Lab Report 6

Tier 2 EM Lab: Magnetism and Faraday's Law

Allen Chen (Manager), Javier Santillan (Skeptic), Pablo Castaño (Analyst)

April 5, 2021

1.1 Introduction/Objectives:

In this lab, our main goal is to gain a deeper understanding of electromagnetic induction. To do so, we perform three experiments. First, using the IOLab's magnetometer, we check the magnetic field-vs-distance relationship for a straight current-carrying wire. Then, we will use the Earth's magnetic field and a rotating IOLab to create a generator. Finally, we will use the IOLab's magnetometer and high-gain amplifier to directly test the validity of Faraday's law. After acquiring all the necessary data for all these experiments, we will analyze it and make sure it fits with the predicted theory of Faraday's Law.

1.2 Theory:

For a current carrying wire, the magnetic field produced by the wire can be derived from Ampere's Law to be

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$$

where $\mu_0 = 4\pi$ x 10^{-7} $T\frac{m}{A}$, $\hat{\theta}$ is the direction tangent to the loop of the field, r is the distance from the wire to the point on the loop, and I is the current in the Wire. The direction of the loop is determined by right hand rule: pointing your thumb in the direction of the current, the direction your thumb curls around in is the direction that the magnetic field loops around the wire.
If the current carrying wire is made into a loop, A magnetic dipole is created. The dipole moment vector $\vec{m}$ is a measure of both the strength and direction of the dipole, the direction of which is determined by curling your fingers in the direction that current flows in the wire, and the direction your thumb points in is the direction of $\vec{m}$. The magnitude is determined by $|\vec{m}| = NIa$. The magnitude of the magnetic field along the axis of the dipole moment vector is then

$$|\vec{B}| = \frac{\mu_0 m}{2\pi r^3}$$

According to Faraday's Law, changing magnetic flux through a loop will induce an EMF

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

For a coil of N loops and area a in a uniform magnetic field. $\Phi_B = Na(\hat{n} \cdot \vec{B}) = NBacos\theta$, where $\hat{n}$ is the direction normal to the loop. The source of the change in flux does not matter for Faraday's law to hold, as it can come from a chang in N, B, or a. As such, in the case where the z component is approximately uniform across the cross section of the loop,

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d(B_z a)}{dt} == Na\frac{dB_z}{dt}$$

and for the case where the field is uniform and the angle between $\hat{n}$ and the field changes with angular velocity ω,

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d(B_z acos\theta)}{dt} == NBa\omega sin\theta$$

.

1.3 Methods:

The equipment necessary for this lab, so for all three experiments, includes:

- IOLab
- N35 Magnet
- 4.5V Battery Holder 3 AAA batteries
- 9V Battery Holder and 1 9V battery
- Scissors and/or sandpaper (to strip magnet wire leads)
- Kit box, taped shut, scotch tape
- Dowel
- Magnet wire, 22 AWG Cu wire with coating
- Breadboard and hookup wires
- Tape measure or ruler or calipers
- Paper
- Spring, long from IOLab kit

Because this lab relies heavily on the IOLab's built in magnetometer, we must calibrate the IOLab to the surroundings where the experiments will be conducted. To do this, we follow the steps provided in the IOLab app under Settings $\longrightarrow$ Calibration $\longrightarrow$ Accel-Gyro-Magnet. After the IOLab has been calibrated, then the experiments can be conducted.

1.3.1 Methods for Experiment 1:

In this experiment we measure the magnetic field produced by the current carrying wire at different distances from the magnetometer on the IOLab. We will need a decently large current so we will connect the 6 ft of magnetic wire across a 9V battery. The overall procedure is:

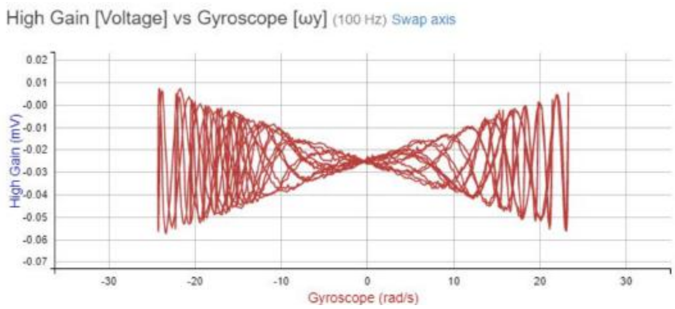
1. Place wire approximately 2.1 cm above the surface that the IOLab is on (using a book or small box) so as to keep it level with the IOLab's magnetometer. Use non magnetic supports or tape to keep the wire in place
2. Use the alligator clip wires to attach the ends of the wire to the breadboard.
3. Attach the battery pack to the breadboard, connecting both ends to the wire. Disconnect one of the ends so as to not drain the battery.
4. Select the Magnetometer on the IOLab. Take data for the magnetic field produced by the current for at least 8 and preferably 10 or 11 different distances to the wire. Try to keep the IOLab's orientation fixed so that the wire is parallel to the front surface of the IOLab.

1.3.2 Methods for Experiment 2:

In this experiment, our goal is to use the magnetic field of the earth as a fixed, uniform external magnetic field and induce an EMF in a coil by changing the coil's relative orientation. This will let us test the validity of Faraday's Law on the classical system described below. To test Faraday's law directly with the IOLab's magnetometer, we must first construct an experimental setup and measure some quantitites. The procedure is shown below:

1. Screw in the IOLab eyebolt. Attach a hook up wire to the 6' magnet wire.
2. Place the hook up wire pin in either G+ or G-, then wrap the wire around the IOLab, following the crack in the x-y plane. Once this is complete, wrap the wire in tape to make sure it is well attached.

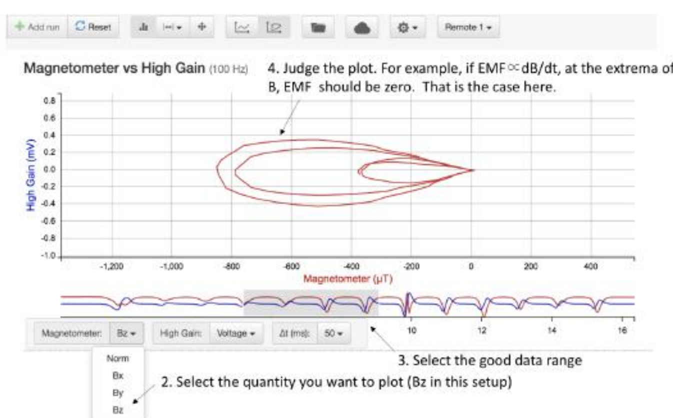
3. Hang your IOLab vertically from a spring hooked on a dowel. Make sure to keep the entire setup away from electronics, and keep the IOLab's magnetometer isolated by placing the dowel on the Lab Kit Box.
4. Let the IOLab rotate/oscillate about the vertical, similar to the torsional pendulum, this will create a rotating coil that can have a magnetic flux.
5. Start recording the magnetometer, gyroscope, and High-Gain as sensors on the IOLab. Do so in a standard way by twisting the IOLab around its vertical a few times and release. Let the oscillation run for at least a full minute. Repeat, until the graphs the IOLab produces are similar to those shown below.



1.3.3 Methods for Experiment 3:

In this experiment, we will be verifying Faraday's Law by using concepts from the previous experiments and building on them with a new experimental setup. The procedure for this experiment is given by the steps below:

1. Make a coil for the EMF measurement using the magnet wire by coiling the wire 20 to 35 times around the dowel. Take the ends of the wire and plug into the G+ and G-inputs on the IOLab.
2. Record the number of times you wind the wire, secure the loop together with tape, and measure its size with a ruler, tape measure, or calipers. Make sure the lengths are accurate, so do several measurements and take an average.
3. Place the coil in the same position as where the IOLab's magnetometer is.
4. Use the kitmagnet, coil, and magnetometer to measure the magnetic field and EMF with your setup.
5. Record the high gain amplifier and magnetometer while moving the magnet. To make data clean, put the magnet directly above the coil, and move it vertically so the signal is in the z direction.



Calculations and Data Reduction/Analysis:

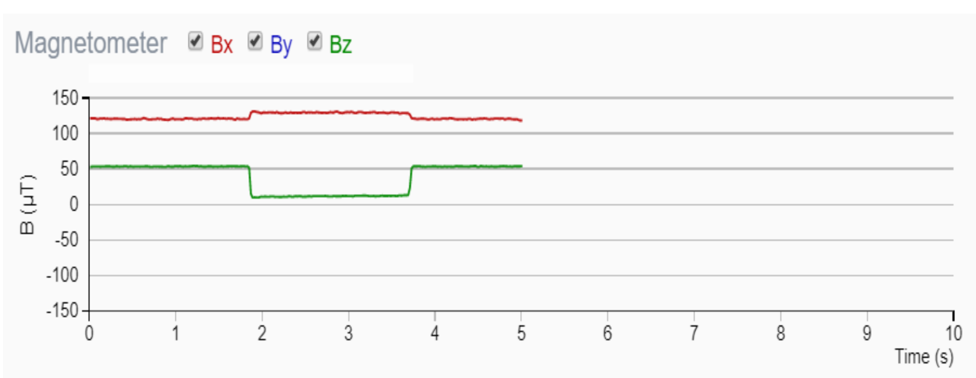
In this section, we analyze the data that we collected over the course of the 3 experiments. In all 3 experiments, we elected to use Allen's data, since he suffered the least technical failures over the course of the lab.

Analysis for Experiment 1:

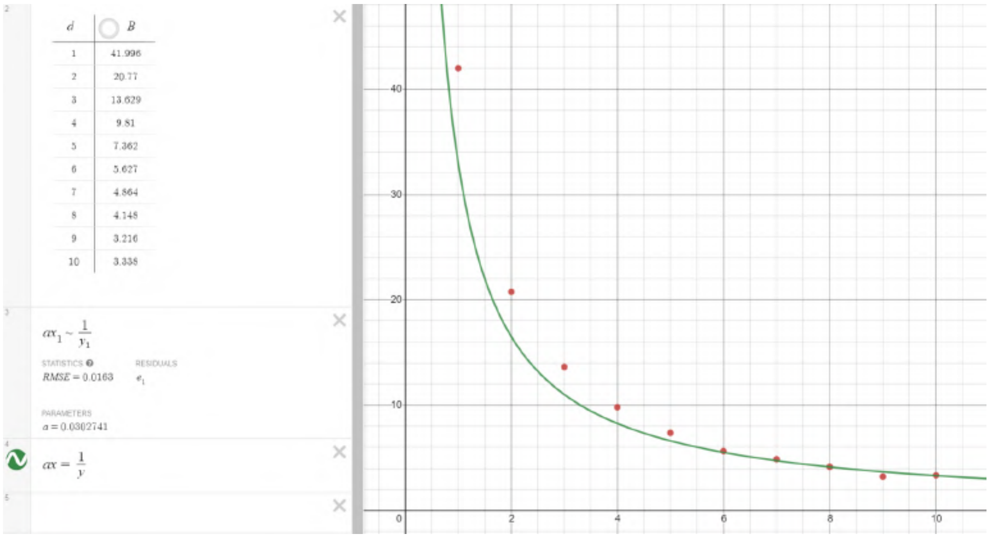
In this experiment, using the IOLab's magnetometer, we checked the magnetic field-vs-distance relationship for a straight current-carrying wire. We obtained some data for the different distances and magnetic fields with the IOLab. In this section, we will be answering some questions about this data and analysing its consequences. These are the questions we have to answer:

- Look at your magnetometer data and assess whether you had a good setup and data run. Explain. Be sure to compare if/how the baseline magnetic field (the ambient field when the current is off) varies with distance.[Hint: Based on the geometry, the magnetic field produced by the wire should be primarily in the z-direction assuming the magnetometer is at the same height as the wire.] What do we do with this? - Create a data table of distance (between the magnetometer and the wire) and the z-component of the magnetic field due to the wire. To get this data you will need to process your raw data, including offsets due to the magnetometer's position in the IOLab and the background magnetic field. - Plot the magnetic field vs the distance and do an appropriate fitting model of the measured field as a function the distance.If linearizing, change the x-or y-axes appropriately and provide a plot with the linearized variables. Your graph should include error bars. From the fitting, determine the value of the current in the wire and the error associated with it.

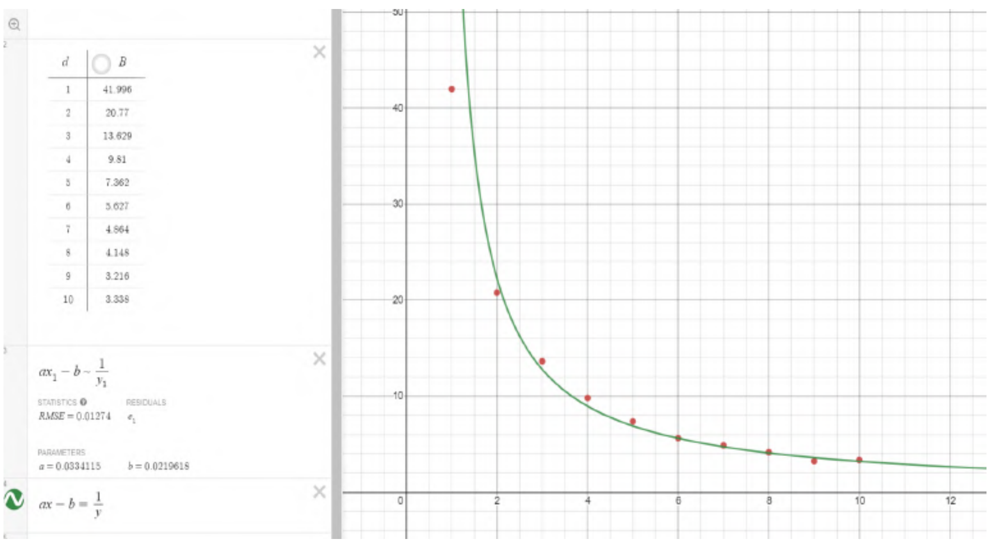
We took the magnetic field at 0 to 9 cm away from the wire at 1 cm intervals. However, we had to offset this value by 1 cm to account for the position of the magnetometer inside the IOLab. We present one reading below, in which the IOLab was 0 cm away from the wire and the magnetometer was 1 cm away from the wire. We did not include the other 9 readings, since they were extremely similar and would be redundant.



We put our data into a chart and fitted it to $\alpha r = 1/B_z$. The results are presented below:



Although there is nothing alarmingly badly fit, this result does not feel very satisfactory. Because of this, we additionally performed the fit $\alpha r - b = 1/B_z$, where the extra $-b$ term is in order to adjust for uncertainty in the approximate location of the magnetometer in the IOLab:

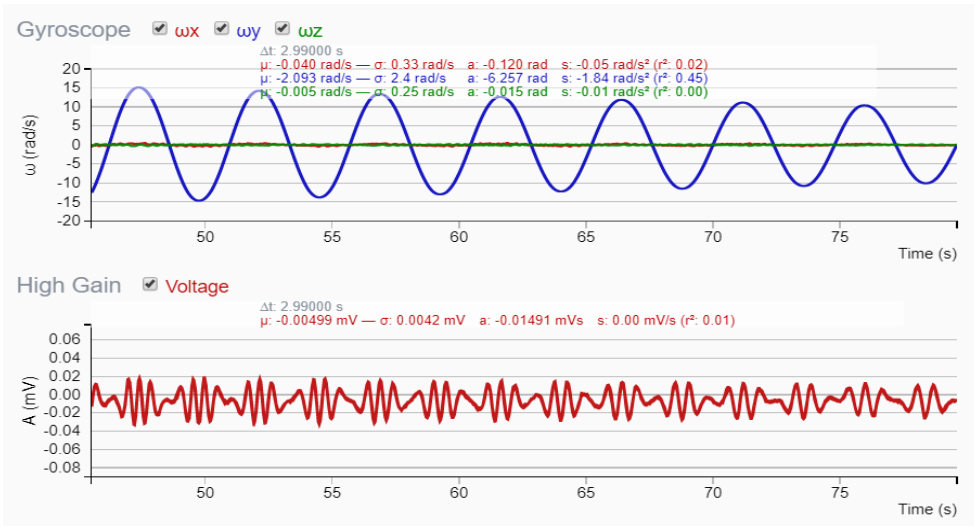


This is marginally better. We proceed with both fits. We can now determine the current running through the wire. We use $\alpha = 2\pi/\mu_0 I$. Adjusting for units, the first fit gives us $I = 1.65$ A and the second fit gives us $I = 1.50$ A.

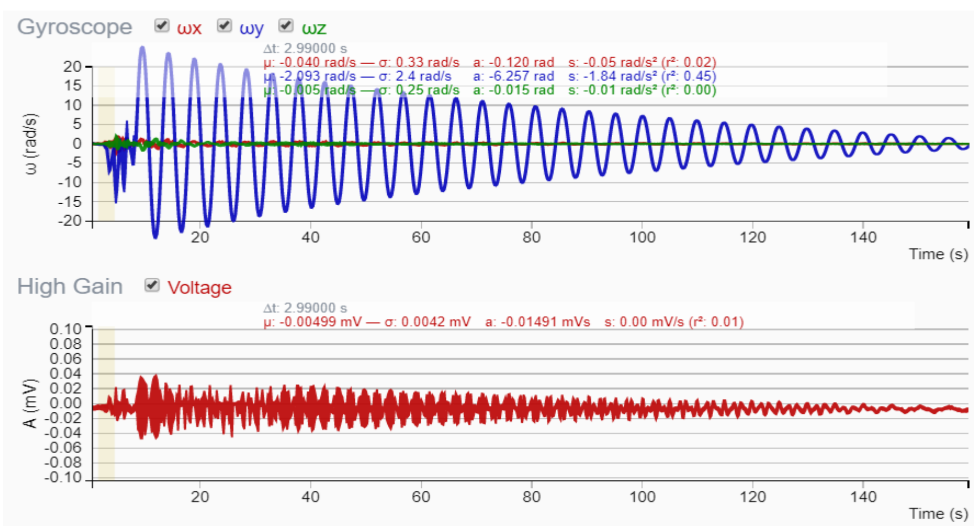
Analysis for Experiment 2:

In this experiment, we will use the Earth's magnetic field and a rotating IOLab to create a generator. We obtained some data, and in this section, we will be answering some questions about this data and analysing its consequences.
These are the questions we have to answer:
-First, do a quick qualitative check. Your High Gain graph should look like a quick oscillation in the envelope of a slower oscillation and decaying amplitude. Qualitatively, how do the times with maximum amplitude of oscillation compare to the angular rotation speed? How do the times with minimum amplitude of oscillation compare? - Next create a parametric plot of High Gain vs Gyroscope. Be sure to only select times after you

started the rotation.This will create a “bow-tie”graph of the type shown below. Use the parametric plot and determine the slope of the lines defining the envelope. From this, determine the magnitude of the component of magnetic field parallel to Earth’s surface at your location. - To compare with the actual value, you can use NOAA’s magnetic field calculator,https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtmligrfwmm. Just enter your latitude and longitude and read the “Horizontal Intensity” (which is given in nT).
The data we collected is summarized visually below:

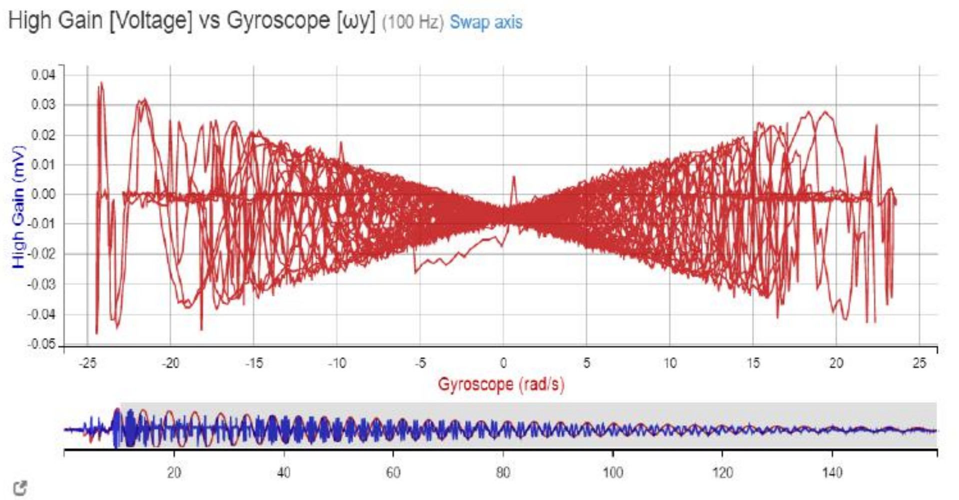


As expected, we can see that the voltage in the wire takes the form of a quick oscillation in the envelope of a slower oscillation. We can additionally look at this on a larger scale to confirm that the amplitude of oscillation decays over time:



Qualitatively, we observe that the oscillation of the amplitude of the oscillation of the voltage through the wire occurs in step with the rotation of the IOLab. In other words, the amplitude of the oscillation of the voltage through the wires is directly tied to the current magnitude of the IOLab’s rotational velocity. The maximum amplitude of oscillation of the voltage through the wire occurs at the maximum angular speed, and the minumum amplitude of oscillation of the voltage through the wire occurs when the angular speed is zero. Furthermore, although it is not very important, we note that the "inner" oscillation occurs 4 times for every "outer" oscillation.

We additionally performed an analysis of the parametric plot of this data. This plot is presented below:



As expected, we see the "bow-tie" shape, although there are slight aberrations which we attributed to random noise. Using a image editing software, we can determine that the slopes of the lines which bound this "bow-tie" are $\pm 1.825 \cdot 10^{-3}$ mVs/rad. Then, $\mathcal{E}_{\text{Amp}} = 1.825 \cdot 10^{-6}\omega$. We also know the number of loops around the IOLab was 6.5. In order to find the area enclosed by the loop, or the area of the face of the IOLab, we measure its sides and the radius of curvature of its rounded corners. Without considering the rounded corners, the IOLab is 13.00 cm by 7.50 cm. The corners have radius approximately 1.3 cm. Then, the area of the face of the IOLab is 96.05 cm^2. Putting these values together into

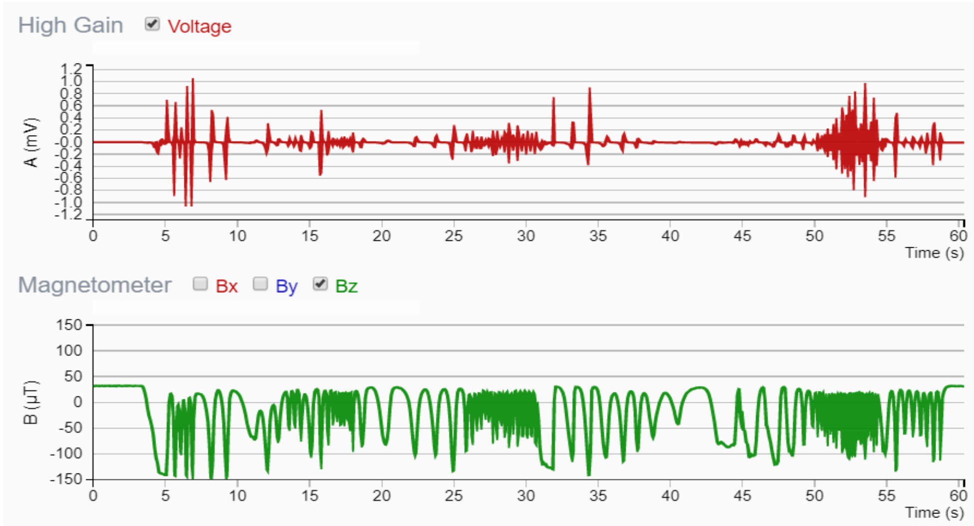
$$\mathcal{E}_{\text{Amp}} = NBa\omega$$

we find that $B = 29,231$ nT. This is reasonably close to the expected value of $23,104$ nT; both values have the same order of magnitude and are roughly similar. However, the two values fail to agree strongly, suggesting that there was some error in the way the experiment was performed. We hypothesize that this was due to capacitance between different loops in the coil.

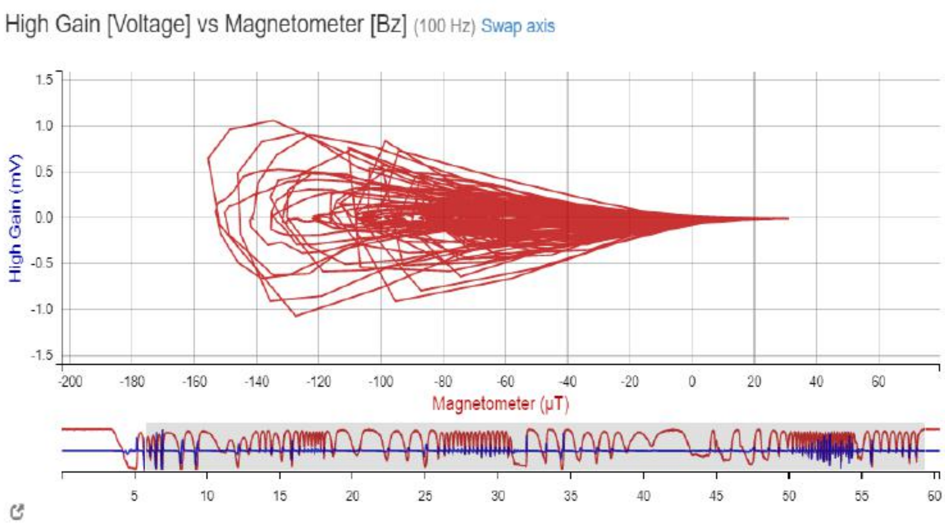
Analysis for Experiment 3:

In this section, we were asked to plot EMF as a function of dB/dt and to perform a linear regression. We were also asked to compare the experimental and theoretical slope of this linear regression.

The data we collected is presented graphically below:



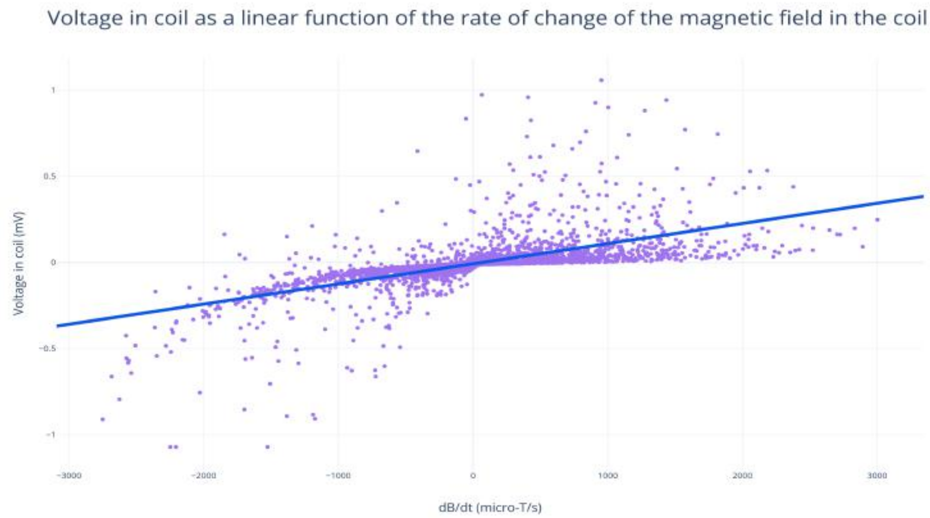
We also present the data in a parametric form below:



We qualitatively confirm that the voltage is 0 as the magnetic field reaches its extremes. Then, we can move on to the linear regression. The results of our regression are presented below:

LINEAR MODEL		
	BEST FIT PARAMETERS	
	m	1.17E-04
	c	-7.69E-03
	COMMON UNCERTAINTY	
	a_cu	7.84E-02
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	1.90E-06
	UNCERTAINTY IN c	1.01E-03
	DELTA	1.03E+13

Our R^2 was 0.386, suggesting that our model was an extremely poor fit for the data. This is confirmed by the plot, which is below:



We can visually see that this fit is very poor. The data does seem to follow some kind of pattern, however, the noise is so severe that it diverges strongly from being linear. Although there is not much hope of our experimental and theoretical slopes agreeing, we proceed with the calculation of the theoretical slope. The coil we used had 116 loops and was 0.75 cm in diameter. From the formula

$$V_{coil} = N\frac{d\Phi}{dt} = NA\frac{dB}{dt}$$

we can calculate the expected slope to be $5.12 \cdot 10^{-3}$. As expected, this is extremely far from our experimental value of $1.17 \cdot 10^{-4}$. These values do not agree, suggesting that there was some error in the way the experiment was performed. The error could be caused by any number of influences: There could have been capacitance between the loops of the magnetic wire, there could have been errors in the way the magnet was moved, there could have been sensor errors due to the magnetometer drifting away from calibration, the coils could have moved around over the magnetometer over time, etc. Ultimately, if we could repeat this experiment we would perform it in a more controlled setting, with a proper solenoid instead of the homemade one we used.

Conclusion:

In this lab, we used a magnetic wire and the magnetoscope on the IOLab to confirm a few laws of magnetism. We first tested Ampere's Law by measuring the magnetic field at several points around a wire with a large current running through it. We were fairly successful, creating two different fits which appeared to give reasonable results. We then tested the Earth's magnetic field by rotating a coil about the z-axis. Our results were roughly accurate, however they did not hold up under close scrutiny. Similarly, our results in the third experiment, in which we tested Faraday's Law using a coil, were not satisfactory. We hypothesize that these experiments would be more successful in a more controlled setting.