NRQCD

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Introduction

The decay rate into a quarkonium can be written as the sum of partial decay rates, to perform $B's$ decay into charmonium $c\bar{c}$ we have:

$$\Gamma[n] = \Gamma_0 \Big[C^2[1,8] f[n](\eta)(1+\delta_P[n]) \;\; +$$

$$\frac{\alpha_s(\mu)}{4\pi} \left(C_{[1]}^2 g_1[n](\eta) + 2C_{[1}C_{[8]} g_2[n](\eta) + C_{[8]}^2 g_3[n](\eta) \right) \Big] \left\langle O^H[n] \right\rangle \quad (2.13)$$

$$\Gamma_0 = \frac{G_F^2 |V_{bc}|^2 m_b^3}{432\pi m_c} \quad (2.14)$$

Colour singlet channels

The Strict NLO calculations for ${}^1S_0^{(1)}, {}^3S_1^{(1)}, {}^3P_1^{(1)}$ leads to a negative and meaningless decay rate.

One should add to the term of order $\alpha_s C_{[1]} C_{[8]}$ all terms of order $\alpha_s^2 C_{[8]}^2$. Instead of it, it is approximated by adding:

$$\Gamma_0 \left\langle O^H[n] \right\rangle \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 C_{[8]}^2 \frac{g_2[n]^2}{f[n]} \quad (3.10)$$

We will consider just the *improved* calculation: The NLO calculation with the term (3.10) added, but without the g_1 term.

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David calculations

By using the threshold expansion and the covariant projectors method, we found:

$$\Gamma_{0D} = \frac{G_F^2 |V_{cb}|^2 m_b^3}{257\pi m_c}$$

And the penguin corrections:

David	Maltoni	n	
$\frac{1,96(3(C_3-C_5)+C_4-C_6)}{C_1}$	$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^1S_0^{(1)}$	$\delta_p[n]$
$\frac{1,96(3(C_3+C_5)+C_4+C_6)}{C_1}$	$\frac{2(3(C_3+C_5)+C_4+C_6)}{C_1}$	${}^3S_1^{(1)}$	
$\frac{1,96(3(C_3-C_5)+C_4-C_6)}{C_1}$	$\frac{2(3(C_3-C_5)+C_4-C_6)}{C_1}$	${}^3P_1^{(1)}$	
$\frac{2,4(C_4-C_6)}{C_8}$	$\frac{4(C_4-C_6)}{C_8}$	${}^1S_0^{(8)}$	
$\frac{2,4(C_4+C_6)}{C_8}$	$\frac{4(C_4+C_6)}{C_8}$	${}^3S_1^{(8)}$	
$\frac{2,4(C_4-C_6)}{C_8}$	$\frac{4(C_4-C_6)}{C_8}$	${}^3P_1^{(8)}$	

Mathematica Implementation

We made a Mathematica code to compute in a systematical way the branching fractions

$$Br(B \to H + X) = N \sum_n \langle O^H[n] \rangle \Big[C^2_{[1,8]} f[n](\eta)(1 + \delta_P[n]) \;+$$

$$\frac{\alpha_s(\mu)}{4\pi} \Big(C^2_{[1]} g_1[n](\eta) + 2C_{[1]}C_{[8]} g_2[n](\eta) + C^2_{[8]} g_3[n](\eta) \Big) \Big] \qquad (4.3)$$

With $N = Br^{exp}_{SL} \frac{\Gamma_0}{\Gamma^{th}_{SL}} \;=\; 3.0 \cdot 10^{-2}\ GeV^{-3}$

And $N_D = Br^{exp}_{SL} \frac{\Gamma_{0D}}{\Gamma^{th}_{SL}} \;\approx\; 5.0 \cdot 10^{-2}\ GeV^{-3}$

Mathematica Implementation

We compute 3 cases:

- *Our Maltoni*: We reproduce Maltoni results, using the parameters that he gave in his paper.
- *David*: We implement the overall correction that David had found at LO in the LO and NLO. We also take into a count the penguin corrections and the N_D
- *David-Maltoni*: We tried to implement the David's correction just a LO and the Γ from Maltoni at NLO, in order to make a comparison. Nevertheless, these results were meaningless.

Branching fraction computations

1. J Ψ

$Br(B \to \Psi(nS) + X)$	$\left\langle O_1^{\Psi}(^3S_1)\right\rangle$	$\left\langle O_8^{\Psi}(^3S_1)\right\rangle$	$\left\langle O_8^{\Psi}(^1S_0)\right\rangle$	$\left\langle O_8^{\Psi}(^3P_0)\right\rangle/m_c^2$
Maltoni	$\begin{Bmatrix}-0.741\\0.0754\\-0.254\end{Bmatrix}10^{-2}$	0.195	0.342	1.06
Our Maltoni	$\begin{Bmatrix}-0.735\\0.0484\\-0.278\end{Bmatrix}10^{-2}$	0.193	0.338	1.05
Γ_{0D} Lo and NLO	$\begin{Bmatrix}-1.235\\0.07835\\-0.468\end{Bmatrix}10^{-2}$	0.3331	0.5676	1.760
Γ_{0D} Lo and Γ_0 NLO	$\begin{Bmatrix}-0.3272\\0.454\\0.1288\end{Bmatrix}10^{-2}$	0.286	0.4949	1.607

Branching fraction computations

2. η_c

$Br(B \to \eta_c + X)$	$\left\langle O_1^{\eta_c}(^1S_0)\right\rangle$	$\left\langle O_8^{\eta_c}(^1S_0)\right\rangle$	$\left\langle O_8^{\eta_c}(^3S_1)\right\rangle$	$\left\langle O_8^{\eta_c}(^1P_1)\right\rangle/m_c^2$
Maltoni	$\begin{Bmatrix} -1.19 \\ 0.250 \\ -0.210 \end{Bmatrix} 10^{-2}$	0.342	0.195	-0.0468
Our Maltoni	$\begin{Bmatrix} -1.18 \\ 0.125 \\ -0.331 \end{Bmatrix} 10^{-2}$	0.338	0.193	-0.0464
Γ_{0D} LO and NLO	$\begin{Bmatrix} -1.989 \\ 0.203 \\ -0.560 \end{Bmatrix} 10^{-2}$	0.5676	0.333	-0.0780
Γ_{0D} LO and Γ_0 NLO	$\begin{Bmatrix} -0.4469 \\ 0.8576 \\ 0.4032 \end{Bmatrix} 10^{-2}$	0.4949	0.286	-0.0464

Branching fraction computations

3.1 χ_{c0}

$Br(B \to \chi_{c0} + X)$	$\left\langle O_1^{\chi_{c0}}(^3P_0)\right\rangle / m_c^2$	$\left\langle O_8^{\chi_{c0}}(^3S_1)\right\rangle$
Maltoni	-0.0148	0.195
Our Maltoni	-0.0147	0.193
Γ_{0D} LO and NLO	-0.0247	0.333
Γ_{0D} LO and Γ_0 NLO	-0.0147	0.286

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Branching fraction computations

3.2 χ_{c1}

$Br(B \to \chi_{c1} + X)$	$\langle O_1^{\chi_{c1}}(^3P_1)\rangle / m_c^2$	$\langle O_8^{\chi_{c1}}(^3S_1)\rangle$
Maltoni	$\begin{Bmatrix} -2.14 \\ -0.783 \\ -1.21 \end{Bmatrix} 10^{-2}$	0.195
Our Maltoni	$\begin{Bmatrix} -2.12 \\ -0.855 \\ -1.28 \end{Bmatrix} 10^{-2}$	0.193
Γ_{0D} LO and NLO	$\begin{Bmatrix} -3.571 \\ -1.441 \\ -2.157 \end{Bmatrix} 10^{-2}$	0.333
Γ_{0D} LO and Γ_0 NLO	$\begin{Bmatrix} -1.249 \\ 0.0174 \\ -0.408 \end{Bmatrix} 10^{-2}$	0.286

Branching fraction computations

3.3 χ_{c2}

$Br(B \rightarrow \chi_{c2} + X)$	$\left\langle O_1^{\chi_{c2}}(^3P_2)\right\rangle / m_c^2$	$\left\langle O_8^{\chi_{c2}}(^3S_1)\right\rangle$
Maltoni	-0.0120	0.195
Our Maltoni	-0.0119	0.193
Γ_{0D} LO and NLO	-0.0199	0.333
Γ_{0D} LO and Γ_0 NLO	-0.0119	0.286

Branching fractions χ_{CJ}

As we see before the branching fraction of the X_{cJ} production $B \to \chi_{cJ} X$ are expressed, in general, as:

- $$Br(B \to \chi_{c0} + X) = A \frac{\left\langle O_1^{\chi_{c0}}(^3P_0)\right\rangle}{m_c^2} + B \left\langle O_8^{\chi_{c0}}(^3S_1)\right\rangle$$
- $$Br(B \to \chi_{c1} + X) = C \frac{\left\langle O_1^{\chi_{c1}}(^3P_1)\right\rangle}{m_c^2} + D \left\langle O_8^{\chi_{c1}}(^3S_1)\right\rangle$$
- $$Br(B \to \chi_{c2} + X) = E \frac{\left\langle O_1^{\chi_{c2}}(^3P_2)\right\rangle}{m_c^2} + F \left\langle O_8^{\chi_{c2}}(^3S_1)\right\rangle$$

Branching fractions χ_{CJ}

Spin symmetry relations for the χ_{cJ}:

- $$O_1 \equiv \left\langle O_1^{\chi_{c0}}(^3P_0) \right\rangle / m_c^2$$
- $$O_8 \equiv \left\langle O_8^{\chi_{c0}}(^3S_1) \right\rangle$$
- $$\left\langle O_1^{\chi_{cJ}}(^3P_J) \right\rangle / m_c^2 = (2J+1) O_1$$
- $$\left\langle O_8^{\chi_{cJ}}(^3S_1) \right\rangle = (2J+1) O_8$$

Branching fractions χ_{CJ}

We can write the Branching fractions for χ_{c1} , χ_{c2} in terms of the same operators of χ_{c0}:

For χ_{c1}:

- $$\left\langle O_1^{\chi_{c1}}(^3P_J)\right\rangle / m_c^2 = 3 \cdot O_1$$
- $$\left\langle O_8^{\chi_{c1}}(^3S_1)\right\rangle = 3 \cdot O_8$$

For χ_{c2}:

- $$\left\langle O_1^{\chi_{c2}}(^3P_J)\right\rangle / m_c^2 = 5 \cdot O_1$$
- $$\left\langle O_8^{\chi_{c2}}(^3S_1)\right\rangle = 5 \cdot O_8$$

$\mathbf{B} \to \chi_0 + \mathbf{X}$ LAL

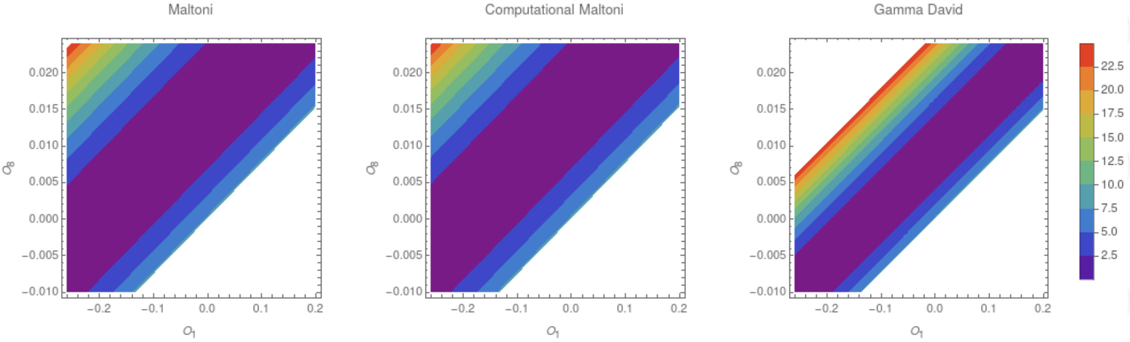

Figure: X^2 of χ_0 for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data in LAL.

$\mathbf{B} \to \chi_0 + \mathbf{X}$ New Data

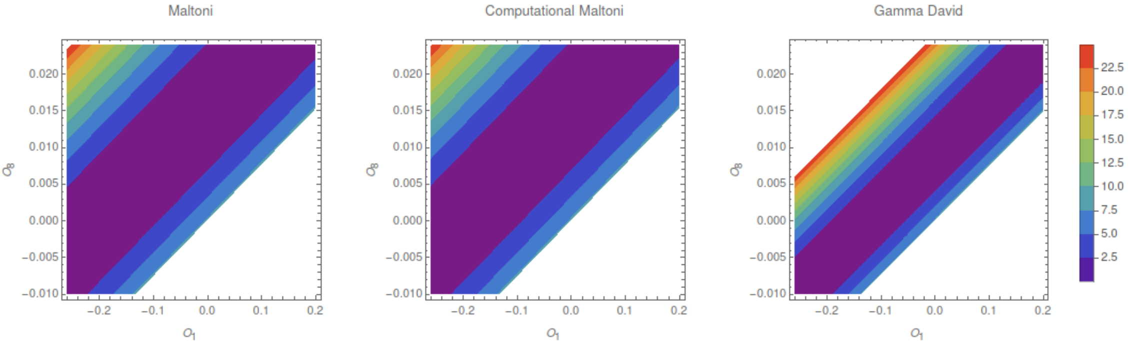

Figure: X^2 of χ_0 for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data with the new experimental data .

$\mathbf{B} \to \chi_1 + \mathbf{X}$ LAL

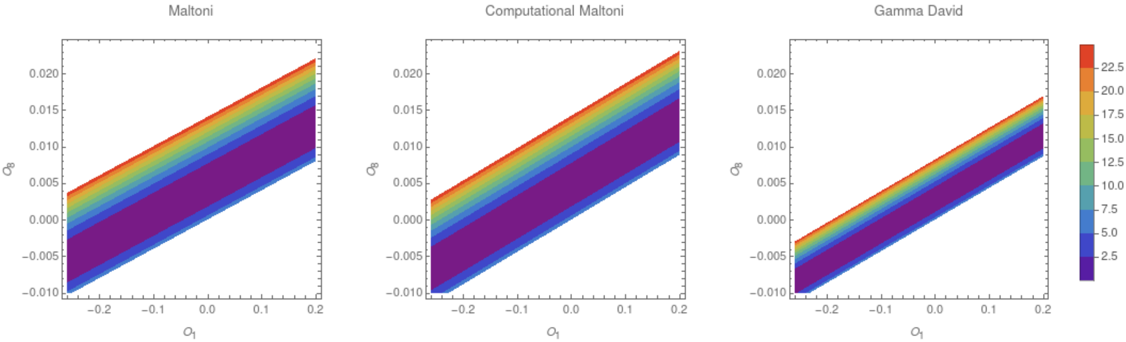

Figure: X^2 of χ_1 for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data in LAL.

$\mathbf{B} \rightarrow \chi_1 + \mathbf{X}$ New Data

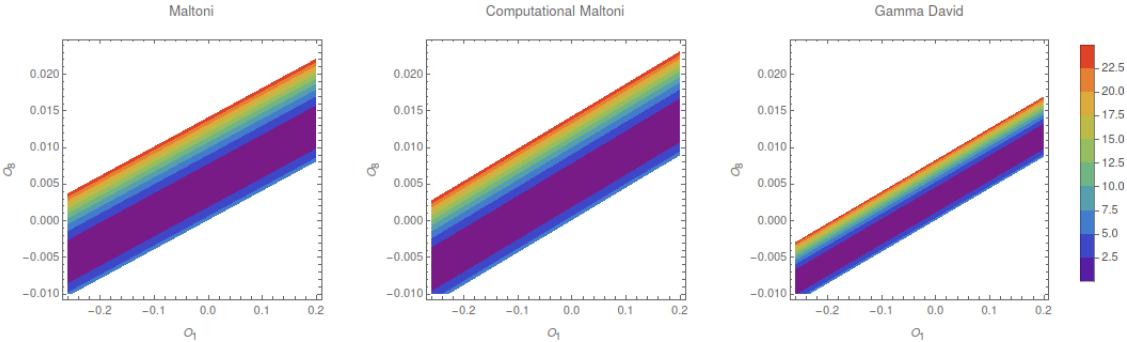

Figure: X^2 of χ_1 for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data with the new experimental data .

$\mathbf{B} \longrightarrow \chi_2 + \mathbf{X}$ LAL

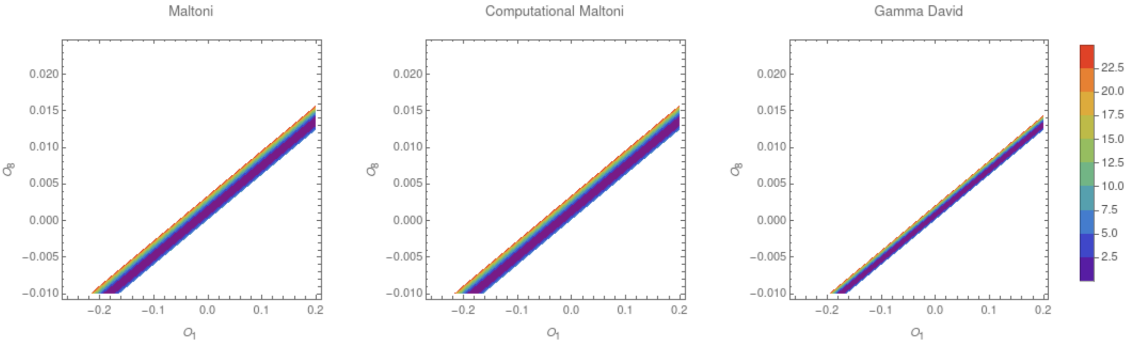

Figure: X^2 of χ_2 for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data in LAL.

$\mathbf{B} \rightarrow \chi_2 + \mathbf{X}$ New Data

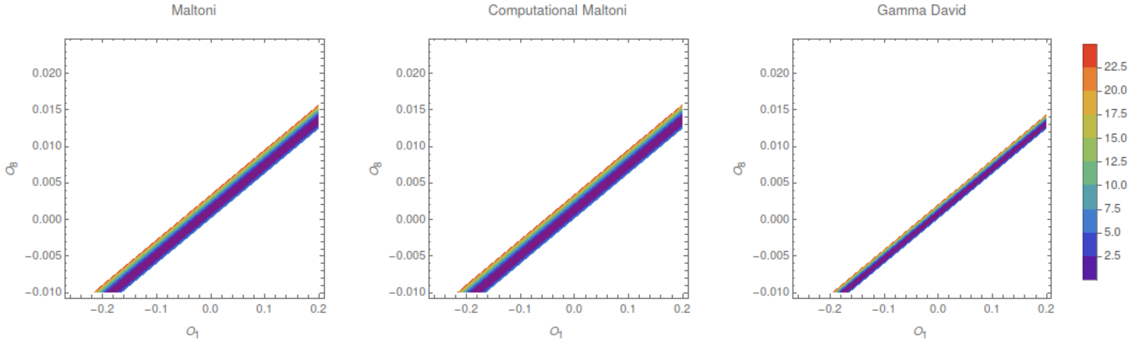

Figure: X^2 of χ_2 for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data with the new experimental data .

$\mathbf{B} \longrightarrow \chi_{\mathbf{c}} + \mathbf{X}$ $\langle|\chi^2|\rangle$ LAL

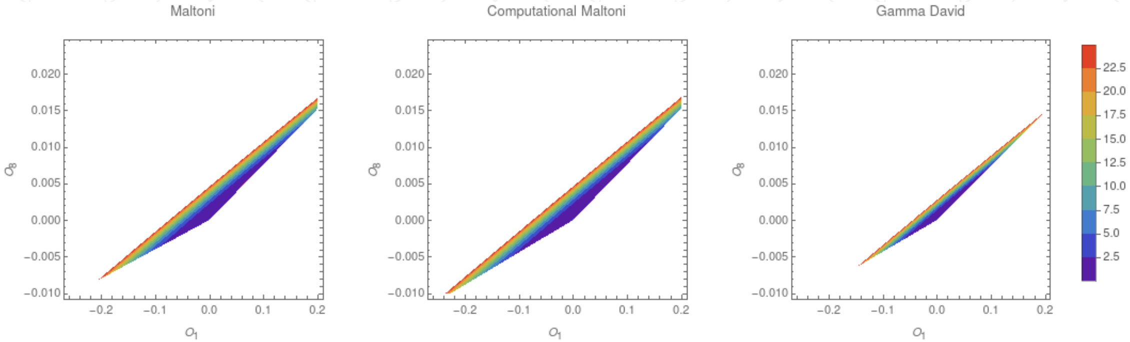

Figure: $\langle|\chi^2|\rangle$ of χ_c for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data in LAL.

$\mathbf{B} \rightarrow \chi_{\mathbf{c}} + \mathbf{X}$ $\langle | \chi^2 | \rangle$ New Data

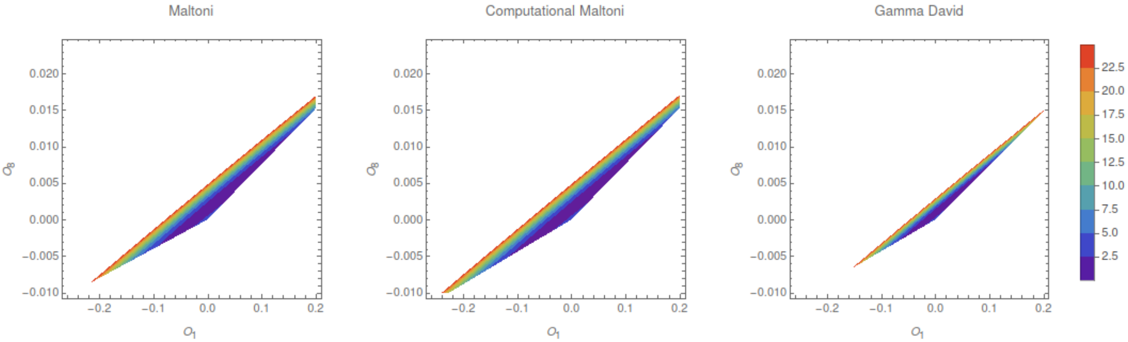

Figure: $\langle | \chi^2 | \rangle$ of χ_c for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data with the new experimental data .

$\chi_{\mathbf{c}}$ $\langle | \chi^2 | \rangle$ Minimum

χ_c						
LAL Data				New Data		
	minimum	O_1	O_8	minimum	O_1	O_8
Maltoni	2.42237	0.0974	0.007057	2.84925	0.1016	0.00759
Our Maltoni	2.52425	0.10451	0.007527	2.9641	0.1083	0.00804
David	2.42986	0.06489	0.004626	2.85442	0.0674	0.00495

$\mathbf{B} \to \frac{\chi_1 + \mathbf{X}}{\chi_0}$ LAL

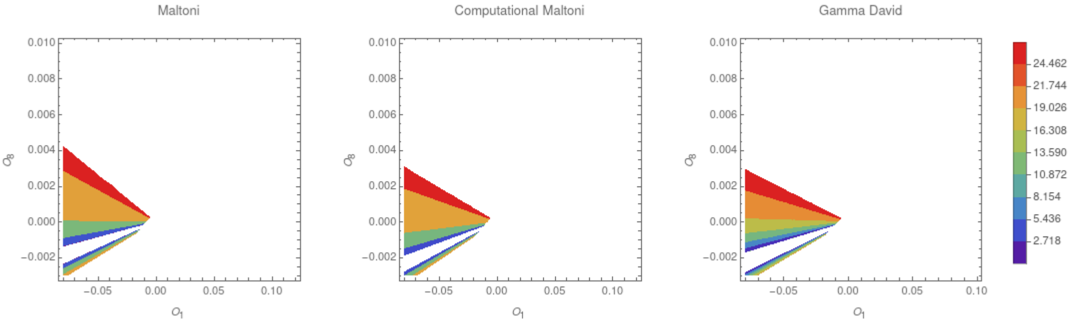

Figure: X^2 of $\frac{\chi_1}{\chi_0}$ for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data in LAL.

$\mathbf{B} \rightarrow \frac{\chi_1 + \mathbf{X}}{\chi_0}$ New Data

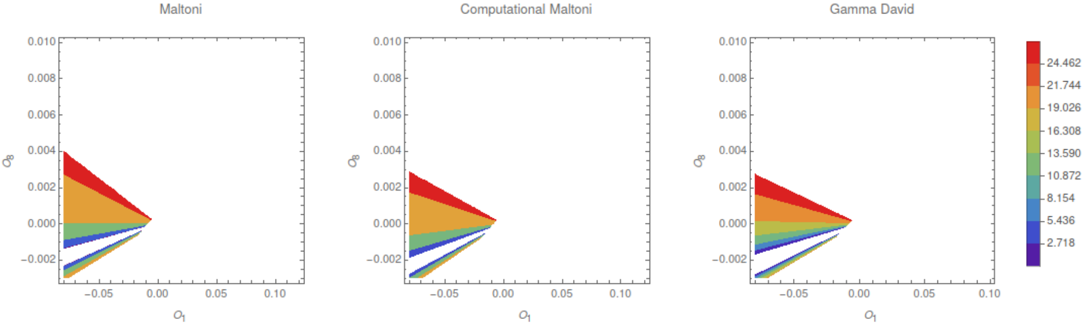

Figure: X^2 of $\frac{\chi_1}{\chi_0}$ for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data with the new experimental data .

$\mathbf{B} \to \frac{\chi_2+\mathbf{X}}{\chi_0}$ LAL

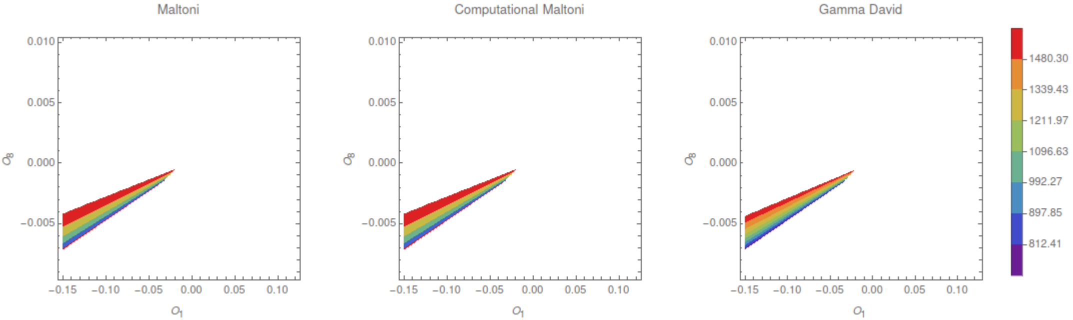

Figure: X^2 of $\frac{\chi_2}{\chi_0}$ for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data in LAL.

$\mathbf{B} \rightarrow \frac{\chi_2 + \mathbf{X}}{\chi_0}$ New Data

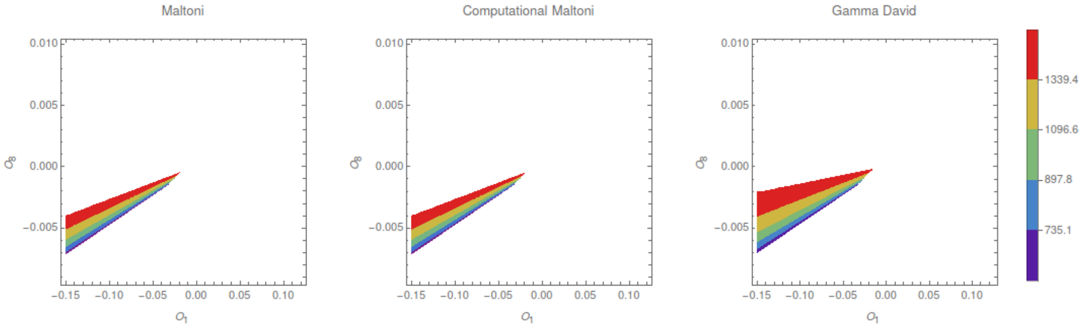

Figure: X^2 of $\frac{\chi_2}{\chi_0}$ for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data with the new experimental data .

$\mathbf{B} \to \frac{\chi_c + \mathbf{X}}{\chi_0}$ $\langle | \chi^2 | \rangle$ LAL

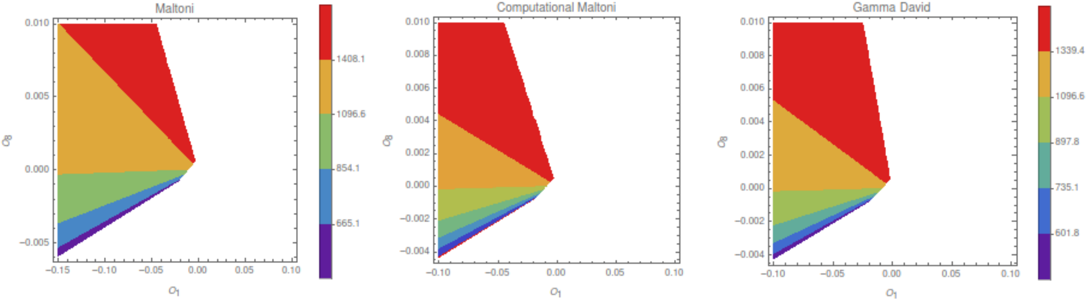

Figure: $\langle | \chi^2 | \rangle$ of $\frac{\chi_c}{\chi_0}$ for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data in LAL.

$\mathbf{B} \rightarrow \frac{\chi_c + \mathbf{X}}{\chi_0}$ $\langle | \chi^2 | \rangle$ New Data

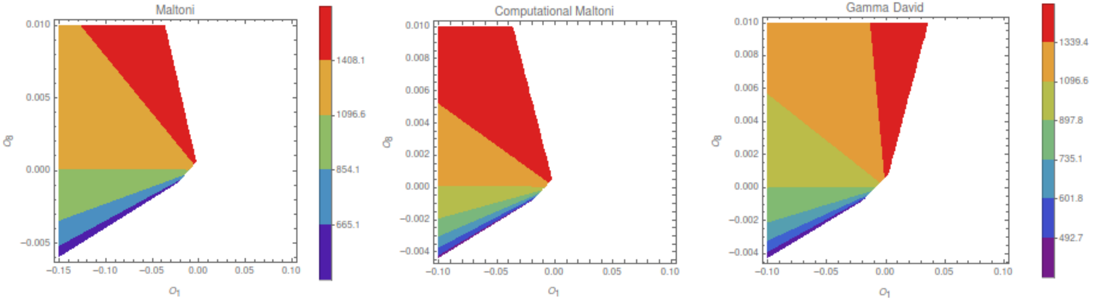

Figure: $\langle | \chi^2 | \rangle$ of $\frac{\chi_c}{\chi_0}$ for the Maltoni's description, the computational Maltoni and the implementation of David calculations with the experimental data with the new experimental data .

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References

S. Barsuk, E. Kou, and A. Usachov.
Test of nrqcd with charmonium production in inclusive b-hadron decays.
2019.

M. Beneke, F. Maltoni, and I. Z. Rothstein.
Qcd analysis of inclusive b decay into charmonium.
Physical Review D, 59:054003, 1999.