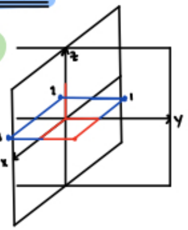
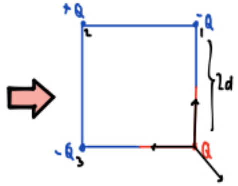
Problem 1

(a)





$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4d^2}(-\hat{x}) = \frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(-\hat{x})$$

$$\vec{F}_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4d^2}(-\hat{y}) = \frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(-\hat{y})$$

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{8d^2}\left(\sin\theta\,(\hat{x}) + \cos\theta\,(\hat{y})\right) = \frac{\sqrt{2}}{64\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(\hat{x}+\hat{y})$$

$$\vec{F}_{TOT} = \vec{F}_1+\vec{F}_2+\vec{F}_3 = \frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(-\hat{x}) + \frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(-\hat{y}) + \frac{\sqrt{2}}{64\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(\hat{x}) + \frac{\sqrt{2}}{64\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(\hat{y}) = \frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}\left[-\hat{x}-\hat{y}+\frac{\sqrt{2}}{4}\hat{x}+\frac{\sqrt{2}}{4}\hat{y}\right]$$

$$\vec{F}_{TOT} = -\frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(0.646)\hat{x} - \frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(0.646)\hat{y} \rightarrow |\vec{F}_{TOT}| = \sqrt{2\left(-\frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(0.646)\right)^2} = \frac{\sqrt{2}}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(0.646)$$

Therefore, the direction, given by $\frac{\vec{F}_{TOT}}{|\vec{F}_{TOT}|}$ componentwise, is: $\vec{F}_{dir} = -\frac{1}{\sqrt{2}}\hat{x} - \frac{1}{\sqrt{2}}\hat{y} = -\frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$ is the direction

(b)

$$F_{TOT} = -\frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(0.646)\hat{x} - \frac{1}{16\pi\varepsilon_0}\cdot\frac{Q^2}{d^2}(0.646)\hat{y} = \frac{Q^2}{16\pi\varepsilon_0 d^2}\left(\sqrt{2}-\frac{1}{2}\right) \qquad F_y = qE_y = q\cdot|E|$$

We only contemplate $\hat{y}$ $\vec{F}$ or $\vec{E}$, since we are looking at the xz plane. Also, we take Q to be $\frac{Q}{2}$ when we divide by Q in $E = F/q$ since we look at half.

$E_T = \frac{2Q}{16\pi\varepsilon_0 d^2}\left(\sqrt{2}-\frac{1}{2}\right) \rightarrow$ Now, we need to change the distance d, since we are looking at Q for all $y>0$. We define the distance R to be $R = \sqrt{(x-d)^2+d^2+z^2}$

$E_T = \frac{Q}{8\pi\varepsilon_0 R^2}\left(\sqrt{2}-\frac{1}{2}\right)$. Now, we must look for the $\hat{y}$ component, the others cancel out due to symmetry. $E_y = \frac{Q}{2\pi\varepsilon_0 R^2}\frac{d}{(x-d)^2+d^2+z^2}$.

We can rewrite this as: $E_y = \frac{Qd}{2\pi\varepsilon_0((x-d)^2+d^2+z^2)^{3/2}}$. Now, since $\sigma = \varepsilon_0 E_y$, we have that $\sigma = \frac{Qd}{2\pi}\left(((x-d)^2+d^2+z^2)^{-3/2}\right)$.

Nevertheless, this only takes into account $\vec{E}$ for $y>0$. To account for $y<0$, we substract an extra term of similar form to get: $\sigma = \frac{Qd}{2\pi}\left(((x+d)^2+d^2+z^2)^{-3/2} - ((x-d)^2+d^2+z^2)^{-3/2}\right)$

(c) Now, we integrate σ from (b) for the space in half the plane:

$$Q_{\substack{half\\plane}} = \int_{-\infty}^{\infty}\int_{0}^{\infty}\frac{Qd}{2\pi}\left(((x+d)^2+d^2+z^2)^{-3/2} - ((x-d)^2+d^2+z^2)^{-3/2}\right)dx\,dz = \frac{Q}{2\pi}\int_0^{\infty}du\int_{-\infty}^{\infty}dv\left[((u+1)^2+1+v^2)^{-3/2} - ((u-1)^2+1+v^2)^{-3/2}\right] = 2\int_0^{\infty}\frac{du}{(u+1)^2+1} - 2\int_0^{\infty}\frac{du}{(u-1)^2+1} = \left(\frac{\pi}{2}-\frac{3\pi}{2}\right)\frac{Q}{2\pi} = -\frac{Q}{2}\ \checkmark$$

(d) We sum over different pairs:

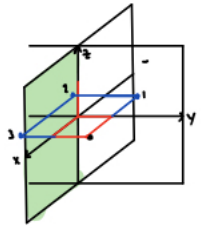
$$U = \frac{Q^2}{4\pi\varepsilon_0}\left(\substack{\text{all the pairs}\\\text{of charges}}\right) = \frac{Q^2}{4\pi\varepsilon_0 d}\left(\frac{1}{\sqrt{2}}-2\right)$$

Ran out of time :)

By doing the integral:

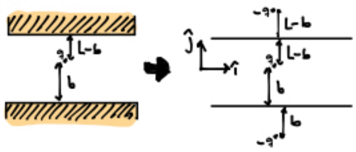
$$\int_{\infty}^{d\sqrt{2}} -\vec{F}\,d\vec{s} = \int_{\infty}^{d\sqrt{2}}\frac{Q^2\left(\sqrt{2}-\frac{1}{2}\right)}{8\pi\varepsilon_0 s^2}ds = \frac{Q^2}{4\pi\varepsilon_0 d}\left(\frac{1}{\sqrt{2}}-2\right)\cdot\frac{1}{4}$$

, so our results are different. This is because the first method counts for all the pairs 'relative' to eachother, while the integral looks at pairs of charges w/ respect to the charge we integrate about.



Problem 2

(a)



The field in this case due to the top $-q$ is: $\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{4(L-b)^2}\hat{j}$.

Similarly, the field from the bottom charge is $\vec{E}_{bottom} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{4b^2}\hat{j}$

Note that there are infinite posibilities for the image charge config.

I chose this one because the distances are simpler to find.

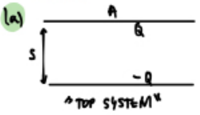
(b) $\vec{E}_{TOT} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{4b^2}\hat{j} - \frac{1}{4\pi\varepsilon_0}\frac{q}{4(L-b)^2}\hat{j} = \frac{q}{16\pi\varepsilon_0}\left(\frac{-1}{b^2} - \frac{1}{(L-b)^2}\right)\hat{j} \longrightarrow \vec{F} = q\vec{E}_{TOT} = \frac{q^2}{16\pi\varepsilon_0}\left(\frac{-1}{b^2} - \frac{1}{(L-b)^2}\right)\hat{j}$. If $b \ll L$, we have the Taylor Expansions $\left\{ \begin{array}{l} \frac{1}{(L-b)^2} \approx \frac{1}{L^2(1-\frac{2b}{L})} \\ \frac{1}{b^2} \approx 1-2(b-1) \end{array} \right\}$.

Therefore, the approximation for $\vec{F}$ is, for $b \ll L$:

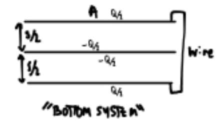
$$\vec{F} \approx \frac{q^2}{16\pi\varepsilon_0}\left(-1+2(b-1) - \frac{1}{L^2(1-\frac{2b}{L})}\right)\hat{j} = \frac{q^2}{16\pi\varepsilon_0}\left(-3+2b-\frac{1}{L^2-2bL}\right)\hat{j}$$

Problem 3

(a)



In this setup, we know the capacitance to be C. We can assign charges Q and $-Q$ to the two conductor plates, as shown.



However, in this case, since 2 plates are connected by a wire, they are the same "piece" of conductor. Therefore, each of them has charge $Q/2$.

From symmetry, the central conductor must have $-Q/2$ on each side to satisfy a total charge of $-Q$ in the central conductor and a net charge of 0 in each of the 2 capacitors.

In the bottom system, $V = \varphi_1 - \varphi_2 = E \cdot s/2$. The field in the bottom capacitor is half of the field in the initial configuration without the middle conductor because σ is half. Also, the separation is half. Therefore, V in the bottom setup is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ of V in the top setup. We have:

$Q = C_{\substack{\text{TOP} \\ \text{SYSTEM}}} \cdot V$ and $Q = C_{\substack{\text{BOTTOM} \\ \text{SYSTEM}}} \cdot (V/4)$. Since Q is the same in both cases,

C in the bottom system is $4C_{\text{TOP}}$.

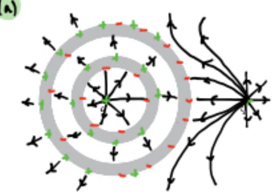
(b) For our planes, we know that $A_b = A_{a}$ and the separation in the bottom system is half of the separation in the top system. $s_b = \frac{1}{2} s_a$. Therefore, using eq. 3.15, we know that $C = \frac{\epsilon_0 A}{s}$ multiplying this equation out for the top and the bottom system yields $C_a = \frac{1}{4} C_b$, which is our result in part (A) if B refers to the bottom system and A to the top one.

Problem 4 (Purcell 3.32)

In Fig 3.8, as the problem says, the box doesn't 'block' the field, it just allows the charges to set up a compensating field that ends up cancelling out with the external field, creating an apparent 'zero' field. However, with gravitational force, there's nothing capable of 'cancelling out' gravity. This is because mass can only be positive, so the sign of the field can only be positive. No negative gravitational field exists, so there's no cancelling out, hence why this doesn't work. If we had 'negative' mass, this could be possible by having the surface of the box have negative mass.

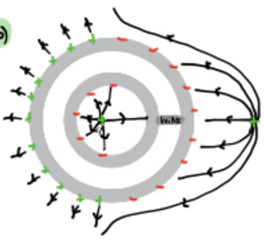
Problem 5 (Purcell 3.33)

(a)



Each ring has different sides and charge distributions to it, depending on the external charge to it. Therefore, since the whole outer ring has $Q_{ext} = +q$, it's going to have negative charges on its outer surface and positive ones on the inner surface. For the inner ring, we have a similar case, but the other way around. Also, the surface charge density is <u>spherically symmetrical</u>, because it feels no field from the external and internal charges since, as problem (4) suggests, the electric field is 'blocked'.

(b)



In this case, we have a similar setup to (a), but the charges are distributed as if the two rings and the wire were all one unique conductor. However, due to Gauss' Law, the charge distribution is still <u>spherically symmetric</u>.

Problem 6 (Purcell 3.41)

There are many Gaussian surfaces that can be used. The flux into the surface is $(Q/2)/\epsilon_0$ because half of the flux passes through a sphere (the Gaussian surface I chose) around Q. Now, we can apply Gauss' Law since we have symmetry, and we realise that there must be a charge of $-Q/2$ inside. This can only happen on the surface of the plane. Using σ and Gauss' Law, we must find R such that $-Q/2 = \int_0^R \sigma \cdot 2\pi r\,dr \longrightarrow \frac{1}{2} = \int_0^R \frac{hr\,dr}{(r^2+h^2)^{3/2}} = 1 - \frac{h}{\sqrt{R^2+h^2}}$

Therefore, we have:

$$\frac{h}{\sqrt{R^2+h^2}} = \frac{1}{2} \rightarrow R = \sqrt{3}\,h$$

Problem 7

In this problem, reasoning (B) is correct. This is because plate 3 has a lower potential than plate 1, so there will be charge flow. Reasoning (A) goes wrong when it states that the potentials of the two planes are equal. It is wrong to assume that because the potential differences of the two capacitors are the same, the potentials of the two planes are equal. By saying that $\phi_{plate1} = 0$, we can easily prove this and see reasoning (A)'s flaw.

Problem 8

To solve this problem, we can find $\vec{E}$ between the cylinders, so basically $\vec{E}$ between two line charges of different radii. The field between them is found to be $\vec{E} = \frac{Q}{2\pi\epsilon_0 L r}$. Therefore, the magnitude of $\Delta\phi$ is $\int_a^b E\,dr = \frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{a}{b}\right)$. Since $C = Q/\Delta\phi$, the capacitance is:

$C = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{a}{b}\right)}$. We can use the Taylor series for ln, and we find $C \approx \frac{2\pi\epsilon_0 bL}{a-b}$. Since $2\pi bL$ is the area of the cylinder we are

about is $2\pi bL$, so $C = \frac{\epsilon_0 A}{s}$, which agrees with what we expected.