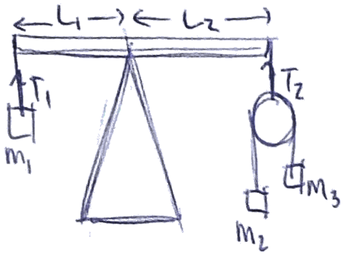
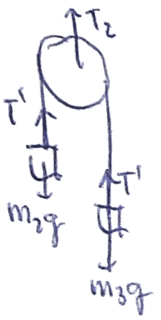
PROBLEM 1 (KK 7.12)



If system is in equilibrium, the net ~~force~~ torque is 0.

Therefore, $T_2 l_2 = +T_1 l_1 \rightarrow +m_1 g l_1 = T_2 l_2$ (1)

Now, use kinematics, where we look at the pulley:



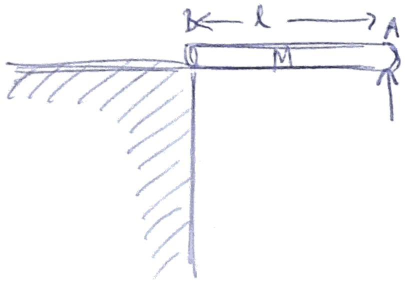
$$\begin{cases} T_2 = 2T' \rightarrow T_2 = 2\left(\dfrac{2m_3 m_2 g}{m_2+m_3}\right) & (2) \\ \left.\begin{array}{l} T' - m_3 g = m_3 a \\ -T' + m_2 g = m_2 a \end{array}\right\} \dfrac{T' - m_3 g}{m_3} = \dfrac{-T' + m_2 g}{m_2} \rightarrow T'(m_2+m_3) = 2m_2 m_3 g \end{cases}$$

$$\therefore T' = \frac{2m_3 m_2 g}{(m_2+m_3)}$$

Now, pluging eq. (1) into eq. (2), we get:

$$\left.\begin{array}{l} T_2 = 2\left(\dfrac{2m_3 m_2 g}{m_2+m_3}\right) \\ T_2 = \dfrac{m_1 g l_1}{l_2} \end{array}\right] \frac{4m_3 m_2 \cancel{g}}{m_2+m_3} = \frac{m_1 \cancel{g} l_1}{l_2} \rightarrow \boxed{\frac{4m_3 m_2}{m_1(m_2+m_3)} = \frac{l_1}{l_2}}$$

PROBLEM 2 (KK 7.14)



(a) $\boxed{\tau = Mg\frac{l}{2}}$ since the center of mass is at $\frac{l}{2}$.

(b) $\tau = I\alpha \rightarrow \frac{Mgl}{2} = I\alpha$. Since it is a rod and we are measuring I around the R_{cm}, $I = \frac{1}{3}Ml^2$

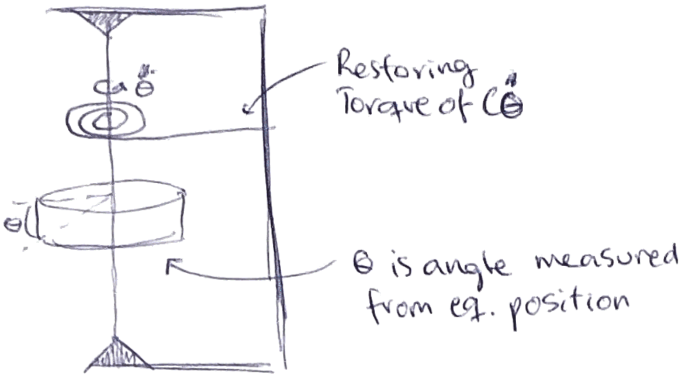
$$\frac{\cancel{M}g\cancel{l}}{2} = \frac{1}{3}\cancel{M}l^{\cancel{2}}\alpha$$

Therefore, $\boxed{\alpha = \frac{3g}{2l}}$

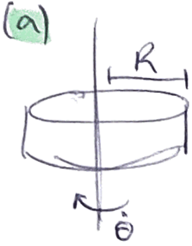
(c) $a = \alpha \cdot r \rightarrow a = \frac{3g}{2l} \cdot \frac{l}{2} = \boxed{\frac{3g}{4}}$

(d) $Ma = Mg - F \rightarrow M\frac{3g}{4} = Mg - F \rightarrow F = Mg\left(1 - \frac{3}{4}\right) = \boxed{\frac{1}{4}Mg}$

PROBLEM 3 (KK 7.19)



(a)



Moment of inertia of disc = $I = \frac{1}{2}MR^2$

$\tau_{disc} = I\alpha = \frac{1}{2}MR^2\ddot{\theta}$

Restoring torque = τ_{disc} $\rightarrow \frac{1}{2}MR^2\ddot{\theta} = C\theta \rightarrow \ddot{\theta} = \frac{2C}{MR^2}\theta$

This is a SHO differential equation of type $\ddot{x} = \omega^2 x$, so $\boxed{\omega = \sqrt{\frac{2C}{MR^2}}}$

(b) (1) $I_{\substack{RING\\PUTTY}} = MR^2 \rightarrow I_{\substack{RING\\PUTTY}} + I_{DISC} = \frac{3}{2}MR^2$

$\tau_{\substack{TOTAL\\MASS}} = \frac{3}{2}MR^2\ddot{\theta}$ = Restoring Torque $\rightarrow C\theta = \frac{3}{2}MR^2\ddot{\theta} \rightarrow \ddot{\theta} = \frac{2C}{3MR^2}\theta \rightarrow \boxed{\omega' = \sqrt{\frac{2C}{3MR^2}}}$

(2) When the putty hasn't been dropped, just before it's dropped: $(t_1 = \frac{\pi}{\omega})$

$\theta = \theta_0 \sin(\omega t_1) = \theta_0 \sin\pi = 0$

$\dot{\theta} = \omega\theta_0\cos(\omega t_1) = \omega\theta_0 \cos\pi = -\omega\theta_0$ } Therefore, $L = I\omega = I \cdot \omega\theta_0$

Since before putty is dropped, the only mass is a disc, $I_1 = \frac{1}{2}MR^2$

Therefore, $L = \frac{1}{2}MR^2\omega\theta_0$. Also, AM is conserved, since $\sum\tau = 0$.

Angular momentum after putty is dropped is: $L' = \frac{3}{2}MR^2\omega'\theta_0'$

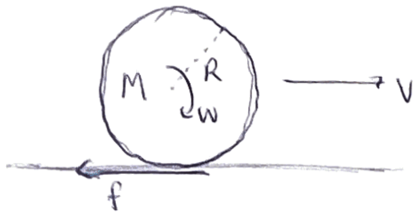
C.O.A.M:

$$\frac{1}{2}\cancel{MR^2}\omega\theta_0 = \frac{3}{2}\cancel{MR^2}\omega'\theta_0' \rightarrow \omega\theta_0 = 3\omega'\theta_0'$$

In part (b)(1), we found that $\omega' = \sqrt{\frac{2C}{3MR^2}} = \frac{\omega}{\sqrt{3}}$

$\therefore, \cancel{\omega}\theta_0 = \theta_0' \frac{\cancel{\omega}}{\sqrt{3}} \rightarrow \theta_0 = \frac{\theta_0'}{\sqrt{3}} \rightarrow \boxed{\theta_0' = \frac{\theta_0}{\sqrt{3}}}$

PROBLEM 4 (kk 7.30)



From the point of view of a point in the bowling alley, Angular momentum is conserved.

$L_i = MRV_0 \quad L_f = MRV + I\omega$

Since the object's a sphere, $I = \frac{2}{5}MR^2$

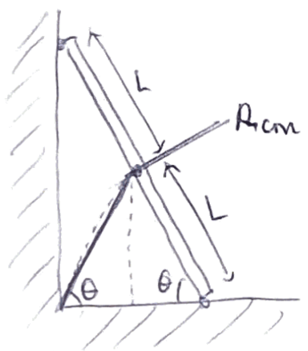
$\therefore L_f = MRV + \frac{2}{5}MR^2\omega$

Since Angular Momentum is conserved,

$\cancel{MR}V_0 = \cancel{MR}V + \frac{2}{5}\cancel{MR^2}\omega \rightarrow V_0 = V + \frac{2}{5}R\omega \; \dots \; \omega = \frac{V}{R}$

$\therefore V_0 = V + \frac{2}{5}V = \frac{7}{5} \longrightarrow \boxed{V = \frac{5}{7}V_0}$

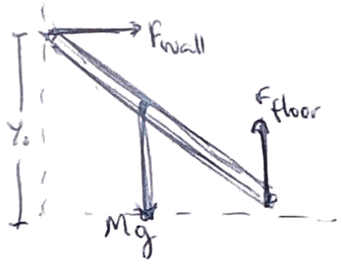
PROBLEM 5 (kk 7.41)



Since we have an isosceles triangle, we can describe the coordinates of the center of mass as:

$\left.\begin{array}{l} x\text{-coordinate} = L\cos\theta \\ y\text{-coordinate} = L\sin\theta \end{array}\right\} x^2 + y^2 = L^2(\cos^2\theta + \sin^2\theta) = L^2$

Therefore, Rcm traces a circle of radius L while the plank is at contact with wall.



When the plank loses contact with the wall, $F_{wall} = 0$. F_{wall} and F_{floor} are perpendicular to the surfaces because $\mu = 0$.

$F_{wall} = 0 \rightarrow \ddot{x} = \frac{F_{wall}}{M} = 0$

If $x = L\cos\theta$, $\ddot{x} = -L\cos\theta\,\dot{\theta}^2 - L\sin\theta\,\ddot{\theta} = 0 \rightarrow \dot{\theta}^2 = -\tan\theta\,\ddot{\theta}$

Now, apply COE, since $F_{ext} = 0$ (non-conservative forces aren't in this problem).

COE: $\cancel{M}gy_0 = \cancel{M}gL\sin\theta + \frac{2}{3}\cancel{M}L^2\dot{\theta}^2 \rightarrow y_0 = L\sin\theta + \frac{2}{3}L^2\dot{\theta}^2$

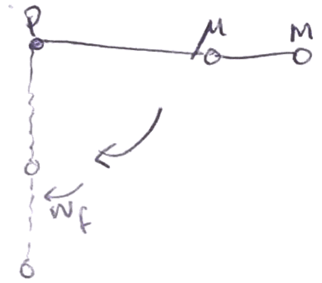
Differentiate previous equation: $0 = L\cos\theta\,\dot{\theta} + \frac{4L^2}{3g}\dot{\theta}\ddot{\theta} \rightarrow \ddot{\theta} = -\frac{3g\cos\theta}{4L}$

Now, substitute:

$\dot{\theta}^2 = +\tan\theta\,\frac{3g\cos\theta}{4L} = \frac{3g}{4L}\sin\theta \longrightarrow Y_0 = L\sin\theta + \frac{2L^2}{3}\cdot\frac{3g}{4L}\sin\theta = \frac{3}{2}\underbrace{L\sin\theta}_{y} = \frac{3}{2}y \rightarrow \boxed{y = \frac{2}{3}y_0}$

PROBLEM 6

(a)



If we consider μ and m to be point masses, I about the pivot P is:

$$I_{pivot} = \mu d^2 + m\ell^2$$

$\therefore$ Apply COE, since $F_{ext}^{non\text{-}cons} = 0$.

$$\therefore E_i = (m+\mu) g\ell$$

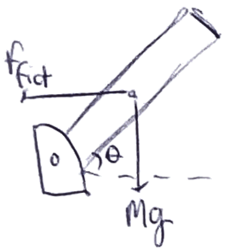
$$E_f = \mu g(\ell - d) + \frac{1}{2}(\mu d^2 + m\ell^2) w_f^2$$

From here, we solve for w_f:

$$(m+\mu) g\ell = \mu g(\ell-d) + \frac{1}{2}(\mu d^2 + m\ell^2) w_f^2 \longrightarrow mg\ell + \cancel{\mu g\ell} = \cancel{\mu g\ell} - \mu g d + \frac{1}{2}(\mu d^2 + m\ell^2) w_f^2$$

$$\therefore \boxed{w_f = \sqrt{\frac{2g(m\ell + \mu d)}{m\ell^2 + \mu d^2}}}$$

PROBLEM 5 (KK 9.1)



(a) Torque about pivot:

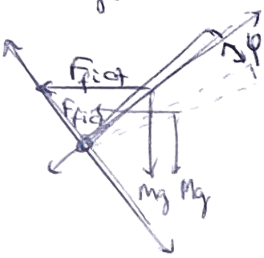
$$\tau_{pivot} = \frac{L}{2}Mg\cos\theta - \frac{L}{2}F_{fict}\sin\theta$$

At equilibrium, $\tau_{pivot} = 0$.

$$\therefore\ 0 = \frac{LMg}{2}\cos\theta - \frac{LMA}{2}\sin\theta \rightarrow \tan\theta = \frac{g}{A}$$

$$\therefore\ \boxed{\theta = \arctan\left(\frac{g}{A}\right)}$$

(b) Change coordinates:



$\tau = I\alpha$. Since M is a rod and the pivot is at the edge,

$$\tau = \frac{1}{3}ML^2\alpha = \frac{1}{3}ML^2\ddot{\varphi}$$

Also, total force on M is $F_{TOT} = W + f_{fict}$

$$\vec{\tau} = \vec{r}\times\vec{F} \rightarrow \vec{\tau} = |\vec{r}||\vec{F}|\sin\varphi = \frac{L}{2}\sqrt{W^2+f_{fict}^2}\,\sin\varphi$$

Now, solve for $\vec{\tau}$ and equal:

$$\begin{cases} \tau = \frac{1}{3}ML^2\ddot{\varphi} \\ \tau = \frac{L}{2}\sqrt{W^2+F_{fict}^2}\,\sin\varphi \rightarrow \text{for small } \varphi \Rightarrow \tau = \frac{L}{2}\sqrt{M^2g^2+M^2A^2}\,\varphi \end{cases}$$

$$\therefore\ \frac{1}{3}ML^2\ddot{\varphi} = \frac{L}{2}\sqrt{M^2g^2+M^2A^2}\,\varphi \rightarrow \frac{1}{3}L\ddot{\varphi} = \frac{1}{2}\sqrt{g^2+A^2}\,\varphi \rightarrow \ddot{\varphi} = \frac{3}{2L}\sqrt{g^2+A^2}\,\varphi$$

This is a differential equation and its solution is known:

$$\varphi = Ce^{kt} + De^{-kt} \quad \text{where } k^2 = \frac{3}{2L}\sqrt{A^2+g^2}$$

Now, suit this solution for the initial conditions:

- At $t=0$, φ is φ_0. Therefore,

$$\boxed{\varphi = \varphi_0 e^{\pm kt} \quad \text{where } k^2 = \frac{3}{2L}\sqrt{A^2+g^2}}$$

PROBLEM 9 (KK 9.2)

Since the truck is accelerating forward with acceleration A, the door can be seen to free fall at a "gravitational field". The center of mass of the door falls freely a distance of $\frac{W}{2}$. This is visualized in the drawings:

(a) Since ~~F~~ $F_{ext}^{\text{non conservative}} = 0$, we apply COE.

$$\frac{1}{2}I\dot{\theta}^2 = MA\frac{W}{2} \rightarrow \frac{1}{2}\left(\frac{1}{3}MW^2\right)\dot{\theta}^2 = MA\frac{W}{2}$$

I of a rod about a end pivot.

$$\therefore \frac{1}{6}\cancel{M}W^{\cancel{2}}\dot{\theta}^2 = \cancel{M}A\frac{\cancel{W}}{2} \rightarrow \frac{1}{3}W\dot{\theta}^2 = A \rightarrow \boxed{\dot{\theta} = \sqrt{\frac{3A}{W}}}$$

(b) Apply equation of motion:

from (a)

$$\therefore F_{TRUCK} - MA = M\frac{W}{2}\dot{\theta}^2 = M\frac{W}{2}\left(\frac{3A}{W}\right) = MA\frac{3}{2}$$

$$\therefore F_{TRUCK} = MA + \frac{3}{2}MA \rightarrow \boxed{F_{TRUCK} = \frac{5}{2}MA}$$

PROBLEM 10 (KK 9.3)

In the accelerating system, the pendulum undergoes an effective gravitational force:

$$g_{eff} = \sqrt{g^2+a^2}$$

Initial Angular displacement is $\arctan\left(\frac{a}{g}\right)$. We can calculate the torque on the pendulum to be:

$$\tau = m\sqrt{g^2+a^2}\,\ell\sin\theta_0 \rightarrow \alpha = \frac{\tau}{I_0} = \frac{m\ell\sin\theta_0\sqrt{g^2+a^2}}{m\ell^2} = \frac{\sqrt{g^2+a^2}\sin\theta_0}{\ell}$$

Since

implies that $\sqrt{g^2+a^2}\sin\theta_0 = a$, $\alpha = \frac{a}{\ell}$.

For the pointing of the pendulum to be towards the center of earth, $\alpha = \frac{a}{R_e}$, which implies that $R_e = \ell$. Knowing this, the period of the pendulum is:

$2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{R_e}{g}}$, so the period is the same, and the pendulum will point down.