

Lab Report 3

Tier 1-EM2: Introduction to AC Circuit Measurements

Allen Chen (Skeptic), Javier Santillan (Manager), Pablo Castaño (Analyst)

March 17, 2021

Introduction/Objectives:

Throughout this lab, we were only interested in analyzing the datasets that we were provided. Our group number is Group 2, so the datasets that we will be analyzing are [Dataset 1](#) and [Dataset 2](#). We were interested in looking at the different defining characteristics of RC (Resistor-Capacitor) and RL (Resistor-Inductor) circuits, and performing an analysis of the data we were given to test our understanding of the Theory and Analysis methods regarding electromagnetic induction and AC circuits. To make sure we did this properly, we have followed the different parts specified in the Lab Manual (Parts 1-4 in the Analysis Section).

Theory:

In this lab we use AC circuits, and we deal with the simplest kind of AC wave, the sine wave:

$$V(t) = V_p \sin(\omega t + \psi)$$

. V_p is the peak voltage, ω is the frequency, and ψ is the phase. The closely related peak to peak voltage is defined as

$$V_{pp} = 2V_p$$

for circuits driven by an AC Voltage source with a sine input, the harmonic driving term is written as $V(t) = V_p \cos(\omega t)$ Because the voltage acts as a simple harmonic oscillator, the current will therefore oscillate with the same frequency ω with some phase shift. To solve for circuits like this, using complex notation can turn out to be convenient. Specifically, we define Voltage and current to be

$$\begin{aligned}\tilde{V}(t) &= \tilde{V} e^{i\omega t} \\ \tilde{I}(t) &= \tilde{I} e^{i\omega t}\end{aligned}$$

This notation is important in AC circuits because the role of resistance is played by a complex impedance, which is defined as

$$Z = \frac{\tilde{V}(t)}{\tilde{I}(t)} = \frac{\tilde{V} e^{i\omega t}}{\tilde{I} e^{i\omega t}}$$

Then, using the fact that for an inductor, $V(t) = L \frac{dI}{dt}$ and that for a capacitor $V(t) = C \frac{dQ(t)}{dt}$ We derive, using the complex notation, for the RL circuit:

$$\tilde{V} = Z_L \tilde{I}$$

and for the RC circuit:

$$\tilde{V} = Z_C \tilde{I}$$

where $Z_L = i\omega L$ and $Z_C = \frac{1}{i\omega C}$ Because we can only measure real quantities with the equipment, we rewrite voltage in terms of its amplitude and phase.

$$\tilde{V}(t) = |\tilde{V}| e^{i\phi} e^{i\omega t}$$

where $\phi = \frac{\text{Im}[\tilde{V}]}{\text{Re}[\tilde{V}]}$ and we then take the real part

$$V(t) = \text{Re}[\tilde{V}(t)] = |\tilde{V}| \cos(\omega t + \phi)$$

because \tilde{V} is relative, it is defined to be the real V_0 So for the RC circuit:

$$\tilde{V}_c = \frac{V_0 e^{i\omega t}}{1 + iR\omega C}$$

and the real part is therefore

$$V_c(t) = V_c \cos(\omega t + \phi)$$

where

$$V_c = \frac{V_0}{\sqrt{1 + \omega^2 R^2 C^2}}$$

and

$$\phi = \tan^{-1}(-\omega RC)$$

Similarly, for the RL circuit, we have:

$$V_L(t) = \frac{V_0 \omega L}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi)$$

and

$$\phi = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

We will fit the data from the experiments using these to equations for voltage in the analysis section.

Since we were given that $\frac{V_c}{V_0} = \frac{A}{\sqrt{1 + \omega^2 \tau^2}}$, and $\tau = RC$, where R is the resistance and C is the capacitance, we can find the errors in τ and A through error propagation. We first identified that A can be written as $A = \frac{V_c \sqrt{1 + \omega^2 \tau^2}}{V_0}$ and τ can be written as RC, which remain constant throughout the experiment in the RC circuits. The error propagation we did to find the errors in A and τ is given by the formulas:

$$\alpha_A = \sqrt{\left(\frac{\partial A}{\partial \tau} \alpha_\tau\right)^2}$$

and for τ ,

$$\alpha_\tau = \sqrt{\left(\frac{\partial \tau}{\partial R} \alpha_R\right)^2 + \left(\frac{\partial \tau}{\partial C} \alpha_C\right)^2}$$

These can now be rewritten one in terms of another, and solving the partial derivatives, as

$$\alpha_\tau = \sqrt{(C \alpha_R)^2 + (R \alpha_C)^2}$$

and

$$\alpha_A = \sqrt{\left(\frac{V_c \omega^2 \tau}{V_0 \sqrt{1 + \omega^2 \tau^2}}\right)^2 (C \alpha_R)^2 + (R \alpha_C)^2}$$

Now, let's do the same process for the RL Circuit, where we have an Inductor instead of a capacitor. Since now we are given that $\frac{V_L}{V_0} = \frac{A \omega \tau}{\sqrt{1 + \omega^2 \tau^2}}$, and $\tau = \frac{L}{R}$, where R is the resistance and L is the impedance, we can find the errors in τ and A through error propagation. We first identified that A can be written as $A = \frac{V_L \sqrt{1 + \omega^2 \tau^2}}{V_0 \omega \tau}$ and τ can be written as $\frac{L}{R}$, which remain constant throughout the experiment in the RL circuits. The error propagation we did to find the errors in A and τ is given by the formulas:

$$\alpha_A = \sqrt{\left(\frac{\partial A}{\partial \tau} \alpha_\tau\right)^2}$$

and for τ ,

$$\alpha_\tau = \sqrt{\left(\frac{\partial \tau}{\partial R} \alpha_R\right)^2 + \left(\frac{\partial \tau}{\partial L} \alpha_L\right)^2}$$

These can now be rewritten one in terms of another, and solving the partial derivatives, as

$$\alpha_\tau = \sqrt{\left(\frac{-L}{R^2} \alpha_R\right)^2 + \left(\frac{1}{R} \alpha_L\right)^2}$$

and

$$\alpha_A = \sqrt{\left(\frac{V_L \omega^3 \tau^2}{V_0 \sqrt{1 + \omega^2 \tau^2}}\right)^2 \left(\frac{-L}{R^2} \alpha_R\right)^2 + \left(\frac{1}{R} \alpha_L\right)^2}$$

We will be using these formulas in the analysis section when we look at the errors, and the actual numerical values will be derived in the Python file, found [here](#).

Methods:

While this lab was not physically performed, we will still outline the equipment and procedures necessary to perform the experiments.

The equipment necessary to carry out the experiment includes:

- Oscilloscope
- Function generator
- one $1.5\text{ k}\Omega$ resistor and one $510\text{ }\Omega$ resistor
- one $1.0\text{ }\mu\text{F}$ capacitor
- one 4.7 mH inductor
- Breadboard
- BNC-to-BNC coaxial cables, two BNC-to-mini grabber coaxial cables, and a “T” BNC adapter

First, to construct the RC Circuit:

1. Connect the circuit using the $1.5\text{ k}\Omega$ resistor and $1.0\text{ }\mu\text{F}$ capacitor.
2. For the function generator, use the “T” BNC connector to split the output signal of the function generator into two.
3. Connect one coaxial cable to CH1 of the oscilloscope by a coaxial cable and the other one to a BNC-to-minigrabber adapter to power the circuit on the breadboard.

Then, to measure the voltages:

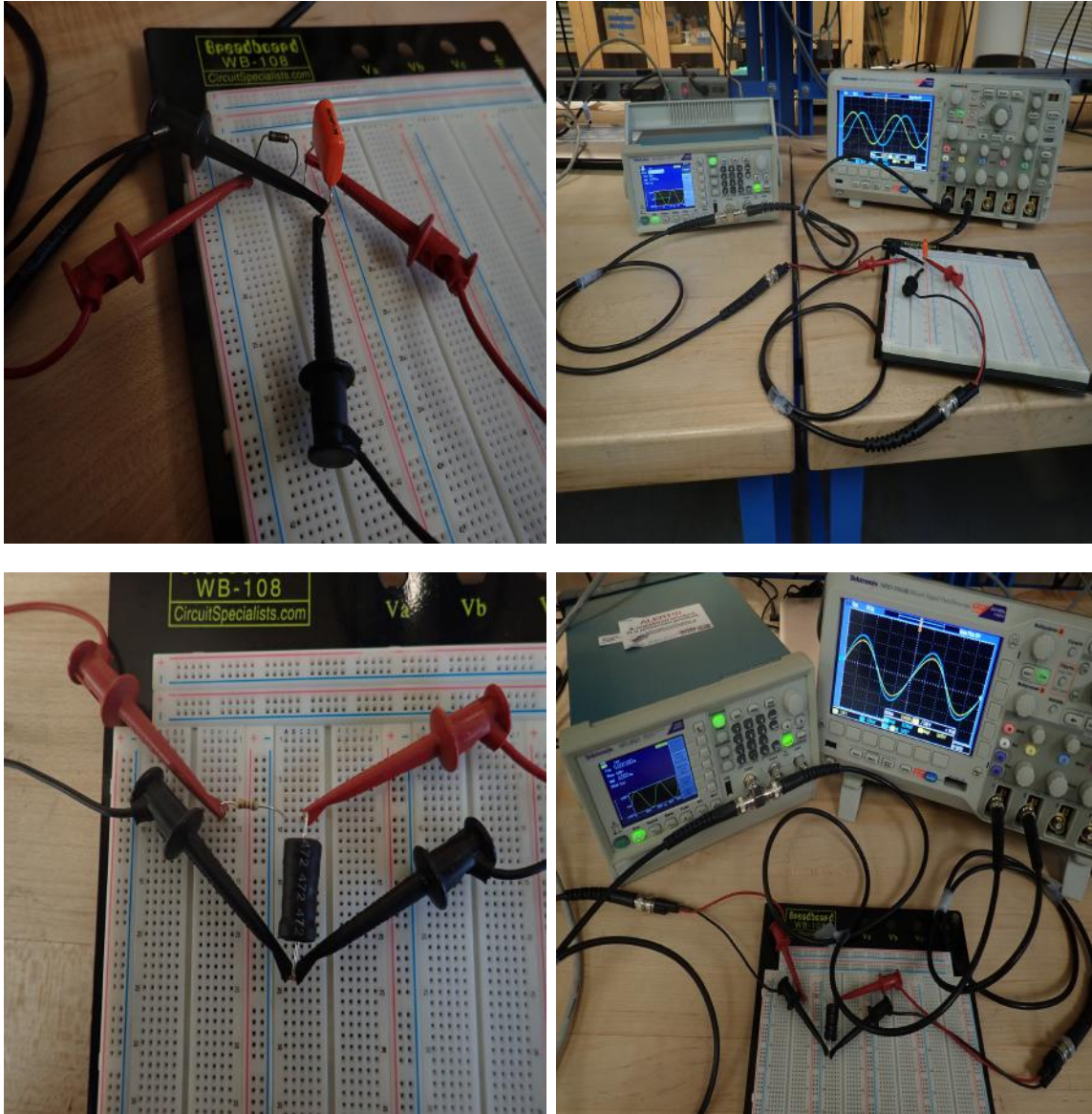
1. Oscilloscopes: set volts/div to be an appropriate value, and the coupling mode switch to DC for CH1 and CH2.
2. Set the function generator output to a 100Hz 4 V_{pp} sine wave.
3. Define a frequency range.
4. Repeat measurement for different frequencies in the range

For the RL circuit, simply replace the capacitor with the inductor and repeat the steps. For both experiments, we measure the difference in peak to peak voltages and the phase shift compared to the generator, which we will use to check whether the data coincides with the theoretical predictions, and, in the case of the RL circuit, find the impedance of the circuit.

Now, we will also provide a brief overview of the steps we took in looking and attempting to gain the best understanding of the data we could:

1. Open the files that belong to the group. In our case, we had these datasets: [Dataset 1](#) and [Dataset 2](#).
2. Look at the Lab Manual and make sure to understand all the different numbers shown in the oscilloscope’s display screen. This will prove useful in the analysis section.
3. Perform error propagation and other necessary calculations, as well as understanding the formulas in the Lab Manual. Now, we are ready to head on to the Data Analysis.

Shown below is the RC Circuit setup and the setup for the RL Circuit respectively:



Calculations and Data Reduction/Analysis:

In this section, we compiled the data given in the datasets and compared them to theory. To review the formulas explained in the Theory Section, for an RC circuit, we expect the voltage across the capacitor V_C , the voltage across the source V_o , and the AC frequency ω to be related by

$$V_C(t) = \frac{V_o}{(1 + \omega^2 R^2 C^2)^{1/2}} \cos(\omega t + \Phi_C)$$

and the phase shift Φ_C and ω to be related by

$$\Phi_C = \tan^{-1}(-\omega RC)$$

Furthermore, for an RL circuit, we expect the voltage across the inductor V_L , the voltage across the source V_o , and the AC frequency ω to be related by

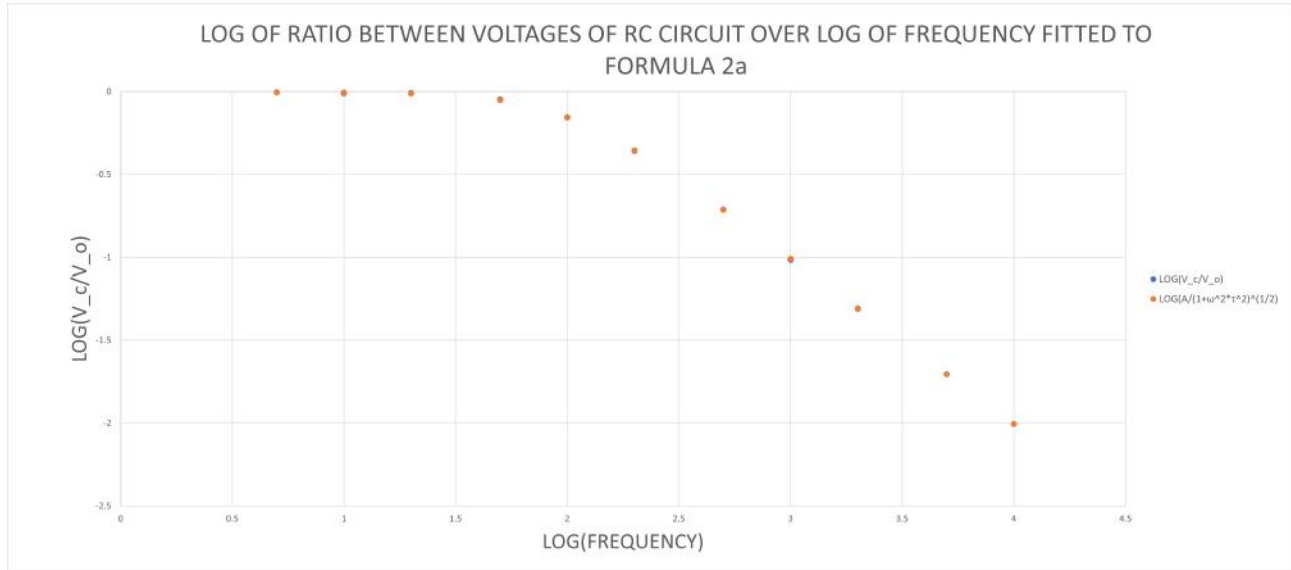
$$V_L(t) = \frac{V_o \omega L}{(R^2 + \omega^2 L^2)^{1/2}} \cos(\omega t + \Phi_L)$$

and the phase shift Φ_L and ω to be related by

$$\Phi_L = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

We compared the data we received against these equations to determine if the experiments conformed to the behavior expected from RC and RL circuits.

We additionally plotted and fitted this in Excel, which is the final log-log scaled plot. This is the same graph as the one above, and it shows our result to the first bullet point in Part 1 of the Analysis. Since the data is also being fitted to equation (1), this also answers the second bullet point in Part 1.

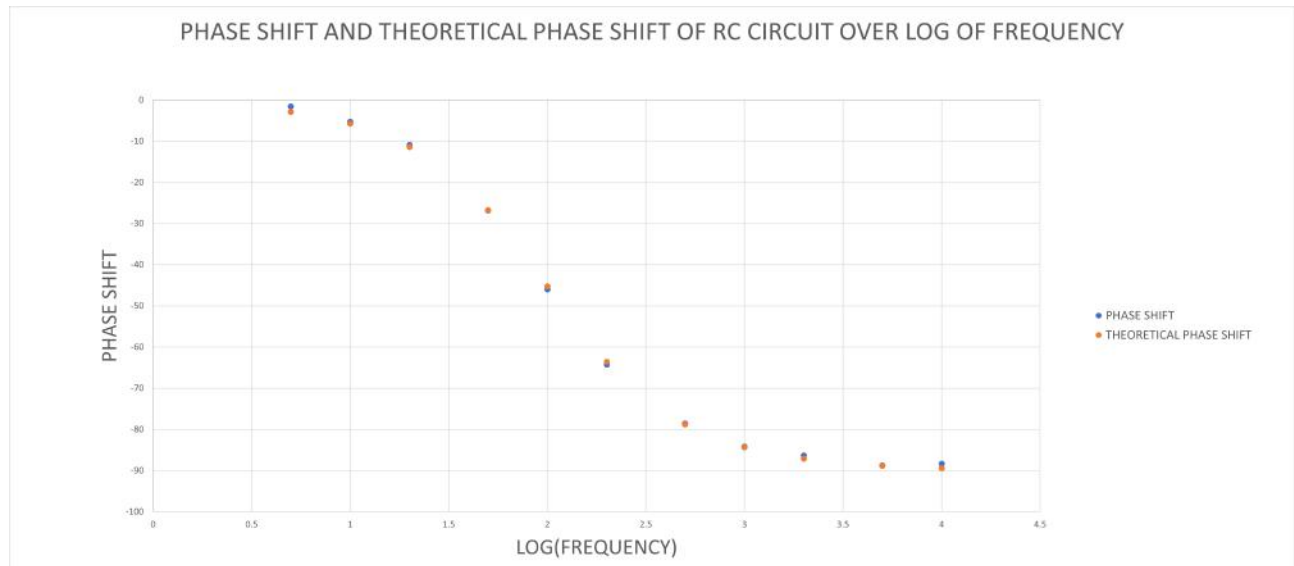
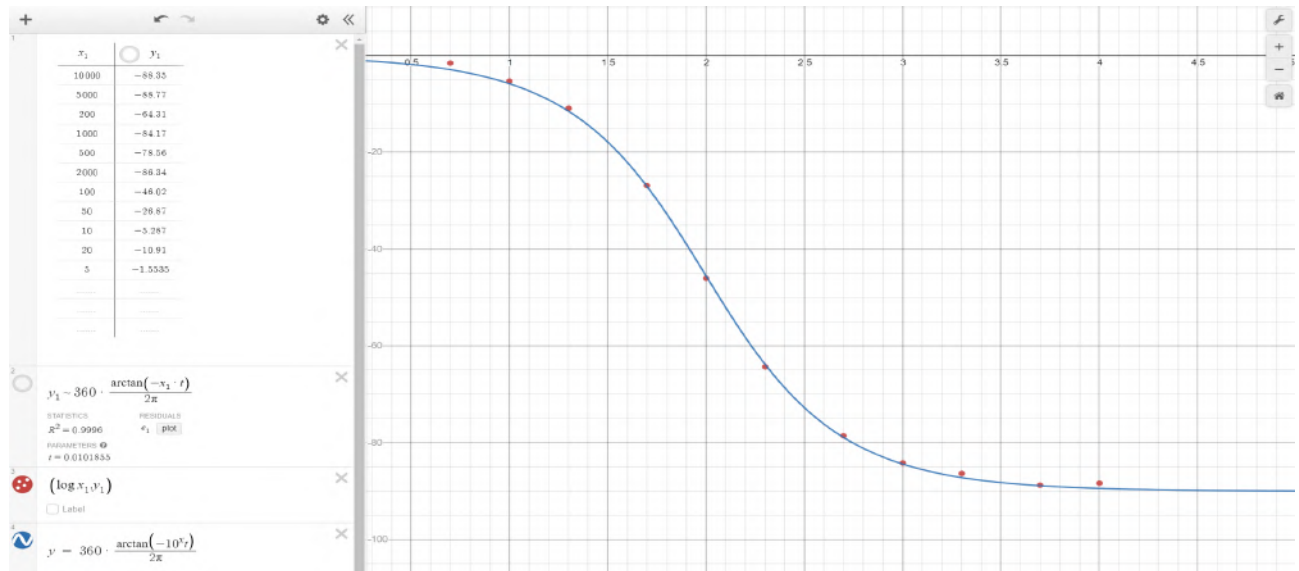


The errors in this fitting were calculated using the Python file, found [here](#). The formulas used for error propagation are explained in the Theory Section, but now we will present our errors for A and τ obtained from Python in the table below:

V_o (V)	V_c (V)	PHASE SHIFT (DEG)	V_c/V_o	Error in A for different V_c/V_o	Error in τ
3.892	0.0384	-88.35	0.0098663926	0.1581675388	0.0001060660172
3.895	0.0768	-88.77	0.01971758665	0.1572492075	0.0001060660172
3.88	1.705	-64.31	0.4394329897	0.04032462818	0.0001060660172
3.892	0.376	-84.17	0.09660842754	0.1291476525	0.0001060660172
3.891	0.7556	-78.56	0.1941917245	0.09355148722	0.0001060660172
3.89	0.19	-86.34	0.04884318766	0.1487410058	0.0001060660172
3.978	2.774	-46.02	0.6973353444	0.01736229366	0.0001060660172
3.96	3.552	-26.87	0.896969697	0.005579016441	0.0001060660172
4.048	3.964	-5.287	0.9792490119	0.0002552659084	0.0001060660172
4.007	3.933	-10.91	0.9815323184	0.001002479397	0.0001060660172
4.01	3.971	-1.553	0.9902743142	6.33E-05	0.0001060660172

Visually, these fits are extremely good, a fact which is confirmed not only by the extremely small errors, but also by our low χ^2 of -0.0056 . This suggests that the experimental setup conforms very well to the behavior expected of an RC circuit. We found τ to be 0.01 and A to be 0.99. We found the average errors α_τ and α_A to be 0.000106 and 0.07514439097, respectively. These errors are extremely small, and show how our analysis and the data are very close to a perfect fit. Now, using the data from the table above, we know the different values of the phase shift for the different frequencies too.

We used two different graphing programs to plot the phase shift and the theoretical phase shift over the log of the frequency. This way, we can analyze the differences between our expectations and our reality.



The theoretical and experimental phase shifts fit very strongly, suggesting, again, that the experimental setup conforms well with our expectations for an RC circuit. Furthermore, from our experimental Φ , we determined τ to be 0.0102, which agrees strongly with the value we obtained using the ratio between the voltages. Visually, from the computer fit above, the theory and our values seem to match for most values. All this suggests that, essentially, nothing went catastrophically wrong in this part of the experiment.

Analysis Part 2

In this part, we were expected to answer some questions about Part 1 based on our understanding of the basic concepts about AC Circuits. The questions are shown below:

1. For a series RC circuit, the time constant is expected to be $\tau = RC$. Does the τ calculated from the circuit elements values agree with the τ obtained from the fit? Explain, including the results from an acceptance test.
2. Is the V_C/V_o fit good at all frequencies? Write down possible reasons if that is not the case.
3. Is the ϕ_C fit good at all frequencies? Write down possible reasons if that is not the case

Our answers to these questions are shown below:

1. Since $\tau = RC$, the theoretical value for τ must be 0.0015. However, in our fitting in part 1, we found τ to be 0.001. If we perform the acceptance test using the average error for τ in Part 1, we find that the two τ s don't agree, and they are off by a factor of 10^{-1} . We used the formula below to come up with this result:

$$|\tau_{exp} - \tau_{theo}| < 2\sqrt{\alpha_{\tau_{exp}}^2 + \alpha_{\tau_{theo}}^2}$$

However, at the same time, our fit was extremely accurate to the experimental data, suggesting that there was nothing significantly wrong with the experiment. We hypothesize that the values for the resistance and capacitance of the circuit parts given in the lab manual were incorrect.

2. From our fit, we can see that the line fits all the data points for the different frequencies almost perfectly. Even though we did find an error for A and τ , these were incredibly small, and they can be attributed to small errors and deviations in the system's potential, resistance, or capacitance, when referring to the RC circuit.
3. From the graphs in Part 1, the fit for the phase shift isn't perfect. However, it is very good in middle-ranged frequencies. As visible on the graph, the fit stops being accurate for more extreme frequencies, and the theoretical and factual data start to diverge. We believe this is because of several reasons. Firstly, the oscilloscope could be entering resonant states at some specific frequencies, reducing the quality of the readings. Second, the oscilloscope has a reduced sensitivity, meaning that it is unable to perform well under too small or too large frequencies. However, we discard this option as not too likely. Lastly, we believe that the higher frequencies imply more readings per unit time, so the net quality of each reading must be decreased to maintain the oscilloscope working. These three errors combined might lead to the results we have, plus other random or systematic errors in our system.

Analysis Part 3

Throughout this section and the one immediately following it, we will make the substitution $\tau = \frac{L}{R}$. We will also be using the adjusted formula

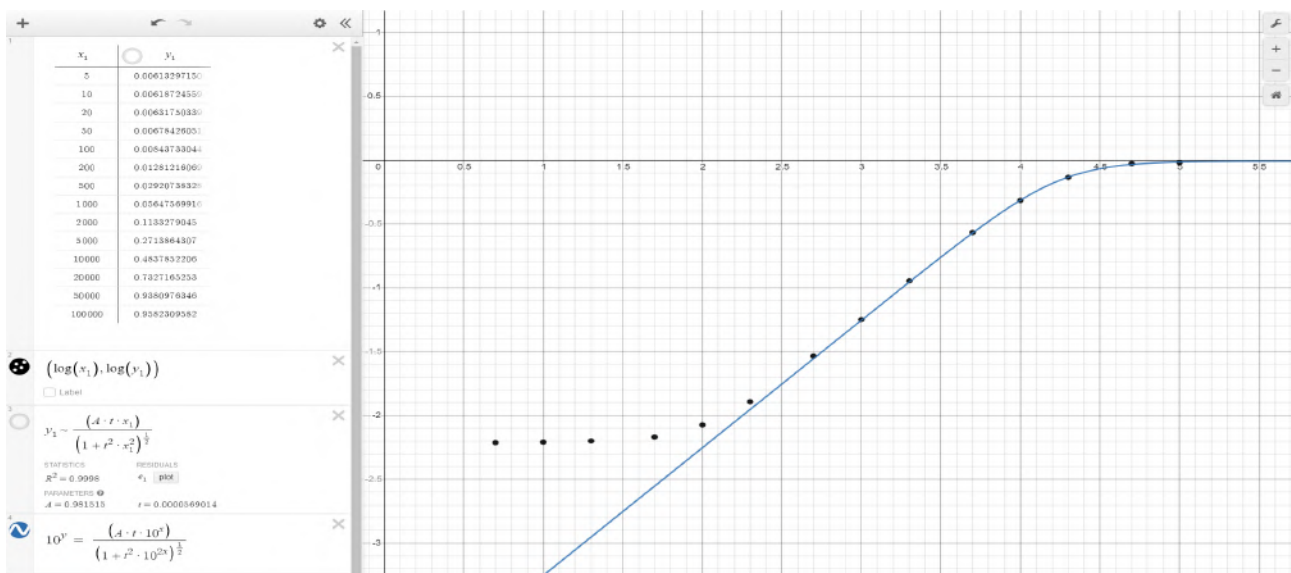
$$\frac{V_L}{V_o} = \frac{A\omega\tau}{(1 + \omega^2\tau^2)^{1/2}} \quad (2)$$

which relates the ratio between the amplitudes of the voltages V_L and V_o through the inclusion of an arbitrary constant A . The first thing we were asked to do was plotting $\frac{V_L}{V_o}$ as a function of ω in a log-log scale chart. We have included our data for this plot in the chart below. The fit parameter values for Part 3 are shown in the yellow box. We fitted this data to formula (2) using a graphing software. We graphed it in log-log scale,

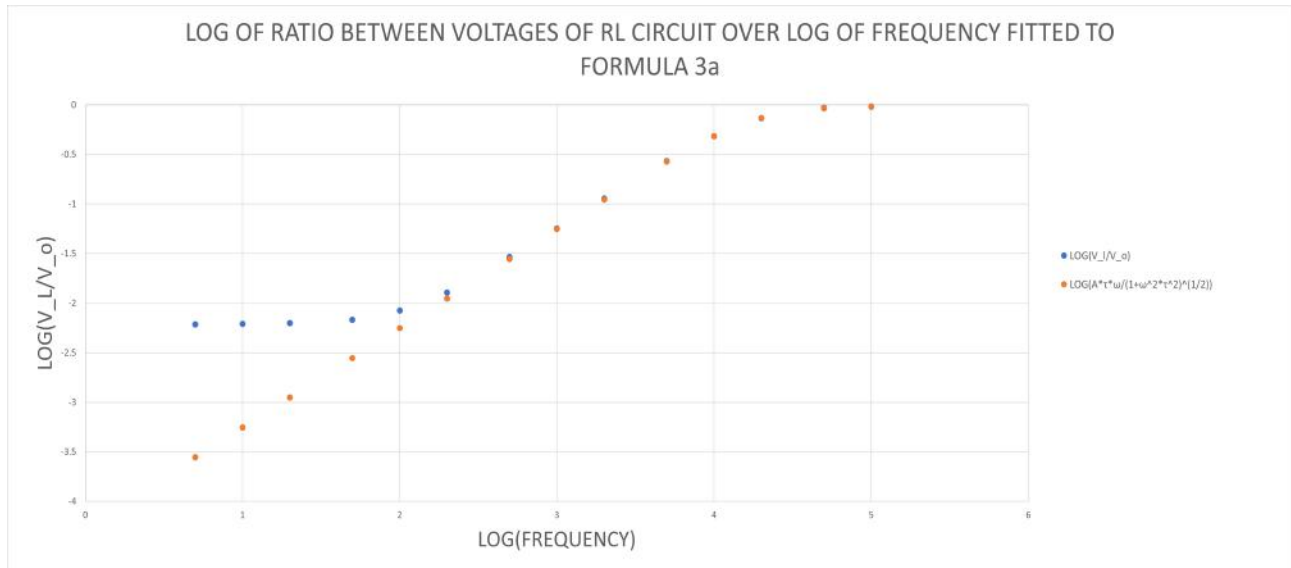
FREQ (Hz)	V _o (V)	V _L (V)	PHASE SHIFT (DEG)	V _L /V _o	LOG(FREQUENCY)	LOG(V _L /V _o)
5	3.685	0.0226	3.306	0.006132972	0.698970004	-2.212329053
20	3.685	0.02328	10.02	0.006317503	1.301029996	-2.199454516
50	3.685	0.025	24	0.006784261	1.698970004	-2.168497484
100	3.685	0.0228	5.083	0.006187246	2	-2.208502645
500	3.684	0.1076	76.29	0.029207383	2.698970004	-1.53450735
1000	3.686	0.0311	42.67	0.00843733	3	-2.073794942
5000	3.729	1.012	72.6	0.271386431	3.698970004	-0.566411871
20000	3.684	0.0472	61.17	0.012812161	4.301029996	-1.892377623
100000	3.762	1.82	59.92	0.483785221	5	-0.315347403
200000	3.684	0.4175	80.56	0.113327904	6	-0.945663142
1000000	3.683	0.208	80.85	0.056475699	7	-1.248138384
2000000	3.891	2.851	40.6	0.732716525	8	-0.135064013
10000000	4.07	3.9	6.332	0.958230958	9	-0.018529802
50000000	3.974	3.728	16.62	0.938097635	10	-0.027751959

A	0.981515	LOG(A*τ*ω/(1+ω²*τ²)^(1/2))	THEORETICAL PHASE SHIFT	RESIDUALS FOR VOLTAGE RATIO	CHI-SQ
τ	5.69014E-05	-3.55401012	89.98369895	-1.341681067	-1.11035562
		-2.951950393	89.93479583	-0.752495876	
		-2.55401186	89.83698994	-0.385514377	
		-3.252980177	89.9673979	-1.044477532	
		-1.5541858	88.37033458	-0.01967845	
		-2.252987138	89.67398251	-0.179192196	
		-0.570911747	74.11857629	-0.004499876	
		-1.951978232	89.34798613	-0.05960061	
		-0.313887511	60.35951783	0.001459892	
		-0.954744347	83.50751195	-0.009081205	
		-1.253682044	86.74330171	-0.00554366	
		-0.132351649	41.30619322	0.002712364	
		-0.014708266	9.967523852	0.003821536	
		-0.033397685	19.3658202	-0.005645726	

and the final graph is visible with the table with the fit parameters. We additionally plotted and fitted this



in Excel, which is the final log-log scaled plot. This is the same graph as the one above, and it shows our result to the first bullet point in Part 1 of the Analysis. Since the data is also being fitted to equation (2), this also answers the second bullet point in Part 3. The graphs above show the results to the first and second

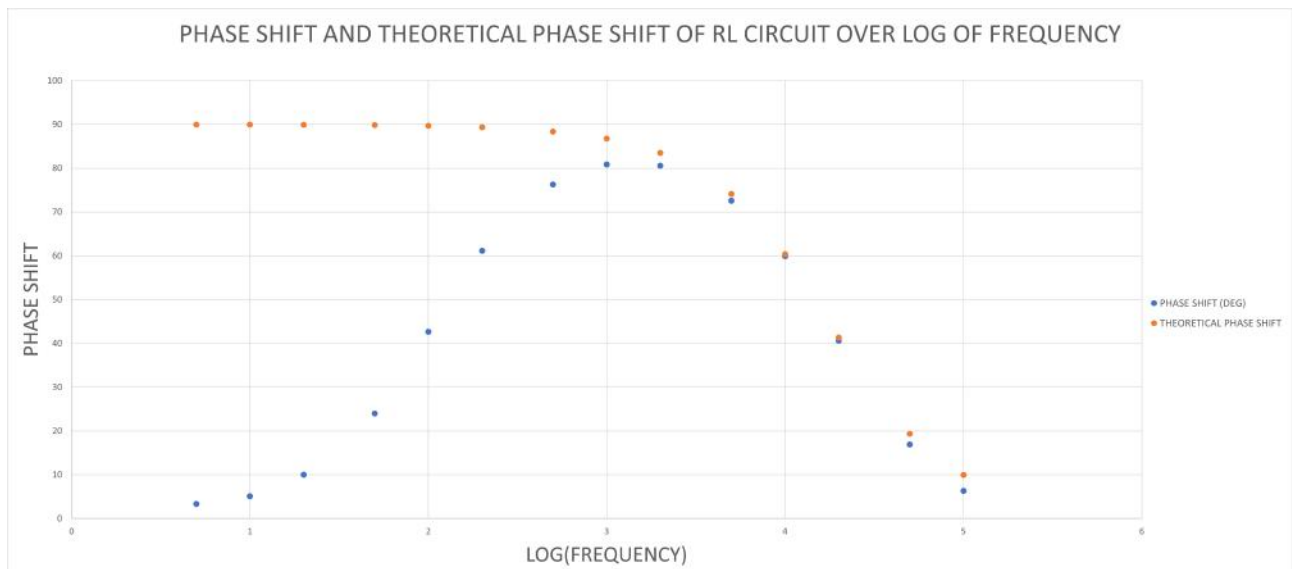
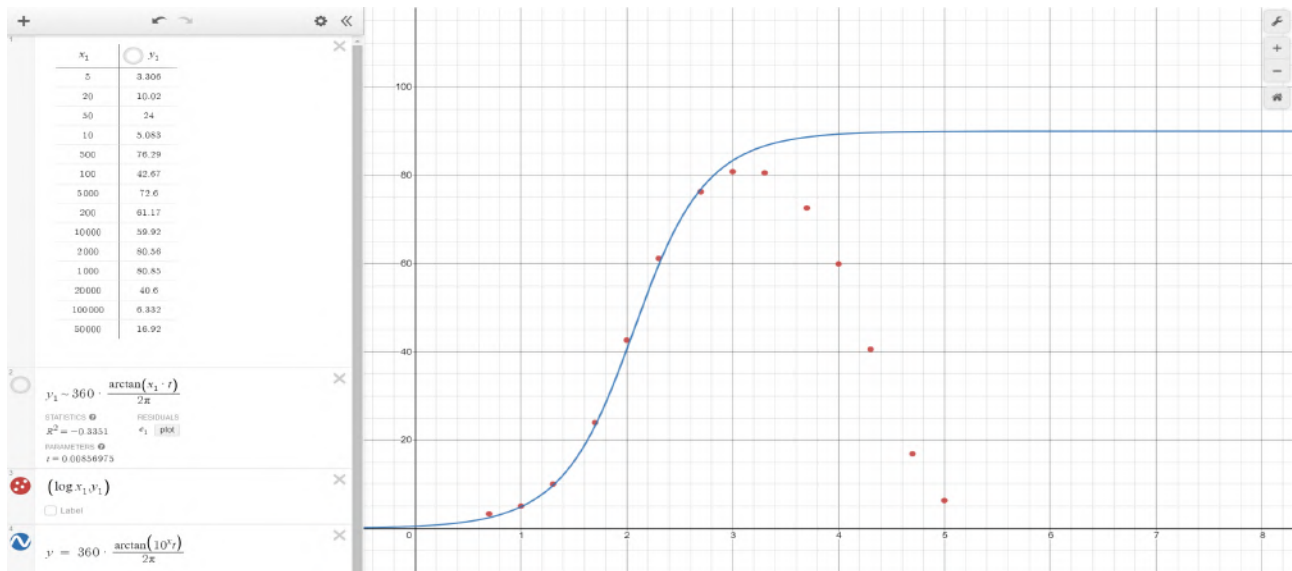


bullet points in the Lab Manual. However, we are missing the errors. Like in Part 1, we performed error regression. The mathematics we used follow the same principles as those in Part 1, but they are adapted to this case. The final error formulas are shown below, and so is the table with the final errors we obtained for the different V_L/V_o values. These were found using our Python file, found [here](#).

V _o (V)	V _L (V)	PHASE SHIFT (DEG)	V _L /V _o	Error in A for different V _L /V _o	Error in Tau
3.685	0.0226	3.306	0.006132971506	2.64E-18	3.50E-07
3.685	0.0228	5.083	0.00618724559	2.13E-17	3.50E-07
3.685	0.02328	10.02	0.006317503392	1.74E-16	3.50E-07
3.685	0.025	24	0.006784260516	2.92E-15	3.50E-07
3.686	0.0311	42.67	0.00843733044	2.90E-14	3.50E-07
3.684	0.0472	61.17	0.01281216069	3.53E-13	3.50E-07
3.684	0.1076	76.29	0.02920738328	1.26E-11	3.50E-07
3.683	0.208	80.85	0.05647569916	1.94E-10	3.50E-07
3.684	0.4175	80.56	0.1133279045	3.12E-09	3.50E-07
3.729	1.012	72.6	0.2713864307	1.17E-07	3.50E-07
3.762	1.82	59.92	0.4837852206	1.66E-06	3.50E-07
3.891	2.851	40.6	0.7327165253	2.01E-05	3.50E-07
3.974	3.728	16.92	0.9380976346	0.0003984441508	3.50E-07
4.07	3.9	6.332	0.9582309582	0.003144913286	3.50E-07

In both programs, the fit is good at higher values of $\log(\omega)$ but becomes noticeably inaccurate at smaller values. Furthermore, the residuals appear to be in a pattern, suggesting that there is some kind of systemic error affecting the experiment. We note that the voltage across the inductor is higher than we expect. We hypothesize that this is caused by the parasitic capacitance present in all real conductors. At low frequencies, the capacitive reactance of this capacitance is extremely high, even through the reactance of the inductor is low, causing the voltage across the inductor to be higher than we expect.

Now, using the data from the table above, we know the different values of the phase shift for the different frequencies too. We used two different graphing programs to plot the phase shift and the theoretical phase shift over the log of the frequency. This way, we can analyze the differences between our expectations and our reality.



Interestingly, the two different analyses yielded different results. The first program used a separate analysis to fit the data to our formula, and the second program used the τ that we obtained from fitting $\frac{V_L}{V_o}$. Both programs gave extremely inaccurate fits. The second one gives a phase shift which is lower than expected at small frequencies, and the first one gives a phase shift which is lower than expected at high frequencies. This probably stems from the same parasitic capacitance we outlined above.

Our analysis of the phase shift gave us a τ of 0.00857. This is extremely different from the value we got from our analysis of $\frac{V_L}{V_o}$, which was $\tau = 5.69 \cdot 10^{-5}$. Again, we hypothesize that this comes from the behavior of the inductor at low frequencies.

Analysis Part 4

In this part, we were expected to answer some questions about Part 1 based on our understanding of the basic concepts about AC Circuits composed of a resistor and an inductor. The questions are shown below:

1. For a series RL circuit, the time constant is expected to be $\tau = L/R$. Does the τ calculated from the circuit elements' values agree with the obtained from the fit? Explain, using an acceptance test.
2. Is the V_L/V_o fit good at all frequencies? Write down possible reasons if that is not the case. (hint: all inductors are not perfect.)
3. Is the ϕ_L fit good at all frequencies? Write down possible reasons if that is not the case. (hint: all inductors are not perfect.)
4. Can you propose an inductor model to explain the phase shift pattern? Hint: look in the literature. The measured inductor resistance is 3.3 ohms.

Our answers to these questions are shown below:

1. The value of τ calculated from the circuit components is $\tau = 9.22 \cdot 10^{-6} \pm 1.38 \cdot 10^{-6}$. The value of τ that we obtained from our analysis of the ratio between the voltages was $\tau = 5.69 \cdot 10^{-5}$. These two values are so far apart that we frankly don't need to perform an agreement test. For rigor, though, we use

$$|\tau_{exp} - \tau_{theo}| < 2\sqrt{\alpha_{\tau_{exp}}^2 + \alpha_{\tau_{theo}}^2}$$

and confirm that these two values do not agree. As before, the values given for the circuit parts were probably not accurate. Additionally, however, the inductor displayed non-ideal behavior at low frequencies, causing our measured voltages to be higher than expected the lower ω was.

2. The $\frac{V_L}{V_o}$ fit is not good at all frequencies. Like we have discussed throughout this section, it underestimates the ratio between the voltages at low frequencies. We hypothesize that this is because of the parasitic capacitance present in all real inductors.
3. The ϕ_L fit is not good at all frequencies. Like we have discussed throughout this section, it overestimates the phase shift at low frequencies. We hypothesize that this is because of the parasitic capacitance present in all real inductors.
4. We propose that the the inductor model be replaced theoretically with an ideal resistor in series with an ideal capacitor and an ideal inductor in parallel.

Summary and Conclusion:

In this experiment, we compared the theoretical and experimental behavior of RC and RL circuits. We used an oscilloscope to measure the voltages across various points of each circuit at different AC frequencies. We determined that the theoretical model for RC circuits describes experimental results with high accuracy. We also became reasonably sure that the values given for the circuit components in the lab manual are incorrect. We determined that the theoretical model for RL circuits fail to describe the experimental results with high accuracy at lower frequencies. We hypothesize that this is because of the increased parasitic capacitance present in all real inductors at low AC frequencies.