Lab Report 5

Tier 2 Mechanics Lab: Standing Waves

Allen Chen (Skeptic), Javier Santillan (Analyst), Pablo Castaño (Manager)

April 5, 2021

1.1 Introduction/Objectives:

In this lab, there were two main parts. In the first one, we had to build an experimental setup that would allow us to experience the physics behind what we will be studying. Also, we will be using the videos that were provided to us to gain a better understanding of standing waves. In the second part, we will explore how the tension and the length of the string affect the formation of the harmonics. By the end, we hope to have performed experiments and analyzed data, ultimately gaining a solid understanding of standing waves.

1.2 Theory:

Throughout this lab, we are looking to observe the behavior of standing waves on a string under different variables. Firstly, it might be helpful to understand what standing waves are. Standing waves on a string are the result of interference from the reflection of the incident wave. This means that since one end of the vibrating string is fixed, the wave bounces back and interferes with the coming wave, creating an interesting pattern. For fixed length and tension of the string, certain frequencies known as ”harmonic frequencies” interfere in just the right way so as to produce standing wave patterns. This is because for some wavelengths, the wave cancels at some points, creating nice patterns. A wave traveling in one direction and its reflected wave can be described by the functions

$$y_i(t) = Asin(\frac{2\pi x}{\lambda} - \omega t) \text{ and } y_r(t) = Asin(\frac{2\pi x}{\lambda} + \omega t),$$

respectively. The resulting wave formed by their interference is therefore their addition, which equals

$$y_c(t) = y_i(t) + y_r(t) = Asin(\frac{2\pi x}{\lambda})cos(\omega t)$$

From this equation, we see that not only is there a time dependence for the resulting wave, but the points where amplitude is 0 (called nodes) occur at $x = \pm(\frac{\lambda}{2})(2k)$, $k = 1, 2, ...$ and the points where amplitude is max occur t $x = \pm(\frac{\lambda}{4})(2k = 1)$, $k = 1, 2, ...$ As such, standing waves occur when the length equals half of the wavelength, or when

$$L = n\frac{\lambda}{2}$$

where n is the number of the harmonic. The speed of any wave is given by $v = f\lambda$. For a string stretched by tension T and with mass-density μ ($\mu = \frac{M}{l}$), the wave speed is given by $v = \sqrt{\frac{T}{\mu}}$, where $\mu = \frac{M}{l}$ and T can be determined from the mass hanging from the string and acceleration due to gravity g. Combining these two equations with the one relating wavelength and length of the string, we can derive the equation $n = 2fl\sqrt{\frac{\mu}{T}}$, where we can see a clear relationship between frequency, string length, and string tension that ultimately affects the number of nodes. By studying this relationship and the others above, we can achieve a full understanding of standing waves, which we will be doing in this lab. However, before going forward, we will show a graphical representation of standing waves that will ease understanding of standing waves in the near future:

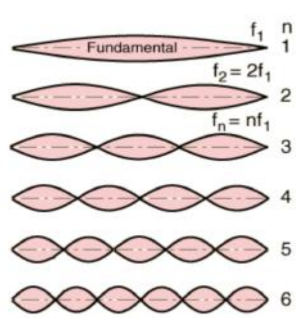


Figure 2: First six harmonics

Methods:

For this lab, we only physically conducted Experiment 0. However, we will still describe the procedure for how we would have conducted each part, including the materials required. Overall, the experimental materials we will need for our setups and/or analysis are for experiment 0:

- IOLab
- String (6m embroidery string or 1m fishing line
- Toothbrush motor
- Tape measurer
- 4.5 V battery holder and ther AAA (1.5 V) batteries
- Table or chair to attach string to or secure mount for dowel
- Dowel (if planning to use with secure mount)
- Wooden block or other mounting for string.
- Breadboard
- Wires
- 2 100 Ω resistors or 1 330 Ω resistor
- Dark background
- Tape
- Weight set or other stabilizer (optional

And for experiments 1-3:

- Tektronix AFG2021 Function Generato
- Pasco String Vibrator
- Pole mount for Pasco String Vibrator
- Meter stick (2m)
- Pulley mounted on a plate
- String (stretchy)
- V arious masses and hangers

Methods for Experiment 0 (Pictures of Setups are included in Appendix):

For this experiment, we measure the tension in the string that results in the maximum amplitude of different harmonics by pulling on the IOLab and observing which value of force results in the greatest amplitude for the standing wave that is created for that much tension in the string. Procedure:

1. Choose between the embroidery string or fishing line. We chose the embroidery string.
2. Securely attach one end of the string to a chair (or dowel) We used a chair because we did not have. secure mount for the dowel.
3. Place dark background so as to clearly see the string oscillate.
4. Attach string to motor by wrapping the string around it.
5. If using embroidery string measure 60 to 70 cm from the from the motor and cur it, or 80 cm if using the fishing line.
6. Place wooden block or some other piece to support the motor and keep it elevated.

7. Use the breadboard to connect the 4.5 V battery pack to the motor, using the 330 Ω resistor or two 100 Ω resistors. We used the 330 Ω resistor.
8. attach the remaining end of the string to the IOLab hook.
9. Turn the battery pack on and pull and push on the IOLab until standing wave patterns are observed.
10. Record using the Force sensor on the IOLab, holding the IOLab steady for several seconds once the maximum amplitude for the observed wave pattern is reached.

Methods for Experiment 1:

For this experiment, the goal is to find the harmonic frequencies for the first 6 or 7 harmonics by varying the frequency until the harmonic frequency has been located, and then recording the frequency and its associated uncertainty. Procedure:

1. Attach one end of the string to the vibrator and the other end to the hanger over the pulley. The mass pulling down on the string should not be more than 200 g and the length from the vibrator to the center of the pulley should be less than 1 m.
2. Connect the signal generator to the vibrator using a long BNC cable and a banana plug-BNC adapter. The adapter should fit perfectly into the vibrator's input plug.
3. Turn on the signal generator, select the sine wave, with the output impedance set to 50 Ω, the amplitude to 10 V_{pp} and the frequency close to 10 Hz.
4. Vary the frequency of the string vibrator until you locate the first 6-7 harmonics. Make sure you record these frequencies and their associated uncertainties.
5. Ensure that the nodes for each harmonic are located at the end with the vibrator and the end at the pulley
6. Find the frequency that that produces the greatest amplitude while maintaining the nodes at proper locations.

Methods for Experiment 2:

Now we will again observe the behavior of standing waves, this time for varying mass while holding the length of the string and the frequency fixed. The goal is to attempt to measure the the tension in the string that produces different standing wave patterns. To do so we would use the same setup as in experiment one, only this time the frequency will be fixed at 30 Hz and the length of the string will be between 1 and 2 m. Procedure:

1. Create same setup as in Experiment 1, this time setting frequency to 30 Hz and the length of the Sting to between 1 and 2 m. Make sure to accurately measure the length of the string to a reasonable level of uncertainty.
2. Estimate the mass needed to produce the second harmonic.
3. Place that mass on the hanger and observe whether or not it does produce the second harmonic. If not, adjust the mass until the second harmonic is produced
4. repeat for the next 5 harmonics.

Methods for Experiment 3:

For this Experiment, the goal is now to measure which lengths produce which harmonics for some fixed tension and frequency. To do so, we again use the setup from the previous two experiments, choosing some reasonable frequency and mass to use that will allow all harmonics to be observed, and then shorten the string by moving the vibrator closer to the pulley until all harmonics are observed. At each harmonic, the length and its associated error should be recorded.

Calculations and Data Reduction/Analysis:

In this section, we analyze the data we collected from experiment 0 as well as the data that was provided in experiments 1, 2, and 3. Therefore, the results we discuss for Experiment 0 are unique to our group, while the other experiments can be standardised. For each section, we will consider the data, and explain why we chose that data.

Analysis for Experiment 0:

This experiment involved using the IOLab to roughly determine the relationship between the tensions in the string which corresponded with each harmonic. All three of us used the embroidery string instead of the fishing line. We collected three datasets, which are summarized below in terms of the string tension, length, and resistance.

A	B	C	D	E	F	G
	RESISTANCE	LENGTH OF STRING (cm)	H-1 TENSION (N)	H-2 TENSION (N)	H-3 TENSION (N)	H-4 TENSION (N)
JAVIER	330	65	2.406	0.938	0	N/A
PABLO	330	70	0.991	0.84	0.486	0.23
ALLEN	200	65	0.881	0.86	0.559	0.909

Visually, out of the three datasets, we chose to analyze and look for patterns in Pablo's dataset, because it had the least noise. Furthermore, the regions where the harmonics were happening where slightly clearer, so it would be easier to find a correlation between n and frequency for this dataset. However, we must take into account that each of us had different lengths and slightly different setups, so we also thought that Pablo's setup resembled the one described in the Lab Manual the most, and decided to go with it for the analysis. Having said this, in the table above we find the different string tensions for the different harmonic numbers (n). When $n = 1$, the tension is about 0.991 N; When $n = 2$, the tension is about 0.840 N; When $n = 3$, the tension is about 0.486 N; and when $n = 4$, the tension is about 0.23 N. We can see that there must definitely be an inversely proportional relationship between the harmonic number and the the tension: the smaller the tension, the higher the harmonic number. However, we aren't entirely sure about this claim, because when we applied very small tensions we couldn't measure the harmonics for 5,6, and 7. If we were to roughly estimate the relationship between n and the tension, we would claim that $T \; \alpha \; \frac{1}{n}$, which matches our values for $n = 1, 4$, and is fairly close for other values. Mathematically, this relationship makes sense too.

Analysis for Experiment 1:

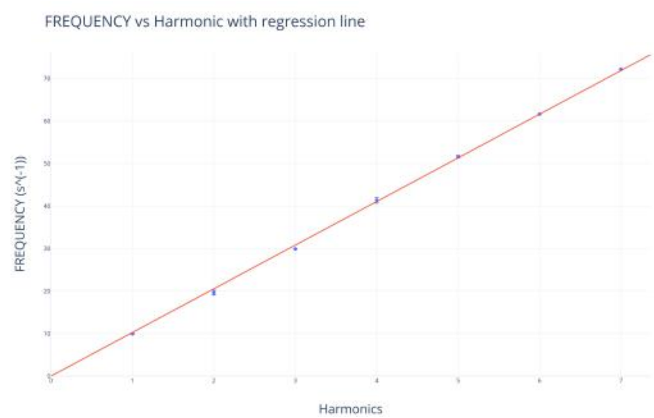
In this part of the analysis, we were asked to look at a video and collect the data from it. Then, we had to analyze that data. Our analysis had to include a graph of the measured resonant frequency vs the harmonic number with a fit to the data using the fact that $n = 2fL\sqrt{\frac{\mu}{T}}$, including error bars, a plot of the normalized residuals, an expected value for the string's linear mass density (and corresponding agreement test), and a chi-squared analysis. Here, we will answer all these different points. Before doing analysis, the table below shows the data that we obtained from the video we were provided with:

Harmonic Number	Min Frequency	Max Frequency
1	9.8	10.1
2	19.1	20.2
3	29.8	30
4	40.7	42
5	51.3	51.9
6	61.6	61.61
7	72	72.31

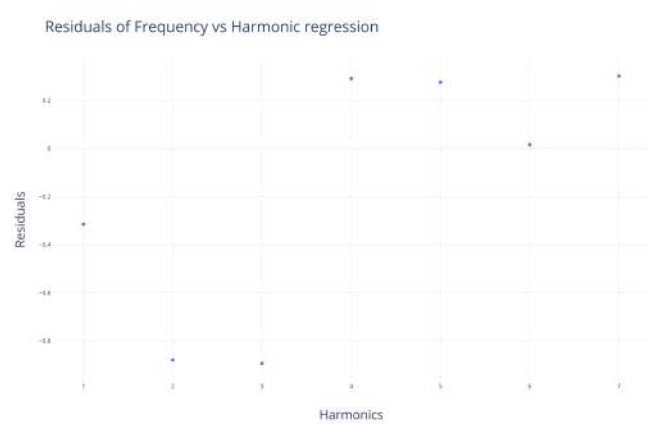
In the data above, we have the maximum and the minimum frequencies at which some harmonic was noted in the experiment. When we plot harmonic number vs resonant frequency, we will actually be plotting harmonic number versus the average between the minimum and the maximum frequency. For example, when n=1, we would have our frequency be 9.95, and there would be some 'error bar' associated to this due to the upper value (10.1) and the lower value (9.8). The results of our regression are shown below:

DIRECT PROPORTIONALITY MODEL		
	BEST FIT PARAMETER	
	m	1.03E+01
	COMMON UNCERTAINTY	
	a_cu	5.66E-01
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	4.79E-02
	CHI-SQUARED	
	CHI-SQUARED	1.03E+02
	REDUCED CHI-SQUARED	1.71E+01

When we plot this using the previously mentioned equation, we find what's shown below:



In the graph above, not only can we see that the errors are relatively small in comparison to the line, but we can also see that the error bars indicate that some frequencies might be above or below the one given by the line of best fit. By measuring this 'deviation' that a value has from the line of best fit, we can find the residuals plot, which is shown below:



In the graph above, we can see that the residuals aren't always the same, they seem to be randomly distributed. We thought that this is perhaps because the more nodes we have, that is the higher the harmonic

number, the higher the frequency, so the errors given by the line of best fit will be larger. Another reason for this is that nothing is intrinsically wrong in the experiment, and all the errors are systemic errors, and we can't change this fact. Now that we have completed the plots, we are ready to take a less graphical and more analytical approach to Experiment 1. To do this, we will first find the expected value for the string's linear mass density (and corresponding agreement test) based on the data we have. We find the theoretical value for the string's mass density using the data we are given in the data sheet: we are given a total length of 292.3 ± 0.14 cm and a total mass of 13.63 ± 0.027 g. Then, the string's theoretical linear mass density is $\mu_{\text{th}} = 4.663 \cdot 10^{-3} \pm 9.5 \cdot 10^{-6}$ kg/m.
Our experimental μ_{exp} and its associated error can be calculated by

$$\mu_{\text{exp}} = \frac{T}{4m^2L^2} \quad \text{and} \quad \alpha_\mu = \sqrt{\left(\frac{T\alpha_m}{2L^2m^3}\right)^2 + \left(\frac{T\alpha_L}{2L^3m^2}\right)}$$

We find that our experimental linear mass density is $\mu_{\text{exp}} = 4.13 \cdot 10^{-3} \pm 4.2 \cdot 10^{-5}$. Now, using the agreement test, we see that these two quantities do not agree. Unfortunately, since we did not perform the experiment, we cannot determine why. However, the residuals do not appear to take a pattern, so the error is not likely to be systematic.

Lastly, we must perform a reduced chi-squared analysis. Our chi-squared was 103 and our reduced chi-squared was 17.1. This is too large to be acceptable. This suggests that we underestimated our errors, or rather, the person who performed the lab underestimated the errors.

Analysis for Experiment 2:

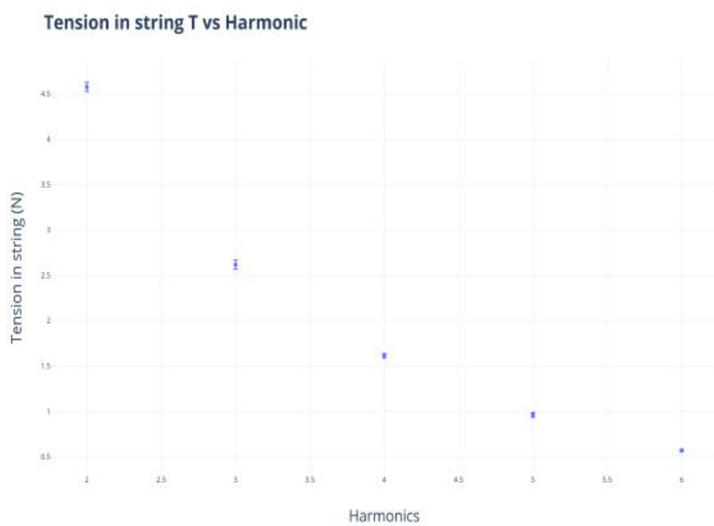
In this experiment, we were given data for the weights associated with each harmonic. The experiment was looking at weights and pulleys, looking for a correlation between the string's tension and the harmonics. In fact, we already guessed this correlation in Experiment 0, but now we will be arriving at mathematically true conclusions. We know the formula

$$n = 2fL\sqrt{\frac{\mu}{T}}$$

However, we readjusted the data to better fit this formula for regression, in order to take into account new parameters such as the weights and pulley. We ultimately performed linear regression on $x = n$ and $y = \sqrt{1/T}$. To do this, we used the formulas

$$y = \sqrt{\frac{1}{9.8 \cdot 10^{-3}(m + m_{\text{str}})}} \quad \text{and} \quad \alpha_y = \left| -\frac{1}{2}(9.8 \cdot 10^{-3}(m + m_{\text{str}}))^{-\frac{3}{2}}\, 9.8 \cdot 10^{-3}\alpha_m \right|$$

Now, if we were to graph the tension against the harmonic number using the equation from the previous experiments, we would get the graph below:

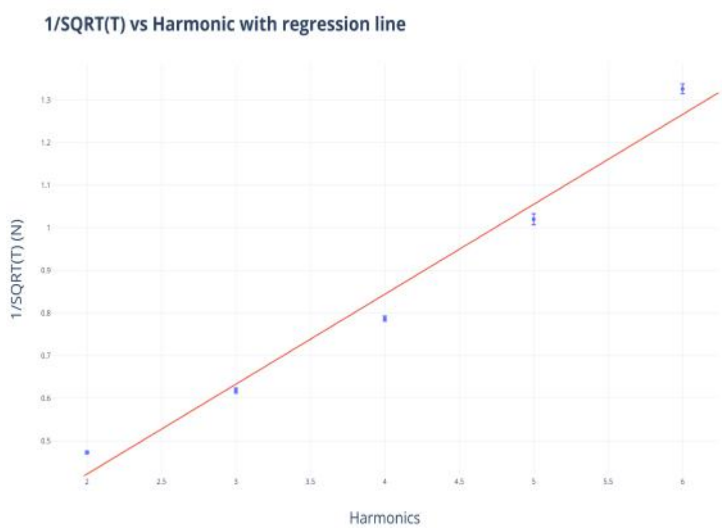


In the graph above, we decided to make our points very small so that the error bars would be visible. However, our real data doesn't have visible error bars. As we mentioned before, in this experiment we will be using different fittings, so our adjusted data is presented in the table below:

n	VARIANCE IN n (NOT USED)	sqrt(1/T)	VARIANCE IN sqrt(1/T)
2		0.473	0.0026
3		0.618	0.0058
4		0.787	0.006
5		1.02	0.013
6		1.326	0.0114

The results of our regression using a direct proportionality model are presented in the purple table below. However, since this might be hard to understand at first glance, we also provide an easy-to-consume graph below the table, as we did for experiment 1.

DIRECT PROPORTIONALITY MODEL		
	BEST FIT PARAMETER	
	m	2.11E-01
	COMMON UNCERTAINTY	
	a_cu	5.22E-02
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	5.50E-03
	CHI-SQUARED	
	CHI-SQUARED	5.14E+02
	REDUCED CHI-SQUARED	1.28E+02



In the model shown above, the fitting parameter m is equivalent to $\frac{1}{2fL\sqrt{\mu}}$. As shown, we found this value to be equal to $0.211 \pm .0055$. The string's theoretical linear mass density was $\mu_{\text{th}} = 4.663 \cdot 10^{-3} \pm 9.5 \cdot 10^{-6}$, as we calculated previously. Our experimental μ_{exp} and its associated error can be calculated by the equations

$$\mu_{exp} = \left(\frac{1}{2fLm}\right)^2 \quad \text{and} \quad \alpha_\mu = \sqrt{\left(\frac{\alpha_m}{2m^3f^2L^2}\right)^2 + \left(\frac{\alpha_L}{2L^3f^2m^2}\right)^2}.$$

We find that $\mu_{\text{exp}} = 3.856 \cdot 10^{-3} \pm 2.0 \cdot 10^{-4}$. This unfortunately does not agree with the theoretical value. Visually, the fit is convincing. However, our χ^2 value was relatively high, and our residuals appear to take a pattern. Unfortunately, since we did not perform the experiment, there is no way for us to determine where the errors came from. One theory is that the tension stretched the string, causing L to increase as the mass used increased. This would decrease the linear mass density.

Analysis for Experiment 3:

In this experiment, we were given data for the string lengths associated with each harmonic. We know the formula

$$n = 2fL\sqrt{\frac{\mu}{T}}$$

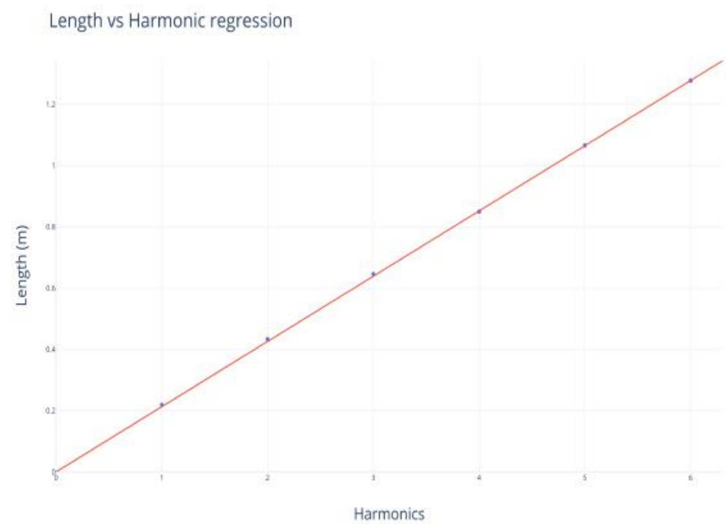
We do not need to perform any adjustments to our data to perform linear regression on this.
The data is presented below:

n	VARIANCE IN n (NOT USED)	LENGTH (m)	VARIANCE IN LENGTH (m)
6		1.2775	0.0018
5		1.066	0.004
4		0.85	0.002
3		0.646	0.0014
2		0.4335	0.0011
1		0.219	0.0007

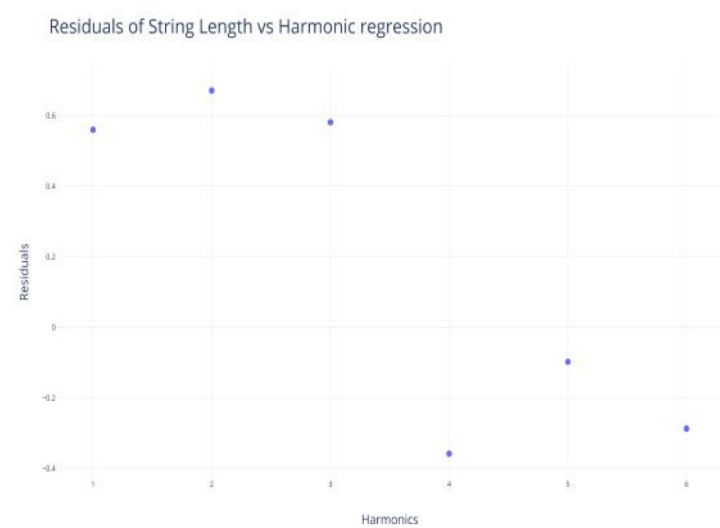
The results of our regression are presented below:

DIRECT PROPORTIONALITY MODEL		
	BEST FIT PARAMETER	
	m	2.13E-01
	COMMON UNCERTAINTY	
	a_cu	5.14E-03
	UNCERTAINTY IN BFPs	
	UNCERTAINTY IN m	5.39E-04
	CHI-SQUARED	
	CHI-SQUARED	1.24E+02
	REDUCED CHI-SQUARED	2.49E+01

These are also presented in a graph below:



The residuals of our regression are also presented in a graph below:



The residuals appear to be randomly distributed, meaning that if there were any errors in our data, they were probably not systematic.
In this model, the fitting parameter m is equivalent to $\frac{1}{2f}\sqrt{\frac{T}{\mu}}$. As shown, we found this value to be equal to $0.213 \pm .0005$. The string's theoretical linear mass density was $\mu_{\text{th}} = 4.663 \cdot 10^{-3} \pm 9.5 \cdot 10^{-6}$, as we calculated previously. Our experimental μ_{exp} and its associated error can be calculated by

$$\mu_{exp} = \frac{T}{4m^2 f^2} \qquad \text{and} \qquad \alpha_\mu = \frac{T\alpha_m}{2m^3 f^2}$$

We were not given errors for the mass or frequency, so our error is not as sophisticated as we would have liked. Nonetheless, we find $\mu_{\text{exp}} = 3.87 \cdot 10^{-3} \pm 1.82 \cdot 10^{-5}$. Unfortunately, this fails to agree with our expected value. Since we did not perform the experiment, there is no way for us to find out what went wrong. However, we can say that the randomness of the residuals suggests that there was no systemic error in the experiment.

Conclusion:

In this lab, we used a motor and the harmonic frequencies of a string to explore the behavior of standing waves. We first built a basic setup to qualitatively confirm a few basic facts about the relationship between the tension in the string and the harmonics at which it vibrated. In this section, we faced many technical problems with setting up the experiment. However, we were able to collect one stable dataset and indeed found that the harmonics were approximately related to the inverse square root of the tension. In the rest of the experiments, we received data sets which showed the relationship between several variables and the harmonics. In these, we used the density of the string as a standard to check how well the experiments conformed to theoretical expectations. However, we were not able to obtain satisfactory results: our experimental values slightly underestimated the correct densities each time. Since we did not do the experiment, we could not figure out what went wrong. However, we hypothesize that the string stretched under the tension, reducing the observed density in all situations. In order to improve the experiment, we propose using a string with less elasticity.

Appendix:

Setups for Experiment 0:   Node Pictures for Experiment 0: