PROBLEM 1 (PURCELL 7.24)

To solve all the parts of this problem, I found an expression for the force. If we let b be the length of the side in the field $\vec{B}$ and P be the total ~~diamet~~ perimeter of the frame, the frame's current is found to be $I = \mathcal{E}/R = \frac{Bbv}{R}$. Since $R = \frac{\rho l}{A} = \frac{\rho P}{\pi r^2}$, we have that $I = \frac{Bbv\pi r^2}{\rho P}$. Now, since the magnetic force on the side b is $F = IBb$, we have that the force exerted by the person must be $F = \frac{B^2 b^2 v \pi r^2}{\rho P}$.

Now, we can answer the questions:

- From above, we see that twice the force $\Leftrightarrow$ twice the velocity. A force of 2N will therefore pull the frame in $\frac{1}{2}$ seconds.
- If we double ρ, we divide F by 2. Therefore, pulling a brass frame would require $\frac{1}{2}$ Newtons.
- Doubling r increases F by r^2, so a 1cm frame will be pulled in 1 second by a force of 4N.

PROBLEM 2 (PURCELL 7.26)

(a) The velocity of the bar at some moment in time is v. Since $\mathcal{E} = Bbv$ and $I = \mathcal{E}/R$, where $R \equiv$ resistance, $I = \frac{Bbv}{R}$. Since Force on a bar is $F = IBb$, $F = \frac{B^2b^2v}{R}$.

Since F opposes motion, F=ma gives:

$$-\frac{B^2b^2v}{R} = m\frac{dv}{dt} \longrightarrow -\int_0^t \frac{B^2b^2}{Rm}\,dt = \int_{v_0}^{v} \frac{1}{v}\,dv \rightarrow v = v_0 e^{-t/\alpha}, \text{ where } \alpha = \frac{Rm}{B^2b^2}$$

In a perfect scenario, the bar never stops moving (exponential decay).

(b) To find the total distance, we just take the limit $t \to \infty$ above:

$$x = \int_0^\infty v\,dt = \int_0^\infty v_0 e^{-t/\alpha}\,dt = v_0\alpha = \frac{v_0 Rm}{B^2b^2}$$, so it travels a finite distance!

(c) We know that the initial KE of the rod is $\frac{1}{2}mv_0^2$, so for energy conservation, the energy dissipated in the resistor through $t \to \infty$ must equal $\frac{1}{2}mv_0^2$:

$$\int_0^\infty IR^2\,dt = \int_0^\infty \frac{Bb}{R}\cdot v_0 e^{-t/\alpha}\cdot R^2\,dt = \frac{B^2b^2v_0^2}{R}\int_0^\infty e^{-2t/\alpha}\,dt = \frac{1}{2}mv_0^2.$$

Therefore, energy is conserved, but dissipated!

PROBLEM 3 (PURCELL 7.27)

(a) We know that $\vec{B}$ in a solenoid is $\mu_0 n I(t)$, so using the value of $I(t)$, we have: $\vec{B}(t) = \mu_0 n I_0 \cos(\omega t)$. Using Faraday's law, $\mathcal{E} = -\frac{d\Phi}{dt} = -\pi r^2 \frac{dB}{dt} = \pi r^2 \mu_0 n I_0 \omega \sin(\omega t)$.
Therefore, the induced current by an upward $\vec{B}$ (from RHR), is given by:
$I_{ind}(t) = \frac{\mathcal{E}}{R} = \frac{\pi r^2 \mu_0 I_0 n \omega}{R} \sin(\omega t)$

(b) The force on a little piece of the loop is $\vec{F} = I_{ind}(t)\, d\vec{l} \times \vec{B}$. Since I is counter-clockwise and B is up, F is radial. $F(t) = \frac{\pi r^2 \mu_0^2 I_0^2 \omega n^2 dl}{R} \sin(\omega t)\cos(\omega t)$.
Therefore, $\vec{F}$ is maximum outward when $\omega t = \frac{\pi}{4} + n\pi$, and the maximum inward happens when $\omega t = \frac{3\pi}{4} + n\pi$.

(c) Since F is only horizontal, it can only stretch or shrink the ring. If the ring is rigid, which we take it to be, this is negligible.

PROBLEM 4 (PURCELL 7.33)

We know that the flux through the ring at any time is $\Phi = \pi a^2 B$. From Faraday's Law, $\mathcal{E}_{ind} = \pi a^2 \left(\frac{dB}{dt}\right)$. However, $\mathcal{E}$ is also, $\mathcal{E} = \int \vec{E} \cdot d\vec{s}$, so the tangential field is:
$\int \vec{E} \cdot d\vec{s} = 2\pi a E = \pi a^2 \left(\frac{dB}{dt}\right) \rightarrow E = \mathcal{E}/2\pi a = \frac{a}{2}\frac{dB}{dt}$. We know that some dq feels a tangential force dF such that $dF = E\,dq$. Therefore, the torque is $Ea\,dq$. Therefore, the total torque τ is $Eaq = \frac{qa^2}{2} \cdot \left(\frac{dB}{dt}\right)$. Therefore, the angular momentum L is:
$L = \int_0^\infty \tau\, dt = \int_0^\infty \frac{qa^2}{2}\frac{dB}{dt} dt = \frac{qa^2}{2}\int_{B_0}^{0} dB = -\frac{qa^2 B_0}{2}$. Here, the rotation will happen in the same direction as B_0 if q is positive. Now, since $L = I\omega = (Ma^2)\omega$ (for a ring), then
$\omega = \frac{L}{ma^2} = \frac{qB_0}{2m}$, which is the final result of our proof.

PROBLEM 5 (PURCELL 7.35)

From Purcell, we know that the $\vec{B}$ field along the axis of a ring of radius a, a distance b from the center is $\vec{B} = \frac{\mu_0 I a^2}{2(a^2+b^2)^{3/2}}$. For $b \gg a$, $\vec{B} \approx \frac{\mu_0 I a^2}{2b^3}$. Therefore, the flux through the other ring is $\Phi = \pi a^2 B = \pi a^2 \cdot \frac{\mu_0 I a^2}{2b^3}$. Therefore, the mutual inductance $\left(\frac{\Phi}{I}\right)$ is given by: $\frac{\Phi}{I} = \frac{\pi \mu_0 a^4}{2b^3}$.

PROBLEM 6 (PURCELL 7.36)

(a) From part (a) of the figure, if I_2 increases, the upward flux trough the top circuit increases. This will induce $\mathcal{E}_1$, which will create current that will make a downward flux. Therefore, bothe equations for $\mathcal{E}_1$ and $\mathcal{E}_2$ should be:

$$\mathcal{E}_1 = -L_1 \frac{dI_1}{dt} - M\frac{dI_2}{dt} \quad \text{and} \quad \mathcal{E}_2 = -L_2 \frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

(b) In part (b) of the figure, we have a new current $I = I_1 + I_2$, and a new value for the electromotive force, $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$. Therefore, we can add the equations for $\mathcal{E}_1$ and $\mathcal{E}_2$:

$$\mathcal{E} = -L_1 \frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt} = (-L_1 - L_2 - 2M)\frac{dI}{dt}$$

Therefore, for Fig 7.40 (b), the self inductance $L' = +L_1 + L_2 + 2M$.

In part (c) of the figure, however, $I = I_1 = -I_2$, and $\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2$. Therefore:

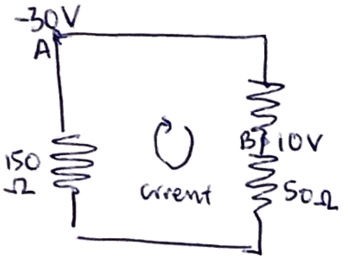
$$\mathcal{E} = -L_1\frac{dI}{dt} - M\frac{d(-I)}{dt} + L_2\frac{d(-I)}{dt} + M\frac{dI}{dt} = -(L_1 + L_2 - 2M)\frac{dI}{dt}$$

Therefore, for Fig 7.40(c), the self-inductance $L'' = L_1 + L_2 - 2M$. Since $M > 0$, $L'' < L'$.

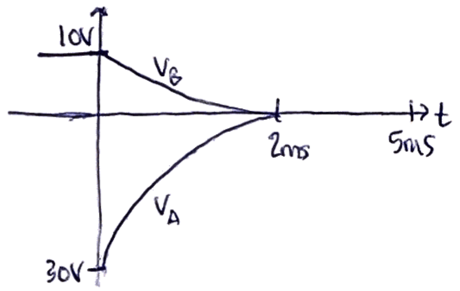
(c) If L is negative, then Lenz' law is no longer stable, because this would break energy conservation. We must have $L'' \geq 0$, so $L_1 + L_2 - 2M \geq 0$. Therefore, $L_1 + L_2 \geq 2M$ for any pair of circuits.

PROBLEM 7 (PURCELL 7.41)

We know that 10V is the initial voltage accross each branch of the circuit. The currents accross the ~~15~~ 150Ω and 50Ω resistors are 0.067 A and 0.2A, respectively. Right after the switch is opened, we can sketch the circuit to be:



At A, we know $V = -30$ volts because $V = -(0.2A)(150\Omega)$. We know that the current is 0.2A clockwise because $\mathcal{E}$ can't be infinite. Now we have an RL circuit, so $I(t) = I_0 e^{-(R/L)t}$. Since we know the values, we can find the current and the voltage as a function of time to be what's shown below:



Between 2ms and 5ms, the voltage is virtually 0, so we ignore it.

PROBLEM 8 (PURCELL 7.46)

We know that the magnetic field at the center of a ring is $B = \frac{\mu_0 I}{2r}$, so in this case, $B = \frac{\mu_0 I}{a}$. Therefore, the stored energy in the system is:

$U = \frac{B^2}{2\mu_0} \cdot V$, where $V \equiv$ Volume of ring $= \pi a^2 \cdot a$. Therefore, $U = \frac{\mu_0 \pi a I^2}{2}$.

Since $R = \frac{\pi}{a\sigma}$, the energy disipation due to resistance is $IR^2 = \frac{\pi I^2}{a\sigma}$. Therefore, $\tau = \frac{U}{I^2 R} = \mu_0 a^2 \sigma$

Since $a = 3000$ km and $\sigma = 10^6$, $\tau = 1 \cdot 10^{13}$ seconds, or 300 millenia, or 3000 centuries. QED.