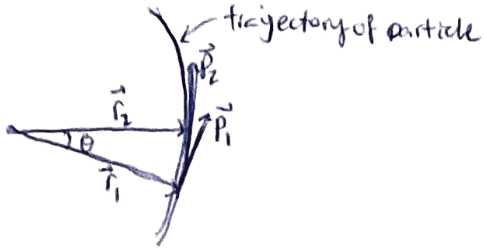
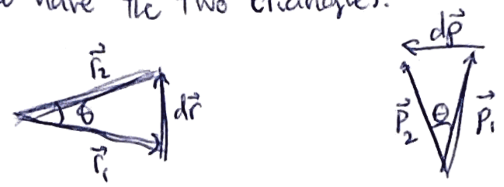
PROBLEM 1 (PURCELL 6.29)

We know that the magnitude of the magnetic force is $F = qvB$, and this force doesn't change the magnitude of the momentum of the particle. Since $F = \frac{dp}{dt}$, where p is the momentum, $dp = Fdt = qvBdt$. Also, the momentum is given to be $p = \gamma m v$. Now, let's look at a motion in a curve:



Now, if we draw a triangle with $\vec{p}_1$ and $\vec{p}_2$ as sides, we have the two triangles:

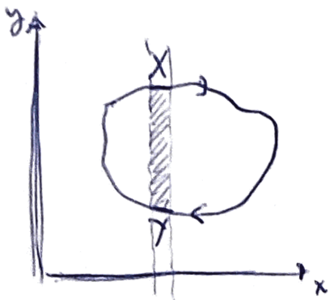


Since in the sketch above p is perpendicular to r, both triangles to the right have angle θ. They're similar triangles. → Therefore, $\frac{|dr|}{|r|} = \frac{|dp|}{|p|} \rightarrow \frac{v\,dt}{R} = \frac{qvB\,dt}{\gamma m v}$

From here, we find that $R = \frac{\gamma m v}{qB}$. Since B is perpendicular to v, when B is uniform, v is constant, so R remains constant, hence the path is a circle. The time for one revolution is:

$$t = \frac{2\pi R}{v} = \frac{2\pi}{v} \cdot \frac{\gamma m v}{qB} = \frac{2\pi \gamma m}{qB}$$

PROBLEM 2 (Purcell 6.34)



We know that the force on a small piece in the loop is $d\vec{F} = I d\vec{l} \times \vec{B}$. Since $\vec{B}$ is perpendicular to the x-axis, it's z-component lies in the x-y plane and doesn't contribute to the torque on the loop about x. Therefore, we only care about the y-component, so $B = B_y \hat{y}$. Therefore, $d\vec{F}_z = (dl \times B_y \hat{y}) I$. We solve for $d\vec{F}$:

$d\vec{F} = I(d\vec{l} \times B_y \hat{y}) = I(dl B_y \sin\theta)\hat{z}$. Here, θ is the angle between dl and y-axis, so $dl \sin\theta = dx$. We now have: $d\vec{F}_z = I B_y dx$. Now, the torque is just $y F_z$, so $I B_y y\, dx$ = Torque on dx.

Now, to find total torque on all the loop, we integrate. On the sketch to the left, our procedure to integrate will be to add up all the thin rectangles such as the one between segments X and Y to find the total area of the loop. However, $y\,dx$ is the area of the loop when we integrate it over the loop! Therefore, Torque on dx becomes Total torque $= I B_y A$, where A is the area of the loop.

Now if we define the magnetic moment to be $m = IA(-\hat{z})$ ← comes from Right Hand Rule, we can

PROBLEM 2 (CONTINUED):

we can write the Total Torque as $\vec{m} \times \vec{B}$ because, ~~...~~ since $\vec{B}$ is perp. to x, $\vec{B} = B_y\hat{y} + B_z\hat{z}$. Therefore,

$$\vec{m} \times \vec{B} = (-IA\hat{z}) \times (B_y\hat{y} + B_z\hat{z}) = -IAB_y\hat{z} \times \hat{y} = IAB_y\hat{x}.$$ This matches the result before.

Note that it's in the $\hat{x}$ direction because of the Right Hand Rule by looking at the flow of I on the loop.

Now, to find the force on the loop, since $\vec{B}$ is uniform, $\int_{Loop} dF = \left[I \int d\vec{\ell}\right] \times \vec{B}$. However, for a closed loop, $\int d\ell = 0$. Therefore, the net force is 0.

PROBLEM 3 (PURCELL 6.39)

If we have a wire of radius R, ~~...~~ let $r < R$. If I_r is the current at some radius r, then we have from Ampere's law that: $B \cdot 2\pi r = \mu_0 I_r \rightarrow B = \frac{\mu_0 I_r}{2\pi r}$. If we want $\vec{B}$ to have constant magnitude inside the wire, it must be independent of r. For this to happen, I_r must be proportional to r, so they cancel out.

$$I_r = \int J(r)\, da = \int_0^r J(r) \cdot (2\pi r\, dr)$$

Observe that when J is proportional to $\frac{1}{r}$, we have the integral $\int_0^r 2\pi C\, dr$ for some constant C. This integral is just $2\pi r C$. By plugging it into B as the value of I_r, we have:

$$B = \frac{\mu_0 (2\pi r C)}{2\pi r} = \mu_0 C,$$ which is constant. Therefore, the current density must be proportional somehow to $1/r$ for $\vec{B}$ to be constant magnitude.

PROBLEM ~~...~~ 4 (Purcell 6.40)

What prevents all of the electrons from clustering at the rod's axis is that doing so would accumulate all the ~~positive~~ negative charges at the center, creating an imbalance in the charges. This would create a inward radial $\vec{E}$ field that pushes the electrons radially outwards. Therefore, what happens is that the electric force done by $\vec{E}$ on the electrons balances the magnetic force. Since $\vec{E}_{radial} = \vec{v}\vec{B}_{radial}$, we can find that for the forces to be balanced, $\frac{r_{\text{of negative charges}}}{r_{TOTAL}} = \frac{v^2}{2c^2}$, where v is the drift velocity. Note that in conductors, $\frac{v^2}{c^2} \ll 1$, so this effect is not noticeable. However, in insulators like glass, ~~...~~ This might be greater, so even detectable. To detect this effect, I would look at a potential difference in the wire. To do this, I would use a Hall Probe and put the wire through the whole.

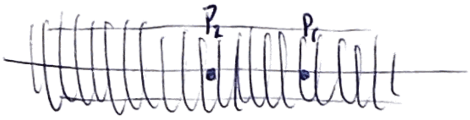
Problem 7 (Purcell 6.61)

Like for all toruses, we have heavy symmetry about the central axis. This requires that the field lines are all circular about this same axis, as if we had a curved solenoid. To prove this, we consider two such circles or loops, around the axis. If they weren't circular, all their components that are parallel to the radius wouldn't cancel, and this would mean there isn't a uniform field. However, if we let the loops be actual circles, there isn't parallel component and it all works out. Therefore, all the field lines within the torus are circular. Furthermore, by Ampère's Law, $\int B \cdot d\vec{s} = \mu_0 I$, since we have circles, we can say that $2\pi r \cdot |B| = \mu_0 I$. For $r >$ Radius of Torus, $I = 0$, so $|\vec{B}|$ must equal 0. Hence, $|\vec{B}| = 0$ outside the torus. Lastly, we are told that the torus has N coils, so the current enclosed is NI. Therefore, plugging in above, the field is $|B| = \frac{NI\mu_0}{2\pi r}$ for r inside the torus.

PROBLEM 8 (Purcell 6.63)

(a)

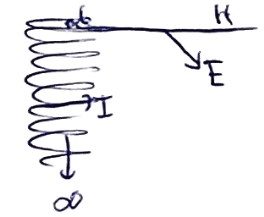
We have this original solenoid. To show that the field at P_2 is either more than half the field at P_1 or slightly less, we add a similar solenoid to the ~~far~~ left of this one.



Now, since P_2 is at the extreme of the added solenoid, the field at P_2 is doubled. Now, fields at P_2 and P_1 are approximately equal, but since P_2 is closer to the center, it has a slightly higher field. Therefore, in the initial solenoid, the field at P_2 is slightly more than half of the field at P_1.

(b) If we let the line going from GH to infinity have a vertical component too in search of a proof by contradiction, ~~then~~ we must join two ~~in~~ finite solenoids.

Solenoid 1



Solenoid 2

In solenoids 1 and 2 the current flows in the same direction. If we join them to create an infinite solenoid, we'd still have a vertical component left after superposition.

However, the field outside an infinite solenoid must be 0, so we have arrived at a contradiction. Therefore, the field can't be vertical, and must be horizontal in $\overline{GH}$.

(c) Using the same argument as in part (a), we find that the axial component of the field at any point on the end face must be $B/2$, where B is the uniform field inside the solenoid. This is because adding another semi-infinite solenoid at the end face should make it B, so it's $B/2$ for one solenoid. Since we only need the field to find the flux, the flux is ~~double~~ half at the end face than in the middle part.

(d) Since we have a tube solenoid where the cross-section is always a circle, we have that $\pi r_1^2 = 2\pi r_0^2$, so $r_1 = \sqrt{2}\, r_0$. We obtain this from the reasoning in (c), since the flux tube ~~must~~ should become thinner as B becomes $B/2$. We now find that it becomes thinner by this factor. Note that $r_1 = \sqrt{2}\, r_0$ holds for $r_0 < R/\sqrt{2}$ where R is the radius of the solenoid; because the field line wouldn't reach the end otherwise.

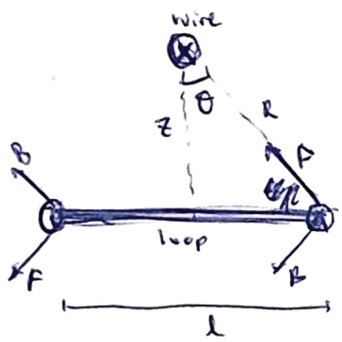
PROBLEM 5 (Purcell 6.44)

From Eq 6.53, we know that B at the axis of the ring is $B_z = \frac{\mu_0 I b^2}{2(b^2+z^2)^{3/2}}$, where b is the radius and z is the distance away from the center on the z axis, assuming that the ring is in the xy plane. Therefore, the line integral on the axis is:

$$\int_{-\infty}^{\infty} B_z\, dz = \frac{\mu_0 I b^2}{2} \int_{-\infty}^{\infty} \frac{dz}{(b^2+z^2)^{3/2}} = \frac{\mu_0 I b^2}{2} \cdot \frac{z}{b^2(b^2+z^2)^{1/2}} \Big|_{-\infty}^{\infty} = \mu_0 I$$

This confirms Eq 6.97, which states that $\int \vec{B} \cdot d\vec{s} = \mu_0 I$. Now, since we integrated over ∞ to $-\infty$, the path begins at a radius of infinity, and it ends there. The entire return path can be ommited, because it goes through a radius of ∞, where $\vec{E} = 0$ and it won't affect the integral's ~~eq~~ result.

PROBLEM 6 (Purcell 6.54)



From Purcell, we are given that the force on a wire is given by $F = I_2 B \sin\theta\, l$, where θ is the angle shown in the diagram ~~between F and the vertical~~. Since there's force being done on the ~~loo~~ wire from both sides of the loop, we have $F = 2I \cdot B \sin\theta\, l$. Using trig, $\sin\theta = \frac{l/2}{R}$. We plug back in to find that $F = 2I\left(\frac{l}{2R} \cdot B\right) l$.

Since we also know that the field due to an infinite straight wire is $B = \frac{\mu_0 I_{wire}}{2\pi R}$, we find that:

$$F = 2I_{loop}\left(\frac{l}{2R} \cdot \frac{\mu_0 I_{wire}}{2\pi R}\right) l = \frac{\mu_0 I_{wire} I_{loop}\, l^2}{2\pi R^2}.$$