Problem 1 (Purcell 9.18)

Since $\vec{E} = E_0(\hat{x}+\hat{y})\sin[(\frac{2\pi}{\lambda})(z+ct)]$, the wave travels in the $-\hat{z}$ direction. We know that $\vec{B}$ is perpendicular to $-\hat{z}$ and to $\vec{E}$. Therefore, $\vec{E}$ points in the $\pm(\hat{x}-\hat{y})$ direction. Using the right hand rule, $\vec{B}$ points in the $(\hat{x}-\hat{y})$ direction. Since $|\vec{B}| = \frac{1}{c}|\vec{E}|$, we have that $\vec{B}$ is:

$\vec{B} = \frac{E_0(\hat{x}-\hat{y})}{c}\sin[\frac{2\pi}{\lambda}(z+ct)]$ Now, taking $E_0 = 20$ V/m, we find that $B_0 = \frac{1}{c}E_0 = 6.67 \cdot 10^{-8}$ T.

Problem 2 (Purcell 9.20)

We are told that $\vec{E}$ follows Eq 9.28, so $\vec{E} = \frac{E_0\hat{y}}{1+\frac{(x+ct)^2}{\ell^2}}$. We are given that $E_0 = 100$ kV/m and $\ell = 1$ ft. Since $F = \frac{dp}{dt}$, we can find the momentum acquired by the proton during the pulse to be:

$$p_y = \int_{-\infty}^{\infty} eE_y\,dt = \frac{e\ell E_0}{c}\left(\arctan(t)\right)\Big|_{-\infty}^{\infty} = \frac{e\ell E_0\pi}{c}$$

Therefore, the velocity is $v_y = \frac{p_y}{m}$, so $\frac{\pi e\ell E_0}{mc} \approx 3.1 \cdot 10^4$ m/s. Therefore, since we know the velocity, we can find the displacement after one microsecond to be $v_y t = (3.1\cdot 10^4)(10^{-6}) = 3.1$ cm. Therefore, the proton is 3.1 cm away from where it was a microsecond after the pulse.

Problem 3 (Purcell 9.24)

Since the leaving signal has 1000 km in diameter, it sweeps an area of $8\cdot 10^{11}$ m^2. Since we know the power of this signal, the power density is $10^4/8\cdot 10^{11}$, so the power density is roughly 10^{-8} W/m^2. Since $10^{-8} = \frac{\overline{E}^2}{377\,\Omega}$, E at the receptor is roughly 0.002 V/m at the receiving end!

Problem 4 (Purcell 9.25)

We have that the energy density is $4\cdot 10^{-14}$ J/m^3. From section 9.6, $E_{rms}^2 = \frac{U}{\epsilon_0}$, where U is the average energy density. Therefore, $E_{rms}^2 = 4.5\cdot 10^{-3}$ V^2/m^2, so $E_{rms} = 0.067$ V/m. Now, if our 1 kW radio transmitter is "spherically symmetrical", then the power density at a radius R is just $10^3\,\text{W}/4\pi R^2$. Therefore, $U = \frac{1}{c}\cdot\frac{10^3\,\text{W}}{4\pi R^2}$, so $R = 2600$ m.

Problem 5 (Purcell 9.29)

To find the $\vec{B}$ field inside the capacitor, we integrate $\nabla\times\vec{B} = \mu_0\vec{J} + \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$ over a disk. Using Stoke's Theorem, we have:

$$\int \vec{B}\cdot d\vec{s} = \mu_0 I - \epsilon_0\mu_0\frac{\partial E}{\partial t}A \xrightarrow{\text{over a disk}} B = \frac{\mu_0 I}{2\pi r} - \frac{\epsilon_0\mu_0 r}{2}\frac{\partial E}{\partial t}.$$

Since $I = \epsilon_0(\pi b^2)\frac{\partial E}{\partial t}$, $B = \frac{\epsilon_0\mu_0}{2}\frac{dE}{dt}(\frac{b^2}{r}-r)$. For all $r<b$ inside the capacitor, $B>0$. Since B points into the page on the right side of the capacitor, $\vec{E}$ points downwards, and $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$ points away. Therefore, energy flows out away from the wire. Since $\vec{E}\perp\vec{B}$, we have that $|S| = \frac{E\epsilon_0}{2}\frac{dE}{dt}(\frac{b^2}{r}-r)$. Now, since the total energy flowing out of a cylinder is $S\cdot 2\pi r h$, the power is given by

$$P = S\cdot 2\pi r h = \frac{d}{dt}\left(\frac{\epsilon_0 E^2 h}{2}(\pi b^2 - \pi r^2)\right).$$ Therefore, the rate of flux of the Poynting vector is the

as the energy stored in the field. We can just confirm this by taking limiting cases $r=0$ or $r=b$.

Problem 6 (Purcell 10.17)

We know that the maximum field is just $5.5 \cdot 10^8$ V/m , after converting from kV/mm. Since the capacitance of the Mylar-filled capacitor is $\frac{K\epsilon_0 A}{s}$, and the energy stored in it is $\frac{1}{2}C\phi^2$, the maximum energy density is $\frac{\text{energy}}{\text{volume}} = \frac{1}{2}K\epsilon_0 E^2 = 4.4 \cdot 10^6$ J/m³. Therefore, the max. energy per kg of Mylar is 3100 J/kg. To determine how high the capacitor could elevate, we take $E = mgh$, where m is the total mass of the capacitor and plug in to find that $h \approx 240$ m. The cell in problem 4.41 had about 60 times more energy than this capacitor, but the Mylar capacitor can "transfer" its energy a lot quicker!

Problem 7 (Purcell 10.18)

The second capacitor is no more than two capacitors in series. Both capacitors have separation $s/2$ and area A. The capacitance for each half, with vacuum or with dielectric, is $C_{vacuum} = \frac{\epsilon_0 A}{s/2}$ and $C_{dielectric} = k\frac{\epsilon_0 A}{s/2}$. Taking $C_0 = \frac{\epsilon_0 A}{s}$, $C_v = 2C_0$ and $C_d = 2kC_0$. Since capacitors add in series the total capacitance is : $\frac{1}{C_{TOT}} = \frac{1}{C_v} + \frac{1}{C_d} = \frac{2k}{k+1}C_0$

The third capacitor is two capacitors in parallel. Both capacitances are now $C_v = \frac{A\epsilon_0}{2s}$ and $C_d = \frac{kA\epsilon_0}{2s}$, so $C_d = \frac{k}{2}C_0$ and $C_v = \frac{1}{2}C_0$. Therefore, $C_{TOTAL} = \frac{1+k}{2}C_0$