The U Manifold and Ontological Differentiation

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Part I.
Abstract

A new manifold notion ($\mathbf{U}$) is proposed along with an operation (OD) to calculate distanced between the points in such manifold. In this work the different properties of the manifold and the operation are shown as long as the results of defining it alongside functions. Finally it is applied in a language model and it is discussed how it could be applied into physical models.

Part II.
Introduction

The concept of manifold was born out of the need of describing the intuitive idea of space, that is, the distribution of different points and their relations. This idea was developed during the 19th and 20th century specially, it has since then been a central part of many disciplines and theories both in mathematics and physics. While many kinds of manifolds are possible, only a few remain well known and daily used. In this work we propose a manifold which should be a further notion from its regular version.

We will present a manifold, which we call $\mathbf{U}$, whose elements are continuously related, every point may be defined according to other points, so every point is as relative as any other. The motivation behind the creation of such a manifold is to represent the inherent relative and related structure of ideas. It is rare, or rather impossible, to have ideas which are not defined and explained through the use of others, be it mathematical, physical or philosophical. The aim of the creation for such a manifold is precisely to give a formulation to this situation.

Since it is also important to be able to operate on manifolds, we will also present here an operation, which we call Ontological Differentiation, in order to understand the relations between the points in $\mathbf{U}$, given their inter-defined nature.

Part III. Definitions and Properties

1. U Manifold

Definition 1. $\mathbf{U}$ *is a set of elements* $\{P_1, ..., P_n\}$ *of size* $n \in \mathbb{N}$*, for which every element* $P_i \in \mathbf{U}$ *is itself a set of size* $m \in \mathbb{N}$ *of elements in* $\mathbf{U}$*, that is* $P_i = \{P_{i_1}, ..., P_{i_m}\}$ *where every* $P_{i_j} \in \mathbf{U}$*.*

2. Ontological Differentiation

Definition 2. *We call* R *the read function, an homomorphism such that* $\forall\, P_i \in \mathbf{U}$, $R^n(P_i) : \mathbf{U} \to \mathbf{U}$*, whose purpose is to expand the sets* n *times.*

That is, if we have as input some element $P_i \in \mathbf{U}$ whose content is given as $P_i = \{P_{i_1}, ..., P_{i_m}\}$, the read function R^1 will produce an output: $R^1(P_i) = P_{i_1}, ..., P_{i_m}$. The n parameter specifies how many times this expansion operation is to be carried out, being $n = 0$ just the reading $R^0(P_i) = P_i$. For instance, $R^2(P_i) = R^1(P_{i_1}), ..., R^1(P_{i_m})$. This can be generalized as $R^n(P_i) = R^{n-1}(P_{i_1}), ..., R^{n-1}(P_{i_m})$, and furthermore as $R^n(P_i) = R^{n-1}(R^{n-2}(...R^0(P_i)...))$.

Definition 3. *Let us have a subset* $\mathcal{A}$ *of* $\mathbf{U}$ *of size* k*. Let us call* $\mathcal{B}$ *another subset of* $\mathbf{U}$ *which contains* ω *orders of the Read Function operated on* $\mathcal{A}$*. That is, if we have that* $\mathcal{A} = \{P_1, ..., P_k\}$*, then we have that* $\mathcal{B} = \{R^0(P_1), ..., R^0(P_k), ..., R^\omega(P_1), ..., R^\omega(P_k)\}$*.*

In order to indicate up to what order of R^n in $\mathcal{A}$ we refer to, we write a sub-index n in $\mathcal{A}_n \subseteq \mathbf{U}$, therefore $\mathcal{A}_n \subseteq \mathcal{A}$, and of course we have that $\mathcal{A}_\omega = \mathcal{A}$.

Definition 4. *A cancellation rule* κ *determines how an element* $\alpha \in R^n(P_s) \in \mathcal{B}$ *gets cancelled.*

Definition 5. *A termination rule* τ *determines, based on the results of applying a cancellation rule* κ*, the value of* ω*.*

Definition 6. *A cancellation function equipped with a cancellation and a termination rule is expressed as* $F_{\kappa\tau}$*. Given some* $\mathcal{A} \subseteq \mathbf{U}$ *of size* k*, based on* κ *and* τ*,* $F_{\kappa\tau}$ *will act on* $\mathcal{A}$ *determining what elements get cancelled and the value of* ω*. Formally,* $F_{\kappa\tau} : \mathbf{U}^{k\omega} \to \mathbf{U}$*.*

Definition 7. *The result quantification of a cancellation function is given by* $\sum_{n=0}^{\omega} nF_{\kappa\tau}(\mathcal{A}_n)$*.*

Definition 8. *The distance between the sets in some subset* $\mathcal{A}$ *of* $\mathbf{U}$ *is given by an Ontological Differentiation (OD), which is a cancellation function* $F_{\kappa\tau}$ *with its result quantification* $\sum_{n=0}^{\omega} nF_{\kappa\tau}(\mathcal{A}_n)$*, and it is expressed as* $OD(\mathcal{A})$*.*

2.1. Types

In this subsection we will present some types of Ontological Differentiations which seem to the author to be the most intuitively straight forward and which will be used later on applied cases. In order to give examples, we will first present the Sample Set, a set example of **U** for us to illustrate the different OD's:

Example 1. *The Sample Set which we will use for some of the examples in this work:*

1:	*[2, 6]*
2:	*[6, 1]*
3:	*[4, 5]*
4:	*[5, 3]*
5:	*[3, 4]*
6:	*[1, 2]*
7:	*[6, 8]*
8:	*[9, 10]*
9:	*[10, 11]*
10:	*[11, 12]*
11:	*[12, 8]*
12:	*[10, 9]*
13:	*[14, 15]*
14:	*[15, 13]*
15:	*[13, 14]*
16:	*[17, 18]*
17:	*[18, 16]*
18:	*[16, 17]*
19:	*[18, 20]*
20:	*[21, 22]*
21:	*[22, 19]*
22:	*[21, 20]*

2.1.1. Weak Ontological Differentiation

The Weak Ontological Differentiation (WOD) is the most simple and less restrictive type of the here to be presented. It is given by the following cancellation and termination rules.

Definition 9. *The WOD cancellation rule is expressed as: If $\alpha \in R^n(P_s) \wedge \beta \in R^h(P_r) \wedge \alpha = \beta$, where $n, h \in (0, \omega)$ and $r, s \in (1, k)$, then $\alpha \in F_{\kappa\tau}(\mathcal{A})$.*

Definition 10. *The WOD termination rule is expressed as: ω is reached when $\forall \alpha \in R^n(P_s) \, \exists \beta \in R^h(P_r)$, where $n, h \in (0, \omega)$ and $r, s \in (1, k)$, such that $\alpha = \beta$.*

This means that in WOD, elements get cancelled when they are repeated at least once in any $R^n(P_s)$, that is, any element that is appears at least twice anywhere, at any order of the read function, across any set in $\mathcal{A}$, will get cancelled. The WOD terminates when there is at least one $R^n(P_s) \in \mathcal{B}$ which has all of its elements cancelled.

Let us illustrate this with an example from the Sample Set:

Example 2. $WOD(16, 20)$*:*

Level 0: *16; 20*

Level 1: *[17, 18]; [21, 22]*
16; 20
17, 18; 21, 22

Level 2: *[18, 16, 16, 17]; [22, 19, 21, 20]*
16(c); 20(c)
17(c), 18(c); 21(c), 22(c)
18(c), 16(c), 16(c), 17(c); 22(c), 19, 21(c), 20(c)

Terminating at Level 2 because one side is fully canceled.

Total distance: 18.

Let us explain the format a bit so it is easier to understand. The levels here refer to the order of the read function as one can easily identify. What we have done after every level is update every cancellation status of every level (every cancelled element is marked with "(c)"), until we have found one side of the differentiated sets (which we separate by the symbol ";") to be fully cancelled. We then count how many cancelled elements there are for every level and we multiply that number by the level number.

The purpose of this format is to offer a visual aid to the process, but we can certainly be more effective and optimized in our format. Let us call U_{nm} the set of uncancelled elements of level n and of differentiated set/side m, and R_{nm} the set of repeated (cancelled) elements of level n and of differentiated set/side m. It will stop when any U_{nm} becomes empty after updating after a new level of the read function, which is the exact equivalent of stopping when one side of any level has all of its elements fully cancelled. We place the number of times the element is repeated in its respective R_{nm} so the total distance can be easily calculated. The result shown is the total update when the calculation is terminated, not the step by step update as we had in the previous format. This optimized version is then:

Example 3. $WOD(16, 20)$*:*

Level 0:
Side 1 - U_0_1: {}, R_0_1: {'16': 1}

Side 2 - U_0_2: {}, R_0_2: {'20': 1}

Level 1:
Side 1 - U_1_1: {}, R_1_1: {'18': 1, '17': 1}
Side 2 - U_1_2: {}, R_1_2: {'22': 1, '21': 1}

Level 2:
Side 1 - U_2_1: {}, R_2_1: {'16': 2, '18': 1, '17': 1}
Side 2 - U_2_2: {'19'}, R_2_2: {'22': 1, '20': 1, '21': 1}

Termination condition met at Level 2 *because U_0_1, U_0_2, U_1_1, U_1_2, U_2_1 became empty.*

Total distance: 18

2.1.2. Strong Ontological Differentiation

The Strong Ontological Differentiation (SOD) is a stricter version of the WOD, while the termination rule is the same, the cancellation rule only applies to elements that are not found in the same differentiating set/side. So if two elements are found to be repeated across any level of the same side, they will not be cancelled unless they are also found on the other side.

Definition 11. *The SOD cancellation rule is expressed as: If* $\alpha \in R^n(P_s) \wedge \beta \in R^h(P_r) \wedge \alpha = \beta$, *where* $n, h \in (0, \omega)$ *and* $r, s \in (1, k)$ *but* $r \neq s$, *then* $\alpha \in F_{\kappa\tau}(\mathcal{A})$.

Definition 12. *The SOD termination rule is expressed as:* ω *is reached when* $\forall \alpha \in R^n(P_s) \, \exists \beta \in R^h(P_r)$, *where* $n, h \in (0, \omega)$ *and* $r, s \in (1, k)$ *but* $r \neq s$, *such that* $\alpha = \beta$.

Example 4. $SOD(16, 20)$*:*

Level 0: *16; 20*

Level 1: *[17, 18]; [21, 22]*
16; 20
17, 18; 21, 22

Level 2: *[18, 16, 16, 17]; [22, 19, 21, 20]*
16; 20
17, 18; 21, 22
18, 16, 16, 17; 22, 19, 21, 20

Level 3: *[16, 17, 17, 18, 17, 18, 18, 16]; [21, 20, 18, 20, 22, 19, 21, 22]*
16; 20
17, 18(c); 21, 22
18(c), 16, 16, 17; 22, 19, 21, 20
16, 17, 17, 18(c), 17, 18(c), 18(c), 16; 21, 20, 18(c), 20, 22, 19, 21, 22

Level 4: *[17, 18, 18, 16, 18, 16, 16, 17, 18, 16, 16, 17, 16, 17, 17, 18]; [22, 19, 21, 22, 16, 17, 21, 22, 21, 20, 18, 20, 22, 19, 21, 20]*
16(c); 20
17(c), 18(c); 21, 22
18(c), 16(c), 16(c), 17(c); 22, 19, 21, 20
16(c), 17(c), 17(c), 18(c), 17(c), 18(c), 18(c), 16(c); 21, 20, 18(c), 20, 22, 19, 21, 22
17(c), 18(c), 18(c), 16(c), 18(c), 16(c), 16(c), 17(c), 18(c), 16(c), 16(c), 17(c), 16(c), 17(c), 17(c), 18(c); 22, 19, 21, 22, 16(c), 17(c), 21, 22, 21, 20, 18(c), 20, 22, 19, 21, 20

Terminating at Level 4 because one side is fully canceled.

Total distance: 113.

As for the optimized format of the SOD calculation we have:

Example 5. $SOD(16, 20)$*:*

Level 0:
Side 1 - U_0_1: {}, R_0_1: {'16': 1}
Side 2 - U_0_2: {'20'}, R_0_2: {}

Level 1:
Side 1 - U_1_1: {}, R_1_1: {'18': 1, '17': 1}
Side 2 - U_1_2: {'21', '22'}, R_1_2: {}

Level 2:
Side 1 - U_2_1: {}, R_2_1: {'18': 1, '16': 2, '17': 1}
Side 2 - U_2_2: {'22', '19', '21', '20'}, R_2_2: {}

Level 3:
Side 1 - U_3_1: {}, R_3_1: {'18': 3, '16': 2, '17': 3}
Side 2 - U_3_2: {'21', '20', '22', '19'}, R_3_2: {'18': 1}

Level 4:
Side 1 - U_4_1: {}, R_4_1: {'16': 6, '17': 5, '18': 5}
Side 2 - U_4_2: {'22', '19', '21', '20'}, R_4_2: {'16': 1, '17': 1, '18': 1}

Termination condition met at Level 4 *because U_0_1, U_1_1, U_2_1, U_3_1, U_4_1 became empty.*

Total distance: 113

2.1.3. Great Ontological Differentiation

The Great Ontological Differentiation (GOD) is a more extensive approach than the past two types. The cancellation rule is the same as in WOD, but the termination rule requires that the calculation stop when no new element appears with a new level of the read function, that is, every single future read function will produce repeated elements. This OD will be used later on to detect those those sets which can loop towards infinity in an OD (it will depend if **U** is finite or infinite size, or if it's SOD or WOD), and it will be also used to find all the related sets to a set.

Definition 13. *The GOD cancellation rule is expressed as: If* $\alpha \in R^n(P_s) \wedge \beta \in R^h(P_r) \wedge \alpha = \beta$*, where* $n, h \in (0, \omega)$ *and* $r, s \in (1, k)$*, then* $\alpha \in F_{\kappa\tau}(\mathcal{A})$*.*

Definition 14. *The GOD termination rule is expressed as:* $\forall P_i \in \mathcal{A} \forall \alpha \in R^n(P_i) \exists \beta \in R^{n-1}(R^{n-2}(...R^0(P_i)...))$ *such that* $\alpha = \beta$*.*

Example 6. $GOD(16, 20)$*:*

Level 0:
Already Existing Elements - Repeated: ; Uncanceled:
New Elements in This Iteration: 16, 20
Updated Repeated Elements:
Updated Uncanceled Elements: 16, 20

Level 1:
Already Existing Elements - Repeated: ; Uncanceled: 16, 20
New Elements in This Iteration: 17, 18
Updated Repeated Elements:
Updated Uncanceled Elements: 16, 17, 20, 18

Level 2:
Already Existing Elements - Repeated: ; Uncanceled: 16, 17, 20, 18
New Elements in This Iteration:
Updated Repeated Elements: 16, 17, 18
Updated Uncanceled Elements: 20

Stopping at Level 2 because no new elements were found.

As we can see this format is different from the previous ones, as it is adapted to be more explicit and focused on the appearing of new elements, essential factor of this OD, that is why the total distance is not shown, but it certainly could be.

2.1.4. Eigen Ontological Differentiation

An Eigen Ontological Differentiation (EOD) is not defined by any cancellation or termination rule, but by the size of the $\mathcal{A}$ input, which is 1. EOD performs an OD of one set onto itself. There can be Weak, Strong or Great EOD's, we show here a $WEOD$:

Example 7. $WEOD(16)$*:*

Level 0: [16]

Level 1: [17, 18]
16
17, 18

Level 2: [18, 16, 16, 17]
16(c)
17(c), 18(c)
18(c), 16(c), 16(c), 17(c)

Stopping at Level 2 because the set is fully canceled.
Total distance: 10

An *EOD* will be useful later on to explain the concept of measure in **U**.

2.1.5. Others

As one can easily imagine, there can be as many variations of the rules as one wishes, mixing types, alternating them, perturb them, add to them and so on. The scope of this work is not to show a wide spectrum of the infinite possibilities, but to present the general concept and the most intuitive versions of it along their applications.

3. Properties

3.1. Topology

3.1.1. Manifold

In this subsection we will show that **U** is a topological manifold and can therefore be treated as such.

First, we will establish the concept of neighbourhood in topology, that is, a set of points that are within a distance k from one same point, so each point belongs to every one of its neighbourhoods. Here the analogous notion for a neighbourhood of P_i is presented:

Definition 15. *Neighbourhood of P_i:*

$$\mathrm{Nbd_k} P_i = \{P_j : OD(P_i, P_j) \leq k \in \mathbb{N}\}. \tag{1}$$

Now we will state the manifold definition and prove each of these statements for our case.

Definition 16. *A topological manifold is a second-countable, Hausdorff, locally Euclidean space. It has dimension n if it is locally Euclidean of dimension n.*

Definition 17. *A Hausdorff space is a space in which for any two distinct points, there exists neighbourhoods of each which are disjoint from each other.*

Lemma 8. $\mathbf{U}$ *is a Hausdorff space.*

Proof. We must show that for any two points P_i and P_j, there are Nbd's of them which are disjoint. This is easily proved by Nbd_0 of both P_i and P_j: $\mathrm{Nbd}_0 P^i = P_i$ and $\mathrm{Nbd}_0 P_j = P_j$, since for any P_x, if $D(P_x, P_y) = 0$, then $P_x = P_y$. Therefore, since $P_i \neq P_j$, $\mathrm{Nbd}_0 P_i \cap \mathrm{Nbd}_0 P_j = \emptyset$. □

This states that P's are separated.

Definition 18. *A second-countable space is a topological space with a countable base. T is a second-countable space if there is some countable collection $\mathcal{C} = \{C_i\}_{i=1}^{\infty}$ of open subsets of T such that any open subset of T can be written as a union of elements of some subfamily of $\mathcal{U}$.*

Lemma 9. $\mathbf{U}$ *is a second-countable space.*

Proof. We have to show that $\mathbf{U}$ is a second-countable space. Let us call $\{P_i\}_{i=1}^{\infty}$ our countable collection of open subsets of $\mathbf{U}$. They are open because for any $P_i = \{P_{i_1}, ..., P_{i_m}\}$, we have that its elements $\{P_{i_1}, ..., P_{i_m}\}$ (of any level from the read function), its points they are all within $\mathrm{Nbd}_1 P_i$ (or for any k for $\mathrm{Nbd}_\mathrm{k} P_i$ if we want to take another level). Now, let us take some open subset of $\mathbf{U}$, let us take some $\mathcal{A}$ as we have been doing, that is $\mathcal{A} = \{P_1, ..., P_k\}$ for some k. We have that $\mathcal{A}$ is a union of the elements $\{P_1, ..., P_k\}$, which all of them are elements in $\{P_i\}_{i=1}^{\infty}$. Therefore $\mathbf{U}$ must be second-countable. □

Definition 19. *A locally Euclidean space is a topological space $\mathcal{M}$ of dimension n if every point p in $\mathcal{M}$ has a neighbourhood $\mathcal{N}$ such that there is a homeomorphism ϕ from $\mathcal{N}$ onto an open subset of $\mathbb{R}^n$.*

Lemma 10. $\mathbf{U}$ *is a locally Euclidean space.*

Proof. We have to show that $\mathbf{U}$ is locally Euclidean. For that we have to show that every point P_i in $\mathbf{U}$ has a neighbourhood $\mathrm{Nbd}_\mathrm{k} P_i$ (for some $k \in \mathbb{N}$) which is homemorphic to an open subset of $\mathbb{R}^n$. Let us have that $\mathrm{Nbd}_\mathrm{k} P_i = \{P_1, ..., P_s\}$ for some $s \in \mathbb{N}$. Let us call ϕ the association of every subindex of the members in the neighbourhood with a natural number, up to s: $\{P_{\phi(1)}, ..., P_{\phi(s)}\}$. Since we have mapped with ϕ every member of the neighbourhood with a natural number and $\mathbb{N} \subset \mathbb{R}$, we have that $\mathbf{U}$ is locally Euclidean. □

Corollary 10.1. $\mathbf{U}$ *is a topological manifold.*

There is one rather remarkable aspect that can be obtained from mapping elements of $\mathbf{U}$ to $\mathbb{N}$. As we will see later on, in the section where we apply OD to a language model, a set of words or letters can be used as a $\mathbf{U}$ manifold. Let us imagine we want to show that this set up is also locally Euclidean. Let us have a function ϕ which maps every letter of the Latin alphabet to a base of ten with the exponent being the place the letter occupies in the alphabet (e.g. Owl=$P^{10^{15}10^{23}10^{12}}$, where the numbers do not represent a multiplication but just an abbreviation so $10^2 10^3 = 1001000$). Since numbers, any number, is a word made out of letters (e.g. 32,5 = thirty two comma five), we have that ϕ can map any $\alpha \in \mathbb{R}$ onto some $\beta \in \mathbb{N}$.

3.1.2. Non-Euclidean Duality

Let us take from the Sample Set the sets 16, 20 and 21. The triangle inequality states that $d(p_1, p_3) \leq d(p_1, p_2) + d(p_2, p_3)$, so we should have that $OD(16, 20) \leq OD(16, 21) + OD(21, 20)$. If we take the SOD of each and we substitute into the inequality we find that $113 \leq 42 + 3$, which clearly breaks the inequality. Therefore, $\mathbf{U}$ can be a manifold which is locally Euclidean, and at the same time it can be non-Euclidean and have their distances behave in probability-like relations.

3.2. Measure

We say a set $A \subseteq \mathbf{R}$ is Lebesgue measurable if for any other set $B \subseteq \mathbf{R}$, the following condition applies:

$$m(B) = m(B \cap A) + m(B \cap A^c). \tag{2}$$

Let us have our $\mathbf{U}$ to be the sets formed by $P_i = \{P_i + 1, P_i + 2\}$ (which we will later see that this is ordered after $f(x) = x + 1$, since $x = \{f(x), f(f(x))\}$. Let us take from those just $1 = [2, 3]$ and $2 = [3, 4]$, which we can denote as A_1 and A_2. Let us see if the Lebesgue measure condition is fulfilled for A_2:

$$m(A_2) = m(A_2 \cap A_1) + m(A_2 \cap A_1^c) = m([3, 4] \cap [2, 3]) + m([3, 4] \cap (\infty, 2) \cup (\infty, 3)) = m(3) + m([3, 4]). \tag{3}$$

Up to this stage, we can say that it is Lebesgue measurable, since we will have:

$$m(A_2) = m([3, 4]) = 4 - 3 = 1 = m(3) + m((3, 4]) = 0 + 1. \tag{4}$$

However, we also know that these sets are interdefined, that is, their definition can go beyond the first level from the read function. That means, initially, that since $3 = [4, 5]$ we would have:

$$m([3, 4]) = m(3) + m([3, 4]) = m([4, 5]) + m((3, 4]) = 2. \tag{5}$$

At this second stage the Lebesgue measure is not fulfilled, therefore $\mathbf{U}$ would not be Lebesgue measurable. This second stage directly states that: $m(2) = m(2) + m(3)$. This would not necessarily have to end here, we can also further develop the expression:

$$m(2) = m([4, 5]) + m((3, 4]) = m([5, 6] \cup [6, 7]) + m([4, 5] \cup [5, 6]) = m([5, 7]) + m([4, 6]) etc, \tag{6}$$

since the measure of $m([a, b, c]) = m([a, c])$. So at this stage we would have that the measure is of sets given by the ordering $f(x) = x + 2$ instead of $f(x) = x + 1$, no longer the same function, and as one keeps using the read function (vaguely noted here as "etc"), one would be raising $f(x) = x + k$ one by one.

So for A_n and A_{n-1} we have that:

- First stage: $m(A_n) = m(A_n \cap A_{n-1}) + m(A_n) \cap A^c_{n-1}$.
- Second stage: $m(A_n) = m(A_n) + m(A_{n+1})$.
- Third stage: Measure of sets given by increasing function of $y = x + k$.

And of course we have the zeroth stage, which is the trivial case where we have that the measure of any point $m(a) = 0$, so $m(A_n) = 0$. These so called stages here are the direct implication of the intrinsic read function.

We can also selectively alternate different stages in the same expression, arriving at situations like:

$$m([3,4]) = m(3)+m([3,4]) = m([4,5])+m([4,5]\cup[5,6]) = m([4,5])+m([4,5,6]) = m([4,5])+m([4,5]\cup \tag{7}$$

So if we decided to stop here, we could have the expression for the measure of any A_n and A_{n-1} as:

$$m(A_n) = m(A_{n+1}) + m(A_{n+1} \cup A_{n+4}) \tag{8}$$

All these situations here described for A_n and A_{n-1} can be generalized for A_n and A_{n-k}, therefore we have an infinite spectacle of possible options for measures of a single set in **U**, this means that **U** can be chosen to be a "normal" and Lebesgue measurable if we just stay with first level given by the read function, or it can be chosen not to be and display any kind of measure.

A more solid approach to calculate the measure of a set in **U** is to calculate its EOD, since it aims more appropriately at the question behind measure, which can be put to be "Given the elements of a set, where does it finish and what size is that from its beginning?". An $SEOD$, like any SOD, can either finish or not finish (depending on its $GEOD$ and if it is a finite or infinite size **U**). An $SEOD$ which does not finish has infinite size.

3.3. Algebra

Definition 20.

3.4. Connectivity

The elements in **U** are defined, or better said, they are interdefined, its interdefinition is a connection between sets. Its connectivity is therefore an interesting matter to study.

The connectivity among sets in $\mathbf{U}$ is a consequence of the fulfillment or violation of any of two conditions:

Condition 1: Every element of a set is a set itself (contains other elements): $\forall \alpha \in P_i \, \exists \, P_j \in \mathbf{U}$ such that $\alpha = P_j$.

Condition 2: Every set is itself an element of a different set: $\forall P_i \in \mathbf{U} \, \exists \, P_j \in \mathbf{U}$ such that $P_i \in \; P_j$.

Options:

1.- Fulfillment of both conditions, which is the same as fulfilling only Condition 2.
2.- Fulfillment of Condition 1, but not of Condition 2. OD does not work for those sets which don't fulfill condition 2. We can identify them, and then stop the operation, or it will either produce an error or run forever, applying infinitely the read function on a P_i which shows nothing.
3.- Violation of both conditions, which is the same as violation of Condition 1. In this case there will be sets for which OD does not work either, it will just produce blankness. We can identify these operations beforehand by identifying said sets, or we can let them grow to infinity.

We then give the following terminology to the type of sets which violate some condition:
- Unicularity: A set which violates Condition 2, it contains elements but it is itself not contained anywhere.
- Phantom: A set which violates Condition 1, and therefore condition 2, it is contained in set or sets, but it does not contain other elements, other sets.

Now let us define some more notions to study the connectivity of $\mathbf{U}$:

Definition 21. *The Island Finder Operator $I(P_i)$ takes an element P_1 from $\mathbf{U}$, it performs a Great Eigen Ontological Differentiation (GEOD) on it, and every P_i produced in that GEOD constitutes the set $GEOD(P_1)$. Then for every element $\alpha \in GEOD(P_1)$, it picks those $P_i \in R^1((GEOD(P_1))^c)$, where $(GEOD(P_1))^c$ is the complement of $GEOD(P_1)$, such that $\alpha \in P_i$. Let us have the set of those which fulfill the condition to be $\{P_2, ..., P_k\}$. Then it carries the following operation $\overset{k}{\underset{i=2}{I}}(P_i)$ iteration after iteration until the following condition is met: $\nexists \, \alpha \in GEOD(P_i)$ such that $\alpha \in R^1((GEOD(P_i))^c)$.*

Definition 22. *The set which contains all the elements from the initial $GEOD(P_1)$ until the final one $GEOD(P_f)$ when $I(P_i)$ finishes, is called an Island.*

Definition 23. *The SOD-able Condition is defined as: Let $\mathcal{A}$ be a subset of* $\mathbf{U}$ *as usual, let n be the maximum level (R^n) that $GOD(\mathcal{A})$ takes to finish, if $SOD(\mathcal{A})$ finishes within R^n, then $\mathcal{A}$ is SOD-able.*

Definition 24. *A Sub-Island is a subset of an Island whose elements fulfill the SOD-able Condition.*

The concept of an Island refers to those sets that are part of other sets, it establishes a "walkable" relation between them, that is, one can walk through some OD between them, they are reachable within some OD process. However, if one of the elements in an Island is a unicularity, the unicularity can walk to the non-unicularity elements, but not otherwise. On the opposite case, if a phantom is present in an Island, the other elements can walk to the phantom (and stop there), but not otherwise.

A Sub-Island is the set of elements in an Island which can be performed an SOD among them. If Islands are the most basic notion of connection between sets, Sub-Islands are so far the strictest.

This is of course for a $\mathbf{U}$ of finite size, if we were to consider a $\mathbf{U}$ of infinite size, then even if there were no unicularities or phantoms, many OD's of well-behaved P_i's would never finish since its $GEOD$ would never stop, for the read function is always adding up new elements after every level (as we will see in the recursive functions sub-section).

Example 11.

Part IV. Functions and Order

In this part we will deal with how the order or structure of the sets/elements in $\mathbf{U}$ can be arranged and what role some regular functions play in it, as well as randomness. We will then see how OD can operate on functions, since a $\mathbf{U}$ arranged according to some function $f(x)$ is just a set of its values. Let us first define what order is:

Definition 25. *For some $x_{0_0} \in \mathbf{U}$, for some function f, the order of its elements $x_{0_0} = [x_{0_1}, ..., x_{0_k}]$ for some $k \in \mathbb{N}$ is given by a structure S: $\{S(f(x_{0_i}))\}_{i=1}^{k}$.*

Here we have used x instead of P in the notation just so it is visually more familiar to what we are used to when describing functions.

Example 12. *Let us have the function $f = x + 1$ and the structure $S(f(x_{0_{n-1}})) = x_{0_n}$, then $x_{0_0} = [x_{0_1}, ..., x_{0_k}]$ will be given by $x_0 = [f(x_{0_0}), .., f(x_{0_{k-1}})]$. So if we take $x_{0_0} = 1$ then we have $1 = [2, 3, 4, ...]$ up to size k.*

4. Finite Functions

In this sub-section we will deal with a finite size **U** and functions which will consider this finiteness into its calculation. The fact that our set **U** is finite imposes a non-recursive nature into the operations, making it impossible to operate in $\mathbb{R}$ (which we will do when get to recursive functions sub-section).

For a same function $f(x) = x + k$, we will select just the one following structure S:

$$x_{0_n} = \begin{cases} x_{0_0} + 1 & \text{if } n = 1 \\ f(x_{0_{n-1}}) & \text{if } n \neq 1. \end{cases} \tag{9}$$

Let us see it in the following example:

Example 13. *For a* **U** *of size 10, with elements of size 3, for a function* $f(x) = x+2$*, and the above described structure* S*, we would have the following sets:*

Set 0: [1, 3, 5]
Set 1: [2, 4, 6]
Set 2: [3, 5, 7]
Set 3: [4, 6, 8]
Set 4: [5, 7, 9]
Set 5: [6, 8, 0]
Set 6: [7, 9, 1]
Set 7: [8, 0, 2]
Set 8: [9, 1, 3]
Set 9: [0, 2, 4]

As one can easily see, given the finiteness of **U**, when the function result would surpass the maximum set number, it outputs what the result would be taking the sets as a continuation (a circle if you will). So we have that $Set8 : [9, 1, 3]$ where 9 is just $9 + 1$, 1 is just $9 + 2$ (the continuation goes as $9, 0, 1$) and of course 3 is just $1 + 2$.

4.1. $f(x) = x + k$

We will investigate the OD's applied to a finite **U**, ordered for the given structure S, and the function $f(x) = x + k$, having as variable parameters: k in the function, the size of **U** and the size of its elements.

4.1.1. Results

5. Recursive Functions

In this sub-section we will deal with functions in the $\mathbb{R}$ realm. Our structure S will be the same as in (9), with a recursive adaptation in order for the sets to be rightfully calculated. This recursive adaptation consists of the following: If we have x_n and $x_{n_0} = [x_{n_1}, ..., x_{n_k}]$, the following points x_{n+m} will be taken from $\forall m \in x_{n_m}$.

Example 14. *For a* $\mathbf{U}$ *of size 10, with elements of size 2, for a function* $f(x) = \frac{1}{x}$*, and the already given structure* S*, we would have the following sets:*

Set 1:	*[2, 0.5]*
Set 2:	*[3, 0.$\overline{3}$]*
Set 0.5:	*[0.$\overline{6}$, 1.5]*
Set 3:	*[4, 0.25]*
Set 0.$\overline{3}$:	*[0.75, 1.$\overline{33}$]*
Set 0.$\overline{6}$:	*[1.$\overline{6}$, 0.6]*
Set 1.5:	*[0.4, 2.5]*
Set 4:	*[5, 0.2]*
Set 0.25:	*[0.8, 1.25]*
Set 0.75:	*[1.75, 0.57]*

We have obviously rounded up some of these numbers in this example in order to give it a shorter look.

As we can see, there are some phantoms for a $\mathbf{U}$ of this size, what we will do when one is found is expand them until the OD in question is finished. The OD's will only be performed between $Set1$ and the rest.

5.1. $f(x) = \frac{1}{x}$

We will investigate the OD's applied to a recursive/infinite $\mathbf{U}$, ordered for the given structure S, and the function $f(x) = \frac{1}{x}$, where our variable parameters will only be the size of $\mathbf{U}$, the size of the elements will be restricted to 2.

5.1.1. Results

5.2. $f(x) = \sin x$

We will investigate the OD's applied to a recursive/infinite $\mathbf{U}$, ordered for the given structure S, and the function $f(x) = \sin x$, where our variable parameters will only be the size of $\mathbf{U}$, the size of the elements will be restricted to 2.

5.2.1. Results

5.3. $f(x) = \cos x$

5.3.1. Results

5.4. $f(x) = x + k$

We will investigate the OD's applied to a recursive/infinite $\mathbf{U}$, ordered for the given structure S, and the function $f(x) = x + k$ (we have come back to this function for this recursive version), where our variable parameters are: k in the function, the size of $\mathbf{U}$ and the size of its elements.

5.4.1. Results

6. Randomness

In this section we will study $\mathbf{U}$ manifold whose element's order is given by a random process. Their members will be generated randomly (using NumPy) restricted to a size of $\mathbf{U}$ range.

When it comes to the constraints under which these sets are generated, there are many which we will now present here, but all of them have in common that no set can contain itself as fundamental constraint. The different types are:

- Unconstrained: When generating sets, there are no specific rules or restrictions applied. The elements in each set are chosen randomly without any regard for previous selections or any other criteria.
- Regular Uniqueness: Under this constraint, each set generated must be unique in terms of its content compared to the previously generated sets. The sets are checked to ensure that there are no exact duplicates in the order of elements. However, if the same elements appear in different orders, these are considered different sets under this constraint.
- This is a stricter version of Regular Uniqueness. It requires that each set is unique not just in terms of its exact content but also considering the order of elements. That means even permutations of the same elements are not allowed.
- Weighted: In this constraint, certain elements have a higher likelihood of being selected based on a predefined weight. These weights influence the random selection process, making it more likely for some elements to appear in sets than others. The weight factor determines the degree to which these elements are favored during the selection process.
- Fixed Size: In this constraint, all sets are generated with an identical number of elements. For example, if you specify that each set should contain 3 elements, then every set created will have exactly 3 elements, regardless of other factors. This ensures uniformity across all sets, with each one containing the same number of items.

- Irregular Size: Under this constraint, sets are allowed to vary in size within a specified range. Instead of every set having the same number of elements, each set can have a different number of elements, as long as they fall within a defined minimum and maximum range. This introduces flexibility, allowing for some sets to be larger or smaller than others.
- Continuation (True or False): This constraint determines whether the set generation should continue to expand or stop after a fixed number of sets. When continuous=True, after the initial set of elements are generated, the function keeps expanding the sets by adding more elements according to the rules defined by the selected constraint. When continuous=False, the set generation stops after creating a fixed number of sets without further expansion. After the initial set generation, if continuous=True, the code extends the set generation process by generating additional sets (num new sets). The continuous generation function combines existing sets with newly generated sets. The new sets are created similarly to the initial sets but are appended to the existing list. This extension allows the set generation process to evolve beyond the initial configuration. The total number of sets after this process is num sets + num new sets.

6.0.1. Results

Part V. Applications

In this part we will apply both a **U** manifold and Ontological Differentiation to two different models. One for language and one for theoretical physics. In the first one we will obtain graphical results as we did in the functions sections and we will analyze them, and in the second one we will use the results from the functions sections along with both the notion of **U** and OD to be able to provide a new physical model interpretation.

7. Language Model

As anticipated, in this section we will create a **U** manifold for the English language and we will analyze it using OD.

7.1. Construction of U

In order to create our **U** manifold we have taken a list of the 5000 thousand most common words in English from [**?**]. We then have created a code to give definitions to these words. This code's main task is to generate concise and valid definitions

for a list of words by following specific conditions:
- Relevance and Consistency: Each word's definition must consist of exactly three related words, which are either directly connected in meaning or are synonyms. Importantly, these related words must also be part of the original list of words being inspected.
- No Self-Reference: The word being defined should not appear in its own definition. The code actively avoids including the original word in the list of related terms.
- Part of Speech Matching: The words included in the definition should ideally match the part of speech (e.g., noun, verb) of the original word, ensuring the definition is contextually appropriate.
- Completeness: If fewer than three relevant words are initially found, the code searches for additional synonyms or related terms to fill the definition. If it's still not possible to create a valid definition that meets all the criteria, that word is removed from the final output.
- Final Output: The code then cleans up any definitions that don't meet these criteria and saves the valid ones to a text file.

7.2. Results

8. Physical Model

In this section we will go over the results obtained from the functions section and we will use the notions of $\mathbf{U}$ and OD to provide an approach to some of the most basic physical models.

8.1. QFT Manifold

In section 6 where $\mathbf{U}$ had a random distribution, the results showed how OD operated on this type of order produced a landscape of discrete levels. It seems only natural to propose that a manifold $\mathbf{U}$ with a random distribution could be a model for a Quantum Field Theory (QFT) universe.

In QFT, a point is given as x^μ where $\mu = 0, 1, 2, 3$ are the indices representing the 4-dimensional spacetime. Then we have a field $\phi(x)$ (or other field like the spinor or gauge field) which it gives some value to that point. That is, both x^μ and field $\phi(x)$ are separated. What we propose in a $\mathbf{U}$ manifold is that both point value and field value are one and the same. Points give a sense to fields, and fields give a sense to points, and the reason behind is because they are part of the same conglomerate.

A point P_i in $\mathbf{U}$ is defined by its relations with other points P in $\mathbf{U}$. The way to tell apart different points in such a manifold is through OD, it tells you how "different" the are, how distant these points are from each other. Any function, any physical theory is made to tell points and their causes apart, to explain why the difference in points cause whatever different results.

The field properties (like energy) emerge when we start differentiating points from each other, not from applying some exotic function. The exotic function was already applied in its elements distribution, the exotic function explains the order of the elements in the manifold. The energy levels, the gaps, appear when we do a differentiation of points.

U is the spacetime, P are just the manifold points and the point distribution (given by some structure S) creates the field. The difference between points, what makes one point be itself and not other, makes the field properties emerge. So the field properties emerge not from applying a function to points, but by telling points apart.

We have then that the reason for discreteness in Quantum Mechanics (QM) comes from how the points of the manifold are distributed, from how they have their elements ordered.

There is also one more aspect to take into account for considering **U** for a QFT manifold. The absolute non-Euclidean property that **U** can have, as shown in the violation of the triangle inequality for the Sample Set in 3.1.2 (let us contemplate though that **U** can be ordered with a function such that it fulfills the triangle inequality, it is not an intrinsic property of **U**, its possibility is) means that **U** can behave in a probability-like form. The calculation of some $OD(P_i, P_j)$ may not be known, determined from former calculations of other P in the same **U**, but it may only be known once the calculation is done. This means that the nondeterministic nature of QM, the probability interpretation, comes from the configuration given by the structure which is applied onto **U**.

8.2. Gravitation

In our description of the **U** it becomes clear that there is no place, no outer place, for particles in **U**, but particles are part of **U**, as we suggest in the previous sub-section. There is nothing defined outside the spacetime, everything is spacetime.

One of the obvious questions then is how gravity, a force so obviously related to particles, can be understood in the context of a **U** manifold. The answer relies on OD once again. Some P_i will have shorter values of OD with other points (shorter distances) if such P_i is or has elements which are densely repeated across **U**, or in the case that the elements size is irregular, and P_i has a very big size, then it will be easier for it to have closer distances to other points. If we were to understand the cardinality, the size, of the read function (e.g. at level 1) as the mass, the content of the point, then we could see it as an analogy of gravitational attraction.

In an irregular element size set up, the more mass P_i has, the shorter it will be to the other points. In a weighted set up, the denser the frequencies of the elements of P_i, the shorter the distance to the other points. That analogy of mass, of density of P_i is the gravitational attraction. It's a point mass which attracts the others closer to it.

Part VI.
Online Resources

We have created a repository on Github () where one can find the different codes used to calculate the different OD's with their different set ups, as well as the application on the language model.

9. References

@articledavies, title=A Frequency Dictionary of Contemporary American English: Word Sketches, Collocates and Thematic Lists, author=Mark Davies and Dee Gardner, year=2010, edition=1st, publisher=Routledge, address=New York,