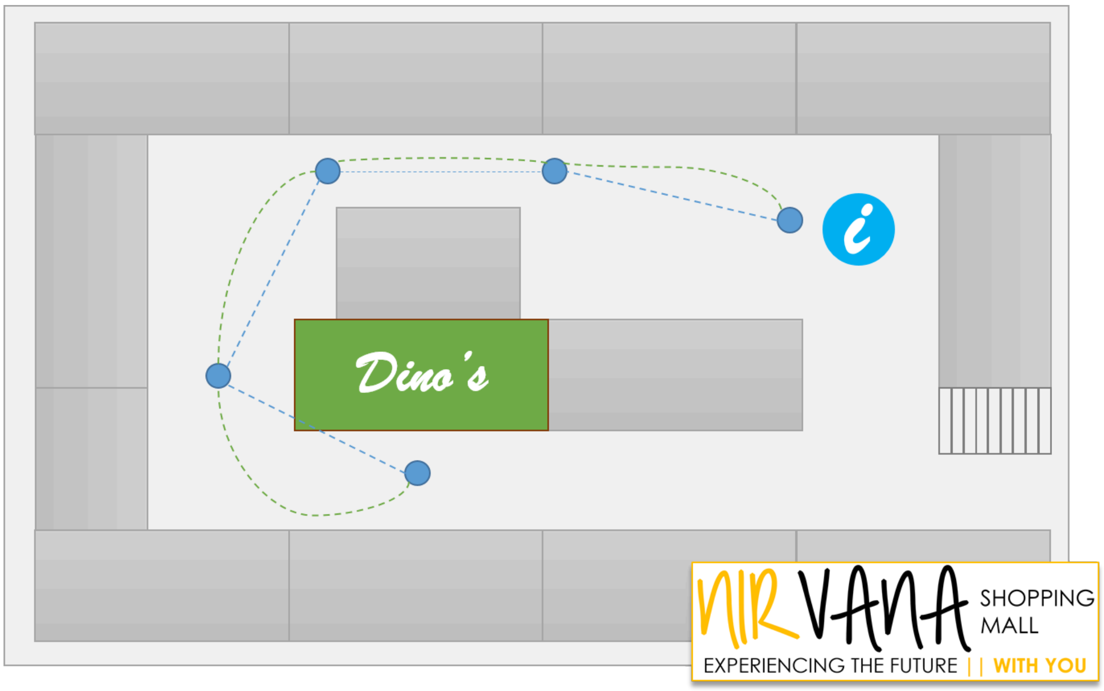
8.2 Motion Planning with Potential Fields - Moving in the mall

After implementing the SLAM algorithm, the robots provided by **UMA-MR** are able to simultaneously build maps of the malls and localize themselves within them. However, the **managers at Nirvana** are looking for a fully operational robot, and something is still missing: the navigation between any two points in the malls. These points could be, for example, an information point, a shop entrance or a shop counter, a rescue point, a restaurant, etc.

From previous developments, our team has an algorithm able to find a sequence of waypoints between the start point and the goal one, that is, to plannify a **global navigation**. So **our mission here** is to develop an algorithm able to command the robot to safely navigate from a start waypoint to a (close) goal one, that is, to carry out **local reactive navigation**.

The image below shows an sketch of the restaurants area in the **Nirvana mall**, along with an example of global navigation (blue waypoints and dotted lines) between the information point and the *Dino's* restaurant. In that example, the green dotted lines correspond to the trajectory followed by a local reactive navigation avoiding obstacles in the waypoints path.



8.2.1 Formalizing the problem

The **reactive navigation** (or **local navigation**) has the objective of moving towards a close destination while avoiding obstacles. For that, it is available sensor data within a specific *look-ahed* as well as the goal point (**inputs**), being the reactive navigation method in charge of producing motor commands (**outputs**) to safely reach such goal.

In this way, reactive navigation methods **does not require neither any kind of map of the environment nor memory of previous observations**. In practice, the last requirement usually arises since in some situations it could be useful to also consider the last sensor observations (e.g. while crossing a door).

Finally, reactive navigation techniques **must run very fast** (i.e. real time or close to it) in order to safely reach the goal point. If not, dynamic obstacles or deprecated motion commands could lead the robot to crash!

In summary:

```
reactive_navigation(current_location,target_location,sensor_readings)
    # Method computations ... so fast!
    return (v_l,v_r) # Motor actuation
```

8.2.2 Potential Fields

Potential Fields is a popular and simple technique for carrying out reactive navigation. Imagine the robot and the objects in its environment (like obstacles, the goal, etc.) are surrounded by invisible fields, similar to magnetic fields around magnets. These fields exert virtual forces on the robot, guiding its movement.

To do this, it consist of defining a **potential (energy) function** over the free space in the robot workspace, which has a **global minimum** at the goal and a maximum at obstacles. Then, in each iteration of the algorithm, the robot moves to a lower energy configuration, similar to a a ball rolling down a hill. To carry out such navigation the robot applies a force proportional to the **negated gradient of the potential field** (recall that the gradient always go in the direction in which the signal increases, and the robot pursues a lower energy, so it has to use the negated gradient).

The **potential (energy) function** defines a **potential field** over the workspace. For each robot position p in such workspace, the energy function is computed as:

$$U(p) = U_{att}(p) + U_{rep}(p)$$

where:

- $U_{att}(p)$ is the **atractive potential field** representing the squared Euclidean distance to the goal, which is retrieved by:

  $$U_{att}(p) = \frac{1}{2}K_{att}d^2_{goal}(p)$$

  being d_{goal} said distance from the robot to the goal: $d^2_{goal}(p) = ||p - p_{goal}||^2$ and K_{att} a given gain, so this potential is higher for far distances,

- and $U_{rep}(q)$ is the **repulsive potential field**, which generates a barrier around obstacles, computed as:

$$U_{rep}(p) = \begin{cases} \frac{1}{2}K_{rep}(\frac{1}{d(p)} - \frac{1}{d_{max}})^2 & \text{if } d(p) \leq d_{max} \\ 0 & \text{if } d(p) > d_{max} \end{cases}$$

being d_{max} a given distance threshold, so obstacles far away from the robot does not influence the potential field, and $d(p)$ the distance from the robot to the object so $d^2(p) = ||p - p_{obj}||^2$.

Having defined such potential field, it can be computed a **force field** at the robot position $F(p)$ (a two-element vector) as the gradient of the previous one:

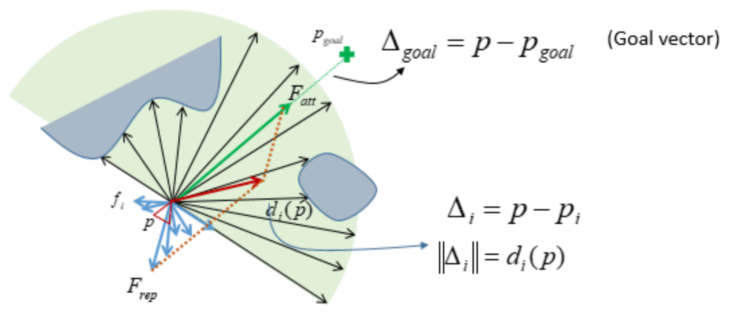
$$F(q) = -\nabla U(p) = -\nabla U_{att}(p) - \nabla U_{rep}(p) = \begin{bmatrix} \partial U/\partial x \\ \partial U/\partial y \end{bmatrix}$$

Where:

- $F_{att}(p) = -\nabla U_{att}(p)$ is also called the **attractive force** and
- $F_{rep}(p) = -\nabla U_{rep}(p)$ the **repulsive force**, so
- $F(p) = F_{att}(p) + F_{rep}(p)$.

Finally, the **robot speed** $[v_x, v_y]$ is set proportional to the force $F(p)$ as generated by the field.

The picture below illustrates all the elements in the computation of $F(p)$ (F_{total} in the image, colored as a red arrow):



Repulsive force

Unit vector: force direction

$$f_i = \begin{cases} (\frac{1}{d_i(p)} - \frac{1}{d_{max}}) \frac{1}{d_i^2(p)} \frac{\Delta_i}{d_i(p)} & if\ d_i(p) \leq d_{max} \\ 0 & if\ d_i(p) > d_{max} \end{cases}$$

$$F_{rep}(p) = k_{rep} \sum_i f_i$$

Attractive force

$$\boxed{F_{att}(p) = -k_{att}\Delta_{goal}}$$

$$F_{total}(p) = F_{att}(p) + F_{rep}(p)$$

8.2.3 Developing the Potential Fields method for Reactive navigation

It's time to develop our own Potential Fields method! For that, you first need to obtain the sum of the forces that apply at a certain robot position, computing for that the attractive and repulsive forces. Then, the total force can be retrieved, and it can be used to apply velocities to the robot wheels! (recall that $F(p) = F_{att}(p) + F_{rep}(p)$)

In [1]:
```python
# IMPORTS
import numpy as np
from numpy import random
from scipy import linalg
import matplotlib
matplotlib.use('TkAgg')
from matplotlib import pyplot as plt

import sys
sys.path.append("..")
from utils.DrawRobot import DrawRobot
```

ASSIGNMENT 1: Computing the repulsive force

Let's start with the repulsive force (`FRep`) computation, which is the sum of the repulsive forces yielded by each obstacle close to the object. Recall that forces are 2-elements column vectors.

The `repulsive_force()` function below partially implements this computation. Notice that this function also plots a marker over the obstacles that have influence on this force, and store the handler of that plot in `hInfluentialObstacles` .

Recall that:

$$f_i = \begin{cases} \left(\frac{1}{d_i(p)} - \frac{1}{d_{max}}\right)\frac{1}{d_i(p)^2}\frac{p-p_i}{d_i(p)} & \text{if } d_i(p) \leq d_{max} \\ 0 & \text{if } d_i(p) > d_{max} \end{cases}$$

$$F_{rep}(p) = K_{rep}\sum_i f_i$$

In the code below, $p - p_i$ is stored in `p_to_object` , and $d(p)$ in `d` . Notice that for each f_i, the distance from the robot to the object $d_i(p)$ is a number, while $p - p_i$ is a vector.

In [2]:
```python
def repulsive_force(xRobot, Map, RadiusOfInfluence, KObstacles):
    """ Computes the respulsive force at a given robot position

        Args:
            xRobot: Column vector containing the robot position ([x,y]')
            Map: Matrix containing the obstacles coordinates (size 2xN_obstacles
            RadiusOfInfluence: distance threshold for considering that an obstac
            KObstacles: gain related to the repulsive force

        Returns: Nothing. But it modifies the state in robot
            Frep: repulsive force ([rf_x, rf_y]') (Column vector!)
            hInfluentialObstacles: handler of the plot marking the obstacles tha
    """
    p_to_object = xRobot - Map
    d = np.sqrt(np.sum(p_to_object**2, axis=0))
    iInfluential = np.where(d <= RadiusOfInfluence)[0]

    if iInfluential.shape[0] > 0:
        p_to_object = p_to_object[:, iInfluential]
        d = d[iInfluential]
        FRep = np.vstack(KObstacles*np.sum((1/d - 1/RadiusOfInfluence)* (1/(d **

        hInfluentialObstacles = plt.plot(Map[0,iInfluential],Map[1,iInfluential]
    else:
        # Nothing close
        FRep = 0
        hInfluentialObstacles = None # Don't touch this! It is ok :)

    return FRep, hInfluentialObstacles
```

In [3]:
```
# TRY IT!
xRobot = np.vstack([[1],[2]])
Map = np.vstack([[1.1, 2.4, 3.5],[2.2, 1.4, 4.5]])
RadiusOfInfluence = 2
KObstacles = 200

FRep, handler = repulsive_force(xRobot, Map, RadiusOfInfluence, KObstacles)

print ('Repulsive force:\n ' + str(FRep))
```

```
Repulsive force:
 [[ -7117.97589183]
 [-14205.83001107]]
```

Expected output:

```
Repulsive force:
 [[ -7117.97589183]
 [-14205.83001107]]
```

ASSIGNMENT 2: Retrieving the attractive force

Next, **you need to compute** the Attractive Force `FAtt`. Do it in the `attractive_force()` function below, taking into account that:

$$F_{att}(p) = -K_{att} d_{goal}(p)$$

Normalize the resultant Force by $||\Delta_{goal}||$ so its doesn't become too dominant. You can take a look at linalg.norm() for that.

In [4]:
```
def attractive_force(KGoal, GoalError):
    """ Computes the attractive force at a given robot position

        Args:
            KGoal: gain related to the attractive force
            GoalError: distance from the robot to the goal ([d_x d_y]')

        Returns: Nothing. But it modifies the state in robot
            FAtt: attractive force ([af_x, af_y]')
    """
    FAtt = -KGoal*GoalError
    FAtt /= np.linalg.norm(GoalError) # Normalization

    return FAtt
```

In [5]:
```python
# TRY IT!
KGoal = 1.5
GoalError = np.vstack([[2.3],[1.4]])

FAtt = attractive_force(KGoal, GoalError)

print ('Attractive force:\n ' + str(FAtt))
```

```
Attractive force:
 [[-1.28129783]
 [-0.77992042]]
```

Expected output:

```
Attractive force:
 [[-1.28129783]
 [-0.77992042]]
```

ASSIGNMENT 3: Concluding with the Total Force

Finally you can compute the Total Force `FTotal`. **Do it in the main program below**, considering that:

$$F(p) = F_{att}(p) + F_{rep}(p)$$

In [6]:
```python
def main(nObstacles=175,
         MapSize=100,
         RadiusOfInfluence=10,
         KGoal=1,
         KObstacles=250,
         nMaxSteps=300,
         NON_STOP=True):

    Map = MapSize*random.rand(2, nObstacles)

    fig, ax = plt.subplots()
    plt.ion()
    ax.plot(Map[0,:],Map[1,:],'ro', fillstyle='none');

    fig.suptitle('Click to choose starting point:')
    xStart = np.vstack(plt.ginput(1)).T
    print('Starts at:\n{}'.format(xStart))


    fig.suptitle('Click to choose end goal:')
    xGoal = np.vstack(plt.ginput(1)).T
    print('Goal at:\n{}'.format(xGoal))

    fig.suptitle('')

    ax.plot(xGoal[0, 0], xGoal[1, 0],'g*', markersize=10)

    hRob = DrawRobot(fig, ax, np.vstack([xStart, 0]), axis_percent=0.001, color=

    # Initialization of useful vbles
    xRobot = xStart
    GoalError = xRobot - xGoal
    # Simulation
    k = 0

    while linalg.norm(GoalError) > 1 and k < nMaxSteps:

        FRep, hInfluentialObstacles = repulsive_force(xRobot, Map, RadiusOfInflu
        FAtt = attractive_force(KGoal, GoalError)

        # Point 1.3
        # TODO Compute total (attractive+repulsive) potential field

        FTotal = FAtt + FRep
        #FTotal /= linalg.norm(FTotal)

        xRobot += FTotal
        Theta = np.arctan2(FTotal[1, 0], FTotal[0, 0])

        hRob.pop(0).remove()
        hRob = DrawRobot(fig, ax, np.vstack([xRobot, Theta]), axis_percent=0.001

        if NON_STOP:
            plt.pause(0.1)
        else:
            plt.waitforbuttonpress(-1)

        if hInfluentialObstacles is not None:
            hInfluentialObstacles.pop(0).remove()

        # Update termination conditions
        GoalError =  xRobot - xGoal
        k += 1
```

8.2.4 Understanding how the technique performs

As a brilliant engineer, you have to provide some indications to the **managers at Nirvana** about how the technique performs and its limitations, which has to be provided in the next ***Thinking about it***. The following code cells help you to execute the implemented technique with different parameters in order to retrieve the required information.

In [13]:
```
# For considering different gains
main(KGoal=0.2, KObstacles=1)
```

```
Starts at:
[[55.35793   ]
 [63.43868859]]
Goal at:
[[30.76938769]
 [41.65152068]]
```

In [11]:
```
# For considering different number of obstacles
main(nObstacles=175)
```

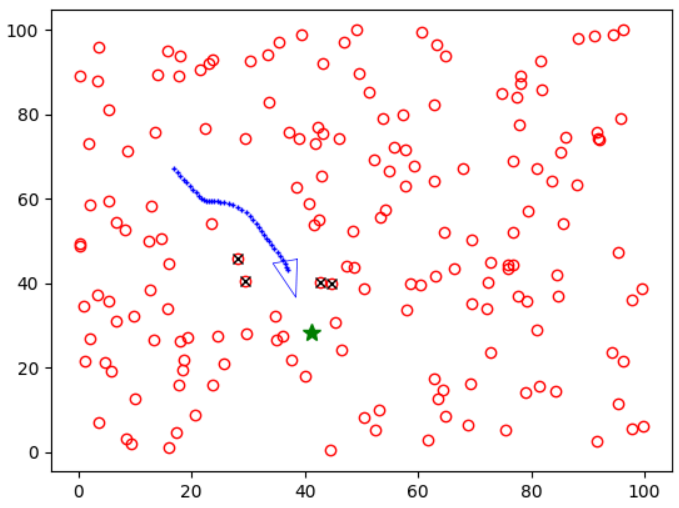
```
Starts at:
[[54.32216416]
 [38.56548947]]
Goal at:
[[12.27622779]
 [73.14967476]]
```

Thinking about it (1)

Address the following points to gain insight into how the developed Potential Fields technique performs. You can include some figures if needed.

- Discuss the meaning of each element appearing in the plot during the simulation of the *Potential Fields reactive navigation*.

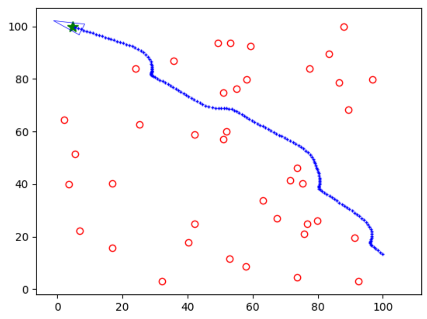
Click to choose end goal:

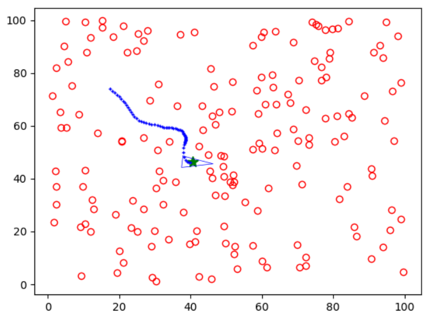


Los puntos rojos son obstáculos en el mapa. La estrella verde es la meta del robot. El triángulo azul representa al robot. Los pequeños puntos azules representan la trayectoria que recorre el robot. Las cruces en los puntos rojos indican obstáculos que están en el radio de influencia y afectamn al cálculo del FRep.

- Run the program setting different start and goal positions. Now change the values of the goal and obstacle gains (`KGoal` and `KObstacles`). How does this affect the paths followed by the robot?

  Examples with different values for such constants:



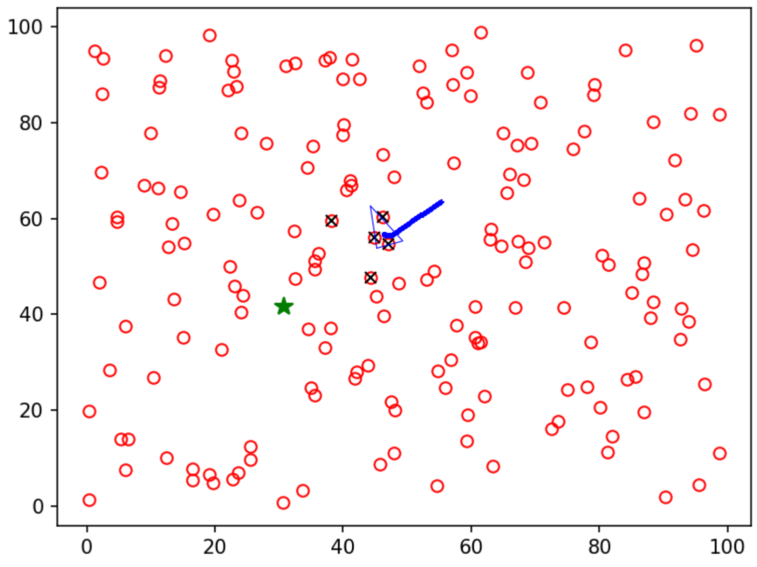


Si fijamos el KObstacles y tomamos un valor alto de KGoal, el robot se mueve mucho más rápido (en algunos casos puede llegar a chocarse con algún obstáculo). Si ponemos un valor bajo de KGoal, el robot se mueve muy despacio y podría quedarse estancado por la fuerza repulsiva de los obstáculos. Si variamos ahora KObstacles, cuanto más pongamos, el robot más alejado se moverá de los mismos de forma que pueda llegar a estancarse. Si ponemos pocos KObstacles, el robot se moverá más rápido entre ellos y puede que pete.

- Play with different numbers of obstacles and discuss the obtained results.

  Cuanto más obstáculos pongams, la posibilidad de que el robot llegue a la meta decrece.

- Illustrate a navigation where the robot doesn't reach the goal position in the specified number of steps. Why did that happen? Could the robot have reached the goal with more iterations of the algorithm? Hint: take a look at the `FTotal` variable.



El robot puede no alcanzar el objetivo ya que las fuerzas repulsivas y atractoras se contrarrestan de forma que FTotal = 0.

```
In [ ]:
```