

6.2 Mapping EFK - The Nirvana shopping mall

The managers behind the **Nirvana shopping mall**, in the scope of their *experiencing the future* plan, has contacted **UMA-MR** (our brand-new mobile robotics company at UMA) looking for mobile robots able to guide their visitors between different points of interest in their facilities, like information points, the entrance to relevant (paying well) shops, rescue points, etc.



These managers have placed identifying marks close to these points of interest, but do not know their exact location in the mall. The image below shows an example of this:



Our mission as engineers at **UMA-MR** is to build a map of the mall containing such points, so the robot can operate within it. We are going to use for that an **Extended Kalman Filter (EKF)**.

Fortunately, our company has developed a system able to provide the exact location of our robots at each time instant, which are also equipped with a range-and-bearing nosiy sensor able to detect the identifying marks and take measurements to them.

6.2.1 Formalizing the problem

Since we are going to build a map of N landmarks, the position of those landmarks in the map m are the random variables to be estimated. In this way, the state vector in the mapping case is defined as:

$$m = [m_1, m_2, \dots, m_N] = [x_1, y_1, \underbrace{x_2, y_2}_{\text{Position of landmark 2}}, \dots, x_N, y_N]^T, \quad \text{len}(m) = 2N$$

In other words, we pursuit the estimation of the probability distribution:

$$p(m|z_{1:t}, x_{1:t})$$

being $z_{1:t}$ the sensor measurements taken until time instant t , and $x_{1:t}$ the robot poses from which those measurements were acquired. Recall that a sensor measurement is related to the pose x and the map m by means of the **observation function**:

$$z = h(x, m) + e \quad e \sim N(0, Q)$$

Two assumptions are made when building maps using EFK.

Assumption 1: each landmark position is estimated independently

For simplifying the problem, it is usual to assume that the estimation of the position of the landmarks is independent one to another, only depending each one on its observations, so:

$$p(m_i|z_{1:t}, x_{1:t}) = p(m_i|z_{1:t}^i, x_{1:t})$$

which results in the following simplification:

$$p(m|z_{1:t}, x_{1:t}) = p(m_1, m_2, \dots, m_N|z_{1:t}, x_{1:t}) = \prod_{k=1}^N p(m_k|z_{1:t}^k, x_{1:t})$$

As a consequence of this, the estimation of, for example, 3 landmarks simultaneously is the same as using three concurrent and independent EKFs for estimating them.

Assumption 2: the map is static

Unlike the localization case, where the robot pose changed over time, in this case it is assumed that the map is static, that is, the landmarks are still. This means that in the state transition model $m_k = m_{k-1}$, that is:

$$m_t = A_t m_{t-1} + B_t u_t + \epsilon_t \quad (A = I, u = 0, \epsilon = 0)$$

So good news here!, there is no need for a prediction step in the EKF, we only have to model the correction (update) one.

6.2.2 Developing the EKF filter for mapping the mall

```
In [1]: %matplotlib widget

import sys
import time

import numpy as np
from numpy import random
from scipy import linalg
import matplotlib
import matplotlib.pyplot as plt
from IPython.display import display, clear_output
import time

sys.path.append("..")
from utils.tcomp import tcomp
from utils.AngleWrap import AngleWrap
from utils.DrawRobot import DrawRobot
from utils.PlotEllipse import PlotEllipse
from utils.Jacobians import J2
from utils.unit6.MapCanvas import MapCanvas
```

The provided robot

Take a look at the following `EKFMappingRobot()` class, which is already provided by your colleagues at **UMA-MR**, modeling **a robot equipped with a range and bearing sensor**, which is able to keep the following information:

- `true_pose` : The exact pose of the robot in the environment, which is perfectly known.
- `Q` : Uncertainty of the range and bearing sensor, which has the form:

$$\Sigma_{r\theta} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$
- `xEst` : Vector with the estimated state, in this case the position of the M observed landmarks: $[x_1, y_1, x_2, y_2, \dots, x_M, y_M]^T$. Its size changes over time with the detection of previously unobserved landmarks.
- `PEst` : uncertainty associated with those predictions, with size $(M \times 2, M \times 2)$. Its size also changes over time.
- `MappedLandmarks` : A vector with length equal to the number of landmarks in the map (N), which elements can take the following values:
 - `-1` if the landmark with that index has not been seen yet.
 - `idx_in_xEst` : an odd number indicating the position of that landmark in `xEst`. For example, if during the robot operation in a map with 5 landmarks, it

first detect the landmark with id 2 (ids starts at 0), and later the one with id 4, the content of this vector would be `MappedLandmarks=[-1,-1,1,-1,3]` .

and to perform the following actions:

- `step()` : Performs a motion command, without noise.
- `observe()` : Returns a range and bearing measurement (in polars) to a given landmark in the map.
- `get_random_observation()` : Returns a range and bearing measurement (in polars) to a random landmark.

```
In [2]: class EFKMappingRobot():
    def __init__(self, true_pose, sigma_r, sigma_theta, n_features):
        # Robot description
        self.true_pose = true_pose
        self.Q = np.diag([sigma_r, sigma_theta])**2

        # Map -- Initially empty
        self.xEst = np.empty((0, 0))
        self.PEst = np.empty((0, 0))
        self.QEst = 1.0*self.Q

        self.MappedLandmarks = -1*np.ones((n_features,1), int)

    def step(self, u):
        self.true_pose = tcomp(self.true_pose, u)

    def observe(self, idx, world, noisy=True):
        """ Generate a observation of a feature in our world

        Args:
            world: Complete map of all landmarks in the world
            idx: Landmark to observe (index in world matrix)
            noisy: Add noise to z, prporportional to self.Q

        Returns:
            z: One range and bearing observation
        """

        z = np.empty((2, 1))
        delta = world[:, [idx]] - self.true_pose[0:2, :]

        # Range
        z[0, :] = np.sqrt(np.sum(delta**2))
        # Bearing
        z[1, :] = np.arctan2(delta[1, 0], delta[0, 0]) - self.true_pose[2, 0]
        z[1, :] = AngleWrap(z[1, :])

        if noisy:
            z = z + np.sqrt(self.Q)@random.randn(2,1)

        return z

    def get_random_observation(self, world, noisy=True):
        iLandmarks = world.shape[1]
        iLandmark = random.randint(iLandmarks)
        z = self.observe(iLandmark, world, noisy)
```

```
return z, [iLandmark]
```

The prediction step

As commented, the map is considered static, so the prediction step is reduced to consider as the predicted landmarks' positions the ones estimated in the previous step. The same holds for the predicted uncertainty, so this steps results in something like this:

```
def ExtendedKalmanFilter( $m_{t-1}$ ,  $\Sigma_{t-1}$ ,  $z_t$ ) :  
    Prediction.  
     $\tilde{m}_t = m_{t-1}$  (1. Map prediction)  
     $\tilde{\Sigma}_t = \Sigma_{t-1}$  (2. Uncertainty of prediction)
```

ASSIGNMENT 1: Implementing the naive prediction step

You are tasked to implement the previous behavior in the following function, which performs the prediction step.

```
In [3]: def prediction_step(robot: EFKMappingRobot):  
        """ Performs the prediction step of the EKF algorithm for mapping  
            robot: Robot base (contains state map: xEst, PEst)  
  
            Returns: Nothing. But it modifies the state in robot  
            xPred: Predicted position of the landmarks  
            PPred: Predicted uncertainty of the landmarks positions  
        """  
  
        # We assume that the map is static  
        xPred = robot.xEst  
        PPred = robot.PEst  
  
        return xPred, PPred
```

You can **test your function** with the next code:

```
In [4]: # TRY IT!  
xVehicleTrue = np.vstack([0.5, 0.7, 0]) # We know the exact robot pose at any mo  
robot = EFKMappingRobot(xVehicleTrue, 1, 0.8, 1)  
robot.xEst = np.vstack([.5, .7])  
robot.PEst = np.diag([1.32, 0.8])  
  
[xPred, PPred] = prediction_step(robot)  
  
print('>> Previous estimation <<')  
print('xEst:\n' + str(robot.xEst))  
print('PEst:\n' + str(robot.PEst) + '\n')  
print('>> Prediction <<')  
print('xPred:\n' + str(xPred))  
print('PPred:\n' + str(PPred))
```

```
>> Previous estimation <<
xEst:
[[0.5]
 [0.7]]
PEst:
[[1.32 0. ]
 [0.   0.8 ]]

>> Prediction <<
xPred:
[[0.5]
 [0.7]]
PPred:
[[1.32 0. ]
 [0.   0.8 ]]
```

Expected output:

```
xPred:
[[0.5]
 [0.7]]
PPred:
[[1.32 0. ]
 [0.   0.8 ]]
```

Observing a landmark for first time

When the sensor onboard the robot detects a landmark for the first time, there is no need to do the EKF update step (indeed, since there is not previously knowledge about the landmark, there is nothing to update). Instead, we have to properly modify 1) the vector of estimated landmark positions, and 2) their associated uncertainties, to accommodate this new information:

1. **Modifying the state vector:** Insert the position of the new observed landmark $[x_{M+1}, y_{M+1}]$, using the sensor measurement $z_k = [r_k, \theta_k]$, at the end of the vector containing the estimated positions `xEst`, so:

$$xEst = [x_1, y_1, \dots, x_M, y_M, x_{M+1}, y_{M+1}]$$

Since the measurment is provided in polar coordinates in the robot local frame, we have to convert them first to cartensians and then to the world frame using the robot pose $[x_v, y_v, \theta_v]'$. The function in charge of doing so can be defined as:

$$f(x_v, z_k) = \begin{bmatrix} x_{M+1} \\ y_{M+1} \end{bmatrix} = \begin{bmatrix} x_v \\ y_v \end{bmatrix} + r_k \begin{bmatrix} \cos \alpha_k \\ \sin \alpha_k \end{bmatrix}, \quad \alpha_k = \theta_k + \theta_v$$

2. **Extending the covariance matrix.** In order to acomodate the uncertainty regarding the position of the new landmark, we have to extend the covariance matrix in the following way:

$$PEst = \begin{bmatrix} [\Sigma_{xy}^1]_{2 \times 2} & \cdots & 0_{2 \times 2} \\ \vdots & \ddots & \vdots \\ 0_{2 \times 2} & \cdots & [\Sigma_{xy}^{M+1}]_{2 \times 2} \end{bmatrix}_{2n \times 2n}$$

Notice that the covariance Σ_{xy}^{M+1} stands for the uncertainty in the measurement expressed in the world cartesian coordinates, retrieved by:

$$\Sigma_{xy}^{M+1} = J \Sigma_{r\theta}^{M+1} J^T$$

being $\Sigma_{r\theta}^{M+1}$ the uncertainty characterizing the sensor measurements (`QEst` in our code), and J (`jGz` in our code) the jacobian of the function $f(x_v, z_k)$ that expresses the measurement in global coordinates, which is:

$$J = \begin{bmatrix} \partial x / \partial r & \partial x / \partial \theta \\ \partial y / \partial r & \partial y / \partial \theta \end{bmatrix} = \begin{bmatrix} \cos \alpha & -r \sin \alpha \\ \sin \alpha & r \cos \alpha \end{bmatrix}$$

Notice that this jacobian is the result of concatenating both, the jacobian of the composition of a pose and a landmark, and the jacobian of the function transforming polar into cartesian coordinates. It is quite similar to the one just transforming polar into cartesian, but in this case the angle α is the sum of the robot orientation and the measured angle (bearing).

ASSIGNMENT 2: Incorporating a landmark detected for first time

Your work here is to:

- Complete the `get_new_landmark_jacobians()` to compute the jacobian J .
- Complete the `incorporate_new_landmark()` method to modify the state vector `xEst` and the covariance matrix `PEst` as explained above. We will make use of the `linalg.block_diag()` function at this point.

```
In [5]: def get_new_landmark_jacobians(Xv, z):
        """ Calculate the jacobian for transforming an observation to the world frame

        Args:
            Xv: True pose of our robot
            z: Observation of a landmark. In polar coordinates from the p.o.v. of

        Returns:
            2x2 matrix containing the corresponding jacobian.
        """
        r, a = z[0,0], z[1,0] + Xv[2,0]
        c, s = np.cos(a), np.sin(a)
        jGz = np.array([
            [c, -r*s],
```

```

        [s, r*c]
    ])
    return jGz

```

Is it working properly? **Try it!**

```

In [6]: # TRY IT!
z = np.vstack([1.2, 0.35])
Xv = np.vstack([2, 2.1, 0])
jGz = get_new_landmark_jacobians(Xv, z)
print('jGz:\n' + str(jGz))

```

```

jGz:
[[ 0.93937271 -0.41147737]
 [ 0.34289781  1.12724726]]

```

Expected output:

```

jGz:
[[ 0.93937271 -0.41147737]
 [ 0.34289781  1.12724726]]

```

```

In [7]: def incorporate_new_landmark(robot: EFKMappingRobot, z, iLandmark, xPred, PPred)
        """ Incorporates the information relative to a new landmark to our system
            robot: Robot base (contains state map: xEst, PEst)
            z: Observation of a landmark
            iLandmark: Index of z in the world map
            xPred: Predicted map
            PPred: Uncertainty of the prediction

            Returns: Nothing. But it modifies the state in robot
        """
        # This is a new feature, so add it to the map

        # The observation is in the local frame of the robot, it has to
        # be translated to the global frame

        r, a = z[0, 0], z[1, 0] + robot.true_pose[2, 0]
        c, s = np.cos(a), np.sin(a)
        xLandmark = robot.true_pose[0:2] + r * np.vstack([c, s])

        # Add it to the current state
        nStates = xPred.size

        if nStates == 0:
            robot.xEst = xLandmark
        else:
            robot.xEst = np.vstack([robot.xEst, xLandmark]) #Each new feature two ne

        # Compute the jacobian
        jGz = get_new_landmark_jacobians(robot.true_pose, z) #Dimension 2x2

        # Build a matrix M incorporating the jacobian to multiply the extended PEst
        if nStates != 0:
            # note we don't use jacobian w.r.t vehicle since the pose doesn't have u
            M = np.vstack([
                np.hstack([np.eye(nStates), np.zeros((nStates, 2))]),
                np.hstack([np.zeros((2, nStates)), jGz])
            ])

```



```

else:
    # First Landmark observed!
    M = jGz

robot.PEst = M@linalg.block_diag(robot.PEst, robot.QEst)@M.T

#This can also be done directly PEst = [PEst,zeros(nStates,2);
# zeros(2,nStates),
# jGz*QEst*jGz']

#remember this Landmark as being mapped: we store its ID for the state vector
robot.MappedLandmarks[iLandmark] = robot.xEst.size-2 #Always an odd number

```

The correction (update) step

Once a landmark has been detected and its provided information (location and uncertainty) has been properly incorporated to our mapping system, such an information can be updated with new measurements of such a landmark. For doing so, the EKF algorithm performs the following steps:

Correction.

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} \quad (3. \text{ Kalman gain})$$

$$m_t = \bar{m}_t + K_t(z_t - h(x_t, \bar{m}_t)) \quad (4. \text{ Map estimation})$$

$$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t \quad (5. \text{ Uncertainty of estimation})$$

return m_t, Σ_t

Notice that the map (landmark locations) is estimated according to the map estimation in the previous time step $t - 1$, the *Kalman gain*, and the error (also called **innovation**) between the observation taken by the sensor (z_t) and the one computed by the observation model given the robot pose and the predicted map, that is $z_t - h(x_t, \bar{m}_t)$.

In such equations, H (`jH` in our code) represents the jacobian of the observation model. The shape of such a jacobian is $(2, M \times 2)$, and has the following form:

$$H = \begin{bmatrix} 0 & 0 & \cdots & \underbrace{\begin{bmatrix} jHxl_{11} & jHxl_{12} \\ jHxl_{21} & jHxl_{22} \end{bmatrix}}_{\text{Jacobian for the observed landmark, } jHxl} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & & \cdots & 0 & 0 \end{bmatrix}$$

$$jHxl = \begin{bmatrix} (x_l - x)/d & (y_l - y)/d \\ -(y_l - y)/d^2 & (x_l - x)/d^2 \end{bmatrix}$$

where $[x_l, y_l]$ is the position of the landmark, $[x, y]$ is the robot location, and $d = \sqrt{(x_l - x)^2 + (y_l - y)^2}$.

ASSIGNMENT 3: Updating the knowledge about an observed landmark

Your job at this point is:

- To implement the function `get_observation_jacobian()` returning the jacobian of the observed landmark, that is, $jHxl$.

- To complete the `update_step()` method that performs the update step of the EKF algorithm.

```
In [8]: def get_observation_jacobian(xPred, xLandmark):
        """ Calculate the jacobian of the observation model.

        Needed to update a landmark we have already seen.
        Hint. Similar to the one described in unit 5 (Localization)

        Args:
            xPred: True pose of our robot.
            xLandmark: Estimated pose of a landmark in our map. World p.o.v in c
                    Does not contain an angle.

        Return:
            jHx1: 2x2 matrix containing the corresponding jacobian.
        """
        xdist = xLandmark[0,0] - xPred[0,0]
        ydist = xLandmark[1,0] - xPred[1,0]
        r = np.sqrt((xdist**2) + (ydist**2))
        r2 = r**2
        jHx1 = np.array([
            [xdist/r , ydist/r],
            [-ydist/r2 , xdist/r2]
        ])
        return jHx1
```

Let's see if your implementation is right:

```
In [9]: # TRY IT!
        xLandmark = np.vstack([2.5,2])
        xPred = np.vstack([2, 2.1, 0])
        jHx1 = get_observation_jacobian(xPred, xLandmark)
        print('jHx1:\n' + str(jHx1))
```

```
jHx1:
[[ 0.98058068 -0.19611614]
 [ 0.38461538  1.92307692]]
```

Expected output:

```
jHx1:
[[ 0.98058068 -0.19611614]
 [ 0.38461538  1.92307692]]
```

```
In [10]: def update_step(robot, z, iLandmark, xPred, PPred):
        """ Performs the update step of EKF
            robot: Robot base (contains state map: xEst, PEst)
            z: Observation of a landmark
            iLandmark: Index of z in the world map
            xPred: Predicted map
            PPred: Uncertainty of the prediction

            Returns: Nothing. But it modifies the state in robot
        """
        # Find out where it is in state vector
        landmarkIndex = robot.MappedLandmarks[iLandmark[0], 0]
```

```

# xLandmark is the current estimation of the position of the
# Landmark "FeatureIndex"
xLandmark = xPred[landmarkIndex:landmarkIndex+2]

# DONE Predicts the observation
zPred = robot.observe(0, xLandmark, noisy=False) # Hint: use robot.observe f

# Get observation Jacobians
jHxf = get_observation_jacobian(robot.true_pose, xLandmark)

# Fill in state jacobian
# (the jacobian is zero except for the observed landmark)
jH = np.zeros((2, xPred.size))
jH[:, landmarkIndex:landmarkIndex+2] = jHxf

#
# Kalman update
#
Innov = z-zPred # Innovation
Innov[1] = AngleWrap(Innov[1])
S = jH @ PPred @ jH.T + np.diag(np.tile(np.diag(robot.QEst), xLandmark.shape
K = PPred@jH.T@linalg.inv(S) # Gain
robot.xEst = xPred + K@Innov
robot.PEst = (np.eye(robot.PEst.shape[0]) - K@jH)@PPred
#robot.PEst = PPred - K@S@K.T # Alternative way

#ensure P remains symmetric
robot.PEst = 0.5*(robot.PEst+robot.PEst.T)

```

ASSIGNMENT 4: Putting all together in the EKF algorithm

Now that you have implemented the building blocks of the EKF filter for mapping the **Nirvana shopping mall**, it is time to write a simple function `EKFMapping()` putting them together. For that, **you have to** call each method with the appropriate parameters.

```

In [11]: def EKFMapping(robot: EKFMappingRobot, z, iLandmark):
        """ EFK algorithm for mapping

        robot: Robot base (contains state map: xEst, PEst)
        z: Observation of a landmark
        iLandmark: Index of z in the world map

        Returns: Nothing. But it modifies the state in robot
        """

        # Do prediction step
        [xPred, PPred] = prediction_step(robot)

        # Check if feature observed is in map
        if robot.MappedLandmarks[iLandmark] > -1:
            update_step(robot, z, iLandmark, xPred, PPred)

        else:
            # This is a new feature, so add its information to the map
            incorporate_new_landmark(robot, z, iLandmark, xPred, PPred)

```

6.2.3 Testing the mapping system

Playing with one landmark

Let's consider that the mall has only one landmark to get things started (`nLandmarks=1`). The following function provides a demo where the robot is commanded to follow a squared trajectory while observing a landmark after each movement.

The **Nirvana** managers are curious about the state and dimensions of the variables storing the estimated positions `xEst` and their associated uncertainties `Pest` , so we show their content after each 5 iterations of the algorithm.

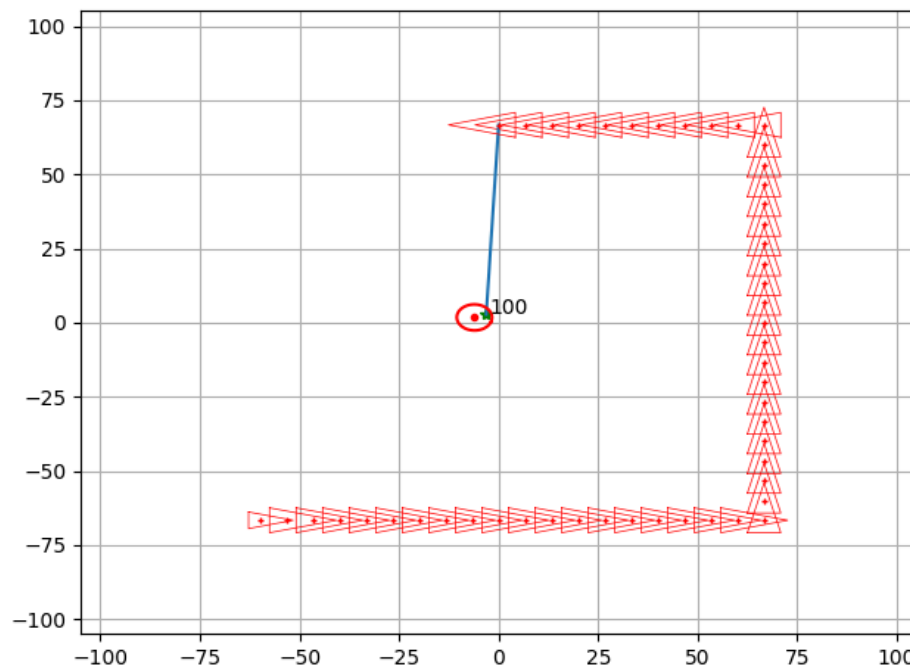


Fig. 1: Example run of the EKF algorithm for mapping (only one landmark).
it shows the true pose (in red),
the real pose of the landmark (as a green star),
and the estimation from the EKF algorithm (pose and confidence ellipse).

```
In [12]: def demo_ekf_mapping(robot,
    Map,
    nLandmarks,
    mode='non_stop',
    logger=None,
    nSteps=100, # Number of motions
    turning= 40, # Number of motions before turning (square pat
    print_each=2):

    %matplotlib widget
    if mode == 'step_by_step':
        matplotlib.use('TkAgg')

    # storing the number of times a landmark has been seen
    # also store the handler to the graphical info shown
```

```

canvas = MapCanvas(nLandmarks)

canvas.ax.plot(Map[0, :], Map[1, :], 'g*')
hObsLine = canvas.ax.plot([0,0], [0,0], linestyle=':')

# Control action
u = np.zeros((3, 1))
u[0] = (2.*MapSize/1.5)/turning
u[1] = 0.

log_xEst = []
log_PEst = []

# Start the Loop!
for k in range(nSteps):
    #
    # Move the robot
    #
    u[2]=0.
    if k%turning == turning-1:
        u[2] = np.pi/2

    robot.step(u) # Perfectly known robot pose

    z, iLandmark = robot.get_random_observation(world=Map)

    # Update the "observedtimes" for the feature and plot the reading
    canvas.increment_observed_times(iLandmark)
    canvas.PlotNumberOfReadings(robot.true_pose, iLandmark, Map)

    EKFMapping(robot, z, iLandmark)

    # Store map evolution each 5 steps
    if not k%5:
        log_xEst.append(robot.xEst)
        log_PEst.append(robot.PEst)

    # Log important values
    if logger is not None:
        logger.log(k, robot, Map)

    # Drawings
    if k%print_each == print_each-1:
        DrawRobot(canvas.fig, canvas.ax, robot.true_pose, 'r')#plot(xVehicleT
        canvas.DoMapGraphics(robot) # Draw estimated poitns (in black) and e
        plt.axis([-MapSize-5, MapSize+5, -MapSize-5, MapSize+5]) # Set Limit
        #plt.draw()
        plt.savefig(str(k)+' .jpg')
        clear_output(wait=True)
        display(canvas.fig)

        if mode == 'step_by_step':
            plt.waitforbuttonpress(-1)
        elif mode == 'visualize_process':
            time.sleep(0.2)
        elif mode == 'non_stop':
            pass # non stop!

#
# Final drawings
%matplotlib inline

```

```

if logger is not None:
    logger.plot()

# Print map evolution each 5 steps
for i in range(0, len(log_xEst)):
    with np.printoptions(precision=3):
        print('Iteration: ' + str(i*5))
        print('Estimated xEst:\n' + str(log_xEst[i]))
        print('Estimated PEst:\n' + str(log_PEst[i]))
        print('-----')

```

```

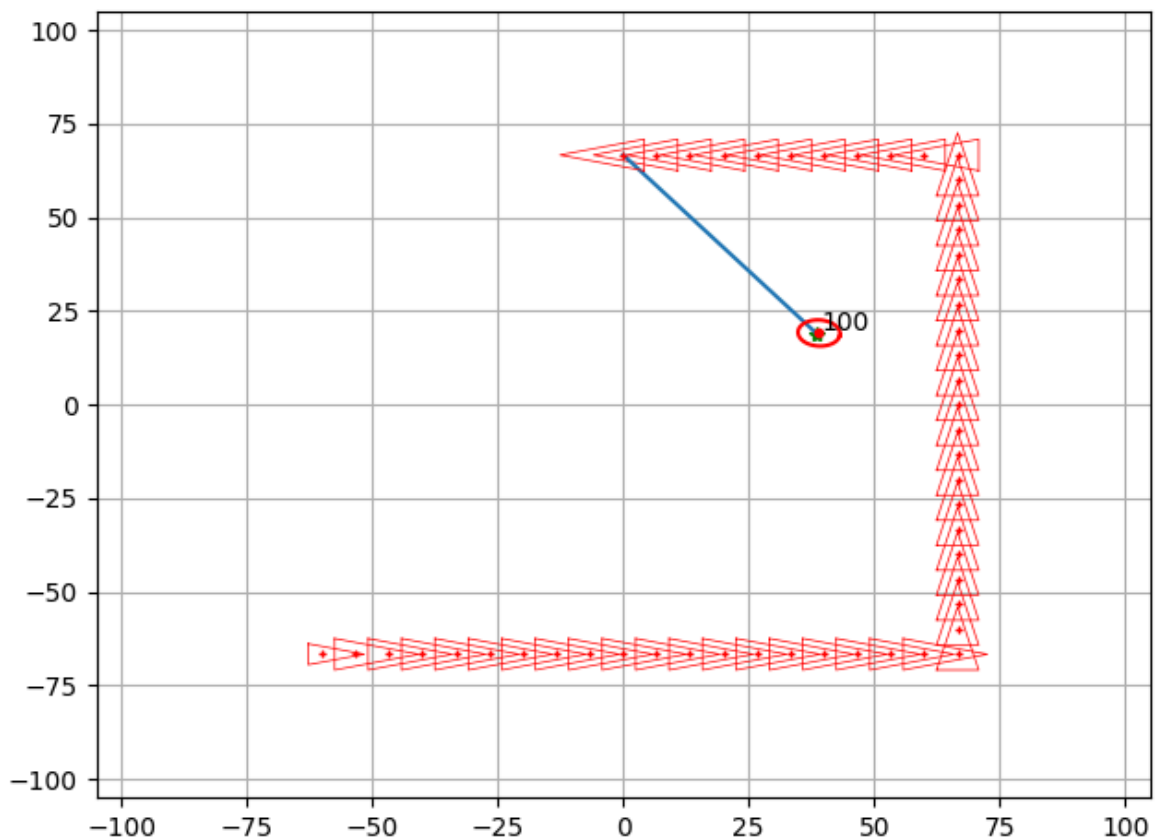
In [13]: #mode = 'step_by_step'
mode = 'visualize_process'
#mode = 'non_stop'

# WORLD MAP
# Num features/landmarks considered within the map
nLandmarks = 1
# Generation of the map
MapSize = 100
Map = MapSize*random.rand(2, nLandmarks)-MapSize/2

# ROBOT
# Covariances for our very bad&expensive sensor (in the system <d,theta>)
Sigma_r = 8.0
Sigma_theta = 8*np.pi/180
# Initial robot pose
xVehicleTrue = np.vstack([-MapSize/1.5, -MapSize/1.5, 0.]) # We know the exact r
robot = EKFMappingRobot(xVehicleTrue, Sigma_r, Sigma_theta, nLandmarks)

demo_ekf_mapping(robot, Map, nLandmarks, mode=mode)

```



Iteration: 0
Estimated xEst:
[[3.614]
[69.953]]
Estimated PEst:
[[376.277 -153.025]
[-153.025 138.987]]

Iteration: 5
Estimated xEst:
[[30.847]
[25.773]]
Estimated PEst:
[[39.127 -20.732]
[-20.732 26.03]]

Iteration: 10
Estimated xEst:
[[39.12]
[23.161]]
Estimated PEst:
[[18.736 -9.953]
[-9.953 13.576]]

Iteration: 15
Estimated xEst:
[[39.114]
[22.016]]
Estimated PEst:
[[12.153 -5.966]
[-5.966 8.416]]

Iteration: 20
Estimated xEst:
[[39.47]
[21.461]]
Estimated PEst:
[[8.736 -3.82]
[-3.82 5.661]]

Iteration: 25
Estimated xEst:
[[38.98]
[21.007]]
Estimated PEst:
[[6.647 -2.506]
[-2.506 4.023]]

Iteration: 30
Estimated xEst:
[[37.937]
[21.318]]
Estimated PEst:
[[5.25 -1.659]
[-1.659 3.009]]

Iteration: 35
Estimated xEst:
[[37.776]
[21.682]]

Estimated PEst:
[[4.302 -1.097]
[-1.097 2.364]]

Iteration: 40
Estimated xEst:
[[38.888]
[20.415]]
Estimated PEst:
[[3.615 -0.72]
[-0.72 1.946]]

Iteration: 45
Estimated xEst:
[[39.288]
[19.953]]
Estimated PEst:
[[3.033 -0.476]
[-0.476 1.666]]

Iteration: 50
Estimated xEst:
[[40.465]
[19.698]]
Estimated PEst:
[[2.492 -0.334]
[-0.334 1.464]]

Iteration: 55
Estimated xEst:
[[40.203]
[19.275]]
Estimated PEst:
[[1.999 -0.281]
[-0.281 1.297]]

Iteration: 60
Estimated xEst:
[[39.564]
[18.745]]
Estimated PEst:
[[1.633 -0.299]
[-0.299 1.099]]

Iteration: 65
Estimated xEst:
[[39.529]
[19.411]]
Estimated PEst:
[[1.431 -0.248]
[-0.248 0.83]]

Iteration: 70
Estimated xEst:
[[39.744]
[19.459]]
Estimated PEst:
[[1.207 -0.12]
[-0.12 0.66]]


```

Iteration: 75
Estimated xEst:
[[39.526]
 [19.24 ]]
Estimated PEst:
[[ 1.041 -0.066]
 [-0.066  0.602]]
-----
Iteration: 80
Estimated xEst:
[[39.095]
 [19.24 ]]
Estimated PEst:
[[ 0.947 -0.052]
 [-0.052  0.572]]
-----
Iteration: 85
Estimated xEst:
[[38.965]
 [19.125]]
Estimated PEst:
[[ 0.864 -0.042]
 [-0.042  0.546]]
-----
Iteration: 90
Estimated xEst:
[[38.929]
 [18.993]]
Estimated PEst:
[[ 0.788 -0.037]
 [-0.037  0.524]]
-----
Iteration: 95
Estimated xEst:
[[38.838]
 [19.158]]
Estimated PEst:
[[ 0.733 -0.036]
 [-0.036  0.502]]
-----

```

Considering a larger number of landmarks

Once our EKF implementation is working with one landmark, let's try it in a scenario with 5 landmarks. Again, the content of the `xEst` and `Pest` is shown after each 5 iterations of the algorithm.

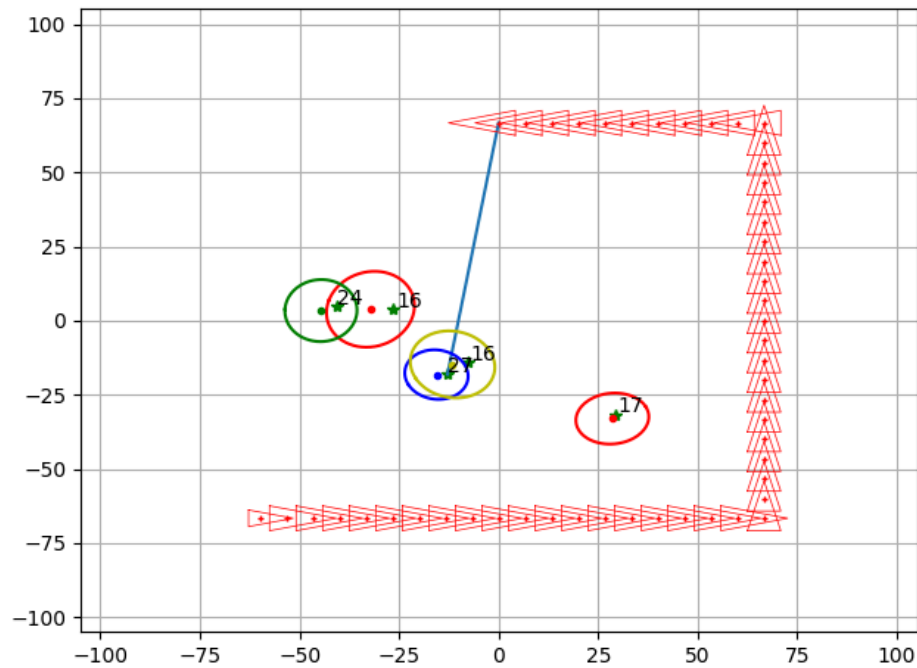
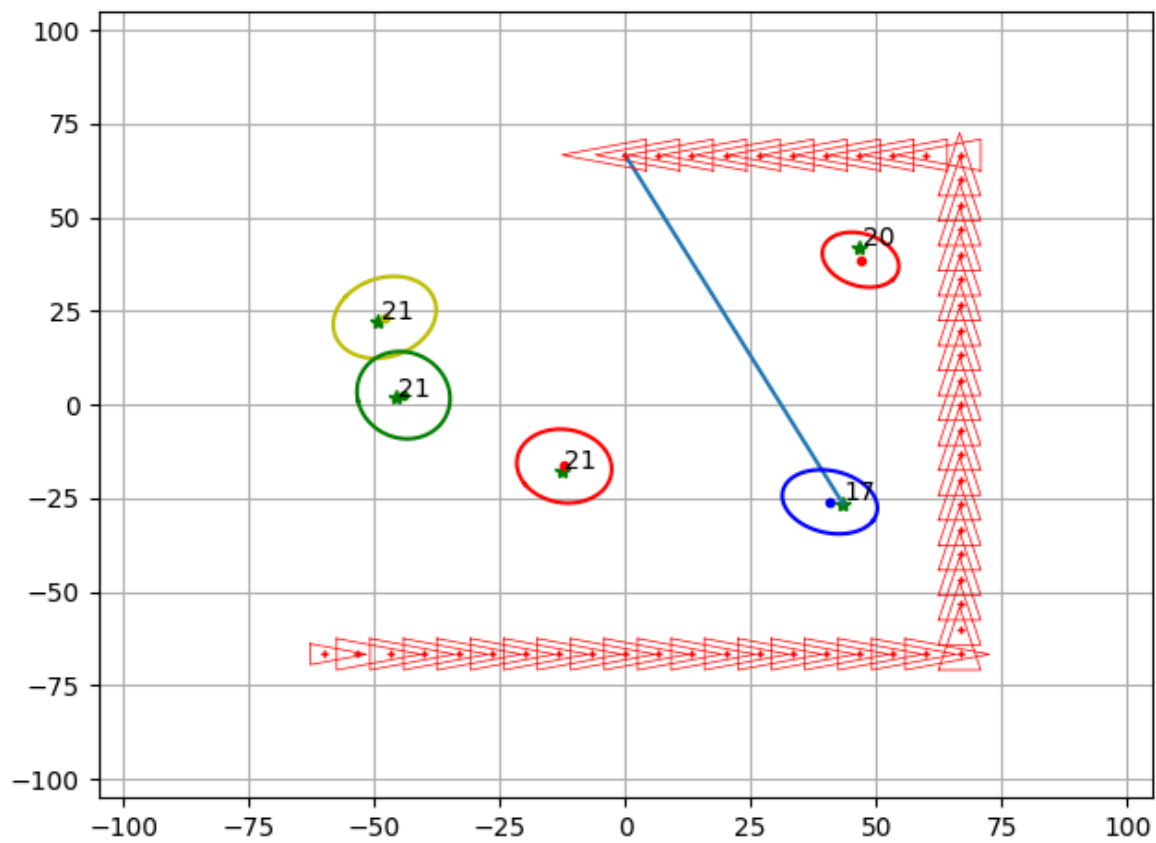


Fig. 2: Execution of the EKF algorithmn for mapping (multiple landmarks).
Same as in Fig 1., each landmark is accompanied by a number of times
observed.

```
In [14]: #mode = 'step_by_step'
mode = 'visualize_process'
#mode = 'non_stop'

# WORLD MAP
# Num features/Landmarks considered within the map
nLandmarks = 5
# Generation of the map
MapSize = 100
Map = MapSize*random.rand(2,nLandmarks)-MapSize/2

# ROBOT
# Covariances for our very bad&expensive sensor (in the system <d,theta>)
Sigma_r = 8.0
Sigma_theta = 7*np.pi/180
# Initial robot pose
xVehicleTrue = np.vstack([-MapSize/1.5, -MapSize/1.5, 0.]) # We know the exact r
robot = EKFMappingRobot(xVehicleTrue, Sigma_r, Sigma_theta, nLandmarks)
demo_ekf_mapping(robot, Map ,nLandmarks, mode=mode)
```



Iteration: 0
 Estimated xEst:
 [[57.582]
 [50.052]]
 Estimated PEst:
 [[236.474 -178.675]
 [-178.675 249.099]]

 Iteration: 5
 Estimated xEst:
 [[56.134]
 [43.288]
 [40.139]
 [-14.054]
 [-42.226]
 [22.512]]
 Estimated PEst:
 [[76.367 -53.146 0. 0. 0. 0.]
 [-53.146 72.692 0. 0. 0. 0.]
 [0. 0. 91.472 -52.288 0. 0.]
 [0. 0. -52.288 163.522 0. 0.]
 [0. 0. 0. 0. 51.935 -3.405]
 [0. 0. 0. 0. -3.405 32.601]]

 Iteration: 10
 Estimated xEst:
 [[56.134]
 [43.288]
 [39.213]
 [-32.927]
 [-43.399]
 [23.694]
 [-42.293]
 [-8.526]
 [-7.527]
 [-0.555]]
 Estimated PEst:
 [[76.367 -53.146 0. 0. 0. 0. 0. 0. 0.
 0.]
 [-53.146 72.692 0. 0. 0. 0. 0. 0. 0.
 0.]
 [0. 0. 42.685 -17.348 0. 0. 0. 0. 0.
 0.]
 [0. 0. -17.348 60.566 0. 0. 0. 0. 0.
 0.]
 [0. 0. 0. 0. 27.751 -0.71 0. 0. 0.
 0.]
 [0. 0. 0. 0. -0.71 16.122 0. 0. 0.
 0.]
 [0. 0. 0. 0. 0. 0. 50.555 -0.53 0.
 0.]
 [0. 0. 0. 0. 0. 0. -0.53 63.979 0.
 0.]
 [0. 0. 0. 0. 0. 0. 0. 0. 75.649
 -5.135]
 [0. 0. 0. 0. 0. 0. 0. 0. -5.135
 66.263]]

 Iteration: 15
 Estimated xEst:

```

[[ 57.314]
 [ 39.256]
 [ 40.506]
 [-28.516]
 [-43.315]
 [ 21.89 ]
 [-40.881]
 [  1.206]
 [ -7.527]
 [ -0.555]]
Estimated PEst:
[[ 54.446 -34.691  0.      0.      0.      0.      0.      0.      0.
   0.      ]
 [-34.691  47.408  0.      0.      0.      0.      0.      0.      0.
   0.      ]
 [  0.      0.      25.078 -6.089  0.      0.      0.      0.      0.
   0.      ]
 [  0.      0.      -6.089  31.414  0.      0.      0.      0.      0.
   0.      ]
 [  0.      0.      0.      0.      22.587  0.258  0.      0.      0.
   0.      ]
 [  0.      0.      0.      0.      0.258  13.007  0.      0.      0.
   0.      ]
 [  0.      0.      0.      0.      0.      0.      19.147 -0.157  0.
   0.      ]
 [  0.      0.      0.      0.      0.      0.      -0.157  21.32  0.
   0.      ]
 [  0.      0.      0.      0.      0.      0.      0.      0.      75.649
 -5.135]
 [  0.      0.      0.      0.      0.      0.      0.      0.      -5.135
 66.263]]

```

Iteration: 20

Estimated xEst:

```

[[ 56.909]
 [ 37.065]
 [ 36.165]
 [-24.438]
 [-43.315]
 [ 21.89 ]
 [-40.881]
 [  1.206]
 [-13.092]
 [-10.16 ]]
Estimated PEst:
[[ 40.533 -22.931  0.      0.      0.      0.      0.      0.      0.
   0.      ]
 [-22.931  31.966  0.      0.      0.      0.      0.      0.      0.
   0.      ]
 [  0.      0.      13.24  -1.011  0.      0.      0.      0.      0.
   0.      ]
 [  0.      0.      -1.011  14.838  0.      0.      0.      0.      0.
   0.      ]
 [  0.      0.      0.      0.      22.587  0.258  0.      0.      0.
   0.      ]
 [  0.      0.      0.      0.      0.258  13.007  0.      0.      0.
   0.      ]
 [  0.      0.      0.      0.      0.      0.      19.147 -0.157  0.
   0.      ]
 [  0.      0.      0.      0.      0.      0.      -0.157  21.32  0.
   0.      ]

```

| | | | | | | | | | |
|----------|----|----|----|----|----|----|----|----|--------|
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 20.877 |
| -1.031] | | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.031 |
| 21.391]] | | | | | | | | | |

Iteration: 25

Estimated xEst:

```
[[ 52.971]
 [ 35.902]
 [ 38.738]
 [-25.756]
 [-44.943]
 [ 23.433]
 [-40.881]
 [ 1.206]
 [-11.2 ]
 [-15.058]]
```

Estimated PEst:

| | | | | | | | | | |
|-----------|---------|--------|--------|--------|--------|--------|--------|--------|--------|
| [[25.803 | -11.525 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. |] | | | | | | | | |
| [-11.525 | 17.429 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. |] | | | | | | | | |
| [0. | 0. | 10.203 | -0.077 | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. |] | | | | | | | | |
| [0. | 0. | -0.077 | 11.557 | 0. | 0. | 0. | 0. | 0. | 0. |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 18.952 | 1.055 | 0. | 0. | 0. | 0. |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 1.055 | 11.165 | 0. | 0. | 0. | 0. |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 19.147 | -0.157 | 0. | 0. |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | -0.157 | 21.32 | 0. | 0. |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 15.588 | 0. |
| -0.658] | | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -0.658 |
| 15.987]] | | | | | | | | | |

Iteration: 30

Estimated xEst:

```
[[ 48.539]
 [ 41.191]
 [ 38.738]
 [-25.756]
 [-45.917]
 [ 24.918]
 [-41.825]
 [ -0.461]
 [-12.485]
 [-16.383]]
```

Estimated PEst:

| | | | | | | | | | | |
|----------|--------|--------|--------|--------|-------|--------|-------|----|----|---|
| [[21.716 | -8.711 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [-8.711 | 13.862 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [0. | 0. | 10.203 | -0.077 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [0. | 0. | -0.077 | 11.557 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [0. | 0. | 0. | 0. | 14.458 | 1.987 | 0. | 0. | 0. | 0. |] |
| [0. | 0. | 0. | 0. | 1.987 | 9.14 | 0. | 0. | 0. | 0. |] |
| [0. | 0. | 0. | 0. | 0. | 0. | 15.762 | 1.072 | 0. | 0. |] |

| | | | | | | | | | | |
|---|----|----|----|----|----|-------|--------|--------|--------|----|
| [| 0. | 0. | 0. | 0. | 0. | 1.072 | 17.311 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 12.702 | -0.262 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | -0.262 | 12.94 |]] |

Iteration: 35

Estimated xEst:

```
[[ 48.539]
 [ 41.191]
 [ 38.738]
 [-25.756]
 [-44.79 ]
 [ 24.605]
 [-42.311]
 [  1.393]
 [-12.701]
 [-18.466]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|--------|-------|--------|--------|-------|-------|----|
| [| 21.716 | -8.711 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -8.711 | 13.862 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 10.203 | -0.077 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.077 | 11.557 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 12.915 | 2.25 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 2.25 | 8.554 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 11.748 | 2.047 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 2.047 | 13.311 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 9.451 | 0.336 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.336 | 9.828 |]] |

Iteration: 40

Estimated xEst:

```
[[ 48.539]
 [ 41.191]
 [ 40.151]
 [-25.252]
 [-44.663]
 [ 24.992]
 [-42.587]
 [  3.1 ]
 [-11.883]
 [-17.991]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|--------|-------|--------|--------|-------|------|----|
| [| 21.716 | -8.711 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -8.711 | 13.862 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 6.651 | -0.726 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.726 | 8.053 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 11.622 | 2.437 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 2.437 | 8.162 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 10.469 | 2.279 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 2.279 | 12.211 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 8.402 | 0.52 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.52 | 8.94 |]] |

Iteration: 45

Estimated xEst:

```
[[ 51.498]
 [ 37.773]
 [ 39.795]
 [-26.07 ]
 [-45.156]
 [ 24.779]
```

```

[-42.587]
[ 3.1 ]
[-13.651]
[-17.813]]
Estimated PEst:
[[17.467 -5.811 0. 0. 0. 0. 0. 0. 0. 0. ]
[-5.811 10.879 0. 0. 0. 0. 0. 0. 0. 0. ]
[ 0. 0. 4.957 -1.17 0. 0. 0. 0. 0. 0. ]
[ 0. 0. -1.17 5.473 0. 0. 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 10.467 2.503 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 2.503 7.829 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 0. 0. 10.469 2.279 0. 0. ]
[ 0. 0. 0. 0. 0. 0. 2.279 12.211 0. 0. ]
[ 0. 0. 0. 0. 0. 0. 0. 0. 7.537 0.612]
[ 0. 0. 0. 0. 0. 0. 0. 0. 0.612 8.227]]
-----

```

Iteration: 50
Estimated xEst:

```

[[ 51.498]
[ 37.773]
[ 40.457]
[-25.953]
[-46.049]
[ 24.345]
[-43.797]
[ 2.349]
[-13.719]
[-17.935]]

```

```

Estimated PEst:
[[17.467 -5.811 0. 0. 0. 0. 0. 0. 0. 0. ]
[-5.811 10.879 0. 0. 0. 0. 0. 0. 0. 0. ]
[ 0. 0. 4.555 -1.164 0. 0. 0. 0. 0. 0. ]
[ 0. 0. -1.164 4.085 0. 0. 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 8.503 2.388 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 2.388 7.262 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 0. 0. 9.178 2.183 0. 0. ]
[ 0. 0. 0. 0. 0. 0. 2.183 11.451 0. 0. ]
[ 0. 0. 0. 0. 0. 0. 0. 0. 6.78 0.601]
[ 0. 0. 0. 0. 0. 0. 0. 0. 0.601 7.612]]
-----

```

Iteration: 55
Estimated xEst:

```

[[ 50.451]
[ 37.812]
[ 40.457]
[-25.953]
[-46.556]
[ 23.993]
[-45.715]
[ 2.365]
[-12.186]
[-16.718]]

```

```

Estimated PEst:
[[12.429 -3.82 0. 0. 0. 0. 0. 0. 0. 0. ]
[-3.82 8.984 0. 0. 0. 0. 0. 0. 0. 0. ]
[ 0. 0. 4.555 -1.164 0. 0. 0. 0. 0. 0. ]
[ 0. 0. -1.164 4.085 0. 0. 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 7.049 2.174 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 2.174 6.77 0. 0. 0. 0. ]
[ 0. 0. 0. 0. 0. 0. 8.112 2.007 0. 0. ]

```


| | | | | | | | | | | |
|---|----|----|----|----|----|-------|--------|-------|------|----|
| [| 0. | 0. | 0. | 0. | 0. | 2.007 | 10.783 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 6.127 | 0.5 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.5 | 7.05 |]] |

Iteration: 60

Estimated xEst:

```
[[ 48.501]
 [ 39.579]
 [ 40.457]
 [-25.953]
 [-46.541]
 [ 23.442]
 [-46.406]
 [  0.747]
 [-12.186]
 [-16.718]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|-------|-------|--------|--------|-------|-------|-------|--------|-------|------|----|
| [| 6.188 | -1.88 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -1.88 | 6.084 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 4.555 | -1.164 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -1.164 | 4.085 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 6.389 | 1.978 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 1.978 | 6.518 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 7.198 | 1.725 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 1.725 | 10.166 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 6.127 | 0.5 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.5 | 7.05 |]] |

Iteration: 65

Estimated xEst:

```
[[ 46.522]
 [ 40.173]
 [ 40.457]
 [-25.953]
 [-46.541]
 [ 23.442]
 [-46.406]
 [  0.747]
 [-11.285]
 [-15.547]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|-------|-------|-------|--------|-------|-------|----|
| [| 4.364 | -1.762 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -1.762 | 4.764 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 4.555 | -1.164 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -1.164 | 4.085 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 6.389 | 1.978 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 1.978 | 6.518 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 7.198 | 1.725 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 1.725 | 10.166 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4.798 | 0.151 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.151 | 5.769 |]] |

Iteration: 70

Estimated xEst:

```
[[ 47.035]
 [ 38.623]
 [ 40.457]
 [-25.953]
 [-47.182]
 [ 22.902]
```

```

[-44.044]
[  1.99 ]
[-10.43 ]
[-15.613]]
Estimated PEst:
[[ 4.028 -1.661  0.      0.      0.      0.      0.      0.      0.      0. ]
 [-1.661  3.504  0.      0.      0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      4.555 -1.164  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.     -1.164  4.085  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      5.793  1.74   0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      1.74   6.251  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      5.834  0.996  0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      0.996  8.96   0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      0.      0.      4.511  0.043]
 [ 0.      0.      0.      0.      0.      0.      0.      0.      0.043  5.448]]
-----

```

```

Iteration: 75
Estimated xEst:
[[ 47.035]
 [ 38.623]
 [ 41.19 ]
 [-25.859]
 [-47.182]
 [ 22.902]
 [-45.535]
 [  2.125]
 [-10.43 ]
 [-15.613]]
Estimated PEst:
[[ 4.028 -1.661  0.      0.      0.      0.      0.      0.      0.      0. ]
 [-1.661  3.504  0.      0.      0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      4.3    -1.057  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.     -1.057  3.841  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      5.793  1.74   0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      1.74   6.251  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      4.334  0.125  0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      0.125  7.228  0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      0.      0.      4.511  0.043]
 [ 0.      0.      0.      0.      0.      0.      0.      0.      0.043  5.448]]
-----

```

```

Iteration: 80
Estimated xEst:
[[ 46.957]
 [ 38.576]
 [ 41.19 ]
 [-25.859]
 [-47.182]
 [ 22.902]
 [-44.971]
 [  2.476]
 [-10.884]
 [-15.742]]
Estimated PEst:
[[ 2.582 -0.531  0.      0.      0.      0.      0.      0.      0.      0. ]
 [-0.531  2.407  0.      0.      0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      4.3    -1.057  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.     -1.057  3.841  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      5.793  1.74   0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      1.74   6.251  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      4.105 -0.03   0.      0. ]

```

| | | | | | | | | | | | |
|---|----|----|----|----|----|----|-------|-------|--------|--------|---|
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.03 | 6.882 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4.124 | -0.164 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -0.164 | 4.882 |] |

Iteration: 85

Estimated xEst:

```
[[ 46.957]
 [ 38.576]
 [ 41.281]
 [-26.255]
 [-47.423]
 [ 22.784]
 [-44.689]
 [  3.061]
 [-11.491]
 [-16.689]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|-------|-------|--------|--------|-------|-------|---|
| [| 2.582 | -0.531 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.531 | 2.407 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 4.151 | -0.977 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.977 | 3.622 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 4.864 | 1.158 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 1.158 | 5.553 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 3.909 | -0.153 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.153 | 6.539 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.97 | -0.24 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -0.24 | 4.633 |] |

Iteration: 90

Estimated xEst:

```
[[ 46.957]
 [ 38.576]
 [ 41.173]
 [-26.571]
 [-47.423]
 [ 22.784]
 [-44.25 ]
 [  2.111]
 [-11.563]
 [-16.518]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|-------|-------|-------|-------|-------|-------|--------|--------|---|
| [| 2.582 | -0.531 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.531 | 2.407 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 4.005 | -0.89 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.89 | 3.42 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 4.864 | 1.158 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 1.158 | 5.553 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 3.586 | -0.31 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.31 | 5.802 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.707 | -0.313 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -0.313 | 4.138 |] |

Iteration: 95

Estimated xEst:

```
[[ 46.658]
 [ 38.987]
 [ 41.485]
 [-26.424]
 [-48.085]
 [ 23.093]]
```

```

[-44.196]
[ 2.525]
[-12.147]
[-16.419]]
Estimated PEst:
[[ 2.438 -0.533 0. 0. 0. 0. 0. 0. 0. 0. ]
 [-0.533 2.274 0. 0. 0. 0. 0. 0. 0. 0. ]
 [ 0. 0. 3.866 -0.804 0. 0. 0. 0. 0. 0. ]
 [ 0. 0. -0.804 3.236 0. 0. 0. 0. 0. 0. ]
 [ 0. 0. 0. 0. 4.518 0.955 0. 0. 0. 0. ]
 [ 0. 0. 0. 0. 0.955 5.182 0. 0. 0. 0. ]
 [ 0. 0. 0. 0. 0. 0. 3.447 -0.349 0. 0. ]
 [ 0. 0. 0. 0. 0. 0. -0.349 5.44 0. 0. ]
 [ 0. 0. 0. 0. 0. 0. 0. 0. 3.584 -0.311]
 [ 0. 0. 0. 0. 0. 0. 0. 0. -0.311 3.899]]
-----

```

Thinking about it (1)

Having completed these trials, you will be able to **answer the following questions**:

In the **one landmark** case:

- Discuss the evolution of the variables `xEst` and `Pest`, including their dimensions.

Como tenemos una única marca en el mapa, observada en la iteración inicial, la variable `xEst` tiene dimensión (1,2) y tiene asociada una matriz de covarianza `PEst` de dimensión (2,2). Conforme más observaciones del robot a dicha marca, su incertidumbre decrece y por tanto, su estimación mejora.

In the **five landmarks** case:

- Why and how the content of the variables `xEst` and `Pest` has change? Discuss their size.

En este caso, dado que no estamos observando los cinco landmarks en el mapa en la primera iteración, estos se consideran en las variables `xEst` y `Pest` una vez que se observan por primera vez y no desde el principio. Ahora las dimensiones son $\text{size}(\text{xEst}) = (2 \cdot n_{\text{obs_landmarks}}, 1)$ y $\text{size}(\text{PEst}) = (2 \cdot n_{\text{obs_landmarks}}, 2 \cdot n_{\text{obs_landmarks}})$. En cada iteración del algoritmo, la estimación respecto al punto de referencia observado tiende a mejorar, mientras que su incertidumbre asociada disminuye (solo la de la observación del punto de referencia observado).

- What structure does the matrix of covariances have? Is there any kind of correlation among the observations of different landmarks?

La matriz `Pest` está compuesta por sub-matrices 2x2 en su diagonal, cada una asociada a un punto de referencia. Además no existe correlación entre las observaciones de diferentes puntos de referencia (ya que el resto de valores son aproximados a 0).

Getting performance results

As is normal, the contracting company requires some information about how well our EFK implementation performs. For that, your colleagues have implemented a logger, which is meant to store some information each loop regarding the method performance and plot it at the end of its execution. **Execute the following code cells and take a look at that plots!**

In [17]: `import pandas as pd`

```
class Logger():
    """ Logs info about the covariance and error of a map.

    Attrs:
        n_features: Number of features in the world.
        log_error: Matrix to store the error in the fitting for each landmark
        log_det: Matrix to store the determinant of the covariance matrix for

    """
    def __init__(self, n_steps, n_features):
        """ Initializes each matrix to log the information

        Args:
            n_steps: Maximum number of steps our robot will take.
            n_features: Number of features in the world.
        """
        self.n_features = n_features
        self.log_error = np.empty((n_steps, n_features))
        self.log_det = np.empty((n_steps, n_features))

    def log(self, k: int, robot: EFKMappingRobot, Map: np.ndarray):
        """ Computes relevant info about the error and covariances.

        It is called once per loop in the demo.

        Args:
            k: Number of iteration we are at. Range: [0, n_steps)
            robot:
            Map:
        """
        for idx in range(self.n_features):
            tid = robot.MappedLandmarks[idx, 0]
            if tid <= -1:
                self.log_error[k, idx] = np.inf
                self.log_det[k, idx] = np.inf
            else:
                self.log_det[k, idx] = np.linalg.det(robot.PEst[tid:tid+2, tid:tid+2])
                self.log_error[k, idx] = np.sqrt(np.sum((robot.xEst[tid:tid+2, 0] - Map[tid:tid+2, 0])**2))

    def plot(self):
        """ Plot all relevant figures. It is called at the end of the demo"""
        fig1, ax1 = plt.subplots(1, 1, sharex=True)
        fig2, ax2 = plt.subplots(1, 1, sharex=True)
        fig2.tight_layout()
        fig1.tight_layout()

        df1 = pd.DataFrame(data=self.log_error, columns=['Landmark {}'.format(i) for i in range(self.n_features)])
        ax1.set_title('Error between map and est')
        df1.plot(ax=ax1)
        df2 = pd.DataFrame(data=np.log(self.log_det), columns=['Landmark {}'.format(i) for i in range(self.n_features)])
        ax2.set_title('Log of determinant of covariance matrix')
        df2.plot(ax=ax2)
```

```
ax2.set_title('Det. of covar.')
df2.plot(ax = ax2)
```

```
#mode = 'step_by_step'
mode = 'visualize_process'
#mode = 'non_stop'

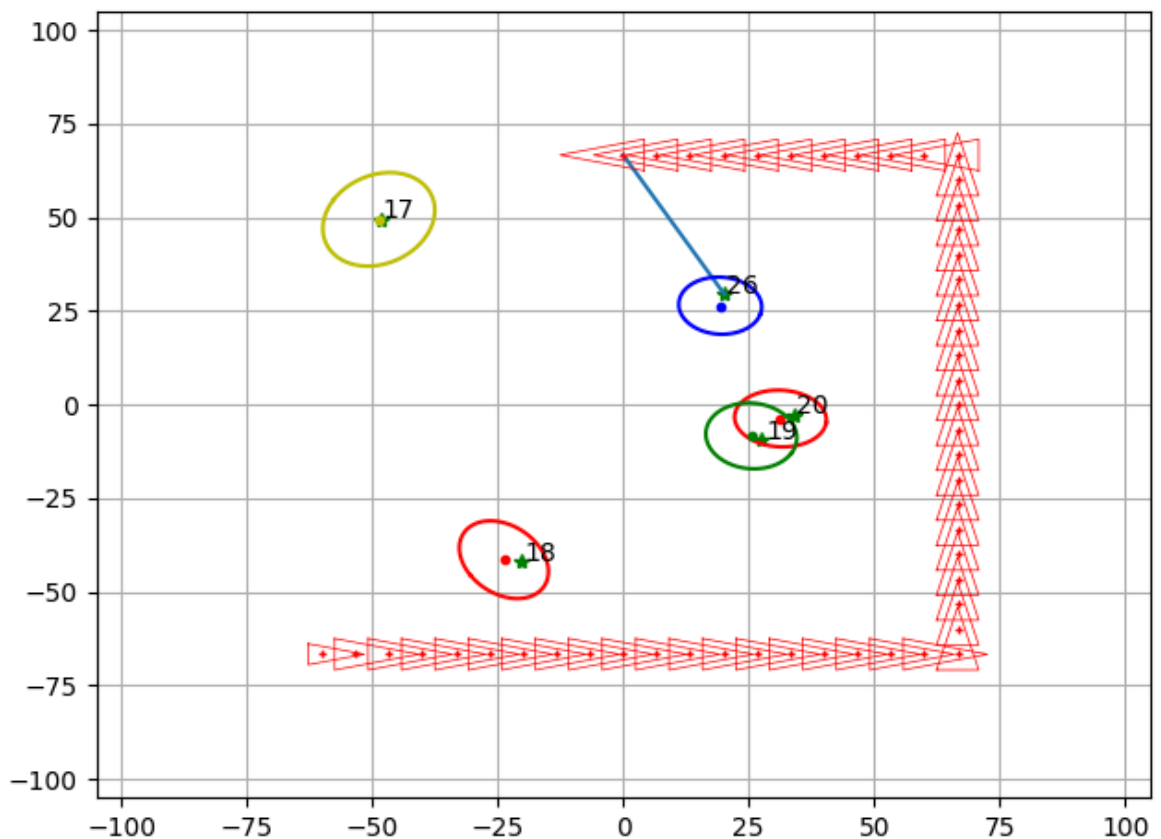
# WORLD MAP
# Num features/Landmarks considered within the map
nLandmarks = 5
# Generation of the map
MapSize = 100
Map = MapSize*random.rand(2,nLandmarks)-MapSize/2

# ROBOT
# Covariances for our very bad&expensive sensor (in the system <d,theta>)
Sigma_r = 8.0
Sigma_theta = 7*np.pi/180
# Initial robot pose
xVehicleTrue = np.vstack([-MapSize/1.5, -MapSize/1.5, 0.]) # We know the exact r

robot = EKFMappingRobot(xVehicleTrue, Sigma_r, Sigma_theta, nLandmarks)

nSteps=100
logger = Logger(n_features=nLandmarks, n_steps=nSteps)

demo_ekf_mapping(robot,
                  Map,
                  nLandmarks,
                  logger=logger,
                  mode='non_stop',
                  nSteps=nSteps)
```



Iteration: 0
 Estimated xEst:
 [[40.922]
 [-18.225]]
 Estimated PEst:
 [[87.662 -50.925]
 [-50.925 173.6]]

 Iteration: 5
 Estimated xEst:
 [[33.019]
 [-2.902]
 [19.721]
 [35.166]
 [-31.835]
 [51.354]]
 Estimated PEst:
 [[43.023 -22.188 0. 0. 0. 0.]
 [-22.188 76.738 0. 0. 0. 0.]
 [0. 0. 59.672 -32.302 0. 0.]
 [0. 0. -32.302 48.585 0. 0.]
 [0. 0. 0. 0. 208.902 -18.21]
 [0. 0. 0. 0. -18.21 66.288]]

 Iteration: 10
 Estimated xEst:
 [[33.019]
 [-2.902]
 [15.4]
 [28.304]
 [-35.475]
 [51.867]
 [32.892]
 [-8.491]]
 Estimated PEst:
 [[43.023 -22.188 0. 0. 0. 0. 0. 0.]
 [-22.188 76.738 0. 0. 0. 0. 0. 0.]
 [0. 0. 42.93 -20.172 0. 0. 0. 0.]
 [0. 0. -20.172 31.324 0. 0. 0. 0.]
 [0. 0. 0. 0. 103.689 -4.953 0. 0.]
 [0. 0. 0. 0. -4.953 32.413 0. 0.]
 [0. 0. 0. 0. 0. 0. 29.506 -11.657]
 [0. 0. 0. 0. 0. 0. -11.657 38.009]]

 Iteration: 15
 Estimated xEst:
 [[33.019]
 [-2.902]
 [19.483]
 [24.442]
 [-35.475]
 [51.867]
 [31.159]
 [-8.646]
 [-23.752]
 [-44.389]]
 Estimated PEst:
 [[43.023 -22.188 0. 0. 0. 0. 0. 0.]
 [0.]
 [-22.188 76.738 0. 0. 0. 0. 0. 0.]

| | | | | | | | | | |
|----------|----|--------|--------|---------|--------|--------|--------|--------|--|
| 0. |] | | | | | | | | |
| [0. | 0. | 25.499 | -9.104 | 0. | 0. | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | -9.104 | 16.544 | 0. | 0. | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 103.689 | -4.953 | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | -4.953 | 32.413 | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 16.964 | -5.287 | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | -5.287 | 19.626 | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 9.173 | |
| -9.234] | | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -9.234 | |
| 62.445]] | | | | | | | | | |

Iteration: 20

Estimated xEst:

```
[[ 31.604]
 [ -5.808]
 [ 19.504]
 [ 24.783]
 [-35.475]
 [ 51.867]
 [ 26.88 ]
 [ -8.512]
 [-24.775]
 [-40.178]]
```

Estimated PEst:

| | | | | | | | | | |
|----------|--------|--------|--------|---------|--------|--------|--------|-------|--|
| [27.264 | -9.113 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [-9.113 | 36.836 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 17.824 | -5.246 | 0. | 0. | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | -5.246 | 11.04 | 0. | 0. | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 103.689 | -4.953 | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | -4.953 | 32.413 | 0. | 0. | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 13.231 | -3.235 | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | -3.235 | 14.834 | 0. | |
| 0. |] | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 7.445 | |
| -4.91] | | | | | | | | | |
| [0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -4.91 | |
| 20.597]] | | | | | | | | | |

Iteration: 25

Estimated xEst:

```
[[ 32.571]
 [ -1.265]
 [ 19.359]
 [ 26.523]
 [-47.685]
 [ 53.472]
```



```

[ 26.096]
[ -8.37 ]
[-23.323]
[-39.534]]
Estimated PEst:
[[18.759 -4.126  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[-4.126 23.031  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      15.344 -3.963  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      -3.963  9.293  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      65.281  3.742  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      3.742 22.422  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      10.415 -2.032  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      -2.032 11.906  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      0.      0.      6.414 -3.552]
[ 0.      0.      0.      0.      0.      0.      0.      0.      -3.552 13.658]]
-----

```

Iteration: 30
Estimated xEst:

```

[[ 32.571]
[ -1.265]
[ 19.359]
[ 26.523]
[-47.685]
[ 53.472]
[ 27.462]
[ -8.016]
[-22.59 ]
[-42.183]]

```

```

Estimated PEst:
[[18.759 -4.126  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[-4.126 23.031  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      15.344 -3.963  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      -3.963  9.293  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      65.281  3.742  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      3.742 22.422  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      5.721 -0.689  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      -0.689 6.769  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      0.      0.      5.695 -2.682]
[ 0.      0.      0.      0.      0.      0.      0.      0.      -2.682 10.779]]
-----

```

Iteration: 35
Estimated xEst:

```

[[ 30.012]
[ -2.421]
[ 18.435]
[ 27.11 ]
[-47.685]
[ 53.472]
[ 27.462]
[ -8.016]
[-22.59 ]
[-42.183]]

```

```

Estimated PEst:
[[ 8.938 -0.704  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[-0.704  9.378  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      13.335 -2.817  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      -2.817  7.963  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      65.281  3.742  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      3.742 22.422  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      5.721 -0.689  0.      0.      0. ]

```

| | | | | | | | | | | | |
|---|----|----|----|----|----|----|--------|-------|--------|--------|----|
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.689 | 6.769 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 5.695 | -2.682 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -2.682 | 10.779 |]] |

Iteration: 40

Estimated xEst:

```
[[ 31.784]
 [ -2.177]
 [ 17.396]
 [ 25.759]
 [-47.685]
 [ 53.472]
 [ 27.462]
 [ -8.016]
 [-21.9 ]
 [-40.62 ]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| [| 7.986 | -0.458 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.458 | 8.219 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 10.646 | -1.412 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -1.412 | 6.291 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 65.281 | 3.742 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 3.742 | 22.422 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 5.721 | -0.689 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.689 | 6.769 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4.715 | -1.756 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.756 | 8.781 |]] |

Iteration: 45

Estimated xEst:

```
[[ 31.784]
 [ -2.177]
 [ 17.778]
 [ 26.452]
 [-46.241]
 [ 49.362]
 [ 27.901]
 [ -7.88 ]
 [-21.9 ]
 [-40.62 ]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|
| [| 7.986 | -0.458 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.458 | 8.219 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 9.574 | -0.975 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.975 | 5.741 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 26.634 | 8.149 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 8.149 | 16.501 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 5.178 | -0.654 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.654 | 6.018 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 4.715 | -1.756 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.756 | 8.781 |]] |

Iteration: 50

Estimated xEst:

```
[[ 31.784]
 [ -2.177]
 [ 18.597]
 [ 26.826]
 [-46.241]
 [ 49.362]
```

```

[ 26.862]
[ -7.442]
[-23.121]
[-39.211]]
Estimated PEst:
[[ 7.986 -0.458  0.      0.      0.      0.      0.      0.      0.      0. ]
 [-0.458  8.219  0.      0.      0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      7.643 -0.504  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.     -0.504  4.913  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      0.      0.     26.634  8.149  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      8.149 16.501  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      4.718 -0.687  0.      0. ]
 [ 0.      0.      0.      0.      0.      0.     -0.687  5.23  0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      0.      0.      4.075 -1.333]
 [ 0.      0.      0.      0.      0.      0.      0.      0.     -1.333  7.58 ]]
-----

```

Iteration: 55

Estimated xEst:

```

[[ 30.566]
 [ -2.894]
 [ 19.458]
 [ 26.645]
 [-48.155]
 [ 49.173]
 [ 26.862]
 [ -7.442]
 [-23.121]
 [-39.211]]

```

Estimated PEst:

```

[[ 6.223 -0.806  0.      0.      0.      0.      0.      0.      0.      0. ]
 [-0.806  4.847  0.      0.      0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      6.225 -0.317  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.     -0.317  4.276  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      0.      0.     21.319  7.395  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      7.395 15.289  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      4.718 -0.687  0.      0. ]
 [ 0.      0.      0.      0.      0.      0.     -0.687  5.23  0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      0.      0.      4.075 -1.333]
 [ 0.      0.      0.      0.      0.      0.      0.      0.     -1.333  7.58 ]]
-----

```

Iteration: 60

Estimated xEst:

```

[[ 29.987]
 [ -2.031]
 [ 19.698]
 [ 26.933]
 [-49.828]
 [ 49.267]
 [ 26.553]
 [ -7.658]
 [-23.132]
 [-39.396]]

```

Estimated PEst:

```

[[ 5.663 -0.671  0.      0.      0.      0.      0.      0.      0.      0. ]
 [-0.671  3.906  0.      0.      0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      5.629 -0.335  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.     -0.335  3.907  0.      0.      0.      0.      0.      0. ]
 [ 0.      0.      0.      0.     17.168  6.396  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      6.396 14.18  0.      0.      0.      0. ]
 [ 0.      0.      0.      0.      0.      0.      4.345 -0.441  0.      0. ]

```

| | | | | | | | | | | | |
|---|----|----|----|----|----|----|--------|-------|--------|---------|---|
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.441 | 4.301 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.852 | -1.252] | |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.252 | 7.151]] | |

Iteration: 65

Estimated xEst:

```
[[ 30.555]
 [ -4.118]
 [ 19.698]
 [ 26.933]
 [-50.067]
 [ 50.008]
 [ 25.901]
 [ -8.025]
 [-23.132]
 [-39.396]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---|
| [| 4.605 | -0.185 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.185 | 2.909 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 5.629 | -0.335 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.335 | 3.907 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 11.799 | 4.634 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 4.634 | 12.284 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 3.983 | -0.263 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.263 | 3.843 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.852 | -1.252] | |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.252 | 7.151]] | |

Iteration: 70

Estimated xEst:

```
[[ 30.555]
 [ -4.118]
 [ 19.365]
 [ 25.598]
 [-50.067]
 [ 50.008]
 [ 25.444]
 [ -8.167]
 [-23.191]
 [-39.998]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---|
| [| 4.605 | -0.185 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.185 | 2.909 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 4.765 | -0.207 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.207 | 3.176 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 11.799 | 4.634 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 4.634 | 12.284 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 3.716 | -0.202 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.202 | 3.594 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.588 | -1.253] | |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.253 | 6.447]] | |

Iteration: 75

Estimated xEst:

```
[[ 30.942]
 [ -3.953]
 [ 19.862]
 [ 26.306]
 [-46.884]
 [ 52.475]
```

```

[ 25.444]
[ -8.167]
[-23.191]
[-39.998]]
Estimated PEst:
[[ 4.275 -0.155  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[-0.155  2.777  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      4.084 -0.088  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.     -0.088  2.795  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      8.617  3.197  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      3.197 10.647  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      3.716 -0.202  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.     -0.202  3.594  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      0.      0.      3.588 -1.253 ]
[ 0.      0.      0.      0.      0.      0.      0.      0.     -1.253  6.447]]
-----

```

Iteration: 80
Estimated xEst:

```

[[ 31.178]
[ -3.509]
[ 19.862]
[ 26.306]
[-46.884]
[ 52.475]
[ 25.472]
[ -7.97 ]
[-23.656]
[-40.171]]

```

```

Estimated PEst:
[[ 3.665 -0.154  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[-0.154  2.472  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      4.084 -0.088  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.     -0.088  2.795  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      8.617  3.197  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      3.197 10.647  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      3.569 -0.215  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.     -0.215  3.422  0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      0.      0.      3.518 -1.274 ]
[ 0.      0.      0.      0.      0.      0.      0.      0.     -1.274  6.112]]
-----

```

Iteration: 85
Estimated xEst:

```

[[ 30.907]
[ -3.227]
[ 19.549]
[ 25.986]
[-48.075]
[ 51.282]
[ 25.472]
[ -7.97 ]
[-23.687]
[-41.305]]

```

```

Estimated PEst:
[[ 3.509 -0.154  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[-0.154  2.384  0.      0.      0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      3.783 -0.043  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.     -0.043  2.654  0.      0.      0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      7.502  2.515  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      2.515  9.664  0.      0.      0.      0.      0. ]
[ 0.      0.      0.      0.      0.      0.      3.569 -0.215  0.      0.      0. ]

```

| | | | | | | | | | | | |
|---|----|----|----|----|----|----|--------|-------|--------|--------|----|
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.215 | 3.422 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.391 | -1.271 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.271 | 5.465 |]] |

Iteration: 90

Estimated xEst:

```
[[ 30.907]
 [ -3.227]
 [ 19.549]
 [ 25.986]
 [-47.536]
 [ 51.26 ]
 [ 25.481]
 [ -8.175]
 [-23.684]
 [-42.027]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|-------|-------|--------|--------|--------|--------|----|
| [| 3.509 | -0.154 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.154 | 2.384 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 3.783 | -0.043 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.043 | 2.654 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 5.979 | 1.608 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 1.608 | 7.934 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 3.428 | -0.207 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.207 | 3.252 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.274 | -1.225 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.225 | 4.876 |]] |

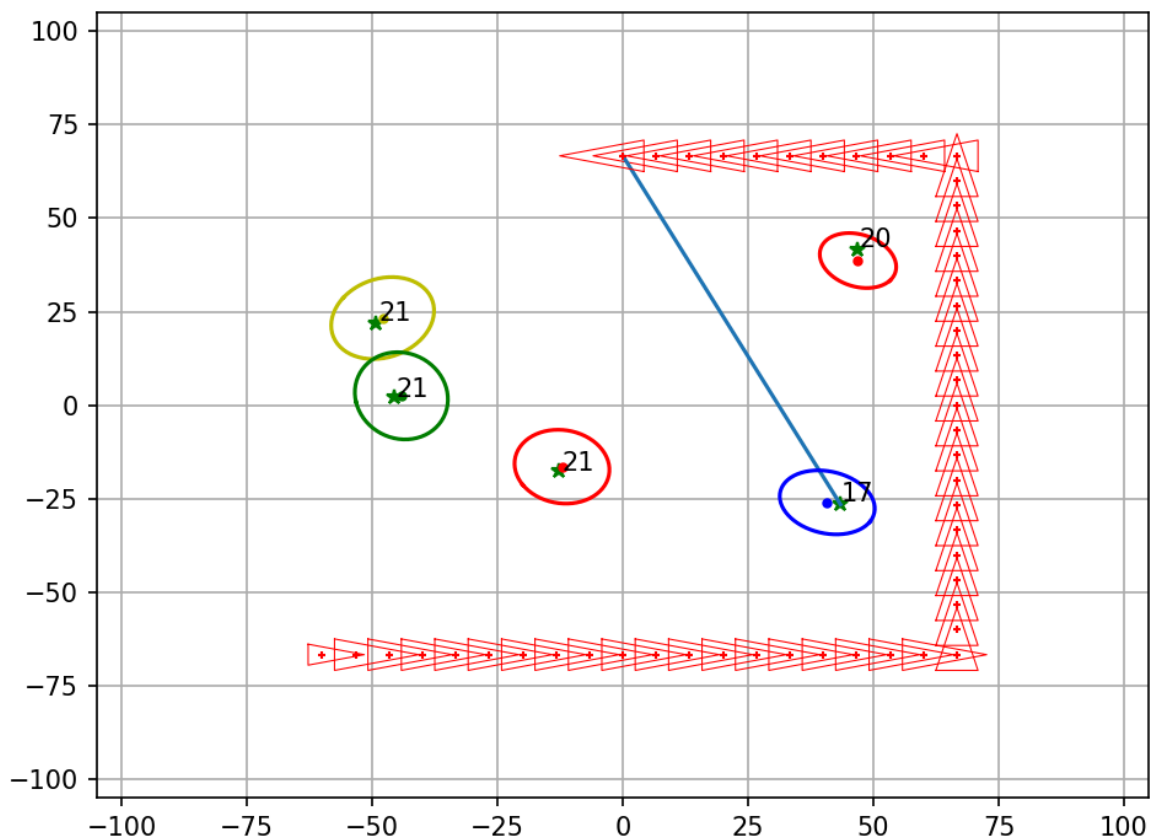
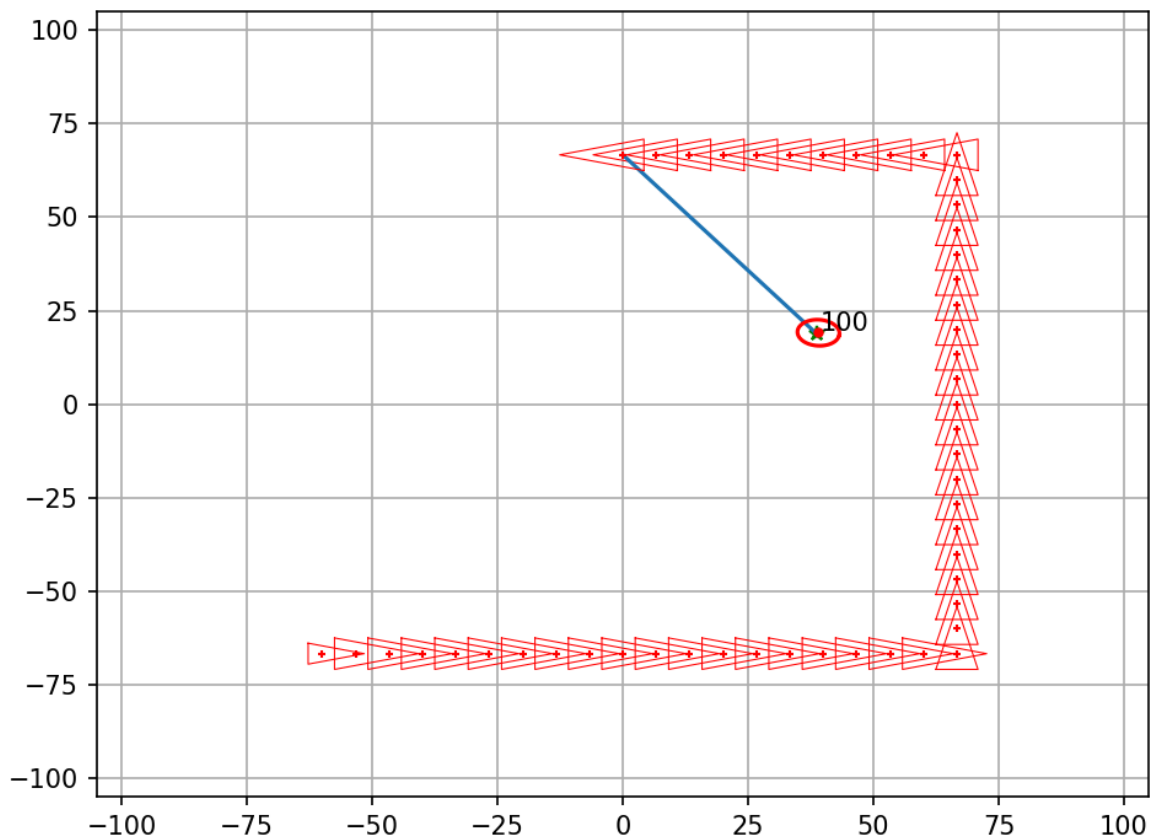
Iteration: 95

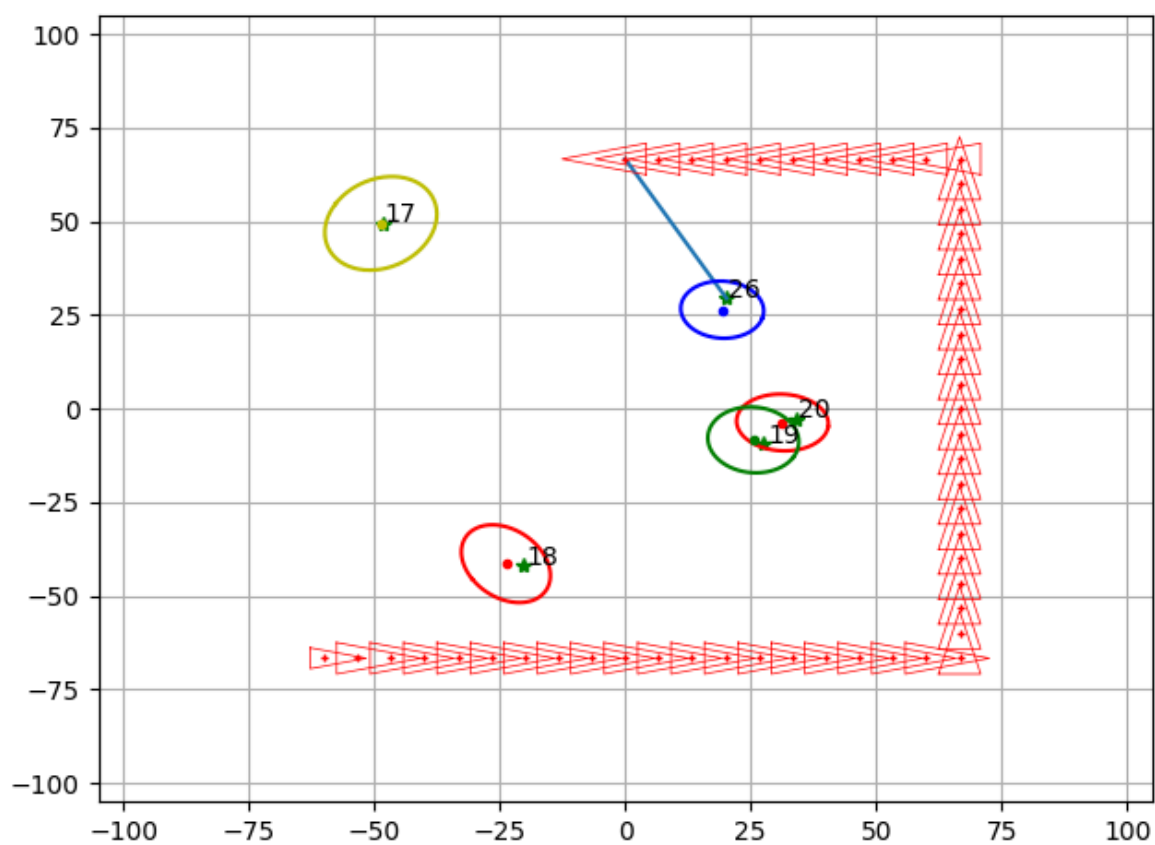
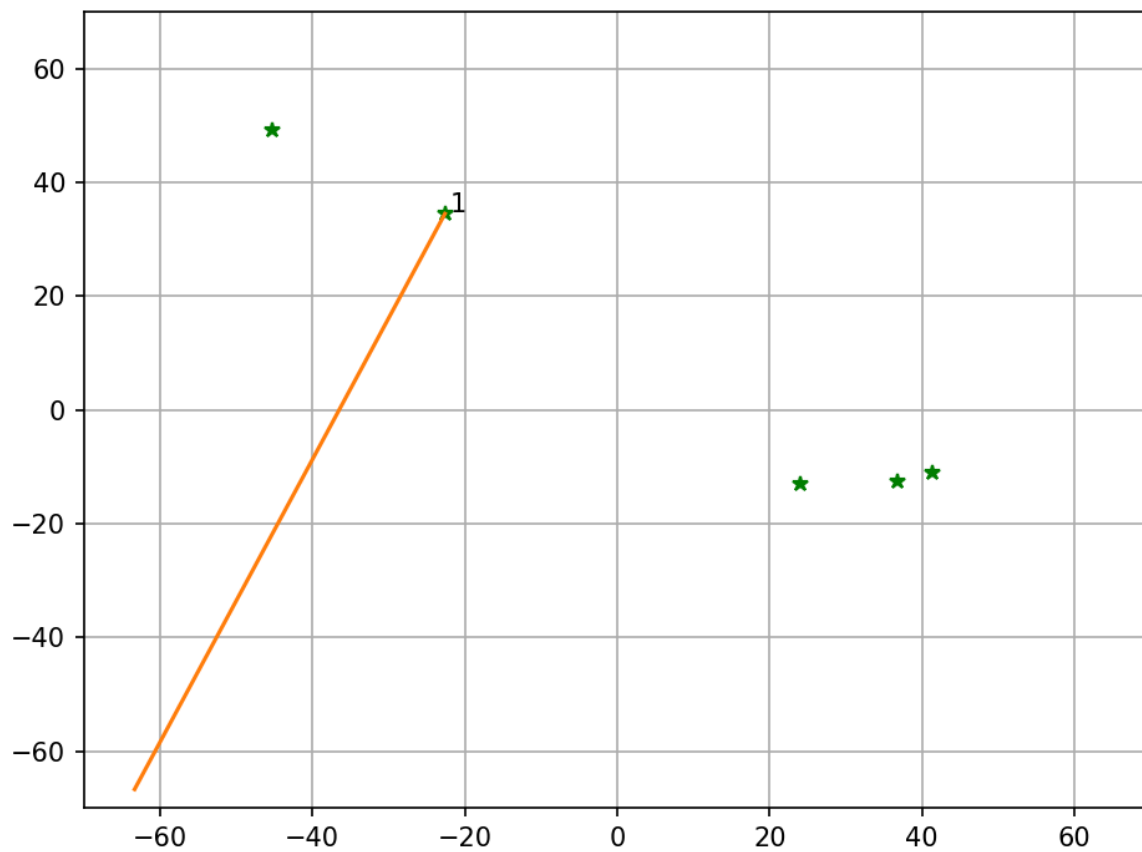
Estimated xEst:

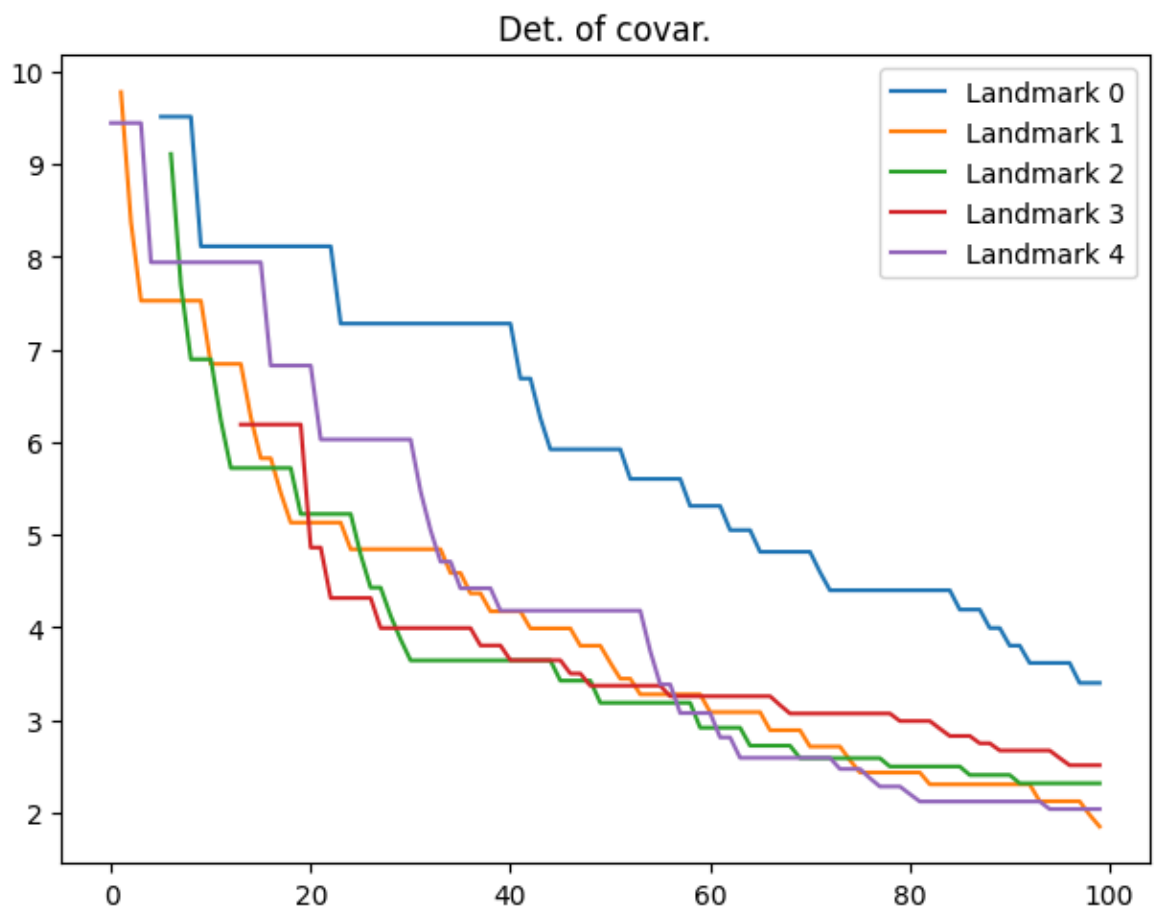
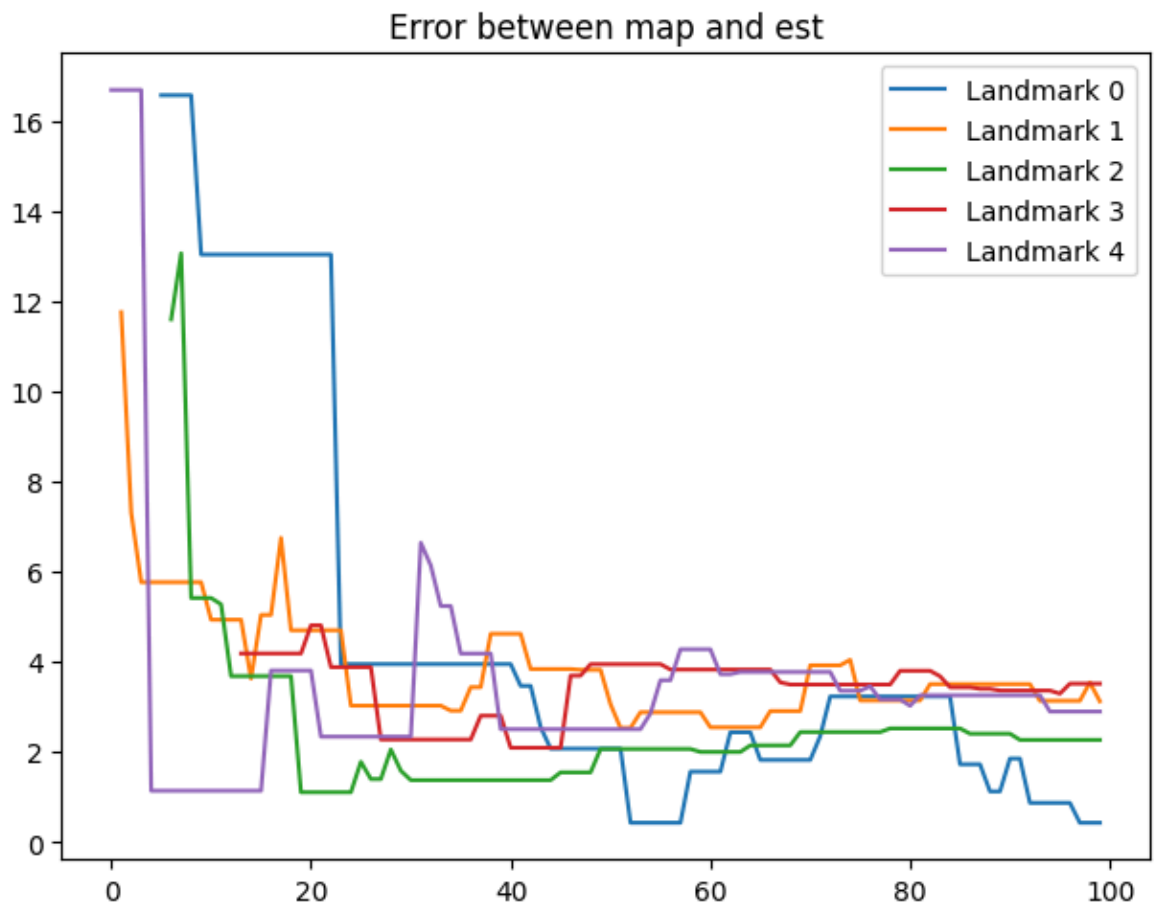
```
[[ 31.351]
 [ -3.711]
 [ 19.981]
 [ 26.303]
 [-47.815]
 [ 50.294]
 [ 25.54 ]
 [ -8.355]
 [-23.583]
 [-41.529]]
```

Estimated PEst:

| | | | | | | | | | | | |
|---|--------|--------|--------|--------|-------|-------|--------|--------|--------|--------|----|
| [| 3.352 | -0.138 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| -0.138 | 2.298 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 3.281 | -0.033 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | -0.033 | 2.549 | 0. | 0. | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 5.438 | 1.311 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 1.311 | 7.157 | 0. | 0. | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 3.292 | -0.189 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | -0.189 | 3.094 | 0. | 0. |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 3.213 | -1.173 |] |
| [| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | -1.173 | 4.58 |]] |







Thinking about it (2)

Having taken a look at the logger and its output, you will be able to **answer the following questions**:

- What information is shown in the figures produced by the logger?

La primera gráfica muestra la diferencia entre las posiciones estimadas de los puntos de referencia y sus posiciones reales. La segunda gráfica es el determinante de la submatriz de covarianza asociada a cada punto de referencia en Pest. Por tanto, lo que nos muestra es que el error con el tiempo decrece.

- The information about the error and the determinant of the covariance is provided for first time at different iterations of the algorithm for each landmark. Is that an error? Why is this happening?

No. Aparecen en la iteración en la que dichos puntos de referencia son observados por primera vez. Antes de eso, el robot no tiene información sobre ellos.

- The error associated to each landmark not always decreases with new observations. Why could this happen?

El error asociado con un punto de referencia no siempre puede disminuir con nuevas observaciones porque el algoritmo puede requerir más observaciones para converger a una estimación precisa. Una vez que un punto de referencia es observado, no siempre será observado en todas las iteraciones posteriores, por lo que la incertidumbre permanecerá constante hasta que sea observado nuevamente.

- On the contrary, the determinant of the covariance matrix associated to each landmark always decreases when new observations are available. Is this an error? Why?

No. Aunque el error pueda empeorar, si combinamos dos piezas de información, la incertidumbre asociada disminuye siempre.

In []: