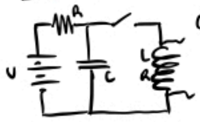
Problem 1 (Purcell 8.16)

At $t=0$, the voltage across the capacitor is $V_0\cos(0)=V_0$. Therefore, the voltage across the inductor must be of $-V_0$ for the net voltage to be 0. Since the charge on the capacitor is known to be $CV_0\cos(\omega t)$, the current is $I(t)=\omega CV_0\sin(\omega t)$. When $t=0$, $I(0)=0$, so none of the energy is stored in the inductor when $t=0$, it is in the capacitor. We know that $E_{capacitor}=\frac{1}{2}CV^2$, so the energy when $t=0$ in the capacitor is $\frac{CV_0}{2}$. When $\omega t=\pi/2$, the voltage in the capacitor is $V_0\cos(\pi/2)=0$, so the voltage in the inductor is also zero. Furthermore, $I\left(\frac{\pi}{2\omega}\right)=\omega CV_0$. Therefore, we can find where the energy is being stored: zero in the capacitor (because voltage is zero), and the energy in the inductor is $\frac{L}{2}(\omega CV_0)^2$, which becomes $\frac{CV_0}{2}$ when $\omega=\frac{1}{\sqrt{LC}}$, showing conservation of energy.

Problem 2 (Purcell 8.19)



(a) We can interpret the circuit as an RLC circuit, since there's a huge difference in the impedances. This allows us to find R and C precisely. Since $\omega=\frac{1}{\sqrt{LC}}$ and 3 cycles happen in 10^{-3} seconds, ω becomes $\omega=\frac{2\pi\cdot 4}{10^{-3}}=2.5\cdot 10^4\ 1/s$.
Therefore,

$$C=\frac{1}{\omega^2 L}=\frac{1}{(2.5\cdot10^4)^2(0.01)}=1.6\cdot10^{-7}\text{ Farads}$$

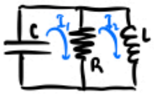
(b) To find R of the coil, we use the fact that decay constant $\alpha=\frac{R}{2L}$, and t for a decrease of $1/e$ is $\frac{2L}{R}$. If we let $t=\frac{2L}{R}$, we find from Fig 8.32 that:

$$R=\frac{2L}{t}=\frac{2(0.01)}{0.5\cdot10^{-3}}=40\text{ Ohms}$$

(c) After a long time, we just have 2 resistors in series with $V=20$ Volts. Therefore, the voltage on the oscilloscope is simply given by

$$V=\left(\frac{40}{10^6+40}\right)(20)=0.008\text{ Volts}$$

Problem 3 (Purcell 8.21)



Here, the resistor is connected in parallel, not in series with the RL circuit. We can solve for the equations using two currents, I_1 and I_2, in the two loops of the circuit. We know from circuit laws that:

$$V=\frac{Q}{C}\qquad V=L\frac{dI_2}{dt}\qquad V=R(I_1-I_2)$$

Using that $I_1=-\frac{dQ}{dt}$, we can write these equations differently, as:

$$\frac{d^2V}{dt^2}=-\frac{1}{C}\frac{dI_1}{dt}\qquad V=\frac{dI_2}{dt}\qquad \frac{dV}{dt}=R\left(\frac{dI_1}{dt}-\frac{dI_2}{dt}\right)$$

We can now merge these equations into one by saying that:

$$\frac{dV}{dt}=R\left(-C\frac{d^2V}{dt^2}-\frac{V}{L}\right)\rightarrow\frac{d^2V}{dt^2}+\frac{1}{RC}\frac{dV}{dt}+\frac{V}{LC}=0$$

Like with any differential equation, we can write the solution to be of an oscillating form, where the exponential decay constant and the oscillatory frequency are given by the equations and solutions:

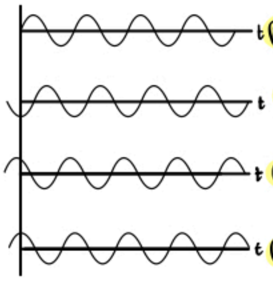
$$\begin{cases}2\alpha\omega-\frac{\omega}{RC}=0\\ \alpha^2-\omega^2-\frac{\alpha}{RC}+\frac{1}{LC}=0\end{cases}\Rightarrow\begin{cases}\alpha=\frac{1}{2RC}\\ \omega^2=\frac{1}{LC}-\frac{1}{4R^2C^2}\end{cases}$$

Now that we have the conditions for α and ω, we can think about what'd happen if R, L, and Quality are the same in this circuit as in a normal RLC series circuit. Since Q (Quality) $=\frac{\omega}{2\alpha}$, we just set this equal in both circuits to find:

$\frac{\omega}{2\alpha}$ are equal in both circuits when $\frac{1}{RC}=\frac{R_{series}}{L}$. Therefore, $R_{series}=\frac{L}{RC}$.

Problem 4 (Purcell 8.26)

The curves look as shown below, and they are labelled there too:



R V_L and V_C are π radians out of phase, and this curve is $\pi/2$.

C This is because V_L is $\pi/2$ ahead from V_R while V_C is $\pi/2$ behind.

E This is because V_L, V_C, and V_R should be $\pi/2$ out of phase and this is the only left one.

L This is because V_L is $\pi/2$ ahead from V_R while V_C is $\pi/2$ behind.

Lastly, it makes mathematical sense that the impedance of inductor is larger than that of the capacitor.

Problem 5 (Purcell 8.27)

We see that the total admittance becomes $\frac{1}{Z} = \frac{1}{R} - \frac{i}{\omega L} + i\omega C$. Since $R = 10^3\,\Omega$, $C = 5 \cdot 10^{-10}\,F$, and $L = 2 \cdot 10^{-3}\,H$, we can find Z for different values of ω:

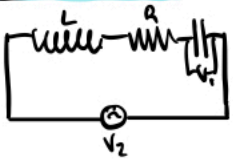
Frequency of 10 kHz $\rightarrow \omega = 2\pi(10^4) = 6.28 \cdot 10^4\ 1/s \rightarrow Z = \frac{1}{10^{-3}(1-7.93i)} = \frac{10^3(1+7.93i)}{1+7.93^2} = (15.7 + 124i)\,\Omega$

Frequency of 10MHz $\rightarrow \omega = 2\pi(10^7) = 6.28 \cdot 10^7\ 1/s \rightarrow Z = \frac{1}{10^{-3}(1+31.4i)} = \frac{10^3(1-31.4i)}{1+31.4^2} = (1.01 - 31.8i)\,\Omega$

Now, since $Z = \frac{1}{1/R + (\omega C - \frac{1}{\omega L})i}$, $|Z| = \frac{1}{\sqrt{(1/R)^2 + (\omega C - \frac{1}{\omega L})^2}}$. $|Z|$ is largest in value when $\omega C = \frac{1}{\omega L}$.

We can solve this to be: $|Z|$ is largest when $\omega = \frac{1}{\sqrt{2 \cdot 10^{-3} \cdot 5 \cdot 10^{-10}}} = 10^6\ 1/s$.

Problem 6 (Purcell 8.32)



Here, we can determine the amplitude of the current to be:

$$\omega = 2\pi(1000) = 6283\ 1/s \rightarrow V_0 = I_0|Z| \rightarrow 15.5 = I_0 \cdot \frac{1}{\omega C} \rightarrow I_0 = 0.0974\ A$$

Since the circuit is in series, I_0 is the current for the entire circuit. Therefore,

$$I_0 = \frac{V_0}{|Z|} \longrightarrow 0.0974 = \frac{10.1}{\sqrt{35^2 + (\omega L - \frac{1}{\omega C})^2}} \rightarrow \omega L - \frac{1}{\omega C} = \pm 97.6\,\Omega$$

Therefore, $L = \frac{1}{\omega^2 C} \pm \frac{97.6}{\omega}$, so L can be 0.041 H or 0.0098 H. If we plug this into $V_0 = I_0 \omega L$ and plug in what we are told in the problem, $V_0 = 25.1\,V$, which is a good amount of 'close' to what we expected. Therefore, it checks out.

Problem 7 (Purcell 8.34)

Since the right inductor and Z_0 are in series, the impedance is $Z_0 + i\omega L$. Since this is then parallel to the capacitor and in series with the left inductor, the impedance becomes:

$$Z = i\omega L + \frac{Z_0 + i\omega L}{Z_0 i\omega C - \omega^2 LC + 1}$$

By taking $Z = Z_0$, we find that $Z_0 = \sqrt{(2-\omega^2 LC)(L/C)}$.

Note that when $\omega = \sqrt{2/LC}$, $Z_0 = 0$, so the impedance is zero. This makes sense because $\omega = \sqrt{2/LC}$ is the resonant frequency of the circuit, so they are resonating without need for an applied voltage, and since $Z = \frac{V}{I}$, if $V=0$, $Z=0$, which is what we obtain.

Problem 8 (Purcell 8.36)

Since $Z \ggg 0$, the current through the resistor equals the current through the inductor. Therefore, $V_0 = \hat{I}(R + i\omega L)$ in terminal A, and $\tilde{V}_1 = \tilde{I}(i\omega L)$ in Terminal B. Now, we can solve for $\left|\frac{\tilde{V}_1}{V_0}\right|^2$ like:

$$\frac{\tilde{V}_1}{V_0} = \frac{i\omega L}{R + i\omega L} \rightarrow \left|\frac{\hat{V}_1}{V_0}\right|^2 = \frac{L^2\omega^2}{R^2 + \omega^2 L^2} = \frac{1}{1 + \left(\frac{R}{\omega L}\right)^2}$$

When $\left(\frac{R}{\omega L}\right)^2 = 9$, the ratio $\left|\tilde{V}_1/V_0\right|^2 = 0.1$. Therefore, when $\omega = 2\pi(100)$, $R/L = 2000$ s to satisfy this. Any R and L that can satisfy this is a valid number. For example, $R = 20000\ \Omega$ and $L = 10\,H$ satisfy this. Now, the signal power is proportional to V^2, so $\left|\hat{V}_1/V_0\right|^2 \approx \frac{\omega^2 L^2}{R^2}$, which is proportional to ω^2. We can see that halving ω reduces power by a factor of $1/4$, proving the claim.

Problem 9 (Purcell 9.14)

Close to the wire, $\vec{B} = \frac{\mu_0 I}{2\pi r}$, so $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$. On maxwell's equation, term with $\vec{J}$ is zero because no current passes through S. To find $\frac{\partial E}{\partial t}$, we know that $E = \frac{Q}{4\pi\varepsilon_0 R^2}$, so $\frac{\partial E}{\partial t} = \frac{I}{4\pi\varepsilon_0 R^2}$. Now, we integrate over a sphere and plug into Maxwell's Equation to obtain that $\mu_0 \varepsilon_0 \frac{I}{4\pi\varepsilon_0 R^2} 4\pi R^2 = \mu_0 I$. Therefore, $\mu_0 I = \mu_0 I$, so it all works out.

Problem 10 (Purcell 9.15)

The integral law can be written as $\int \vec{B} \cdot d\vec{s} = \mu_0 \int (\vec{J}_d + \vec{J}) \cdot d\vec{a}$. Since $s \gg d$, the integral of $\vec{J}_d$ over the area is the current in the wire. The part of I that's in the wire is $\frac{\pi r^2}{\pi b^2}$. Since $\vec{J} = 0$ inside the capacitor, the integral law becomes $B = \frac{\mu_0 I r}{2\pi b^2}$, which completes the proof.