more degrees of freedom you are considering the closer you need $\tilde{\chi}^2$ to be to 1. Hughes and Hase Section 8.4 addresses this. The general guidelines suggested by them are:

- *If $\tilde{\chi}^2 \ll 1$, check your calcuations for the uncertainties in the measurements.*
- *The hypothesis or data is questioned if:*
    - $\tilde{\chi}^2 > 2$ *for* $\nu \approx 10$.

6. Gathering the chi-square values

All of your Intro Lab 4 and Intro Lab 5 plots should include values of chi-square χ^2 for the linear fits. **Record those values in the table below, count the number of degrees of freedom, and calculate the reduced chi-square $\tilde{\chi}^2$ in each case by dividing by the number of degrees of freedom, ν.**

	Chi-square: χ^2	*Number of degrees of freedom: ν*	*Reduced chi-square: $\tilde{\chi}^2 = \chi^2/\nu$*
Spring constant k (from fit to $y = mx$)	7212	5	419.68
Spring constant $\boldsymbol{k}$ (from fit to $\boldsymbol{y = mx+c}$)	37	4	9.24
$\boldsymbol{\mu_s}$ for Cork-Wood	21.18	4	5.29
$\boldsymbol{\mu_s}$ for Sandpaper-Wood	10.51	4	2.63
$\boldsymbol{\mu_s}$ for Cardboard-Wood	27.90	4	6.98

- **Be sure to double check your error calculations if $\tilde{\chi}^2$ is extremely large or small before proceeding!**

7. Interpreting the chi-square values

Determine whether or not the reduced chi-square values you found are too big, too small, or acceptable.

	Reduced Chi-square: $\tilde{\chi}^2$	*Is $\tilde{\chi}^2$ "Too big", "Too small", or "Acceptable"?*
Spring constant $\boldsymbol{k}$ (from fit to $\boldsymbol{y = mx}$)	419.68	Too big
Spring constant $\boldsymbol{k}$ (from fit to $\boldsymbol{y = mx+c}$)	9.24	Too big
$\boldsymbol{\mu_s}$ for Cork-Wood	5.29	Too big
$\boldsymbol{\mu_s}$ for Sandpaper-Wood	2.63	Close enough/ Too big
$\boldsymbol{\mu_s}$ for Cardboard-Wood	6.98	Too big

Now, write an interpretation for each case:

- If the value of $\tilde{\chi}^2$ is too small ($\tilde{\chi}^2 < 0.5$) or too big ($\tilde{\chi}^2 > 2.0$) discuss: What does that mean and why might it have happened for that experiment? For instance, consider whether the fitting function is correct or whether the experimental errors could be too large or small.
- If the value of $\tilde{\chi}^2$ is acceptable, congratulations! Discuss why the fitting function accurately models the experimental situation, and the observed experimental errors.

Spring Constant (both $\boldsymbol{y = mx}$ and $\boldsymbol{y = mx + c}$ fits)

These were too big, probably since, as explained in the IntroLab4, we measured the wrong length of the spring, without taking into account the length of the actual hooks. This completely changed the length we had for the spring, ultimately affecting our value of k. Furthermore, we perhaps had some systematic errors with our measuring devices.

μ_s for Cork-Wood

Even though I didn't carry out this experiment directly, the person who did it told me that the friction was irregular on the surface of the cork. This was weird, and led him to many randomly distributed values, which eventually modified his dataset, and his value for the friction coefficient. We can infer that the experimental errors were too large perhaps, and the final reduced chi squared value was affected by this.

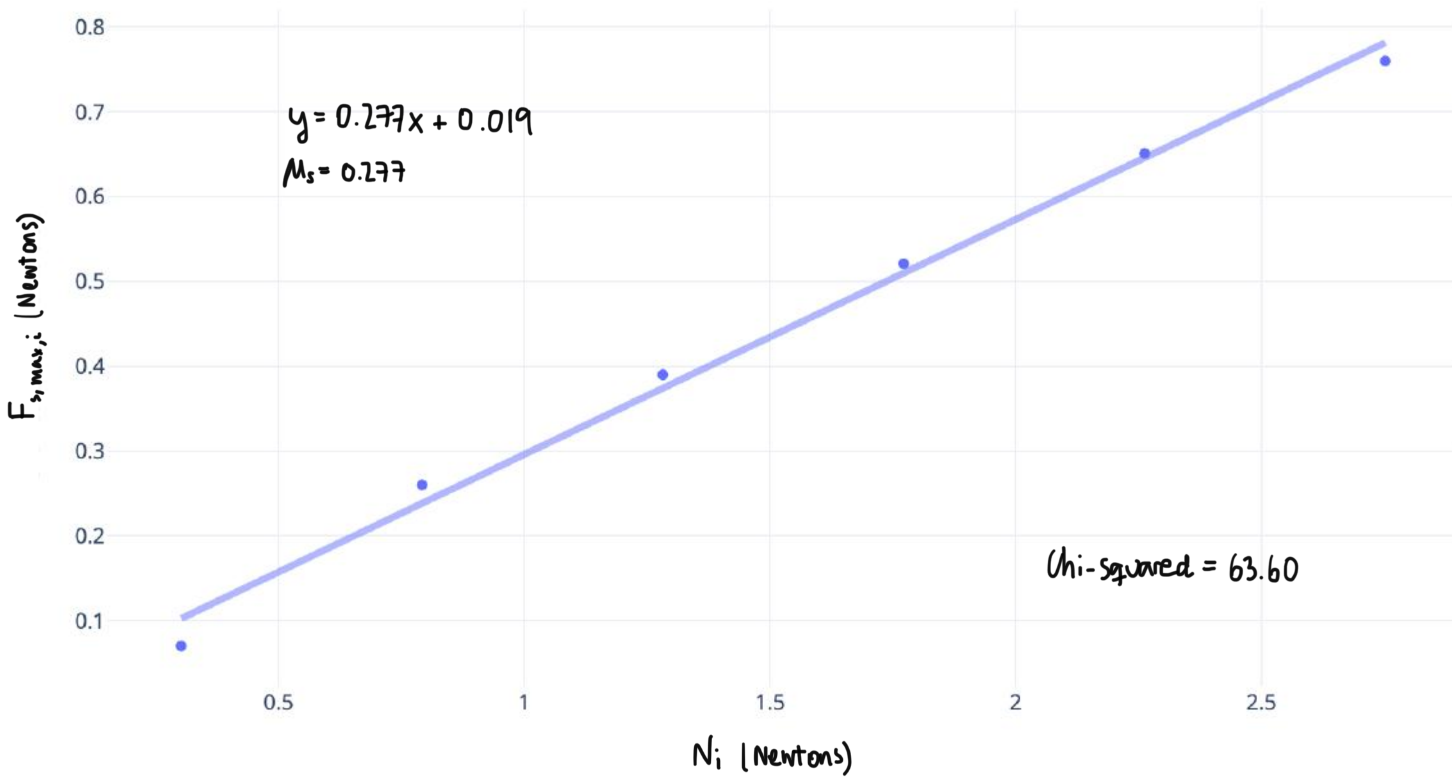
μ_s for Sandpaper-Wood

In this case, it was relatively easy for us to measure mu, since it was clear when the system starts moving, it is easier to tell, and at least predict the precise moment. This reduces systematic errors from reaction time and similar. Furthermore, this case was quite close to a valid value for the reduced chi squared, so perhaps the small error came from approximations in our fitting function.

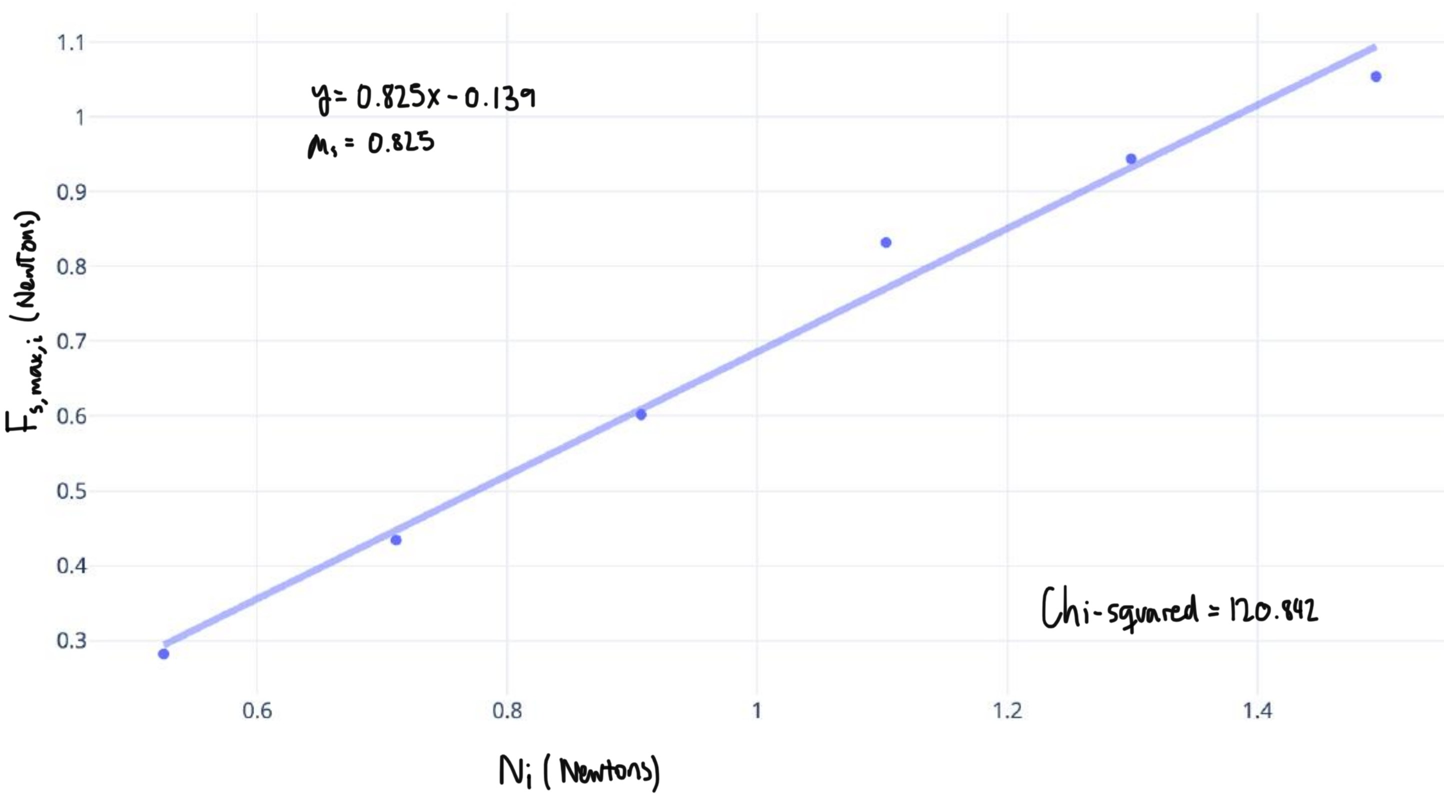
μ_s for Cardboard-Wood

During the experiment, the cardboard kept sliding uncontrollably, so we measured many things wrong. This led to many random errors that eventually affected the fitting matrix and our line of best fit, hence changing our value for the friction coefficient. Something we would've changed about the experiment is probably the reaction time, perhaps using more precise intruments.

Setup A Linear Regression. Curve: y = 0.277~~m~~x + 0.019



Setup B Linear Regression. Curve: $y = 0.825x - 0.139$



Setup C Linear Regression. Curve: y = 0.434x - 0.204