Lab 2

Sources of Error and Error Propagation: Pendulums

Objectives

- Give students practice identifying and correcting for various sources of error
- Learn to check results and adjust setup when necessary
- Introduce error propagation to students
- Introduce modeling error

Lab Overview

You will measure the period of a pendulum, changing the experimental setup as needed for each lab Scenario. Record the group's data on this worksheet or a Google sheet, compute means and errors for each data set, and compare your results to the simple pendulum theory prediction for each setup. As you are collecting data, look for possible sources of error in your experiment! Note these in the space provided, and comment on possible ways you identified and corrected any errors.

Be especially careful to …

- Look for error sources while you measure, and consider what kinds of error they are! (random, systematic, or modeling error)
- Look for ways to reduce errors as you notice them then improve your technique and setup
- Record your data with proper units and the proper number of significant figures
- Write your results in standard format! (*Mean ± Standard Error, or whichever is appropriate*, e.g., 1.1 ± 0.2 kg)
- Get your error propagation classroom checkpoint approved before class ends

What to Turn In

Each group should submit this worksheet as a pdf with measurements, calculations and question answers added to the lab manual. Each student in a group will be responsible for one Scenario's questions and analysis, as managed by the group's Communicator. Decide which group member's data set will be used for each Scenario and justify. The Improved Model analysis can be divided within the group, with attribution.

Measuring the Period of a "Simple" Pendulum

Theory

A simple pendulum consists of **a point mass *m* suspended on a massless string of length *L*.** The string is connected to a fixed point above the mass (the ***pivot***). The only forces on the mass are gravity and the string tension. The period of this simple pendulum for small oscillations is

$$T = 2\pi\sqrt{L/g} \tag{1}$$

In real pendula, the string always has a mass, but as long as the mass of the string is much smaller than the point mass, it will be very close to the idealistic case of the simple pendulum model. For large angles, the period will be different from the predictions of the simple model. At the other extreme when mass is evenly distributed along the pendulum, its period can deviate significantly from a simple pendulum. Then, a more complicated model of physical pendulum would be required.

Equipment

- String – embroidery thread, dental floss or other non-stretchy string you own
- Metal ball, countersunk hole
- Tape
- Experiment Setup A for the ball pendulum (Prelab, Scenarios 1 and 2)
- Stopwatch
- Protractor
- Ruler with hole (if already own)
- Experimental Setup B for the ruler pendulum (Scenario 3)

Experimental Setup

You will be measuring the period of the pendulum for three different scenarios (two distinct setups):

Setup A		**Setup B**
Scenario 1	**Scenario 2**	**Scenario 3**
String Metal ball Amplitude: 10°	String Metal ball Amplitude: 45°	Ruler Amplitude: 10°

For Setup A, the pendulum length measured from the pivot to the ball's *center of mass* should be between 35 and 50 cm, roughly the same length as the ruler.

Experimental setup design details:

Protractor: CAREFULLY remove the manganin wire and 330 ohm resistors from the back, taping to the sandpaper covering to keep from getting lost and label. Trim the protractor's straight edge along the line as evenly as possible.

Pendulum assemblies: each student's experimental setup will be unique. Two different experimental setups will be used in Intro Lab 2, one for the ball pendulum (Prelab, Scenarios 1 and 2) and one for the ruler pendulum (Scenario 3). Students will use kit components and common household materials.

Design considerations for Setup A (string/ball pendulum) layout include:

- String, non-stretchy. Embroidery floss in the kit, dental floss, and thin braided fishing line are all suitable. Thread the string through the ball hole and continue knotting on the countersunk side until the string doesn't slip through the hole with a brisk tug.
- Attachment of the string. The kit dowel held securely in place, a curtain rod, a cabinet knob, a shower curtain rod, two blocks of wood clamped would all work. The setup should be non-flexing, suitable to easy string attachment, and situated for a side view of the pendulum.
- Pivot point. A tight knot held securely in place during the pendulum oscillations works well.

    - The pendulum ball plane can precess during oscillations if pivot movement is possible. You can tape the string around the pivot to its attachment.
- Ball release methodology. Try not to impart an impulse or release unevenly. Takes practice!

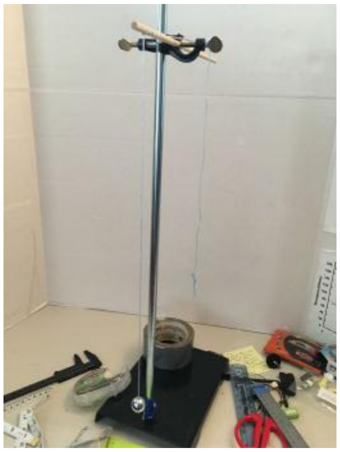

Figure 1a Setup A Example

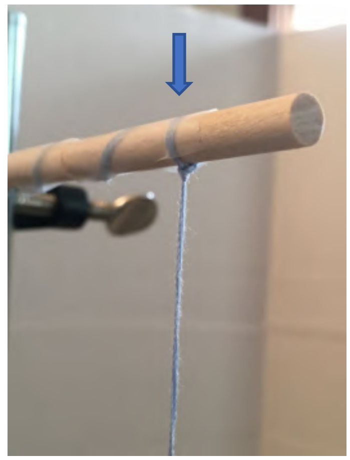

Figure 1b Setup A string pivot point, secured with Scotch tape

Design considerations for Setup B (ruler/rod) layout include:

- Ruler should be rigid with a hole near one end
- Rod supporting the ruler through the hole should let ruler easily rotate. The kit wooden dowel is acceptable, if it fits.

Figure 2 Setup B Example

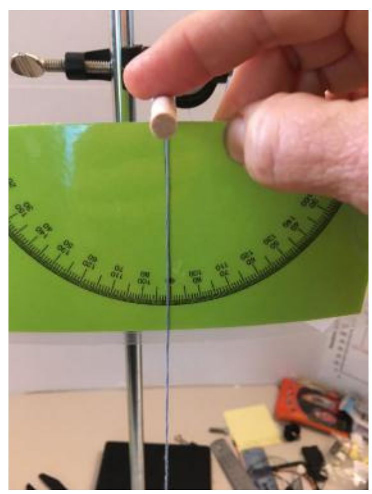

Figure 3 Setup A equilibrium position

Prelab

The prelab introduces you to the Scenario 1 experimental procedure and data collection methodology you'll use in class**.**

Task 1: Construct Setup A and submit a picture
Task 2: For your Setup A, measure the pendulum length as specified, and submit the requested information
Task 3: Measure the Scenario 1 pendulum period as specified, and submit the requested information

Task 4: Measure the Scenario 2 pendulum period as specified, and submit the requested information

Fill the tables in this section with your measurement trials and calculate the requested statistics, then submit the entire prelab as a pdf.

Task 2 detail: Measurement of pendulum length. Measure and record the length of the pendulum from the pivot to the center of the ball. Measure at least 2 times, making sure each measurement is independent, and record in the table below.

Pendulum Length Setup A, Scenario 1 (String; Metal Ball) 10° Amplitude		
Trial #	**Pendulum Length, L_1 (units: ______)**	**Notes**
1		
2		

Mean of length, $\overline{L_1}$ =

Standard Deviation $\sigma =$

Standard Error $\alpha =$

Reported result for length of pendulum with error, L_1=

Task 3 experimental procedure:

a) Let the string/ball pendulum come to rest. This is the equilibrium position.
b) Use the protractor to measure 10° from equilibrium, raise the mass until the ball is at that angle, and release.
c) Wait a couple of oscillations for the initial wobbles to dampen out. Pick one of the extremal positions of the pendulum, and start the timing with your stopwatch. Keep timing for the required number of periods, then stop your stopwatch at the end of the last period.
d) Record this time in the appropriate table
e) Repeat as specified, using the same procedure (start pendulum from rest, same # of skipped initial oscillations) for each repetition.

Task 3 requested data: Measure and record single and multiple periods as follows:

***a) Single periods, Scenario 1 (10° Amplitude*)**:** Using the Task 3 experimental procedure above, measure three distinct single period times, starting from the equilibrium position for each trial. Record the data in the table below.

Single-Period Time Measurements Setup A, Scenario 1 (String; Metal Ball) 10° Amplitude		
Trial #	**Single Period Time, $T_1^{\text{1-period}}$ (units: ______)**	**Notes**
1		

2		
3		

Mean of Single-Period Time, $\overline{T_1^{\text{1-period}}}$ =

Standard Deviation $\sigma =$

Standard Error $\alpha =$

Reported result for single-period time with error, $T_1^{\text{1-period}}$=

b) Multiple periods, Scenario 1 (10° Amplitude): Using the Task 3 experimental procedure above, measure the time it takes the pendulum to swing 10 full periods once.

Ten-Period Time Measurements Setup A, Scenario 1 (String; Metal Ball) 10° Amplitude		
Trial #	**Ten-Period Time,** $T_1^{\text{10-period}}$ **(units: ______)**	**Notes**
1		

Reported result for ten-period time with error, $T_1^{\text{10-period}}$=

Task 4: Multiple periods, Scenario 2 (45° Amplitude): Using the Task 3 experimental procedure above, measure the time it takes the pendulum to swing 10 full periods once.

Ten-Period Time Measurements Setup A, Scenario 2 (String; Metal Ball) 10° Amplitude		
Trial #	**Ten-Period Time,** $T_1^{\text{10-period}}$ **(units: ______)**	**Notes**
1		

Reported result for ten-period time with error, $T_2^{\text{10-period}}$=

Did you encounter any difficulties taking a measurement for Scenario 2, and is so, what?

Measuring the Period of a “Simple” Pendulum – In Class

Experimental Setups – In Class

You will be measuring the period of the pendulum for three different scenarios (two distinct setups).

Setup A		**Setup B**
Scenario 1	**Scenario 2**	**Scenario 3**
String Metal ball Amplitude: 10°	String Metal ball Amplitude: 45°	Ruler Amplitude: 10°

For Setup A, the pendulum length measured from the pivot to the ball’s *center of mass* should be between 35 and 50 cm, roughly the same length as the ruler.

Record instrument precision error. Before doing anything else, record the precision errors of your measurement instruments:

Instrument	**Precision Error**
Ruler	**+- 0.05 cm**
Tape measure	**+- 0.05 cm**
Digital Stopwatch	**+- 0.01 cm**
Protractor	**+- 0.25º**

Procedure and Data Collection

The overall in-class experimental procedure will be the similar for each Scenario and a general outline is provided below. It expands on the Prelab procedures to produce accurate and precise results. Your group will develop and document measurement procedure as implemented for each Scenario.

Determine the experimental procedure. Throughout the Physics 5 lab series, we will ask you to formulate an experimental procedure producing precise and accurate experimental results.

- **A useful trick:** in this class, we’ll use a useful trick to increase the precision - measuring multiple repetitions of the period at one time, then dividing the results by the number of repetitions to obtain your measured value for a single period. ***Be careful to calculate the proper single period time, remembering that the total number of significant figures will stay the same.***

The general experimental procedure for measuring the period throughout Intro Lab 2 *(same as Prelab Task 3, multiple periods)* is:

a) establish the equilibrium position of the pendulum,
b) raise the mass to the appropriate angle (10° or 45°) using the protractor, then release,
c) Wait a couple of oscillations for the initial wobbles to dampen out. Pick one of the extremal positions of the pendulum, and start the timing with your stopwatch. Keep timing for **<u>10 full periods</u>**, then stop your stopwatch at the end of the last period.
d) Record this time in the appropriate table

e) Repeat as specified, using the same procedure (start pendulum from rest, same # of skipped initial oscillations) for each trials.

Do preliminary analysis. Analyzing data as you go can help you to catch mistakes made while collecting data, and save you time if you need to repeat your data collection for some reason. Record data as specified in the scenario, then do a preliminary analysis of your data (mean of the single period, standard error of the single period) when requested.

Scenario 1: String, Metal Ball, 10° Amplitude

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1. **Measure pendulum length.** Measure and record the length of the pendulum from the pivot to the center of the ball. Measure at least 3 times, making sure each measurement is independent, and record in the table below.

Pendulum Length Setup A, Scenario 1 (String; Metal Ball) 10° Amplitude		
Trial #	**Pendulum Length, L_1 (units: cm)**	**Notes**
1	46.55 +- 0.05	
2	46.67 +- 0.05	
3	46.63 +- 0.05	

Mean of length, $\overline{L_1}$ = 46.61 +- 0.05

Standard Deviation $\sigma =$ 0.06 +- 0.05

Standard Error $\alpha =$ 0.04 +- 0.05

Reported result for length of pendulum with error, L_1= 46.61 +- 0.04 cm

2. ***Determine the experimental procedure.*** Throughout the Physics 5 lab series, we will ask you to formulate an experimental procedure. The group should decide on a procedure **each** of you will use to obtain 6 trials of 10-period oscillations, and write it in the space provided below. Take a picture of one setup and insert in the worksheet, appropriately labeled.:

Step 1: Set up the pendulum using the protractor at 10º. Knowing the length of the pendulum and the mass of the ball, the only controlled variable remaining is the angle.
Step 2: Prepare stopwatch. Let go of ball without giving it initial impulse. Let the pendulum stabilise, to reduce the possible effects of a bad release.
Step 3: Once the ball has reached a stable oscillation, start the stopwatch when the ball reaches maximum height or amplitude.
Step 4: 10 periods later, stop the stopwatch. Make sure the oscillations have been counted properly.
Step 5: Once the measurement is done, let the pendulum come to rest until the ball is still in its lowest position. After this, repeat steps 1-5 until there are 6 data points.

3. ***Measure and record.*** Use the stopwatch to time the 6 trials of 10-period oscillations. Record your data in the tables below or on a group Google document while you gather it, along with any experimental notes.
4. ***Do preliminary analysis.*** For each 6 full swing trial in the table, divide the results by 10 to obtain your measured value for a single period. ***Be careful to calculate the proper error on these single periods, remembering that the total number of significant figures stays the same.***

Pendulum Period Data and Results **Setup A, Scenario 1 (String; Metal Ball)** **10° Amplitude**			
Trial #	**Ten-Period Time, $10T_1$** (units: __s__)	**Single-Period Time, T_1** (units: __s__)	**Notes**
1	13.31 +- 0.01	1.331 +- 0.01	
2	13.27 +- 0.01	1.327 +- 0.01	
3	13.28 +- 0.01	1.328 +- 0.01	
4	13.19 +- 0.01	1.319 +- 0.01	
5	13.25 +- 0.01	1.325 +- 0.01	
6	13.21 +- 0.01	1.321 +- 0.01	

Mean of Single-Period Time, $\overline{T_1}$ = 1.325 +- 0.01

Standard Error $\alpha =$ 0.002 +- 0.01

Standard Deviation $\sigma =$ 0.004 +- 0.01

Reported result for single-period time with error, T_1= 1.325 +- 0.002 s

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Scenario 2: String, Metal Ball, 45° Amplitude

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1. ***Measure pendulum length.*** If you have not adjusted or disassembled the setup from Scenario 1, you may simply write "See Pendulum Length Data for Setup A from Scenario 1". If it has been adjusted or disassembled for some reason, you must remeasure the pendulum length here:

Pendulum Length **Setup A, Scenario 2 (String; Metal Ball)** **45° Amplitude**		
Trial #	**Pendulum Length, L_2** (units: ______)	**Notes**
1		
2	See Pendulum Length Data for Setup A from Scenario 1	
3		

Mean of length, $\overline{L_2}$ =

Standard Deviation $\sigma =$

Standard Error $\alpha =$

See Pendulum Length Data for Setup A from Scenario 1

Reported result for length of pendulum with error, L_2=

2. ***Determine the experimental procedure.*** The group should decide on a procedure **each** of you will use for this measurement and write it in the space provided below. Discuss any experimental obstacles encountered that went into formulating the plan. Take a picture of one setup and insert in the worksheet, appropriately labeled. Be sure to view the pendulum from different directions when designing your procedure:

Step 1: Set up the pendulum using the protractor at 45º. Knowing the length of the pendulum and the mass of the ball, the only controlled variable remaining is the angle.
Step 2: Prepare stopwatch. Let go of ball without giving it initial impulse. Let the pendulum stabilise, to reduce the possible effects of a bad release.
Step 3: Once the ball has reached a stable oscillation, start the stopwatch when the ball reaches maximum height or amplitude.
Step 4: 10 periods later, stop the stopwatch. Make sure the oscillations have been counted properly.
Step 5: Once the measurement is done, let the pendulum come to rest until the ball is still in its lowest position. After this, repeat steps 1-5 until there are 6 data points.
Experimental obstacles we encountered: when we were thinking about whether to start the stopwatch right after release or to wait a bit more for the pendulum to stabilise. Since the starting angle was 45º, we decided to give the pendulum an extra two periods to stabilise. Also, we decided to ensure that the pendulum swings in the same plane as in the one we measure the angle with, so that the amplitude was the real one.

3. ***Measure and record the data. Do preliminary analysis.***

Pendulum Period Data and Results Setup A, Scenario 2 (String; Metal Ball) 45° Amplitude			
Trial #	**Ten-Period Time, $10T_2$ (units: s)**	**Single-Period Time, T_2 (units: s)**	**Notes**
1	13.50 +- 0.01	1.350 +- 0.01	
2	13.43 +- 0.01	1.343 +- 0.01	
3	13.48 +- 0.01	1.348 +- 0.01	
4	13.45 +- 0.01	1.345 +- 0.01	
5	13.38 +- 0.01	1.338 +- 0.01	
6	13.45 +- 0.01	1.345 +- 0.01	

Mean of Single-Period Time, $\overline{T_2}$ = 1.345 +- 0.01

Standard Deviation $\sigma =$ 0.004 +- 0.01

Standard Error $\alpha =$ 0.001 +- 0.01

Reported result for single-period time with error, T_2= 1.345 +- 0.001 s

4. ***Compare results to theory.*** Use the measured pendulum length and about $g = 9.80$ m/s^2 to calculate the theoretically expected "simple pendulum" period, then compare to your experimental results. Decide whether your results agree or disagree with the theory. Ignore any error in the theoretical expectation at this point (which results from error in the length)—we will learn how to deal with that in the Analysis section using error propagation.

Pendulum Length: ***Experimental Result from Scenario 2***	**Pendulum Period:** ***Theoretical Expectation*** ($T = 2\pi\sqrt{L/g}$)	**Pendulum Period:** ***Experimental Result (with Error) from Scenario 2***
46.61 +- 0.05 cm	**1.3703 s**	**1.345 +- 0.001 s**

Do they agree?

☐ Yes ☑ No

Scenario 3: Ruler, 10° Amplitude

1. ***Measure pendulum length***, from the pivot to the middle of the ruler (we'll assume this is the center of mass).

Pendulum Length Setup B, Scenario 3 (Ruler) 10° Amplitude		
Trial #	**Pendulum Length, L_3 (units: cm)**	**Notes**
1	**15.25 +- 0.05**	
2	**15.31 +- 0.05**	
3	**15.28 +- 0.05**	

Mean of length, $\overline{L_3}$ = 15.27 +- 0.05

Standard Deviation $\sigma =$ 0.15 +- 0.05

Standard Error $\alpha =$ 0.09 +- 0.05

Reported result for length of pendulum with error, L_3= 15.27 +- 0.09 cm

2. ***Determine the experimental procedure.*** The group should decide on a procedure <u>**each**</u> of you will use for this measurement and write it in the space provided below. Discuss any experimental obstacles encountered that went into formulating the plan. Take a picture of one setup and insert in the worksheet, appropriately labeled. Be sure to view the pendulum from different directions when designing your procedure:

Step 1: Set up the pendulum using the protractor at 10º. Knowing the length of the ruler, the only controlled variable remaining is the angle.
Step 2: Prepare stopwatch. Let go of ruler without giving it initial impulse. Let the pendulum stabilise, to reduce the possible effects of a bad release.
Step 3: Once the ruler has reached a stable oscillation, start the stopwatch when the ruler reaches maximum height or amplitude.
Step 4: 10 periods later, stop the stopwatch. Make sure the oscillations have been counted properly.
Step 5: Once the measurement is done, let the pendulum come to rest until it is still in its lowest position. After this, repeat steps 1-5 until there are 6 data points.
IMPORTANT: Perform the measurement of the angle while looking at the pendulum from an angle directly perpendicular to the plane of oscillation of the plane.

3. *Measure and record the data. Do preliminary analysis*

Pendulum Period Data and Results Setup B, Scenario 3 (Ruler) 10° Amplitude			
Trial #	**Ten-Period Time, $10T_3$** **(units: s)**	**Single-Period Time, T_3** **(units: s)**	**Notes**
1	8.83 +- 0.01	0.88 +- 0.01	
2	8.73 +- 0.01	0.87 +- 0.01	
3	8.78 +- 0.01	0.87 +- 0.01	
4	8.81 +- 0.01	0.88 +- 0.01	
5	8.88 +- 0.01	0.88 +- 0.01	
6	8.76 +- 0.01	0.87 +- 0.01	

Mean of Single-Period Time, $\overline{T_3}$ = 0.879 +- 0.01

Standard Error $\alpha =$ 0.002 +- 0.01

Standard Deviation $\sigma =$ 0.005 +- 0.01

Reported result for single-period time with error, T_3= 0.879 +- 0.002

Scenarios 1-3: Experimental Errors

Throughout the Physics 5 lab series, we will ask you to discuss the observed errors encountered in the experimental design and procedures.

For each Scenario, briefly describe in the space provided possible sources of error you observed during the course of your experiment. Note whether they are random, systematic, or model errors—with a brief explanation why you think so—and describe concrete ways that you adjusted the experimental procedures, experimental design, or the theory to reduce the errors.

Next, identify which error you think was the dominant source of error i.e. random, systematic, or modeling in each Scenario. Answer how did that error manifest itself in your experimental results? (e.g., did it make the uncertainty bigger?) If systematic, would could be done to eliminate it

Scenario 1

What are the possible sources of error encountered, identify the type with an explanation, and describe how you adjusted the design to reduce the errors?

The possible sources of error we encountered are:
1. Giving the ball an initial velocity, that might affect the overall period of motion. This error is a controllable error, and it can be minimized, since it depends on the will of the releaser.
2. Reaction time, not waiting enough time to start the stopwatch, and having a time difference between the actual even and the measured time, which is a controllable error.
3. Measuring the wrong angle, by placing the protractor in a different plane to that of the pendulum's oscillations. This is a controllable error too. We tried to fix all of these by reducing them to the maximum we could. Also, looking at the pendulum from different points of view helped.

What is the dominant source of error and how does it manifest itself in the results?

The main source of error is the stopwatch uncertainty and the reaction time, which manifests itself in the huge variety of results the different people got, meaning that different people have different reaction times.

Scenario 2

What are the possible sources of error encountered, identify the type with an explanation, and describe how you adjusted the design to reduce the errors?

The possible sources of error we encountered are:
1. Giving the ball an initial velocity, that might affect the overall period of motion. This error is a controllable error, and it can be minimized, since it depends on the will of the releaser.
2. Reaction time, not waiting enough time to start the stopwatch, and having a time difference between the actual even and the measured time, which is a controllable error.
3. Measuring the wrong angle, by placing the protractor in a different plane to that of the pendulum's oscillations. This is a controllable error too. We tried to fix all of these by reducing them to the maximum we could. Also, looking at the pendulum from different points of view helped.

What is the dominant source of error and how does it manifest itself in the results?

The main source of error is the stopwatch uncertainty and the reaction time, which manifests itself in the huge variety of results the different people got, meaning that different people have different reaction times.

Scenario 3

What are the possible sources of error encountered, identify the type with an explanation, and describe how you adjusted the design to reduce the errors?

The possible sources of error we encountered are:
1. Giving the ball an initial velocity, that might affect the overall period of motion. This error is a controllable error, and it can be minimized, since it depends on the will of the releaser.
2. Reaction time, not waiting enough time to start the stopwatch, and having a time difference between the actual even and the measured time, which is a controllable error.
3. Measuring the wrong angle, by placing the protractor in a different plane to that of the pendulum's oscillations. This is a controllable error too. We tried to fix all of these by reducing them to the maximum we could. Also, looking at the pendulum from different points of view helped.

What is the dominant source of error and how does it manifest itself in the results?
Here, the main source of error was the fact that our pendulum system wasn't ready for a rotational pendulum. That is, we had to take torque and moment of inertia into account too or our result wouldn't be realistic. Furthermore, the other main source of error is the stopwatch uncertainty and the reaction time, which manifests itself in the huge variety of results the different people got, meaning that different people have different reaction times.

Justification for best group datasets

Discuss which Scenario 1, 2 and 3 dataset(s) will be used for the Analysis parts 2-4, and the reasons for choosing them.

1 is small angle, ruler is inaccurate because of torques and 2 is too big of an angle if we dont take into account the taylor approx for sin(alpha)

Analysis

Propagating error for derived quantities and testing for agreement

1. Symbolically propagate error from theory

The aim of most experiments is to calculate a single quantity of interest. Generally, we measure one or more quantities directly and then use them to calculate the desired quantity. Each of the measured quantities possesses an associated uncertainty, with the error in the final calculated quantity of interest determined through "error propagation". For a function $q(x, y)$ with uncertainties α_x and α_y, the propagated uncertainty for q is given by

$$\alpha_q = \sqrt{\left(\frac{\partial q}{\partial x}\alpha_x\right)^2 + \left(\frac{\partial q}{\partial y}\alpha_y\right)^2}.$$

A full explanation can be found in the Physics 5BL Statistical Review and Hughes and Hase Chapter 4.

So far, we are only considering the period of a simple pendulum for small oscillations:

$$T = 2\pi\sqrt{L/g}, \tag{1}$$

where T is the period, L is the length of pendulum, and g is the acceleration due to gravity. Solve for g as a function of T and L, then use error propagation find the fractional error α_g/g as a function of the period T, length L, and their errors α_T and α_L.

Write your work here (Classroom check):

This is the work I did in class for Part 1 of the Analysis. There is a 4 missing in the 4th line.

From Eq (1), we know:

$$T = 2\pi\sqrt{\frac{L}{g}} \rightarrow \left(\frac{T}{2\pi}\right)^2 = \frac{L}{g} \rightarrow \boxed{g = \frac{L4\pi^2}{T^2}}$$

We also know that

$$\alpha_g = \sqrt{\left(\frac{\partial g}{\partial L}\alpha_L\right)^2 + \left(\frac{\partial g}{\partial T}\alpha_T\right)^2} \quad \cdots\cdots \quad \frac{\partial g}{\partial L} = \frac{4\pi^2}{T^2}$$

$$\alpha_g = \sqrt{\left(\frac{4\pi^2}{T^2}\alpha_L\right)^2 + \left(-\frac{8\pi^2 L}{T^3}\alpha_T\right)^2} \quad \cdots\cdots \quad \frac{\partial g}{\partial T} = -\frac{8\pi^2 L}{T^3}$$

$$\alpha_g = \sqrt{\left(\frac{16\pi^4}{T^4}\alpha_L^2\right) + \left(\frac{64\pi^4 L^2}{T^6}\alpha_T^2\right)} = \sqrt{16\pi^4\left(\frac{\alpha_L^2}{T^4} + \frac{L^2\alpha_T^2}{T^6}\right)}$$

$$= 4\pi^2\sqrt{\frac{1}{T^4}\left(\alpha_L^2 + \frac{L^2\alpha_T^2}{T^2}\right)} = \frac{4\pi^2}{T^2}\sqrt{\alpha_L^2 + \frac{L^2}{T^2}\alpha_T^2}$$

$$= \frac{4\pi^2}{T^3}\sqrt{T^2\alpha_L^2 + L^2\alpha_T^2}$$

Now, find $\alpha_g / g \Rightarrow$

$$\frac{\frac{4\pi^2}{T^3}\sqrt{T^2\alpha_L^2 + L^2\alpha_T^2}}{\frac{4\pi^2}{T^2}L} = \frac{\frac{1}{T}\sqrt{T^2\alpha_L^2 + L^2\alpha_T^2}}{L} = \boxed{\frac{1}{TL}\sqrt{T^2\alpha_L^2 + L^2\alpha_T^2}}$$

2. Compute derived quantity and error from measured quantities

Use Equation (1) to compute g for each of the scenarios from Intro Lab 2. Then use the fractional error $\delta g/g$ you derived in the Analysis Part 1 to compute δg by plugging in g, T, α_T, L, and α_L. Write each result for g in the usual way, as $g \pm \alpha_g$ with proper units and significant figures.

	Scenario 1 (student 1, best group dataset for ball/string)	**Scenario 2 (student 2, same dataset as Scenario 1)**	**Scenario 3 (student 3, best group dataset)**
	String Metal ball Amplitude: 10°	String Metal ball Amplitude: 45°	Ruler Amplitude: 10°
Measured Length (cm)	**44.27 +- 0.12**	**40.8 +- 0.15**	**15.27 +- 0.09**
Measured Period (s)	**1.327 +- 0.007**	**1.35 +- 0.006**	**0.879 +- 0.002**
Derived g_{exp} (m/s^2)	**9.92**	**8.84**	**7.80**
Derived $\alpha_{g_{\text{ex}}}$ (m/s^2)	**0.11**	**0.086**	**0.21**
Result: $g_{\text{exp}} \pm \alpha_{g_{\text{ex}}}$ (m/s^2)	**9.92 +- 0.11**	**8.84 +- 0.086**	**7.80 +- 0.21**
	Results from Allen	**Results from Javier**	**Results from Pablo**

3. Compare derived quantity to accepted value: improved agreement test

Do your experimental values for g match the accepted value using the improved agreement test in equation 2? Given Berkeley's latitude and altitude, the Physics 5BL accepted value for the gravitational acceleration is about $g_{\text{acc}} = 9.80$ m/s^2. Note this has an implied error of $\alpha_{g_{\text{acc}}} = 0.01$ m/s^2, so g_{acc} =9.80 $\pm$ 0.01 m/s^2.

"Agreement Test", improved: Two values $x \pm \alpha_x$ and $y \pm \alpha_y$, which are expected to be equal, are said to reasonably "agree" if

$$|x - y| < 2\sqrt{\alpha_x^2 + \alpha_y^2}, \tag{2}$$

Let's see if our experimental values agree with the accepted value, using the simple pendulum model. Remember $g_{\text{acc}} = 9.80 \pm 0.01$ m/s^2.

	Scenario 1	**Scenario 2**	**Scenario 3**
$\epsilon \equiv \lvert g_{\text{exp}} - g_{\text{acc}} \rvert$ (m/s^2)	**0.12**	**0.96**	**2**
$\alpha_\epsilon \equiv \sqrt{\alpha_{g_{\text{exp}}}^2 + \alpha_{g_{\text{acc}}}^2}$ (m/s^2)	**0.1102**	**0.08657**	**0.21**
Do g_{exp} and g_{acc} agree? (Yes/No) (i.e., is $\epsilon < 2\alpha_\epsilon$?)	**yes**	**no**	**no**

4. Improved model, compare derived quantity to accepted value (split each task between students in group)

Let's investigate whether a refined model for Scenario 2 improves the agreement between $\boldsymbol{g}_{\mathbf{exp}}$ and $\boldsymbol{g}_{\mathbf{acc}}$. Equation (1) is accurate for small pendulum amplitudes only, which is not the case for 45°. The pendulum period correction due to the finite amplitude is (Kleppner, Note 5.1, pg. 199):

$$T = 2\pi\sqrt{L/g}\left(1 + \frac{1}{4}sin^2\left(\frac{\theta}{2}\right) + \cdots\right), \tag{3}$$

Use equation (3) to calculate $g_{\text{exp}}^{\text{corrected}}$(m/s²) in the table. Let $\alpha_{g_{\text{exp}}^{\text{corrected}}} = \alpha_{g_{\text{exp}}^{\text{uncorrected}}}$ (m/s²), to simplify your analysis.

	Scenario 2 String, Metal Ball Amplitude: 45°
Measured period(s)	**1.35 +- 0.006**
$g_{\text{ex}}^{\text{corrected}}$ (m/s²)	**9.50**
$\alpha_{g_{\text{exp}}^{\text{corrected}}}$ (m/s²)	**0.0856**
Result: $g_{\text{ex}}^{\text{corrected}} \pm \alpha_{g_{\text{exp}}^{\text{corrected}}}$ (m/s²)	**9.50 +- 0.0856**
$\epsilon \equiv \left\lvert g_{\text{exp}}^{\text{corrected}} - g_{\text{acc}}\right\rvert$ (m/s²)	**0.3**
$\alpha_\epsilon \equiv \sqrt{\alpha^2_{g_{\text{exp}}^{\text{corrected}}} + \alpha^2_{g_{\text{acc}}}}$ (m/s²)	**0.087**
Do $\boldsymbol{g}_{\mathbf{exp}}^{\mathbf{corrected}}$ and $\boldsymbol{g}_{\mathbf{acc}}$ agree? (Yes/No) (i.e., is $\epsilon < 2\alpha_\epsilon$?)	**No**

Does the improved model resolve the possible ball/string pendulum g discrepancy and if so, why?

Not really, but if it did, it would be because the second order or higher terms account for that discrepancy

Does the "simple pendulum" model adequately describe the ruler pendulum result? If not, what could be done to improve the analysis.

It doesn't, I believe this is because it doesn't take into account the other external forces such as the torques or moment of inertia. To improve this, we could add this to the differential equation where we approximate our found value of g.