Lab Report 1

Tier 1: Hooke's Law Harmonic Oscillator

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Introduction/Objectives:

In this lab, our goal was to measure and explore the Hookean dynamics of a spring-mass system as well as to study harmonic oscillations of different types using the IOLab's force and wheel sensors. Also, we would carry out an analysis of the different types of motion the mass undergoes when under the influence of different angles and frictions.

Theory:

Formulae and Theory for Oscillations:

Usually, when we have a system where one of the components stretches, such as a spring, the system obeys a physical law known as Hooke's Law. This is because the stretching is proportional to the force on the system. In other words, F = -kx for some stretching constant k and some displacement x. In the case we are considering for Part 1 of this report, we consider a system with a mass and a spring. Here, we can use Newton's Law and Hooke's Law to come to the differential equation given by the displacement x:

$$x''(t) + {\omega_0}^2 x(t) = 0$$

The solution to this equation is given by:

$$x(t) = Acos(\omega t + \phi)$$

This equation shows that the motion of the mass under the influence of the spring follows the shape of a wave. However, the original differential equation doesn't take friction into account. When friction is constant, the mass-spring system friction doesn't change as the mass' velocity changes. We are left with the differential equation:

$$x''(t) + {\omega_0}^2 x(t) = \pm \frac{f}{m}$$

Here, the plus sign is taken when the object moves in +x direction, and the negative sign is taken when the object moves in x direction. Since the sign flips every time the mass reverses direction, the solution to the differential equation is given by an arithmetic sequence for the change in amplitude:

$$A_n - A_{n+1} = \frac{2f}{m}$$

In this formula, n is the n-th peak and n+1 is the n+1-th peak in the harmonic motion. Now, let's look at what happens when the friction changes as a function of velocity. In this case, we have the differential equation:

$$x''(t) + \beta x'(t) + {\omega_0}^2 x(t) = 0$$

Here, beta represents the damping constant divided by the mass. Note that this is a homogeneous differential equation, and the solution to it is given by the harmonic function:

$$x(t) = Ae^{-\beta t}cos(\omega t + \phi)$$

where

$$\omega = \sqrt{{\omega_0}^2 - \beta^2}$$

Here, w0 represents the natural frequency of the spring, or how it would oscillate without friction. Now, since we know the x(t) for all three cases, we can derive the acceleration, which is what we will be looking at in this Lab Report. The accelerations are given by:

$$x''(t)_{damped} = Ae^{-\beta t}[(\beta^2 - {\omega_0}^2)cos(\omega t + \phi) + 2\beta\omega sin(\omega t + \phi)]$$

$$x''(t) = -A{\omega_0}^2 cos(\omega_0 t + \phi)$$

Formulae and Theory for Measuring Friction:

Later on in Part 3, we will be measuring friction using a setup we created. To do this, we will give the IOLab some kinetic energy and let it roll on the wood until it stops. The, we will relate the change in kinetic energy to the distance travelled by the Work-Energy theorem, which tells us how much energy is dissipated through friction:

$$-\Delta KE = \int Fxdx = Fx$$

In the formula above, F is the friction force we want to find and x is the distance travelled by the IOLab. As we will later explain, we can find the change in KE thanks to the IOLab's velocity sensors.

Methods:

For the methods section, we thought we would give an overview of all the necessary steps and items to carry out each of the experiments we had to carry out here. Therefore, we will do a methods subsection for each of the 3 experiments.

Method for Part 1

In Part 1, the necessary items from the Lab Kit are the IOLab Device, the long silver spring from the Lab Kit, and the wooden dowel, which we will be using as support for our spring. As a general procedure, we will be using certain steps:

1. Attach the spring to the dowel and make sure the link is solid and doesn't move.
2. Hook the IOLab's hook onto the spring, and make sure that the system rests in equilibrium.
3. Turn the IOLab on, and on your device, select the 'Force' and the 'Wheels' sensors. Make sure to activate the position and acceleration graphics for this sensor.
4. Once the sensors are ready and the system is in equilibrium, press 'Record' on the IOLab online data gathering app.
5. Now, start moving the IOLab device up and down with a constant amplitude and frequency, following harmonic motion.
6. Once enough periods have taken place, stop the 'Record' button and look at the graphs with data given by IOLab. Make sure the data is reliable and has a good form. If not, repeat steps 4-6.

Note that in Part 1, distance is measured by the wheel sensors, and the force of the spring on the mass is measured by the force sensor on the IOLab device. Also, one last part to the method is to gather the best dataset and export it to the online IOLab repository, since this will enable for other group members to analyze the data.

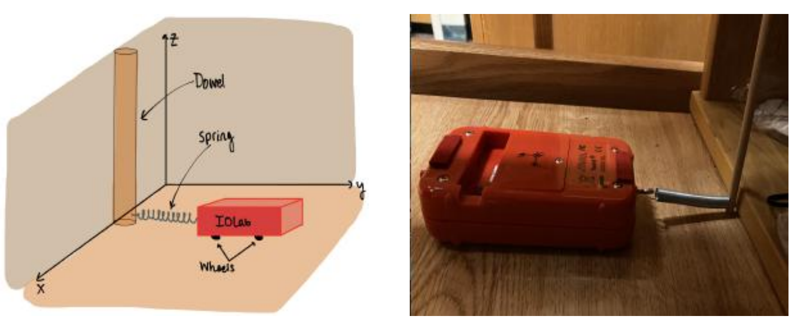

Figure 0.1: Part 1 Diagrams

Method for Part 2

In Part 2, the necessary items from the Lab Kit are the IOLab Device, the long silver spring from the Lab Kit, and the wooden dowel, which we will be using as support for our spring. Furthermore, we will be hanging the IOLab device from relatively high position to perform the vertical harmonic motion analysis, so we must have a good conditioned and safe high place from where to hang the IOLab device. As a general procedure, we will be using certain steps:

1. Attach the spring to the dowel and make sure the link is solid and doesn't move.
2. Hook the IOLab's hook onto the spring, and make sure that the system rests in equilibrium.
3. Turn the IOLab on, and on your device, select the 'Force' and the 'Accelerometer' sensors. Make sure to activate the acceleration graphics for this sensor.
4. Once the sensors are ready and the system is in equilibrium, press 'Record' on the IOLab online data gathering app.
5. Now, pull the IOLab device down a small amplitude downwards from its equilibrium position, and observe it following harmonic motion.
6. Once enough periods have taken place, stop the 'Record' button and look at the graphs with data given by IOLab. Make sure the data is reliable and has a good form. If not, repeat steps 4-6.

Note that in Part 2, we don't use the wheels because the IOLab device is hanging in the air. Instead, we use the IOLab's internal accelerometer, which will also take the gravitational acceleration into account. Therefore, the dataset we have is slightly dependent on where the experiment is performed. Also, one last part to the method is to gather the best dataset and export it to the online IOLab repository, since this will enable for other group members to analyze the data.

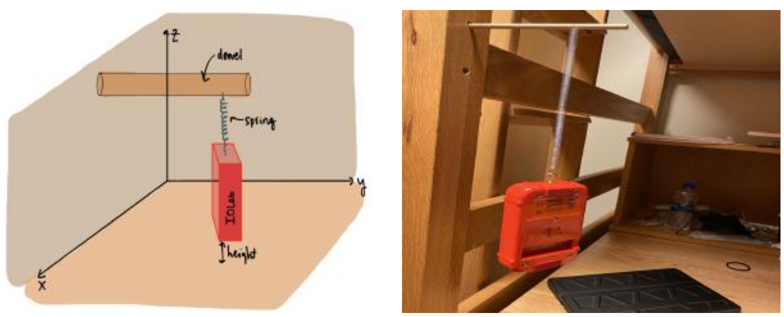

Figure 0.2: Part 2 Diagrams

Method for Part 3

In Part 3, the necessary items from the Lab Kit are the IOLab Device, the long silver spring from the Lab Kit, the wooden dowel which we will be using as support for our spring, and a ramp to have the oscillations be at a 60° angle. In our case, we used a wooden ramp, which was the only thing available for us at the Berkeley dorms. As a general procedure, we will be using certain steps:

1. Prepare the ramp at a 60° angle. To do this, use a protractor, and make sure the measurement is done in the same plane of movement as the ramp's. Then, place the dowel perpendicular to the ramp at the top, ready for attaching the spring to it.
2. Attach the spring to the dowel and make sure the link is solid and doesn't move.
3. Hook the IOLab's hook onto the spring, and make sure that the system rests in equilibrium.

4. Turn the IOLab on, and on your device, select the 'Force' and the 'Wheels' sensors. Make sure to activate the position and acceleration graphics for this sensor.

5. Once the sensors are ready and the system is in equilibrium, press 'Record' on the IOLab online data gathering app.

6. Now, pull the IOLab device down a small amplitude down the ramp from its equilibrium position, and observe it following harmonic motion. It is important that the initial amplitude we give to the oscillating IOLab is small enough to ensure that the IOLab never hits the dowel.

7. Once enough periods have taken place and the damping is noticeable, stop the 'Record' button and look at the graphs with data given by IOLab. Record the oscillations 10 times. Make sure the data is reliable and has a good form. If not, repeat steps 4-6.

Note that in Part 3, acceleration is measured by the wheel sensors, and the force of the spring on the mass is measured by the force sensor and accelerometers on the IOLab device's core and wheels. Also, one last part to the method is to gather the best dataset and export it to the online IOLab repository, since this will enable for other group members to analyze the data.

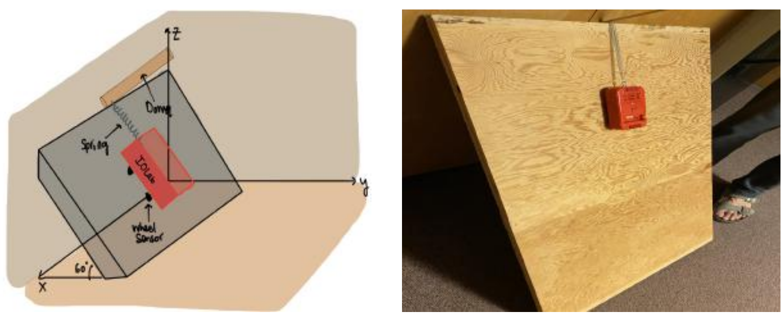

Figure 0.3: Part 3 Diagrams

However, for Part 3, we still have something left to do after we have obtained all the oscillation periods, since this will ease the analysis later on. What we must do is measure the friction between the wheels and the IOLab. Once we later on plot the damped harmonic motion with constant friction model for this part, we will notice that this doesn't give either a good description of the motion or a good prediction for k. Since it isn't scientific to just state that the constant friction model isn't 'good enough', we thought of measuring the friction f to find the spring constant and show how this new value of k is further off the accepted value than the ones obtained through other models of motion. To find f, we had to somehow measure the friction coefficient between the wheels and the body of the IOLab device. To do this, we devised an experimental setup consisting of a wooden ramp placed horizontally on the ground, the IOLab device, and someone to push the IOLab. Also, we will be using the velocity sensor in the IOLab. Once the instruments are gathered, the procedure is as follows:

1. Place the wooden board horizontally on the ground and a smaller. A diagram of the setup is shown below.

2. Place the IOLab at rest at some predetermined point on the wooden board.

3. Push the IOLab device, and measure how far it rolls down the horizontal wooden board and the velocity it rolls with.

4. Apply conservation of energy and the formulae provided in the Theory section. This will give a rough estimate for the friction force.

5. Using the known mass of the IOLab device (202 grams), find the normal force and the friction coefficient.

Figure 0.4: Setup to find friction

Calculations and Data Reduction/Analysis:

In this section, we will be discussing the analysis of the data we obtained. In other words, we will perform all the calculations that we learned during the Intro Labs and discuss our results. Similarly to the Methods section in this report, we will perform the analysis and discussion part by part, since the experimental setups are different. For each part, we will be considering and answering questions such as why we chose a specific dataset, how our results match the accepted values, whether the behaviour of our mass fits the expected behavior given by the formulae developed in the Theory section, and why all the possible discrepancies could happen.

Analysis for Part 0

Here, we were asked to answer some questions in the Lab Manual. The questions were referring to Part 0, where we have to hang the weightset into the IOLab's hook. The questions and our answers go as follows:

- Using the IOLab data analysis tools, evaluate the force in Newtons caused by adding the weight set. Data average along a quiet section for the best results: We found Approximately 2.473 N.
- Do you obtain the correct weight result compared to the nominal value? Yes, this is close to the nominal weight of 2.45 N.
- Why is the accelerometer measurement not zero when the device is held steady? It measures the acceleration on it, which includes acceleration due to gravity.

Analysis for Part 1

For this part, we decided to use Allen's data, because he performed the most repetitions of the loading and unloading of the spring. We thought this would be a good idea since he had a wider and better variety of data to choose from and therefore a higher likelihood of achieving a good final result. We used a simple linear regression to fit our data to the Hookean equation F $= -kx$, and our data took the form below:
From this data, we can see that $k = 13.7 \pm 0.024\,\mathrm{N/m}$, where the error comes entirely from the standard error, since we determined that the IOLab was precise enough as to not affect the error. The accepted value for the spring constant of the spring we used is $k = 12 \pm 10\%\,\mathrm{N/m}$. Therefore, our result did not agree with the accepted value. We hypothesized that we might have moved the IOLab far enough to stretch the spring to its limits, where it does not conform well to the Hookean model. Further examples of things

LINEAR MODEL		
	BEST FIT PARAMETERS	
	m	-1.37E+01
	c	-3.66E-01
	COMMON UNCERTAINTY	
	a_cu	1.09E-01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	2.40E-02
	UNCERTAINTY IN c	2.33E-03
	DELTA	1.27E+05

Figure 0.5: Sheet we used to analyze data

that could've caused the errors are given in the Conclusion, and are directly addressed as sources of error with plausible solutions. This makes sense given that our result was higher than the accepted value, so the spring was further stretched at some points beyond the k of the spring. Once we had analyzed the data we had, we plotted our model as a force vs distance graph, which is found below. The fact that our data has such a straight shape with not many irregularities shows that we had few random errors, and just failed systematically and repeatedly at some points of the procedure, creating an overall shift which led to a higher final value of k.

Figure 0.6: Force vs displacement linear regression

Analysis for Part 2

For this part, we decided to use Javier's dataset since it seemed the most stable. This is because his dataset was very close to the actual expected data, at least to the naked eye, and we considered it a good choice due to the quality and variety of the data. Once we knew what dataset we were using, we manually fitted our data to a harmonic curve. This procedure was done by adjusting some parameters in the equations for Damped Harmonic Motion (DHM) with velocity-dependent friction, so that our data would be easier to analyze in the future. The table showing the fitting of the data is shown below:

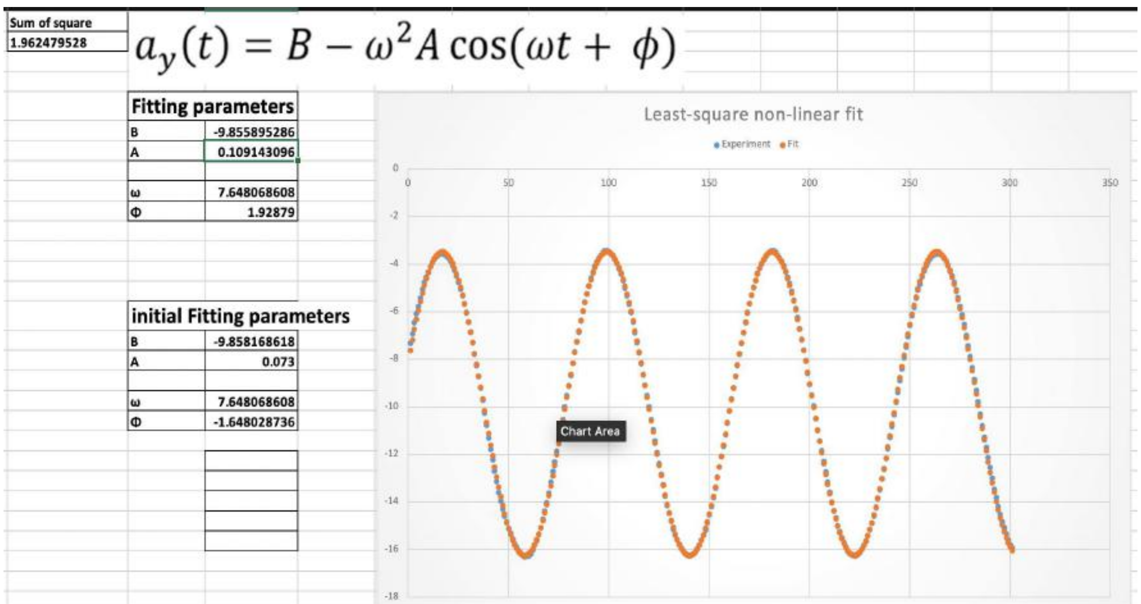

Figure 0.7: Fitting data to Harmonic

From some of these parameters and the overall form of the data, we can see that we can apply the formulae in the Theory section to find an estimated value of k. Therefore, using the formula , $w^2 m = k$ we found k to be $k = 11.82\,\mathrm{N/m}$. Since, as before, we assumed the IOLab to have no standard or significant (at least quantifiable) errors, we assumed our predicted value to be error-free. As before, we took the accepted value for k to be $k = 12 \pm 10\%\,\mathrm{N/m}$, so our measured result really fell within the range of the accepted value. This suggests that, while it is undergoing small oscillations, the spring conforms strongly to the Hookean spring-mass dynamic model.

Additionally, we fitted our data to a damped oscillator under velocity-dependent friction model. Here, we used the same dataset as the one we used in the harmonic oscillator before. Similarly, like before, we fitted the data to the expected graph for the damped harmonic oscillator. The table showing the fitting of the data is shown below:

From some of these parameters and the overall form of the data, we can see that we can apply the formulae in the Theory section to find an estimated value of k. Therefore, using the formula $w^2 = k$, we found k to be $k = 12.06\,\mathrm{N/m}$. Like before, we took the accepted value for k to be $k = 12 \pm 10\%\,\mathrm{N/m}$, so our measured result really fell within the range of the accepted value. Our result agreed strongly with the accepted value. However, the fact that this fit was required was somewhat mysterious to us, since, for sufficiently small , this is essentially the same as the simple harmonic motion model. Then, since there was virtually no damping, the spring was just undergoing simple harmonic motion. We are not really sure why this fit was included in this section, unless of course we are considering the air resistance as a significant factor in this experiment.

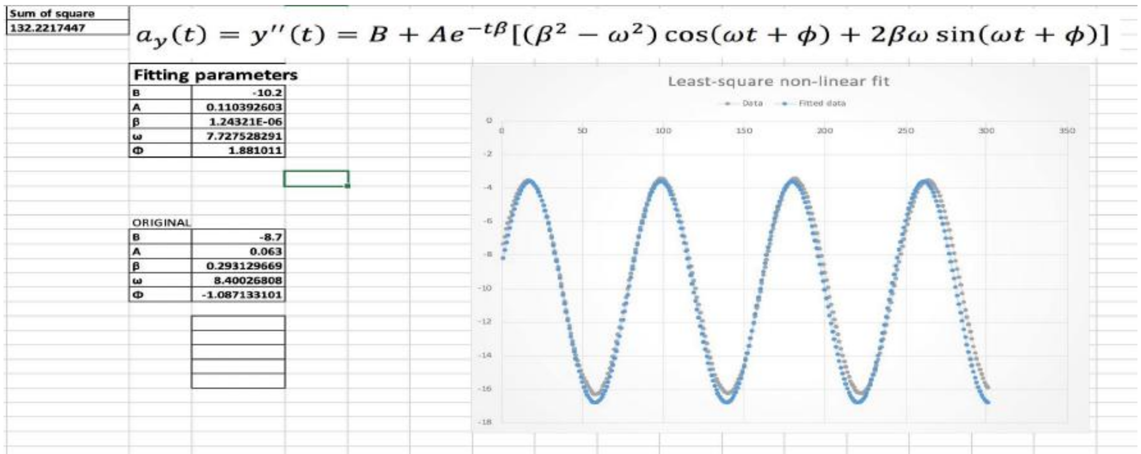

Figure 0.8: Fitting data to Damped Harmonic

Analysis for Part 3

We decided to use Javier's data because he was the only one who had a large, flat ramp readily available. We used the acceleration data from the wheels because the accelerometer on Javier's IOLab was very unstable, and gave many spikes in the data. We manually fitted our data to a harmonic curve using the Excel spreadsheet provided:

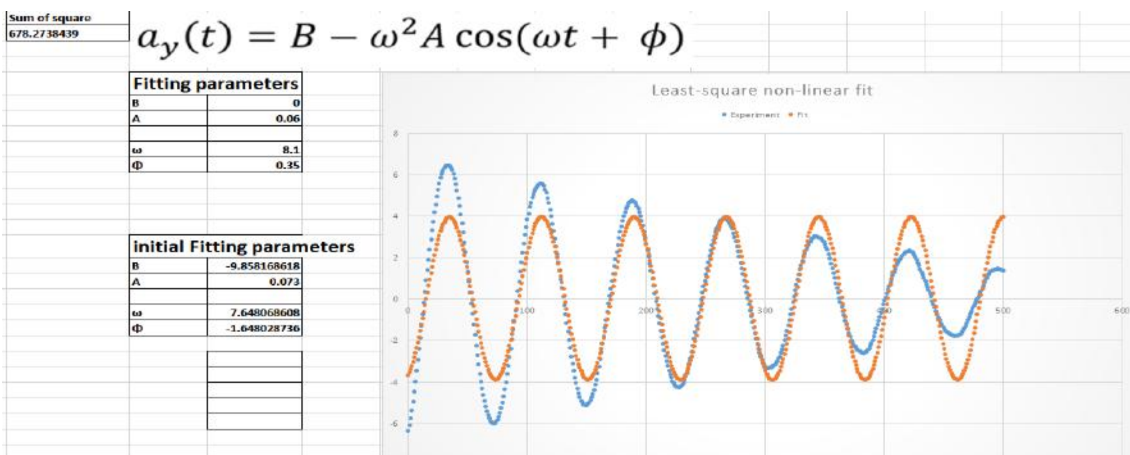

Figure 0.9: Fitting data to Harmonic

This was an extremely unsuccessful fit, which makes sense considering that the motion of the spring was damped and could not possibly be fit into a simple harmonic model. The motion of the spring would be much better approximated by another model. We additionally manually fitted our data to a damped harmonic oscillation under velocity-dependent friction model:
Using the formula $2m = k$, we found $k = 13.32$. The accepted value for the spring constant of the spring we used is $k = 12 \pm 10\%$ N/m. Our result came close but did not agree with the accepted value. We hypothesized that the IOLab might have been massive enough to stretch the spring to its limits, where it does not conform well to the Hookean model. The material of the spring itself would start to create more resistance against further stretching, making the inward force much higher than it would be under a perfect Hookean model. This makes sense given that our result was higher than the accepted value. We additionally considered a damped harmonic oscillation under velocity-dependent friction model. We found the differences between the peak amplitudes of the spring's motion:

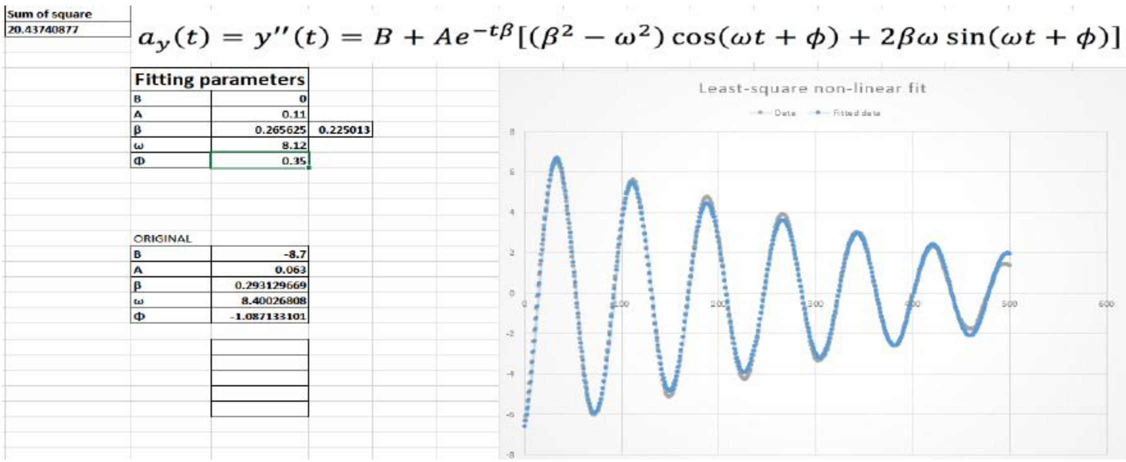

Figure 0.10: Fitting data to Damped Harmonic

A	B	C
DIFFERENCES BETWEEN PEAKS	CALCULATED SPRING CONSTANT k	AVERAGE k
0.0148	8.121621622	8.924061779
0.0144	8.347222222	
0.01379	8.716461204	
0.0136	8.838235294	
0.012	10.01666667	
0.012	10.01666667	
0.014627	8.217679634	
0.014189	8.471351047	
0.0134	8.970149254	
0.01262	9.524564184	

Figure 0.11: Analysis for Constant friction in damped oscillator

From the fact that the differences did not remain constant, and decreased as time went on, it was already fairly obvious that this model would not work. However, for the sake of rigor, we calculated the k that this model would give us. Using the work-force formula, we determined the resistive force f by rolling the IOLab across our wooden board and measuring its velocity and displacement over time. We found the resistive force $f = 0.0601$N. Then, using $A_n - A_{n-1} = 2fk$, we found $k_{av} = 8.924\,\text{N/m}$. The accepted value for the spring constant of the spring we used is $k = 12 \pm 10\%\,\text{N/m}$. Our result did not agree with the accepted value. This was because the model was clearly incorrect.

Summary and Conclusion:

In this section, we will be discussing our overall perspective on this Lab Report and the experiments carried out within it. As originally mentioned, our goal was to understand and measure the Hookean behavior of a spring mass system under different friction or angle-related conditions. To do this, we had three different experimental setups that were described in the Methods section. With these setups, we looked at the data and analyzed it for the three different oscillatory cases described in the Theory section: SHM, DHM with constant friction, and DHM with velocity-dependent friction. For each experiment, we looked at the behavior of the mass under different conditions, and analyzed the data we obtained in the Analysis question.
As a whole, our experiments' results resembled what we expected to obtain for each case, but we couldn't

avoid some discrepancies with the accepted values, shown by the Chi-squared analysis. We believe that we had these small disagreements due to small uncontrollable or systematic errors that we made. The main errors that we could've solved if we had been more aware of them would include:

1. Spring can sometimes not be perfectly horizontal, straight, or parallel to the IOLab's plane of motion.
2. In Part 1 specially, it is important to not move the IOLab to higher amplitudes than the original one for each period. One should be very careful with this since the experiment relies on the amount of force exerted by the person's hand, and applying too much force will invalidate the experiment.
3. Making sure the IOLab doesn't hit the dowel or overcontracts the spring when it reacher maximum height in Parts 2 and 3.
4. This error had to be corrected and took us a lot of time. It was inherent to the spreadsheet we were provided. It had many built-in errors, so we had to be specially careful with how the data was plugged into and processed by the spreadsheet, specially for parts 2 and 3 in the data interpolation sheet.
5. In Part 2 specially, not waiting for the oscillations to stop and the experimental setup to zero between measurements can have severe consequences on the final dataset too.
6. Making sure the ramp in Part 3 is at an angle of 60^{o}, either by performing a good measurement as described in the Methods section, or by making sure we don't move the ramp accidentallly.

Note that we didn't have to correct all of these errors after and throughout the analysis, since some of them were carefully thought of in the Methods section. However, some errors such as errors 2, 3, and 4 were very heavy in terms of work, so we recommend people trying this experiment in the future to take special care of these features. Possible ways to reduce each error is described adjacent to each error in the list above. Leaving these errors aside, the results show that we weren't very far off the accepted values, as previously shown and explained.

Appendix with all Datasets, for Part 0, Part 1, Part 2, and Part 3:

In this section, we will be placing all of the screenshots from the shared IOLab repository that we used for the analysis sections of Part 0, Part 1, Part 2, and Part 3:

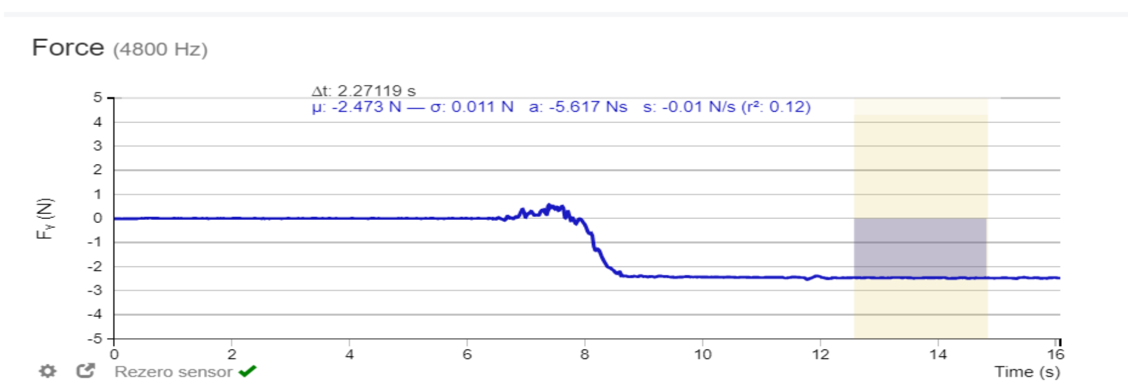

Figure 0.12: IOLab Repository Plot for Part 0

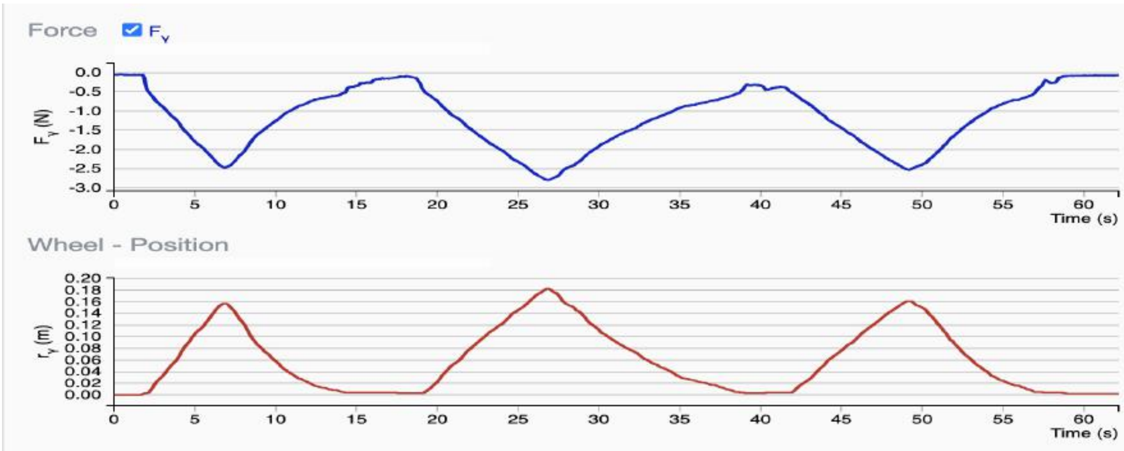

Figure 0.13: IOLab Repository Plot for Part 1

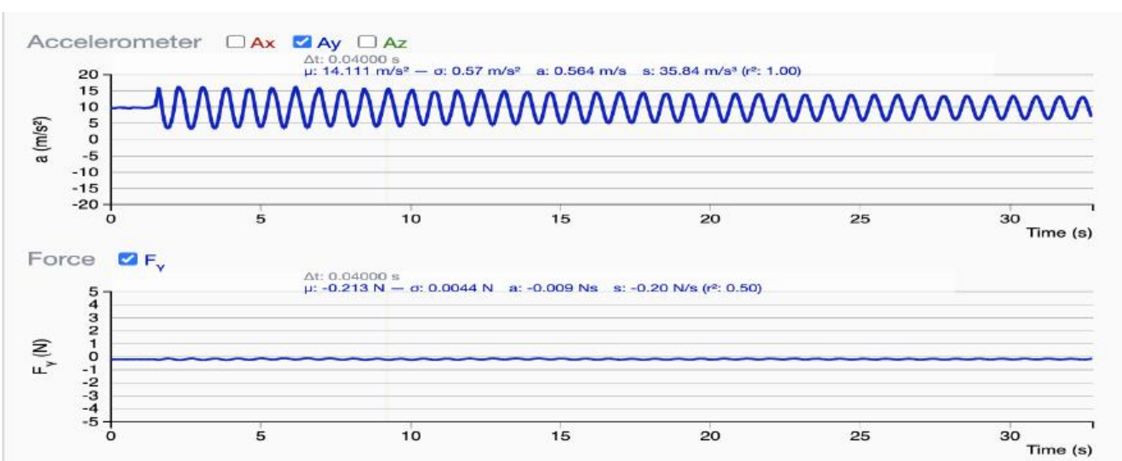

Figure 0.14: IOLab Repository Plot for Part 2

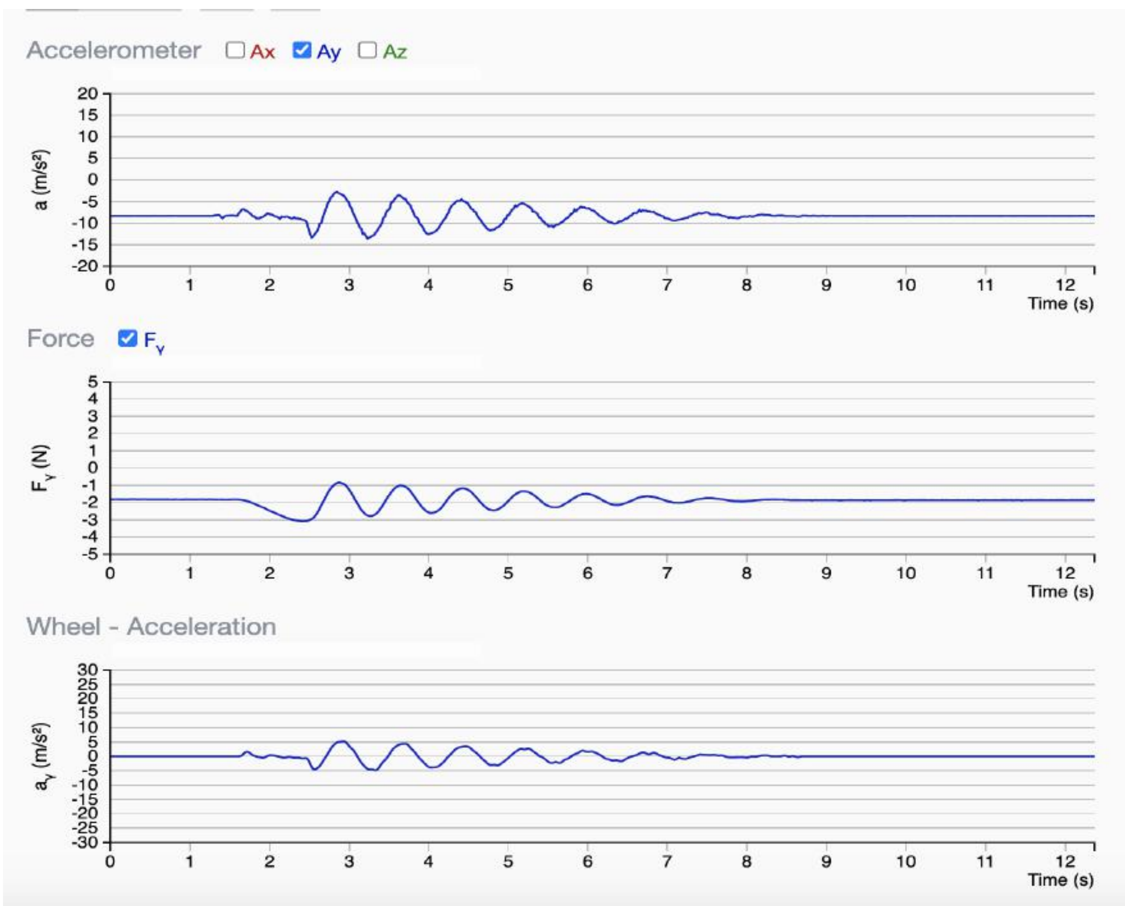

Figure 0.15: IOLab Repository Plot for Part 3