

Proyectors

Comenzamos por correr los paquetes necesarios;

```
In[556]:= $LoadAddOns={"FeynHelpers","FeynOnium"};  
<<FeynCalc`;
```

FeynCalc is already loaded! If you are trying to reload FeynCalc or load FeynArts, TARCER, PHI, FeynHelpers or any other add-on, please restart the kernel.

```
Out[557]= $Aborted
```

Cinematica

```
In[558]:= SP[P,P]=4mc2;  
SP[Pb,Pb]=mb2;  
SP[p,p]=0;  
SP[P,Pb]= $-\frac{1}{2}(m_b^2+4m_c^2)$ ;  
SP[P,p]= $-\frac{1}{2}(m_b^2-4m_c^2)$ ;  
SP[Pb,p]= $-\frac{1}{2}(m_b^2-4m_c^2)$ ;  
LPIPS= $\frac{1-\frac{4m_c^2}{m_b^2}}{8P\cdot i}$ ;
```

Cinematica en d dimensiones

In[565]:=

```

SP[P,P]=4mc2;
SP[p1,p1]=mc2;
SP[p2,p2]=mc2;
SPD[p1,p1]=mc2;
SPD[p2,p2]=mc2;
SPD[P,P]=4mc2;
SP[Pb,Pb]=mb2;
SPD[Pb,Pb]=mb2;
SP[p,p]=0;
SPD[p,p]=0;
SP[P,Pb]= $\frac{1}{2}(m_b^2+4m_c^2)$ ;
SPD[P,Pb]= $\frac{1}{2}(m_b^2+4m_c^2)$ ;
SP[P,p]= $\frac{1}{2}(m_b^2-4m_c^2)$ ;
SPD[P,p]= $\frac{1}{2}(m_b^2-4m_c^2)$ ;
SP[Pb,p]= $\frac{1}{2}(m_b^2-4m_c^2)$ ;
SPD[Pb,p]= $\frac{1}{2}(m_b^2-4m_c^2)$ ;
LIPS= $\frac{1-\frac{4m_c^2}{m_b^2}}{8\pi}$ ;
FCSetDiracGammaScheme["NDR"]

```

Out[582]= NDR

Difinición de los proyectores

■ Proyectores de color

Singlete

```
In[583]:= colorSingletProjector=SUNFDelta[SUNFIndex[m],SUNFIndex[n]]
$$\frac{1}{(8m_c^3 CA)^{\frac{1}{2}}};$$

```

Octete

```
In[584]:= colorOctetProjector=SUNTF[c,m,n]
$$\frac{1}{(4m_c^3)^{\frac{1}{2}}};$$

```

■ Proyectores de spin

spin=0

```
In[585]:= Projector1s0=SpinorVBar[p2,mc].GA[5].SpinorU[p1,mc];
```

Spin=1

```
In[586]:= Projector3s1=SpinorVBar[p2,mc].GA[α].SpinorU[p1,mc];
```

■ Numero de:

■ Colores

Singlete

```
In[587]:= Numcolor1=2 CA;
```

Octete

```
In[588]:= Numcolor8=2 CA CF;
```

■ Polarizaciones

1S0

```
In[589]:= Numpol1S0=1;
```

3P0

```
In[590]:= Numpol3P0=1;
```

3S1

```
In[591]:= Numpol3S1=3;
```

1P1

```
In[592]:= Numpol1P1=3;
```

```
3P1
```

```
In[593]:= Numpol3P1=3;
```

```
3P2
```

```
In[594]:= Numpol3P2=5;
```

Amplitudes con la corriente qb amputada

■ M_{0_1}

```
In[595]:= M01=-c1SUNFDelta[SUNFIndex[i],SUNFIndex[j]]*SpinorUBar[p1,mc].GA[mu].(1-GA[5]).SpinorV[p2,mc];
```

■ M_{0_8}

```
In[596]:= M08=-c8SUNTF[a,i,j]*SpinorUBar[p1,mc].GA[mu].(1-GA[5]).SpinorV[p2,mc];
```

■ M_{0_3}

```
In[597]:= M03=c3SUNFDelta[SUNFIndex[i],SUNFIndex[j]]*SpinorUBar[p1,mc].GA[mu].(1-GA[5]).SpinorV[p2,mc];
```

■ M_{0_5}

```
In[598]:= M05=c5SUNFDelta[SUNFIndex[i],SUNFIndex[j]]*SpinorUBar[p1,mc].GA[mu].(1+GA[5]).SpinorV[p2,mc];
```

■ M_{0_4}

```
In[599]:= M04=c4SUNFDelta[SUNFIndex[i],SUNFIndex[k]]*SUNFDelta[SUNFIndex[j],SUNFIndex[l]]*SpinorUBar[p
```

■ M_{0_6}

```
In[600]:= M06=c6SUNFDelta[SUNFIndex[i],SUNFIndex[k]]*SUNFDelta[SUNFIndex[j],SUNFIndex[l]]*SpinorUBar[p
```

Las correspondientes corrientes son

■ M_{0_1}

```
In[601]:= M01qb=SpinorUBar[p,0].GA[μ].(1-GA[5]).SpinorU[Pb,mb]*SUNFDelta[SUNFIndex[k],SUNFIndex[l]];
```

■ M_{0_8}

```
In[602]:= M08qb=SpinorUBar[Momentum[p],0].GA[μ].(1-GA[5]).SpinorU[Pb,mb]*SUNTF[a,l,k];
```

■ M_{0_3}

```
In[603]:= M03qb=SpinorUBar[Momentum[p],0].GA[μ].(1-GA[5]).SpinorU[Pb,mb]*SUNFDelta[SUNFIndex[k],SUNFIndex[l]];
```

■ M_{0_5}

```
In[604]:= M05qb=SpinorUBar[Momentum[p],0].GA[μ].(1-GA[5]).SpinorU[Pb,mb]*SUNFDelta[SUNFIndex[k],SUNFIndex[l]];
```

■ M_{0_4}

```
In[605]:= M04qb=SpinorUBar[Momentum[p],0].GA[μ].(1-GA[5]).SpinorU[Pb,mb];
```

■ M_{0_6}

```
In[606]:= M06qb=SpinorUBar[Momentum[p],0].GA[μ].(1-GA[5]).SpinorU[Pb,mb];
```

Completez para los vectores de polarización

Comenzamos por el vector de polarización presente en los canales 3S_1 y 1P_1

In[607]:=

$$\begin{aligned} \text{sumpolvector} &= -\text{MT}[\alpha', \alpha] + \frac{\text{FV}[P, \alpha'] \times \text{FV}[P, \alpha]}{4m_c^2}; \\ \text{sumpoltensorJ0} &= \frac{1}{3} \left(-\text{MT}[\alpha, \beta] + \frac{\text{FV}[P, \alpha] \times \text{FV}[P, \beta]}{4m_c^2} \right) \left(-\text{MT}[\alpha', \beta'] + \frac{\text{FV}[P, \alpha'] \times \text{FV}[P, \beta']}{4m_c^2} \right); \\ \text{sumpoltensorJ1} &= \frac{1}{2} \left(\left(-\text{MT}[\alpha, \alpha'] + \frac{\text{FV}[P, \alpha] \times \text{FV}[P, \alpha']}{4m_c^2} \right) \left(-\text{MT}[\beta, \beta'] + \frac{\text{FV}[P, \beta] \times \text{FV}[P, \beta']}{4m_c^2} \right) - \left(-\text{MT}[\alpha, \beta'] + \frac{\text{FV}[P, \alpha] \times \text{FV}[P, \beta']}{4m_c^2} \right) \right. \\ &\quad \left. - \left(-\text{MT}[\alpha', \beta] + \frac{\text{FV}[P, \alpha'] \times \text{FV}[P, \beta]}{4m_c^2} \right) \right); \\ \text{sumpoltensorJ2} &= \frac{1}{2} \left(\left(-\text{MT}[\alpha, \alpha'] + \frac{\text{FV}[P, \alpha] \times \text{FV}[P, \alpha']}{4m_c^2} \right) \left(-\text{MT}[\beta, \beta'] + \frac{\text{FV}[P, \beta] \times \text{FV}[P, \beta']}{4m_c^2} \right) + \left(-\text{MT}[\alpha, \beta'] + \frac{\text{FV}[P, \alpha] \times \text{FV}[P, \beta']}{4m_c^2} \right) \right. \\ &\quad \left. + \left(-\text{MT}[\alpha', \beta] + \frac{\text{FV}[P, \alpha'] \times \text{FV}[P, \beta]}{4m_c^2} \right) \right); \end{aligned}$$

Proyecciones de cada canal

 M_{0_1}

■ Singlete

1S0

In[611]:=

```
SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M01s01=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[613]} = -\frac{\sqrt{2} \, c_1 \, \bar{P}^\mu \sqrt{C_A m_c^3}}{m_c^2}$$

1P1

In[614]:=

```
SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. p1→ Momentum[P/2+q];
%/. p2→ Momentum[P/2-q];
M01p11=FourDivergence[%,FV[q,α]]
```

Out[617]= 0

3S1

```
In[618]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M013s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[619]= } \frac{2\sqrt{2} c_1 \bar{g}^{\alpha\mu} \sqrt{C_A m_c^3}}{m_c} - \frac{c_1 \bar{P}^\alpha \bar{P}^\mu \sqrt{C_A m_c^3}}{\sqrt{2} m_c^3}$$

3P0

```
In[620]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M013p01=%/. Momentum[q]→ 0
```

$$\text{Out[624]= } \frac{i\sqrt{2} c_1 \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha\beta\mu} \bar{P}}{m_c^3}$$

3P1

```
In[625]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M013p11=%/. Momentum[q]→ 0
```

$$\text{Out[629]= } \frac{i\sqrt{2} c_1 \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha\beta\mu} \bar{P}}{m_c^3}$$

3P2

```
In[630]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M013p21=%/. Momentum[q]→ 0
```

$$\text{Out[634]} = \frac{i \sqrt{2} \, c_1 \sqrt{C_A m_c^3} \, \bar{\epsilon}^\alpha \beta_\mu \bar{P}}{m_c^3}$$

■ Octete

1S0

```
In[635]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M011s08=%/. Momentum[p2]→ Momentum[P/2]
```

Out[637]= 0

1P1

```
In[638]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M011p18=FourDivergence[%,FourVector[q,α]]
```

Out[640]= 0

3S1

```
In[641]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M013s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

Out[642]= 0

3P0

```
In[643]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M013p08=%/. Momentum[q]→ 0
```

Out[647]= 0

3P1

```
In[648]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M013p18=%/. Momentum[q]→ 0
```

Out[652]= 0

3P2

```
In[653]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M013p28=%/. Momentum[q]→ 0
```

Out[657]= 0

M_{0_8}

■ Singlete

1S0

```
In[658]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M081s01=%/. Momentum[p2]→ Momentum[P/2]
```

Out[660]= 0

1P1

```
In[661]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M081p11=FourDivergence[%,FourVector[q,α]]
```

Out[663]= 0

3S1

```
In[664]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M083s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

Out[665]= 0

3P0

```
In[666]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M083p01=%/. Momentum[q]→ 0
```

Out[670]= 0

3P1

```
In[671]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M083p11=%/. Momentum[q]→ 0
```

Out[675]= 0

3P2

```
In[676]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M083p21=%/. Momentum[q]→ 0
```

Out[680]= 0

■ Octete

1S0

```
In[681]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M081s08=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[683]} = -\frac{c_8 \sqrt{m_c^3} \bar{P}^\mu \delta^{ac}}{m_c^2}$$

1P1

```
In[684]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M081p18=FourDivergence[%,FourVector[q,α]]
```

Out[686]= 0

3S1

In[687]:=

```
SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M083s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

Out[688]=

$$\frac{c_8 m_c^2 \delta^{ac} \bar{g}^{\gamma \mu}}{\sqrt{m_c^3}} + \frac{c_8 \sqrt{m_c^3} \delta^{ac} \bar{g}^{\gamma \mu}}{m_c} - \frac{c_8 \bar{P}^{\gamma} \bar{P}^{\mu} \delta^{ac}}{2 \sqrt{m_c^3}}$$

3P0

In[689]:=

```
SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M083p08=%/. Momentum[q]→ 0
```

Out[693]=

$$\frac{i c_8 \delta^{ac} \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

3P1

In[694]:=

```
SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M083p18=%/. Momentum[q]→ 0
```

Out[698]=

$$\frac{i c_8 \delta^{ac} \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

3P2

```
In[699]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M083p28=%/. Momentum[q]→ 0
```

$$\text{Out[703]} = \frac{i c_8 \delta^{ac} \bar{\epsilon}^{\alpha \beta \mu} \bar{P}}{\sqrt{m_c^3}}$$

 M_{0_3}

■ Singlete

1S0

```
In[704]:= SUNFSimplify[ColorSingletProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M031s01=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[706]} = \frac{\sqrt{2} c_3 \bar{P}^\mu \sqrt{C_A m_c^3}}{m_c^2}$$

1P1

```
In[707]:= SUNFSimplify[ColorSingletProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M031p11=FourDivergence[%,FourVector[q,α]]
```

$$\text{Out[709]} = 0$$

3S1

```
In[710]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M033s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[711]} = \frac{c_3 \bar{P}^\alpha \bar{P}^\mu \sqrt{C_A m_c^3}}{\sqrt{2} m_c^3} - \frac{2 \sqrt{2} c_3 \bar{g}^{\alpha \mu} \sqrt{C_A m_c^3}}{m_c}$$

3P0

```
In[712]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M033p01=%/. Momentum[q]→ 0
```

$$\text{Out[716]} = -\frac{i \sqrt{2} c_3 \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha \beta \mu} \bar{P}}{m_c^3}$$

3P1

```
In[717]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M033p11=%/. Momentum[q]→ 0
```

$$\text{Out[721]} = -\frac{i \sqrt{2} c_3 \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha \beta \mu} \bar{P}}{m_c^3}$$

3P2

```
In[722]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M033p21=%/. Momentum[q]→ 0
```

$$\text{Out[726]} = -\frac{i\sqrt{2}c_3\sqrt{C_A m_c^3}\epsilon^{\alpha\beta\mu\bar{P}}}{m_c^3}$$

■ Octete

1S0

```
In[727]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M031s08=%/. Momentum[p2]→ Momentum[P/2]
```

Out[729]= 0

1P1

```
In[730]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M031p18=FourDivergence[%,FourVector[q,α]]
```

Out[732]= 0

3S1

```
In[733]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M033s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

Out[734]= 0

3P0

```
In[735]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFDelta[SUNFIndex[n], SUNFIndex[i]]
%/. Momentum[p1] → Momentum[P/2 + q];
%/. Momentum[p2] → Momentum[P/2 - q];
FourDivergence[%, FourVector[q, β]];
M033p08 = %/. Momentum[q] → 0
```

Out[739]= 0

3P1

```
In[740]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFDelta[SUNFIndex[n], SUNFIndex[i]]
%/. Momentum[p1] → Momentum[P/2 + q];
%/. Momentum[p2] → Momentum[P/2 - q];
FourDivergence[%, FourVector[q, β]];
M033p18 = %/. Momentum[q] → 0
```

Out[744]= 0

3P2

```
In[745]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFDelta[SUNFIndex[n], SUNFIndex[i]]
%/. Momentum[p1] → Momentum[P/2 + q];
%/. Momentum[p2] → Momentum[P/2 - q];
FourDivergence[%, FourVector[q, β]];
M033p28 = %/. Momentum[q] → 0
```

Out[749]= 0

M_{0_5}

■ Singlete

1S0

```
In[750]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M051s01=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[752]} = -\frac{\sqrt{2} \, c_5 \, \bar{P}^\mu \sqrt{C_A m_c^3}}{m_c^2}$$

```
In[753]:=
```

M_{051s01}

$$\text{Out[753]} = -\frac{\sqrt{2} \, c_5 \, \bar{P}^\mu \sqrt{C_A m_c^3}}{m_c^2}$$

1P1

```
In[754]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M051p11=FourDivergence[%,FourVector[q,α]]
```

```
Out[756]= 0
```

3S1

```
In[757]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M053s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[758]} = \frac{c_5 \, \bar{P}^\alpha \, \bar{P}^\mu \sqrt{C_A m_c^3}}{\sqrt{2} \, m_c^3} - \frac{2 \sqrt{2} \, c_5 \, \bar{g}^{\alpha \mu} \sqrt{C_A m_c^3}}{m_c}$$

In[759]:=

 M_{053s11}

$$\text{Out[759]} = \frac{c_5 \bar{P}^\alpha \bar{P}^\mu \sqrt{C_A m_c^3}}{\sqrt{2} m_c^3} - \frac{2 \sqrt{2} c_5 \bar{g}^{\gamma \mu} \sqrt{C_A m_c^3}}{m_c}$$

3P0

In[760]:=

```
SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M053p01=%/. Momentum[q]→ 0
```

$$\text{Out[764]} = \frac{i \sqrt{2} c_5 \sqrt{C_A m_c^3} \bar{\epsilon}^\alpha \beta^\mu \bar{P}}{m_c^3}$$

3P1

In[765]:=

```
SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M053p11=%/. Momentum[q]→ 0
```

$$\text{Out[769]} = \frac{i \sqrt{2} c_5 \sqrt{C_A m_c^3} \bar{\epsilon}^\alpha \beta^\mu \bar{P}}{m_c^3}$$

3P2

In[770]:=

```
SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M053p21=%/. Momentum[q]→ 0
```

$$\text{Out[774]} = \frac{i \sqrt{2} c_5 \sqrt{C_A m_c^3} \bar{\epsilon}^\alpha \beta^\mu \bar{P}}{m_c^3}$$

■ Octete

1S0

```
In[775]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M051s08=%/. Momentum[p2]→ Momentum[P/2]
```

Out[777]= 0

1P1

```
In[778]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M051p18=FourDivergence[%,FourVector[q,α]]
```

Out[780]= 0

3S1

```
In[781]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M053s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

Out[782]= 0

3P0

```
In[783]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M053p08=%/. Momentum[q]→ 0
```

Out[787]= 0

3P1

```
In[788]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M053p18=%/. Momentum[q]→ 0

Out[792]= 0
```

3P2

```
In[793]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M053p28=%/. Momentum[q]→ 0

Out[797]= 0
```

M_{0_4}

■ Singlete

1S0

```
In[798]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M041s01=%/. Momentum[p2]→ Momentum[P/2]

Out[800]= 
$$\frac{\sqrt{2} \ c_4 \ \bar{P}^\mu \ \delta_{kl} \ \sqrt{C_A \ m_c^3}}{C_A \ m_c^2}$$

```

1P1

```
In[801]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M041p11=FourDivergence[%,FourVector[q,α]]
```

Out[803]= 0

3S1

```
In[804]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M043s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[805]} = \frac{c_4 \bar{P}^\alpha \bar{P}^\mu \delta_{kl} \sqrt{C_A m_c^3}}{\sqrt{2} C_A m_c^3} - \frac{2 \sqrt{2} c_4 \delta_{kl} \bar{g}^{\alpha \mu} \sqrt{C_A m_c^3}}{C_A m_c}$$

3P0

```
In[806]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M043p01=%/. Momentum[q]→ 0
```

$$\text{Out[810]} = - \frac{i \sqrt{2} c_4 \delta_{kl} \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{C_A m_c^3}$$

3P1

```
In[811]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M043p11=%/. Momentum[q]→ 0
```

$$\text{Out[815]} = - \frac{i \sqrt{2} c_4 \delta_{kl} \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{C_A m_c^3}$$

3P2

```
In[816]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M043p21=%/. Momentum[q]→ 0
```

$$\text{Out[820]} = -\frac{i\sqrt{2}c_4\delta_{kl}\sqrt{C_A m_c^3}\bar{\epsilon}^{\alpha\beta\mu\bar{P}}}{C_A m_c^3}$$

■ Octete

1S0

```
In[821]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M041s08=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[823]} = \frac{2c_4\sqrt{m_c^3}\bar{P}^\mu T_{lk}^c}{m_c^2}$$

1P1

```
In[824]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M041p18=FourDivergence[%,FourVector[q,α]]
```

Out[826]= 0

3S1

```
In[827]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M043s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[828]} = -\frac{2c_4m_c^2\bar{g}^{\alpha\mu}T_{lk}^c}{\sqrt{m_c^3}} - \frac{2c_4\sqrt{m_c^3}\bar{g}^{\alpha\mu}T_{lk}^c}{m_c} + \frac{c_4\bar{P}^\alpha\bar{P}^\mu T_{lk}^c}{\sqrt{m_c^3}}$$

3P0

```
In[829]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M043p08=%/. Momentum[q]→ 0
```

$$\text{Out[833]} = -\frac{2 i c_4 T_{lk}^c \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

3P1

```
In[834]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M043p18=%/. Momentum[q]→ 0
```

$$\text{Out[838]} = -\frac{2 i c_4 T_{lk}^c \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

3P2

```
In[839]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M043p28=%/. Momentum[q]→ 0
```

$$\text{Out[843]} = -\frac{2 i c_4 T_{lk}^c \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

M_{0_6}

■ Singlete

1S0

```
In[844]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M061s01=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[846]} = -\frac{\sqrt{2} \, c_6 \, \bar{P}^\mu \, \delta_{kl} \sqrt{C_A m_c^3}}{C_A m_c^2}$$

1P1

```
In[847]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M061p11=FourDivergence[%,FourVector[q,α]]
```

Out[849]= 0

3S1

```
In[850]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M063s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[851]} = \frac{c_6 \, \bar{P}^\alpha \, \bar{P}^\mu \, \delta_{kl} \sqrt{C_A m_c^3}}{\sqrt{2} \, C_A m_c^3} - \frac{2 \sqrt{2} \, c_6 \, \delta_{kl} \, \bar{g}^{\alpha \mu} \sqrt{C_A m_c^3}}{C_A m_c}$$

3P0

```
In[852]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M063p01=%/. Momentum[q]→ 0
```

$$\text{Out[856]} = \frac{i \sqrt{2} c_6 \delta_{kl} \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{C_A m_c^3}$$

3P1

```
In[857]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M063p11=%/. Momentum[q]→ 0
```

$$\text{Out[861]} = \frac{i \sqrt{2} c_6 \delta_{kl} \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{C_A m_c^3}$$

3P2

```
In[862]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]×SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M063p21=%/. Momentum[q]→ 0
```

$$\text{Out[866]} = \frac{i \sqrt{2} c_6 \delta_{kl} \sqrt{C_A m_c^3} \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{C_A m_c^3}$$

■ Octete

1S0

```
In[867]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M061s08=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[869]} = -\frac{2 c_6 \sqrt{m_c^3} \bar{P}^\mu T_{lk}^c}{m_c^2}$$

1P1

```
In[870]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M061p18=FourDivergence[%,FourVector[q,α]]
```

Out[872]= 0

3S1

```
In[873]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M063s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[874]} = -\frac{2 c_6 m_c^2 \bar{g}^{\alpha \mu} T_{lk}^c}{\sqrt{m_c^3}} - \frac{2 c_6 \sqrt{m_c^3} \bar{g}^{\alpha \mu} T_{lk}^c}{m_c} + \frac{c_6 \bar{P}^\alpha \bar{P}^\mu T_{lk}^c}{\sqrt{m_c^3}}$$

3P0

```
In[875]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M063p08=%/. Momentum[q]→ 0
```

$$\text{Out[879]} = \frac{2 i c_6 T_{lk}^c \epsilon^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

3P1

```
In[880]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M063p18=%/. Momentum[q]→ 0
```

$$\text{Out[884]} = \frac{2 i c_6 T_{lk}^c \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

3P2

```
In[885]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M063p28=%/. Momentum[q]→ 0
```

$$\text{Out[889]} = \frac{2 i c_6 T_{lk}^c \bar{\epsilon}^{\alpha \beta \mu \bar{P}}}{\sqrt{m_c^3}}$$

Contribuciones totales

Singlete

■ $|M_{01}|^2$

1S_0

In[890]:=
$$\text{CM}_{011s01} = \text{ComplexConjugate}[M_{011s01} M_{01qb}] /. \{k \rightarrow g, l \rightarrow h\};$$

$$\text{Simplify}\left[\frac{1}{2m_b \text{Numcolor1 Numpol1S0}} \text{LPIPS} \frac{1}{2} \text{SUNFSimplify}\left[\text{SUNFDelta}[\text{SUNFIndex}[k], \text{SUNFIndex}[g]]\right]\right]$$

Out[891]=
$$\frac{c_1^2 C_A (m_b^2 - 4 m_c^2)^2}{8 \pi m_b m_c}$$

1P_1

In[892]:=
$$\text{CM}_{011p11} = \text{ComplexConjugate}[M_{011p11} M_{01qb}] /. \{k \rightarrow g, l \rightarrow h, \alpha \rightarrow \alpha'\};$$

$$\text{Contract}\left[\text{sumpolvector Simplify}\left[\frac{1}{2m_b \text{Numcolor1 Numpol1P1}} \text{LPIPS} \frac{1}{2} \text{SUNFSimplify}\left[\text{SUNFDelta}[\text{SUNFIndex}[k], \text{SUNFIndex}[g]]\right]\right]\right]$$

Out[893]= 0

3S_1

In[894]:=
$$\text{CM}_{013s11} = \text{ComplexConjugate}[M_{013s11} M_{01qb}] /. \{k \rightarrow g, l \rightarrow h, \alpha \rightarrow \alpha'\};$$

$$\text{Simplify}\left[\text{Contract}\left[\text{sumpolvector} \frac{1}{2m_b \text{Numcolor1 Numpol3S1}} \text{LPIPS} \frac{1}{2} \text{SUNFSimplify}\left[\text{SUNFDelta}[\text{SUNFIndex}[k], \text{SUNFIndex}[g]]\right]\right]\right]$$

Out[895]=
$$\frac{c_1^2 C_A (-48 m_b^2 m_c^4 + m_b^6 + 128 m_c^6)}{24 \pi m_b^3 m_c}$$

3P_0

```
In[896]:= CM013p01=ComplexConjugate[M013p01 M01qb]/.{k →g, l→h, α→ α', β→ β'};
Simplify[Contract[sumpoltensorJ0  $\frac{1}{2m_b \text{Numcolor1 Numpol3P0}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[897]= 0

 3P_1

```
In[898]:= CM013p11=ComplexConjugate[M013p11 M01qb]/.{k →g, l→h, α→ α', β→ β'};
Simplify[Contract[sumpoltensorJ1  $\frac{1}{2m_b \text{Numcolor1 Numpol3P1}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[899]=
$$\frac{c_1^2 C_A (-48 m_b^2 m_c^4 + m_b^6 + 128 m_c^6)}{12 \pi m_b^3 m_c^3}$$

 3P_2

```
In[900]:= CM013p21=ComplexConjugate[M013p21 M01qb]/.{k →g, l→h, α→ α', β→ β'};
Simplify[Contract[sumpoltensorJ2  $\frac{1}{2m_b \text{Numcolor1 Numpol3P2}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[901]= 0

■ $2\text{Re}(M_{01}M_{0i}^*)$ penguins

1S_0

```
In[902]:= CM031s01=ComplexConjugate[M031s01M03qb]/.{k →g, l→h};
CM051s01=ComplexConjugate[M051s01M05qb]/.{k →g, l→h};
CM041s01=ComplexConjugate[M041s01M04qb]/.{k →g, l→h};
CM061s01=ComplexConjugate[M061s01M06qb]/.{k →g, l→h};

2Re[Simplify[ $\frac{1}{2m_b \text{Numcolor1 Numpol1S0}}$  LIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SUNFIndex[k], SUNFIndex

$$\text{Re}\left(\frac{c_1 (c_3 C_A - c_5 C_A + c_4 - c_6) (m_b^2 - 4 m_c^2)^2}{m_b m_c}\right)$$

Out[906]=  $-\frac{1}{4\pi}$ 
```

1P_1

```
In[907]:= CM031p11=ComplexConjugate[M031p11M03qb]/.{k →g, l→h, α→α'};
CM051p11=ComplexConjugate[M051p11M05qb]/.{k →g, l→h, α→α'};
CM041p11=ComplexConjugate[M041p11M04qb]/.{k →g, l→h, α→α'};
CM061p11=ComplexConjugate[M061p11M06qb]/.{k →g, l→h, α→α'};

Contract[sumpolvector Simplify[ $\frac{1}{2m_b \text{Numcolor1 Numpol1P1}}$  LIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SUN
Out[911]= 0
```

3S_1

```
In[912]:= CM033s11=ComplexConjugate[M033s11M03qb]/.{k →g, l→h, α→α'};
CM053s11=ComplexConjugate[M053s11M05qb]/.{k →g, l→h, α→α'};
CM043s11=ComplexConjugate[M043s11M04qb]/.{k →g, l→h, α→α'};
CM063s11=ComplexConjugate[M063s11M06qb]/.{k →g, l→h, α→α'};

Simplify[Contract[sumpolvector  $\frac{1}{2m_b \text{Numcolor1 Numpol3S1}}$  LIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SUNF

$$\text{Out[916]= } -\frac{c_1 (c_3 C_A + c_5 C_A + c_4 + c_6) (m_b^2 - 4 m_c^2)^2 (m_b^2 + 8 m_c^2)}{24 \pi m_b^3 m_c}$$

```

3P_0

```
In[917]:= CM033p01=ComplexConjugate[M033p01 M03qb]/.{k →g, l→h, α→ α', β→ β'};
CM053p01=ComplexConjugate[M053p01 M05qb]/.{k →g, l→h, α→ α', β→ β'};
CM043p01=ComplexConjugate[M043p01 M04qb]/.{k →g, l→h, α→ α', β→ β'};
CM063p01=ComplexConjugate[M063p01 M06qb]/.{k →g, l→h, α→ α', β→ β'};

Simplify[Contract[sumpoltensorJ0  $\frac{1}{2m_b \text{Numcolor1 Numpol3P0}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[921]= 0

 3P_1

```
In[922]:= CM033p11=ComplexConjugate[M033p11 M03qb]/.{k →g, l→h, α→ α', β→ β'};
CM053p11=ComplexConjugate[M053p11 M05qb]/.{k →g, l→h, α→ α', β→ β'};
CM043p11=ComplexConjugate[M043p11 M04qb]/.{k →g, l→h, α→ α', β→ β'};
CM063p11=ComplexConjugate[M063p11 M06qb]/.{k →g, l→h, α→ α', β→ β'};

Simplify[Contract[sumpoltensorJ1  $\frac{1}{2m_b \text{Numcolor1 Numpol3P1}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[926]=
$$-\frac{c_1 (c_3 C_A - c_5 C_A + c_4 - c_6) (m_b^2 - 4 m_c^2)^2 (m_b^2 + 8 m_c^2)}{12 \pi m_b^3 m_c^3}$$

 3P_2

```
In[927]:= CM033p21=ComplexConjugate[M033p21 M03qb]/.{k →g, l→h, α→ α', β→ β'};
CM053p21=ComplexConjugate[M053p21 M05qb]/.{k →g, l→h, α→ α', β→ β'};
CM043p21=ComplexConjugate[M043p21 M04qb]/.{k →g, l→h, α→ α', β→ β'};
CM063p21=ComplexConjugate[M063p21 M06qb]/.{k →g, l→h, α→ α', β→ β'};

Simplify[Contract[sumpoltensorJ2  $\frac{1}{2m_b \text{Numcolor1 Numpol3P2}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[931]= 0

Octete

■ $|M_{08}|^2$

1S_0

In[932]:= $\text{CM}_{081s08} = \text{ComplexConjugate}[M_{081s08} M_{08qb}] /. \{k \rightarrow g, l \rightarrow h\};$
 $\text{SUNSimplify}[\text{Simplify}[\frac{1}{2m_b \text{Numcolor8 Numpol1S0}} \text{LPIPS} - \frac{1}{2} \text{SUNFSimplify}[\text{SUNFDelta}[\text{SUNFIndex}[k], S$

Out[933]= $\frac{c_8^2 (m_b^2 - 4 m_c^2)^2}{16 \pi m_b m_c}$

1P_1

In[934]:= $\text{CM}_{081p18} = \text{ComplexConjugate}[M_{081p18} M_{08qb}] /. \{k \rightarrow g, l \rightarrow h, \alpha \rightarrow \alpha'\};$
 $\text{SUNSimplify}[\text{Contract}[\text{sumpolvector Simplify}[\frac{1}{2m_b \text{Numcolor8 Numpol1P1}} \text{LPIPS} - \frac{1}{2} \text{SUNFSimplify}[\text{SUNFDelta}[\text{SUNFIndex}[k], S$

Out[935]= 0

3S_1

In[936]:= $\text{CM}_{083s18} = \text{ComplexConjugate}[M_{083s18} M_{08qb}] /. \{k \rightarrow g, l \rightarrow h, \alpha \rightarrow \alpha'\};$
 $\text{SUNSimplify}[\text{Simplify}[\text{Contract}[\text{sumpolvector} \frac{1}{2m_b \text{Numcolor8 Numpol3S1}} \text{LPIPS} - \frac{1}{2} \text{SUNFSimplify}[\text{SUNFDelta}[\text{SUNFIndex}[k], S$

Out[937]= $\frac{c_8^2 (-48 m_b^2 m_c^4 + m_b^6 + 128 m_c^6)}{48 \pi m_b^3 m_c}$

3P_0

```
In[938]:= CM083p08=ComplexConjugate[M083p08 M08qb]/.{k →g, l→h, α→ α', β→ β'};
SUNSimplify[Simplify[Contract[sumpoltensorJ0 $\frac{1}{2m_b \text{Numcolor8 Numpol3P0}}$ LPIPS $\frac{1}{2}$ SUNFSimplify[
```

Out[939]= 0

 3P_1

```
In[940]:= CM083p18=ComplexConjugate[M083p18 M08qb]/.{k →g, l→h, α→ α', β→ β'};
SUNSimplify[Simplify[Contract[sumpoltensorJ1 $\frac{1}{2m_b \text{Numcolor8 Numpol3P1}}$ LPIPS $\frac{1}{2}$ SUNFSimplify[
```

Out[941]=
$$\frac{c_8^2 (-48 m_b^2 m_c^4 + m_b^6 + 128 m_c^6)}{24 \pi m_b^3 m_c^3}$$

 3P_2

```
In[942]:= CM083p28=ComplexConjugate[M083p28 M08qb]/.{k →g, l→h, α→ α', β→ β'};
SUNSimplify[Simplify[Contract[sumpoltensorJ2 $\frac{1}{2m_b \text{Numcolor8 Numpol3P2}}$ LPIPS $\frac{1}{2}$ SUNFSimplify[
```

Out[943]= 0

■ $2\text{Re}(M_{08}M_{0i}^*)$ penguins

1S_0

```
In[944]:= CM031s08=ComplexConjugate[M031s08M03qb]/.{k ->g, l->h};
CM051s08=ComplexConjugate[M051s08M05qb]/.{k ->g, l->h};
CM041s08=ComplexConjugate[M041s08M04qb]/.{k ->g, l->h};
CM061s08=ComplexConjugate[M061s08M06qb]/.{k ->g, l->h};

SUNSimplify[2Re[Simplify[ $\frac{1}{2m_b\text{Numcolor8 Numpol1S0}}$  LPIPS $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SUNFIndex
Out[948]=  $-\frac{\text{Re}\left(\frac{(c_4-c_6)c_8(m_b^2-4m_c^2)^2}{m_b m_c}\right)}{4\pi}$ 
```

1P_1

```
In[949]:= CM031p18=ComplexConjugate[M031p18M03qb]/.{k ->g, l->h, alpha->alpha'};
CM051p18=ComplexConjugate[M051p18M05qb]/.{k ->g, l->h, alpha->alpha'};
CM041p18=ComplexConjugate[M041p18M04qb]/.{k ->g, l->h, alpha->alpha'};
CM061p18=ComplexConjugate[M061p18M06qb]/.{k ->g, l->h, alpha->alpha'};

SUNSimplify[Contract[sumpolvector Simplify[ $\frac{1}{2m_b\text{Numcolor8 Numpol1P1}}$  LPIPS $\frac{1}{2}$  SUNFSimplify[ S
Out[953]= 0
```

3S_1

```
In[954]:= CM033s18=ComplexConjugate[M033s18M03qb]/.{k ->g, l->h, alpha->alpha'};
CM053s18=ComplexConjugate[M053s18M05qb]/.{k ->g, l->h, alpha->alpha'};
CM043s18=ComplexConjugate[M043s18M04qb]/.{k ->g, l->h, alpha->alpha'};
CM063s18=ComplexConjugate[M063s18M06qb]/.{k ->g, l->h, alpha->alpha'};

SUNSimplify[Simplify[Contract[sumpolvector $\frac{1}{2m_b\text{Numcolor8 Numpol3S1}}$  LPIPS $\frac{1}{2}$  SUNFSimplify[ SU
Out[958]=  $-\frac{(c_4+c_6)c_8(m_b^2-4m_c^2)^2(m_b^2+8m_c^2)}{24\pi m_b^3 m_c}$ 
```

3P_0

```
In[959]:= CM033p01=ComplexConjugate[M033p01 M03qb]/.{k →g, l→h, α→ α', β→ β'};
CM053p01=ComplexConjugate[M053p01 M05qb]/.{k →g, l→h, α→ α', β→ β'};
CM043p01=ComplexConjugate[M043p01 M04qb]/.{k →g, l→h, α→ α', β→ β'};
CM063p01=ComplexConjugate[M063p01 M06qb]/.{k →g, l→h, α→ α', β→ β'};

Simplify[Contract[sumpoltensorJ0  $\frac{1}{2m_b \text{Numcolor1 Numpol3P0}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[963]= 0

 3P_1

```
In[964]:= CM033p18=ComplexConjugate[M033p18 M03qb]/.{k →g, l→h, α→ α', β→ β'};
CM053p18=ComplexConjugate[M053p18 M05qb]/.{k →g, l→h, α→ α', β→ β'};
CM043p18=ComplexConjugate[M043p18 M04qb]/.{k →g, l→h, α→ α', β→ β'};
CM063p18=ComplexConjugate[M063p18 M06qb]/.{k →g, l→h, α→ α', β→ β'};

SUNSimplify[Simplify[Contract[sumpoltensorJ1  $\frac{1}{2m_b \text{Numcolor8 Numpol3P1}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[
```

Out[968]= $-\frac{(c_4 - c_6) c_8 (m_b^2 - 4 m_c^2)^2 (m_b^2 + 8 m_c^2)}{12 \pi m_b^3 m_c^3}$

 3P_2

```
In[969]:= CM033p21=ComplexConjugate[M033p21 M03qb]/.{k →g, l→h, α→ α', β→ β'};
CM053p21=ComplexConjugate[M053p21 M05qb]/.{k →g, l→h, α→ α', β→ β'};
CM043p21=ComplexConjugate[M043p21 M04qb]/.{k →g, l→h, α→ α', β→ β'};
CM063p21=ComplexConjugate[M063p21 M06qb]/.{k →g, l→h, α→ α', β→ β'};

Simplify[Contract[sumpoltensorJ2  $\frac{1}{2m_b \text{Numcolor1 Numpol3P2}}$  LPIPS  $\frac{1}{2}$  SUNFSimplify[ SUNFDelta[SU
```

Out[973]= 0

NLO corrections

Vertex

■ bq vertex

M_{01} Singlete

```
In[974]:= Mqb01NLO = (-c1*g^2*CF/Sqrt[-1])SUNFDelta[SUNFIndex[i],SUNFIndex[j]]*SUNFDelta[SUNFIndex[k],SUNFIndex[l]]
```

1 S0

```
In[975]:= M011s01
```

$$\text{Out[975]} = -\frac{\sqrt{2} \, c_1 \, \bar{P}^\mu \sqrt{C_A m_c^3}}{m_c^2}$$

```
In[976]:= M011s01 M01qb
```

$$\text{Out[976]} = -\frac{\sqrt{2} \, c_1 \, \bar{P}^\mu \, \delta_{kl} \sqrt{C_A m_c^3} \, \bar{u}(p) \cdot \bar{\gamma}^\mu \cdot (1 - \bar{\gamma}^5) \cdot u(P_b, m_b)}{m_c^2}$$

```
In[977]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[k],SUNFIndex[l]]
SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M011s01M01qb/.{k → g,l → h}%SU(3)
%/. Momentum[p1]→ Momentum[P/2];
%/. Momentum[p2]→ Momentum[P/2];
Simplify[%];
```

$$2\text{Re}\left[\frac{1}{2m_b\text{Numcolor1 Numpol1S0}}\text{LIPS}\frac{1}{2}\times\%\right]$$

$$\begin{aligned} \text{Out[982]} = & -\frac{1}{768 \pi^3} \text{Re}\left(\frac{1}{m_c^3} c_1^2 g^2 C_A m_b C_F (m_b^2 - 4 m_c^2) \left(1 - \frac{4 m_c^2}{m_b^2}\right) \right. \\ & \left(m_c^2 \left(-12 \left(-2 \log\left(\frac{m_b^2}{m_b^2 - 4 m_c^2}\right) + \gamma - \log(4 \pi) \right) \log\left(\frac{\mu^2}{m_b^2}\right) - 24 \text{Li}_2\left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2}\right) + 12 \log^2\left(\frac{m_b^2}{m_b^2 - 4 m_c^2}\right) + \right. \right. \\ & \left. 24 \log(4 \pi) \log\left(\frac{m_b^2}{m_b^2 - 4 m_c^2}\right) - 24 \gamma \log\left(\frac{m_b^2}{m_b^2 - 4 m_c^2}\right) + 6 \log^2\left(\frac{\mu^2}{m_b^2}\right) + 12 \log\left(\frac{4 \pi \mu^2}{m_b^2}\right) + \right. \\ & \left. \left. \pi^2 + 6 \gamma^2 - 12 \gamma + 24 + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) \right) + 6 m_b^2 \log\left(\frac{m_b^2}{m_b^2 - 4 m_c^2}\right) \right) \end{aligned}$$

1 P1

In[983]:= **M_{011p11}**

Out[983]= 0

In[984]:= **0**

Out[984]= 0

3 S1

In[985]:= **M_{013s11}**

$$\text{Out[985]} = \frac{2 \sqrt{2} c_1 \bar{g}^{\mu} \sqrt{C_A m_c^3}}{m_c} - \frac{c_1 \bar{P}^{\alpha} \bar{P}^{\mu} \sqrt{C_A m_c^3}}{\sqrt{2} m_c^3}$$

In[986]:= **SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M_{013s11}M_{01qb}/.{k → g,l → h,α → %/. Momentum[p₁]→ Momentum[P/2];
%/. Momentum[p₂]→ Momentum[P/2];
Simplify[Contract[sumpolvector %]]];
2Re[$\frac{1}{2m_b \text{Numcolor1 Numpol3S1}} \text{LIPS} \frac{1}{2} \times \%$]**

$$\begin{aligned} \text{Out[991]} = & -\frac{1}{2304 \pi^3} \text{Re} \left(\frac{1}{m_b m_c} c_1^2 g^2 C_A C_F (m_b^2 - 4 m_c^2) \left(1 - \frac{4 m_c^2}{m_b^2} \right) \right. \\ & \left(m_b^2 \left(-12 \left(-2 \log \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) + \gamma - \log(4 \pi) \right) \log \left(\frac{\mu^2}{m_b^2} \right) - 24 \text{Li}_2 \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) + \right. \right. \\ & 12 \log^2 \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) + 24 \log(4 \pi) \log \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) + \\ & 6 \log^2 \left(\frac{\mu^2}{m_b^2} \right) + 12 \log \left(\frac{4 \pi \mu^2}{m_b^2} \right) + \pi^2 + 6 \gamma^2 - 12 \gamma + 48 + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) \Big) + \\ & 8 m_c^2 \left(-12 \left(-2 \log \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) + \gamma - \log(4 \pi) \right) \log \left(\frac{\mu^2}{m_b^2} \right) - 24 \text{Li}_2 \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) + 12 \log^2 \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) + \right. \\ & 24 \log(4 \pi) \log \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) + 36 \log \left(\frac{m_b^2}{m_b^2 - 4 m_c^2} \right) + \\ & \left. \left. \left. 6 \log^2 \left(\frac{\mu^2}{m_b^2} \right) + 12 \log \left(\frac{4 \pi \mu^2}{m_b^2} \right) + \pi^2 + 6 \gamma^2 - 12 \gamma + 48 + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) \right) \right) \right) \end{aligned}$$

3P0

In[992]:= $\mathbf{M_{013p01}}$

Out[992]=
$$\frac{i \sqrt{2} c_1 \sqrt{C_A m_c^3} \bar{\epsilon}^\alpha \beta_\mu \bar{P}}{m_c^3}$$

3P1

In[993]:= $\mathbf{M_{013p11}}$

Out[993]=
$$\frac{i \sqrt{2} c_1 \sqrt{C_A m_c^3} \bar{\epsilon}^\alpha \beta_\mu \bar{P}}{m_c^3}$$

3P2

In[994]:= $\mathbf{M_{013p21}}$

Out[994]=
$$\frac{i \sqrt{2} c_1 \sqrt{C_A m_c^3} \bar{\epsilon}^\alpha \beta_\mu \bar{P}}{m_c^3}$$

M₀₈ Singlete

In[995]:= 0

Out[995]= 0

bc vertex

Out[996]= bc vertex

M₀₁ Octete

In[997]:= 0

Out[997]= 0

M₀₈ Octete

```
In[998]:= Mbc08NLO = (-c8*g^2/Sqrt[-1])SUNTF[e,i,r]*SUNTF[a,r,j]*SUNTF[a,l,s]*SUNTF[e,s,k]*ChangeDimensi
```

1 s0

```
In[999]:= M081s08
```

$$\text{Out[999]} = -\frac{c_8 \sqrt{m_c^3} \bar{P}^\mu \delta^{ac}}{m_c^2}$$

```
In[1000]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M081s08M08qb/.{k →
%/. Momentum[p1]→ Momentum[P/2];
%/. Momentum[p2]→ Momentum[P/2];
Simplify[%];
2Re[ $\frac{1}{2m_b \text{Numcolor8 Numpol1S0}}$  LPIPS  $\frac{1}{2}$  *%]
```

$$\begin{aligned} \text{Out[1005]} = & -\frac{1}{512 \pi^3} \text{Re} \left(\left(C_A^2 - 2 \right) g^2 c_8^2 m_b \left(1 - \frac{4 m_c^2}{m_b^2} \right) \right. \\ & \left(-\left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \right. \right. \\ & \left. \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \right. \\ & \left. \left. \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right) \end{aligned}$$

$$\begin{aligned}
& 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 2 \log \left(\frac{m_b^2}{m_c^2} \right) \\
& \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 2 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_b^8 + \\
& \left(4 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \\
& \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \\
& 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) -
\end{aligned}$$

$$\begin{aligned}
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 6 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^2 + \\
& \left(4 \log \left(\frac{\mu^2}{m_b^2} \right) - 6 \log \left(\frac{\mu^2}{m_c^2} \right) + 3 \log \left(\frac{m_b^2}{m_c^2} \right) - 2 \log(\pi) - \log(16) + 2 \gamma - 4 \right) \sqrt{(m_b^2 - 4 m_c^2)^2} \Bigg) \\
& m_b^6 - 4 \left(\left(-3 \log^2 \left(\frac{2(m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - 6 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \right. \\
& \left. \log \left(\frac{2(m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 3 \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - 3 \log^2 \left(\right. \right. \\
& \left. \left. \frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + 3 \log^2 \left(-\frac{2(m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \left. 12 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \left. 6 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(-\frac{2(m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 12 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2 - m_b^2}} \right) - \\
& 12 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 6 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 12 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 12 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 28 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 12 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - 12 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^4 + \\
& \left(6 \log \left(\frac{\mu^2}{m_b^2} \right) - 10 \log \left(\frac{\mu^2}{m_c^2} \right) + 4 \log \left(\frac{m_b^2}{m_c^2} \right) - 4 \log(\pi) - \log(256) + 4 \gamma - 9 \right) \\
& \sqrt{(m_b^2 - 4 m_c^2)^2 - m_c^2} m_b^4 +
\end{aligned}$$

$$\begin{aligned}
& 8 \left(8 \log \left(\frac{\mu^2}{m_b^2} \right) - 12 \log \left(\frac{\mu^2}{m_c^2} \right) - \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 5 \log \left(\frac{m_b^2}{m_c^2} \right) - 4 \log(4 \pi) + 5 \gamma - 13 \right) m_c^4 \\
& \sqrt{(m_b^2 - 4 m_c^2)^2} - 8 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\
& \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \right. \\
& \left. \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \left. 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \left. 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \right. \\
& \left. 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \log\left(\frac{m_b^2}{m_c^2}\right) \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) + \\
& 4 \log(4 \pi) \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) - \\
& 4 \gamma \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) - 4 \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) - \\
& 4 \operatorname{Li}_2\left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) + 4 \operatorname{Li}_2\left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) \Bigg) m_c^6 m_b^2 + \\
& 64 \left(\log^2\left(\frac{2(m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) + 2 \log\left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2}\right) \log\left(\frac{2(m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) - \right. \\
& \log^2\left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2}\right) + \log^2\left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2}\right) - \\
& \log^2\left(-\frac{2(m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) + \\
& \left. 4 \log\left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2}\right) \log\left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) - \right.
\end{aligned}$$

$$\begin{aligned}
 & 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
 & 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
 & 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
 & 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
 & 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
 & 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 4 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
 & 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^8 - \\
 & 32 \left(2 \log \left(\frac{\mu^2}{m_b^2} \right) - 3 \log \left(\frac{\mu^2}{m_c^2} \right) + \log \left(\frac{m_b^2}{4 \pi m_c^2} \right) + \gamma - 3 \right) m_c^6
 \end{aligned}$$

$$\left(\left(C_A m_c \sqrt{(m_b^2 - 4 m_c^2)^2} (m_b^2 - 2 m_c^2)^2 \right) \right)$$

1P1

In[1006]:= 0

Out[1006]= 0

3S1

In[1007]:= M083s18

$$\text{Out[1007]= } \frac{c_8 m_c^2 \delta^{ac} \bar{g}^{\mu}{}_{\mu}}{\sqrt{m_c^3}} + \frac{c_8 \sqrt{m_c^3} \delta^{ac} \bar{g}^{\mu}{}_{\mu}}{m_c} - \frac{c_8 \bar{P}^{\mu} \bar{P}^{\mu} \delta^{ac}}{2 \sqrt{m_c^3}}$$

In[1008]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M083s18M08qb/.{k →

%/ . Momentum[p₁]→ Momentum[P/2];

%/ . Momentum[p₂]→ Momentum[P/2];

Simplify[Contract[sumpolvector %]];

$$2 \text{Re} \left[\frac{1}{2 m_b \text{Numcolor8 Numpol3S1}} \text{LPIPS} \frac{1}{2} \times \% \right]$$

$$\text{Out[1013]= } -\frac{1}{1536 \pi^3} \text{Re} \left(\left(C_A^2 - 2 \right) g^2 c_8^2 \left(1 - \frac{4 m_c^2}{m_b^2} \right) \right)$$

$$\begin{aligned}
& \left(- \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \right. \\
& \quad \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \\
& \quad \left. \log^2 \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \quad \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \quad \left. 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \quad \left. 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \right. \\
& \quad \left. 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 2 \log \left(\frac{m_b^2}{m_c^2} \right) \right. \\
& \quad \left. \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 2 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg] m_b^{10} + \\
& \left(4 \log \left(\frac{\mu^2}{m_b^2} \right) - 6 \log \left(\frac{\mu^2}{m_c^2} \right) + 3 \log \left(\frac{m_b^2}{m_c^2} \right) - 2 \log(\pi) - \log(16) + 2 \gamma - 4 \right) \sqrt{(m_b^2 - 4 m_c^2)^2} - \\
& 4 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\
& \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \right. \\
& \left. \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \left. 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2 - m_b^2}} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 8 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^2 m_b^8 + \\
& 4 \left(\left(11 \log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 22 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \right. \\
& \left. \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - 11 \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 11 \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \\
& 11 \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \\
& 44 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 22 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 44 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 44 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 22 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 44 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) -
\end{aligned}$$

$$\begin{aligned}
& 44 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 116 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 44 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 44 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^4 + \\
& \left(14 \log \left(\frac{\mu^2}{m_b^2} \right) - 14 \log \left(\frac{\mu^2}{m_c^2} \right) + 17 \log \left(\frac{m_b^2}{m_c^2} \right) - 5 \right) \sqrt{(m_b^2 - 4 m_c^2)^2} m_c^6 - \\
& 8 \left(20 \log \left(\frac{\mu^2}{m_b^2} \right) - 31 \log \left(\frac{\mu^2}{m_c^2} \right) + 33 \log \left(\frac{m_b^2}{m_c^2} \right) - 11 \log(\pi) - \log(4 194 304) + 11 \gamma - 41 \right) \\
& m_c^4 \sqrt{(m_b^2 - 4 m_c^2)^2} - 4 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\
& \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \right. \\
& \left. \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 52 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^6 m_b^4 + \\
& 32 \left(-6 \log \left(\frac{\mu^2}{m_b^2} \right) - 3 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 7 \log \left(\frac{m_b^2}{m_c^2} \right) - 6 \log(4 \pi) + 9 \gamma - 29 \right) m_c^6 \sqrt{(m_b^2 - 4 m_c^2)^2} -
\end{aligned}$$

$$\begin{aligned}
& 14 \left(\log^2 \left(\frac{2(m_b^2 - 2m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) \right. \\
& \quad \log \left(\frac{2(m_b^2 - 2m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) + \\
& \quad \log^2 \left(\frac{3m_b^2 - 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) - \log^2 \left(\frac{2(m_b^2 - 2m_c^2)}{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}} \right) + \\
& \quad 4 \log \left(\frac{2m_b m_c}{m_b^2 - 2m_c^2} \right) \log \left(\frac{4m_b m_c}{m_b^2 + 4m_c^2 - \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \\
& \quad 2 \log \left(\frac{3m_b^2 - 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) \log \left(\frac{2(m_b^2 - 2m_c^2)}{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \\
& \quad 4 \log \left(\frac{\sqrt{(m_b^2 - 4m_c^2)^2}}{m_b^2 - 2m_c^2} \right) \log \left(\frac{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{\sqrt{(m_b^2 - 4m_c^2)^2} - m_b^2} \right) + \\
& \quad 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{4m_b m_c} \right) + \\
& \quad 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{4m_b m_c} \right) +
\end{aligned}$$

$$\begin{aligned}
& 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 4 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^8 m_b^2 - \\
& 256 m_c^8 \left(-2 \log \left(\frac{\mu^2}{m_b^2} \right) + \log \left(\frac{\mu^2}{4 \pi m_c^2} \right) + \gamma - 3 \right) \sqrt{(m_b^2 - 4 m_c^2)^2} - \\
& 2 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\
& \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \right. \\
& \left. \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 4 \log(4 \pi) \\
& \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \\
& \left. \right) \left. \right) \left. \right) / \left(C_A m_b m_c \sqrt{(m_b^2 - 4 m_c^2)^2} (m_b^2 - 2 m_c^2)^2 \right)
\end{aligned}$$

In[1014]:=

bcbar vertex

Out[1014]=

bcbar vertex

M₀₁ Octete

In[1015]:= 0

Out[1015]= 0

M₀₈ Octete

In[1016]:= $M_{\text{bcbar}08\text{NLO}} = (-c_8 * g^2 / \text{Sqrt}[-1]) \text{SUNTF}[e, r, j] \times \text{SUNTF}[a, i, r] \times \text{SUNTF}[a, l, s] \times \text{SUNTF}[e, s, k] \times \text{ChangeDimension}$

1S0

In[1017]:= `SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M081s08 M08qb /. {k → p1 → Momentum[p1] → Momentum[P/2];
p2 → Momentum[p2] → Momentum[P/2];
Simplify[%];`

$$2\text{Re}\left[\frac{1}{2m_b \text{Numcolor8} \text{Numpol1S0}} \text{LPIPS} \frac{1}{2} \times \%\right]$$

$$\begin{aligned} \text{Out[1022]} = & -\frac{1}{256 \pi^3} \text{Re} \left(g^2 c_8^2 m_b \left(1 - \frac{4 m_c^2}{m_b^2} \right) \right. \\ & \left(-\left(\log^2 \left(\frac{2(m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2(m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \right. \right. \\ & \left. \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\ & \left. \left. \log^2 \left(-\frac{2(m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right) \end{aligned}$$

$$\begin{aligned}
& 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 2 \log \left(\frac{m_b^2}{m_c^2} \right) \\
& \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 8 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_b^6 + \\
& 2 \left(\left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) + \log^2 \left(\frac{3m_b^2 - 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) - \\
& \log^2 \left(-\frac{2(m_b^2 - 2m_c^2)}{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}} \right) + \\
& 4 \log \left(\frac{2m_b m_c}{m_b^2 - 2m_c^2} \right) \log \left(\frac{4m_b m_c}{m_b^2 + 4m_c^2 - \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \\
& 2 \log \left(\frac{3m_b^2 - 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) \log \left(-\frac{2(m_b^2 - 2m_c^2)}{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4m_c^2)^2}}{m_b^2 - 2m_c^2} \right) \log \left(\frac{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{\sqrt{(m_b^2 - 4m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{4m_b m_c} \right) + \\
& 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{4m_b m_c} \right) + \\
& 4 \log(4\pi) \log \left(\frac{m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{4m_b m_c} \right) -
\end{aligned}$$

$$\begin{aligned}
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 44 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^2 + \\
& 2 \left(\log \left(\frac{\mu^2}{16 m_b^2} \right) - 3 \log \left(\frac{\mu^2}{m_c^2} \right) + 3 \log \left(\frac{m_b^2}{m_c^2} \right) - 2 \log(\pi) + 2 \gamma - 5 \right) \sqrt{(m_b^2 - 4 m_c^2)^2} m_b^4 - \\
& 4 \left(2 \log \left(\frac{\mu^2}{m_b^2} \right) - 2 \log \left(\frac{\mu^2}{m_c^2} \right) - 12 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 13 \log \left(\frac{m_b^2}{m_c^2} \right) + 12 \gamma - 30 \right) m_c^2 \sqrt{(m_b^2 - 4 m_c^2)^2} - \\
& 4 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\
& \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \right. \\
& \left. \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 20 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^4 m_b^2 + \\
& 16 \left(\left(-4 \log \left(\frac{\mu^2}{m_c^2} \right) + 3 \log \left(\frac{m_b^2}{m_c^2} \right) - 4 \log(\pi) - \log(256) + 4 \gamma - 10 \right) m_c^4 \sqrt{(m_b^2 - 4 m_c^2)^2} - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\
& \quad \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \\
& \quad \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \\
& \quad 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& \quad 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& \quad 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& \quad 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& \quad 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) +
\end{aligned}$$

$$\begin{aligned} & 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\ & 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 12 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\ & 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \\ & \left. \left. \left. m_c^6 \right) \right) \right) / \left(C_A m_c \sqrt{(m_b^2 - 4 m_c^2)^2} (m_b^2 - 2 m_c^2) \right) \end{aligned}$$

1P1

In[1023]:= 0

Out[1023]= 0

3S1

In[1024]:= M083s18

Out[1024]=

$$\frac{c_8 m_c^2 \delta^{ac} \overline{g}^{\alpha \mu}}{\sqrt{m_c^3}} + \frac{c_8 \sqrt{m_c^3} \delta^{ac} \overline{g}^{\alpha \mu}}{m_c} - \frac{c_8 \overline{P}^{\alpha} \overline{P}^{\mu} \delta^{ac}}{2 \sqrt{m_c^3}}$$

In[1025]:=

```
SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*
SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M083s18M08qb/.{k →
```

```
%/. Momentum[p1]→ Momentum[P/2];
```

```
%/. Momentum[p2]→ Momentum[P/2];
```

```
Simplify[Contract[sumpolvector %]];

```

$$2\text{Re}\left[\frac{1}{2m_b\text{Numcolor8 Numpol3S1}}\text{LPIPS}\frac{1}{2}\times\%\right]$$

$$\text{Out[1030]} = \frac{1}{768\pi^3} \text{Re} \left(g^2 c_8^2 \left(1 - \frac{4m_c^2}{m_b^2} \right) \right. \\ \left. - \left(\log^2 \left(\frac{2(m_b^2 - 2m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) \log \left(\frac{2(m_b^2 - 2m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \right. \right. \\ \left. \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) + \log^2 \left(\frac{3m_b^2 - 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) - \right. \\ \left. \log^2 \left(-\frac{2(m_b^2 - 2m_c^2)}{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}} \right) + \right. \\ \left. 4 \log \left(\frac{2m_b m_c}{m_b^2 - 2m_c^2} \right) \log \left(\frac{4m_b m_c}{m_b^2 + 4m_c^2 - \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \right. \\ \left. 2 \log \left(\frac{3m_b^2 - 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}}{2m_b^2 - 4m_c^2} \right) \log \left(-\frac{2(m_b^2 - 2m_c^2)}{-3m_b^2 + 4m_c^2 + \sqrt{(m_b^2 - 4m_c^2)^2}} \right) - \right)$$

$$\begin{aligned}
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 2 \log \left(\frac{m_b^2}{m_c^2} \right) \\
& \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 8 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_b^8 + \\
& 2 \left(\left(-3 \log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - 6 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \right. \\
& \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 3 \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - 3 \log^2 \left(\right. \right. \\
& \left. \left. \frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + 3 \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 12 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \\
& 6 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \\
& 12 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) - \\
& 12 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 6 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 12 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 12 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 44 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 12 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - 12 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^2 +
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\log \left(\frac{\mu^2}{16 m_b^2} \right) - 3 \log \left(\frac{\mu^2}{m_c^2} \right) + 3 \log \left(\frac{m_b^2}{m_c^2} \right) - 2 \log(\pi) + 2 \gamma - 5 \right) \sqrt{(m_b^2 - 4 m_c^2)^2} \left(m_b^6 + \right. \\
& 4 \left(8 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \right. \\
& \left. \left. \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \right. \right. \\
& \left. \left. \log^2 \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \right. \\
& \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \left. 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right. \\
& \left. 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \right. \\
& \left. 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \log\left(\frac{m_b^2}{m_c^2}\right) \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) + \\
& 4 \log(4 \pi) \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) - \\
& 4 \gamma \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) - 8 \log\left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c}\right) - \\
& 4 \operatorname{Li}_2\left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) + 4 \operatorname{Li}_2\left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) \Bigg) m_c^4 - \\
& \left(18 \log\left(\frac{\mu^2}{m_b^2}\right) - 14 \log\left(\frac{\mu^2}{m_c^2}\right) + 13 \log\left(\frac{m_b^2}{m_c^2}\right) + 4 \log(\pi) + \log(256) - 4 \gamma + 10\right) \\
& m_c^2 \sqrt{(m_b^2 - 4 m_c^2)^2} \Bigg) m_b^4 - \\
& 16 \left(-16 \log\left(\frac{\mu^2}{m_b^2}\right) - 4 \log\left(\frac{\mu^2}{m_c^2}\right) + 3 \log\left(\frac{m_b^2}{m_c^2}\right) - 20 \log(\pi) - \log(1\,099\,511\,627\,776) + 20 \gamma - 50 \right) \\
& m_c^4 \sqrt{(m_b^2 - 4 m_c^2)^2} - 2 \left(3 \log^2\left(\frac{2(m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}}\right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 6 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 3 \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + 3 \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \\
& 3 \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \\
& 12 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 6 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 12 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 12 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 6 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + 12 \log(4 \pi)
\end{aligned}$$

$$\begin{aligned}
& \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 12 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 12 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \\
& 12 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \Bigg) m_c^6 m_b^2 + \\
& 128 \left(\left(-2 \log \left(\frac{\mu^2}{m_b^2} \right) - 2 \log \left(\frac{\mu^2}{m_c^2} \right) + \log \left(\frac{m_b^2}{256 m_c^2} \right) - 4 \log(\pi) + 4 \gamma - 10 \right) m_c^6 \sqrt{(m_b^2 - 4 m_c^2)^2} - \right. \\
& 2 \left(\log^2 \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 2 \log \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \right. \\
& \left. \log \left(\frac{2 (m_b^2 - 2 m_c^2)}{m_b^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \log^2 \left(\frac{m_b^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) + \right. \\
& \left. \log^2 \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) - \log^2 \left(-\frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + \right. \\
& \left. 4 \log \left(\frac{2 m_b m_c}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{4 m_b m_c}{m_b^2 + 4 m_c^2 - \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \log \left(\frac{3 m_b^2 - 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 m_b^2 - 4 m_c^2} \right) \log \left(- \frac{2 (m_b^2 - 2 m_c^2)}{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) - \\
& 4 \log \left(\frac{\sqrt{(m_b^2 - 4 m_c^2)^2}}{m_b^2 - 2 m_c^2} \right) \log \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{\sqrt{(m_b^2 - 4 m_c^2)^2} - m_b^2} \right) + \\
& 4 \log \left(\frac{\mu^2}{m_b^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 2 \log \left(\frac{m_b^2}{m_c^2} \right) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) + \\
& 4 \log(4 \pi) \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \gamma \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - 4 \log \left(\frac{m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{4 m_b m_c} \right) - \\
& 4 \operatorname{Li}_2 \left(\frac{1}{2} - \frac{m_b^2}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) + 4 \operatorname{Li}_2 \left(\frac{-3 m_b^2 + 4 m_c^2 + \sqrt{(m_b^2 - 4 m_c^2)^2}}{2 \sqrt{(m_b^2 - 4 m_c^2)^2}} \right) \\
& \left. \right) \left. \right) \left. \right) / \left(C_A m_b \sqrt{(m_b^2 - 4 m_c^2)^2} (2 m_c^3 - m_b^2 m_c) \right)
\end{aligned}$$

In[1031]:= qc vertex

Out[1031]= qc vertex

In[1032]:=
$$M_{qc\theta 8NLO} = (-c_8 * g^2 / \text{Sqrt}[-1]) \text{SUNTF}[e, i, r] * \text{SUNTF}[a, r, j] * \text{SUNTF}[a, s, k] * \text{SUNTF}[e, l, s] * \text{ChangeDimens}$$

M_{01} Octete

In[1033]:= 0

Out[1033]= 0

M_{08} Octete

1S0

In[1034]:=

```
SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M081s08M08qb/.{k →
%/. Momentum[p1]→ Momentum[P/2];
%/. Momentum[p2]→ Momentum[P/2];
Simplify[%];

2Re[ $\frac{1}{2m_b \text{Numcolor8 Numpol1S0}} \text{LPIPS} \frac{1}{2} \times \%$ ]
```

$$\begin{aligned} \text{Out[1039]} = & \frac{1}{1536 \pi^3} \text{Re} \left(\frac{1}{C_A (2 m_c^3 - m_b^2 m_c)} c_8^2 g^2 m_b (m_b^2 - 4 m_c^2) \left(1 - \frac{4 m_c^2}{m_b^2} \right) \right. \\ & \left(m_b^2 \left(-12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) + \gamma - 2 - \log(\pi) \right) \log \left(\frac{\mu^2}{m_c^2} \right) - 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + \right. \right. \\ & 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - \\ & 48 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + \pi^2 + 6 \gamma^2 + 24 \gamma - 72 + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) \Big) - \\ & 2 m_c^2 \left(-12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) + \gamma - 2 - \log(\pi) \right) \log \left(\frac{\mu^2}{m_c^2} \right) - 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + \right. \\ & 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \\ & 24 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 48 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + \pi^2 + \\ & \left. \left. \left. \left. \left. 6 \gamma^2 + 24 \gamma - 72 + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) \right) \right) \right) \right) \right) \end{aligned}$$

1P1

In[1040]:=

0

Out[1040]= 0

3S1

In[1041]:=

```

SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFDelta[SUNFIndex[n], SUNFIndex[i]]]
SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M083s18 M08qb /. {k →
%/. Momentum[p1] → Momentum[P/2];
%/. Momentum[p2] → Momentum[P/2];

Simplify[Contract[sumpolvector %]]];

2 Re[ $\frac{1}{2 m_b \text{Numcolor8 Numpol3S1}} \text{LPIPS} \frac{1}{2} \times \%$ ]

```

Out[1046]=

$$\begin{aligned}
& \frac{1}{4608 \pi^3} \text{Re} \left(\frac{1}{C_A m_b (2 m_c^3 - m_b^2 m_c)} c_8^2 g^2 \left(1 - \frac{4 m_c^2}{m_b^2} \right) (4 m_b^2 m_c^2 + m_b^4 - 32 m_c^4) \right. \\
& \left(m_b^2 \left(-12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) + \gamma - 2 - \log(\pi) \right) \log \left(\frac{\mu^2}{m_c^2} \right) - 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + \right. \right. \\
& 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - \\
& 48 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + \pi^2 + 6 \gamma^2 + 24 \gamma - 72 + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) \Big) - \\
& 2 m_c^2 \left(-12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) + \gamma - 2 - \log(\pi) \right) \log \left(\frac{\mu^2}{m_c^2} \right) - 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + \right. \\
& 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \\
& 24 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 48 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + \pi^2 + \\
& \left. \left. \left. \left. \left. 6 \gamma^2 + 24 \gamma - 72 + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) \right) \right) \right) \right) \right) \Big)
\end{aligned}$$

In[1047]:=

qcb̄ vertex

Out[1047]= qcb̄ vertex

In[1048]:=

$$M_{\text{qcb}\bar{0}8\text{NLO}} = (-c_8 * g^2 / \text{Sqrt}[-1]) \text{SUNTF}[e, r, j] * \text{SUNTF}[a, i, r] * \text{SUNTF}[a, s, k] * \text{SUNTF}[e, l, s] * \text{ChangeDime}$$

150

In[1049]:=

```
SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*
SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M081s08M08qb /. {k →
%/. Momentum[p1]→ Momentum[P/2];
%/. Momentum[p2]→ Momentum[P/2];
Simplify[%];

2Re[ $\frac{1}{2m_b \text{Numcolor8 Numpol150}} \text{LPIPS} \frac{1}{2} \times \%$ ]
```

$$\begin{aligned} \text{Out[1054]} = & -\frac{1}{3072 \pi^3} \text{Re} \left(\frac{1}{C_A m_c (m_b^2 - 2 m_c^2)^2} (C_A^2 - 2) g^2 c_8^2 m_b (m_b^2 - 4 m_c^2) \left(1 - \frac{4 m_c^2}{m_b^2} \right) \right. \\ & \left(\left(6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) - \log(\pi) + \gamma - 2 \right) \log \left(\frac{\mu^2}{m_c^2} \right) + 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 12 \right. \right. \\ & \left. \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 36 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \right. \\ & \left. 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) + \pi^2 + 6 \gamma^2 - 12 \gamma + 24 \right) m_b^4 - \\ & 4 \left(6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) - \log(\pi) + \gamma - 2 \right) \log \left(\frac{\mu^2}{m_c^2} \right) + \right. \\ & 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 12 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \\ & 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 36 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + \\ & \left. 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) + \pi^2 + 6 \gamma^2 - 12 \gamma + 18 \right) m_c^2 m_b^2 + \\ & 4 \left(6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) - \log(\pi) + \gamma - 2 \right) \log \left(\frac{\mu^2}{m_c^2} \right) + 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 12 \right. \\ & \left. \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 24 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \right. \\ & \left. 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) + \pi^2 + 6 \gamma^2 - 12 \gamma + 12 \right) m_c^4 \Bigg) \end{aligned}$$

1P1

In[1055]:= 0

Out[1055]= 0

3S1

```

In[1056]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFDelta[SUNFIndex[n], SUNFIndex[i]]
SUNFSimplify[SUNFSimplify[DiracSimplify[FermionSpinSum[ComplexConjugate[M083s18 M08qb /. {k →

%/. Momentum[p1] → Momentum[P/2];
%/. Momentum[p2] → Momentum[P/2];

Simplify[Contract[sumpolvector %]]];

2 Re[  $\frac{1}{2 m_b \text{Numcolor8 Numpol3S1}} \text{LPIPS} \frac{1}{2} \times \% ]$  ]

```

$$\begin{aligned}
\text{Out[1061]} = & -\frac{1}{9216 \pi^3} \text{Re} \left(\frac{1}{C_A m_b m_c (m_b^2 - 2 m_c^2)^2} (C_A^2 - 2) g^2 c_8^2 (m_b^2 - 4 m_c^2) \left(1 - \frac{4 m_c^2}{m_b^2} \right) \right. \\
& \left(\left(6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) - \log(\pi) + \gamma - 2 \right) \log \left(\frac{\mu^2}{m_c^2} \right) + 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 12 \right. \right. \\
& \left. \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) + 36 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \right. \\
& \left. 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) + \pi^2 + 6 \gamma^2 - 12 \gamma + 24 \right) m_b^6 + \\
& 4 \left(6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) - \log(\pi) + \gamma - 2 \right) \log \left(\frac{\mu^2}{m_c^2} \right) + 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 12 \right. \\
& \left. \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \right. \\
& \left. 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) + \pi^2 + 6 \gamma^2 - 12 \gamma - 6 \right) m_c^2 m_b^4 - \\
& 4 \left(42 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 84 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) - \log(\pi) + \gamma - 2 \right) \log \left(\frac{\mu^2}{m_c^2} \right) + \right. \\
& 84 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 84 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 168 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \\
& 168 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 168 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + \\
& \left. 42 \log^2(4 \pi) - 84 \gamma \log(4 \pi) + 168 \log(4 \pi) + 7 \pi^2 + 42 \gamma^2 - 84 \gamma + 60 \right) m_c^4 m_b^2 + \\
& 32 \left(6 \log^2 \left(\frac{\mu^2}{m_c^2} \right) - 12 \left(-2 \log \left(-\frac{4 m_c^2}{m_b^2 - 4 m_c^2} \right) - \log(\pi) + \gamma - 2 \right) \log \left(\frac{\mu^2}{m_c^2} \right) + 12 \log^2 \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \right. \\
& 12 \log \left(\frac{4 \pi \mu^2}{m_c^2} \right) + 24 \log(4 \pi) \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - 24 \gamma \log \left(-\frac{2 m_c^2}{m_b^2 - 4 m_c^2} \right) - \\
& \left. 24 \text{Li}_2 \left(\frac{2 m_c^2}{m_b^2 - 4 m_c^2} + 1 \right) + 6 \log^2(4 \pi) - 12 \gamma \log(4 \pi) + 24 \log(4 \pi) + \pi^2 + 6 \gamma^2 - 12 \gamma + 12 \right) m_c^6 \Bigg)
\end{aligned}$$

Correcciones Reales

Amplitud c con la corriente qb amputada

```
In[1062]:= M_c1 = -c1 Sqrt[-1] (Sqrt[-1] g) SUNTF[b, i, j] ×
           SpinorUBar[p1, mc].GA[μ].(1 - GA[5]).SpinorV[p2, mc]

Out[1062]= c1 g T_ij^b ū(p1, mc).γ̄^μ.(1 - γ̄^5).v(p2, mc)

In[1063]:= M_c8 = -c8 Sqrt[-1] (Sqrt[-1] g) SUNTF[b, i, r] ×
           SUNTF[a, r, j] × SpinorUBar[p1, mc].GA[μ].(1 - GA[5]).SpinorV[p2, mc]

Out[1063]= c8 g T_ir^a T_jr^b ū(p1, mc).γ̄^μ.(1 - γ̄^5).v(p2, mc)
```

Amplitud cbar con la corriente qb amputada

```
In[1064]:= M_cbar1 =
           -c1 Sqrt[-1] (Sqrt[-1] g) SUNTF[b, i, j] × SpinorUBar[p1, mc].GA[μ].(1 - GA[5]).SpinorV[p2, mc]

Out[1064]= c1 g T_ij^b ū(p1, mc).γ̄^μ.(1 - γ̄^5).v(p2, mc)

In[1065]:= M_cbar8 = -c8 Sqrt[-1] (Sqrt[-1] g) SUNTF[a, i, r] ×
           SUNTF[b, r, j] × SpinorUBar[p1, mc].GA[μ].(1 - GA[5]).SpinorV[p2, mc]

Out[1065]= c8 g T_ir^a T_rj^b ū(p1, mc).γ̄^μ.(1 - γ̄^5).v(p2, mc)
```

Amplitud qb

```
In[1066]:= M_c1qb =
           SUNFDelta[SUNFIndex[l], SUNFIndex[k]] × SpinorUBar[p, 0].GA[μ].(1 - GA[5]).SpinorU[Pb, mb]

Out[1066]= δ_kl ū(p).γ̄^μ.(1 - γ̄^5).u(Pb, mb)

In[1067]:= M_c8qb = SpinorUBar[Momentum[p], 0].GA[μ].(1 - GA[5]).SpinorU[Pb, mb] × SUNTF[a, l, k]

Out[1067]= T_lk^a ū(p).γ̄^μ.(1 - γ̄^5).u(Pb, mb)
```

M_c8 c gluon emission

Singlet channel

1S0

```
In[1068]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFInc
%/. Momentum[p1]→ Momentum[P/2];
Mc81s01=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out}[1070]=\frac{c_8\,g\,\overline{P}^\mu\,\delta^{ab}\,\sqrt{C_A\,m_c^3}}{\sqrt{2}\,C_A\,m_c^2}$$

1P1

```
In[1071]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFInc
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
Mc81p11=FourDivergence[%,FourVector[q,α]]
```

Out[1073]= 0

3 S1

```
ln[1074]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFInc
Mc83s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[1075]} = \frac{c_8 g \bar{P}^{\alpha} \bar{P}^{\mu} \delta^{ab} \sqrt{C_A m_c^3}}{2 \sqrt{2} C_A m_c^3} - \frac{\sqrt{2} c_8 g \delta^{ab} \bar{g}^{\alpha \mu} \sqrt{C_A m_c^3}}{C_A m_c}$$

3PJ

```
In[1076]:= SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFInc
%/. Momentum[p1]-> Momentum[P/2+q];
%/. Momentum[p2]-> Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
Mc83p1=%/. Momentum[q]-> 0
```

$$\text{Out[1080]= } -\frac{i\,c_8\,g\delta^{ab}\sqrt{C_A\,m_c^3}\,\bar{\epsilon}^\alpha\beta\,\mu\bar{P}}{\sqrt{2}\,C_A\,m_c^3}$$

Octet channel

150

```
In[1081]:= SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n], SUNFIndex[i]] * SUNFDelta[SUNFIndex[n], SUNFIndex[i]]
%/. Momentum[p1] → Momentum[P/2];
Mc81s08 = %/. Momentum[p2] → Momentum[P/2]
```

$$\text{Out[1083]} = \frac{2 \, c_8 \, g \sqrt{m_c^3} \, \bar{P}^\mu (\text{tr} (T^c.T^b.T^a))}{m_c^2}$$

1P1

```
In[1084]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
Mc81p18=FourDivergence[%,FourVector[q,α]]
```

Out[1086]= 0

3S1

```
In[1087]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
Mc83s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[1088]} = \frac{c_8 g \bar{P}^\alpha \bar{P}^\mu (\text{tr} (T^c.T^b.T^a))}{\sqrt{m_c^3}} - \frac{2 c_8 g m_c^2 \bar{g}^{\alpha \mu} (\text{tr} (T^c.T^b.T^a))}{\sqrt{m_c^3}} - \frac{2 c_8 g \sqrt{m_c^3} \bar{g}^{\alpha \mu} (\text{tr} (T^c.T^b.T^a))}{m_c}$$

3PJ

```
In[1089]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]]];
Mc83pj8=%/. Momentum[q]→ 0
```

$$\text{Out[1093]} = - \frac{2 i c_8 g \bar{\epsilon}^{\alpha \beta \mu \bar{P}} (\text{tr} (T^c.T^b.T^a))}{\sqrt{m_c^3}}$$

M₀₁ cbar gluon emission

Octet channel

1S0

```
In[1094]:= SUNFSimplify[coloroctateprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
Mcbar11s08=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[1096]} = \frac{c_1 g \sqrt{m_c^3} \bar{P}^\mu \delta^{bc}}{m_c^2}$$

1P1

```
In[1097]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
M_c11p18=FourDivergence[%,FourVector[q,α]]
```

Out[1099]= 0

3S1

```
In[1100]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M_c13s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

$$\text{Out[1101]} = -\frac{c_1 g m_c^2 \delta^{bc} \bar{g}^{\alpha\mu}}{\sqrt{m_c^3}} - \frac{c_1 g \sqrt{m_c^3} \delta^{bc} \bar{g}^{\alpha\mu}}{m_c} + \frac{c_1 g \bar{P}^\alpha \bar{P}^\mu \delta^{bc}}{2 \sqrt{m_c^3}}$$

3PJ

```
In[1102]:= SUNFSimplify[ColorOctateProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M_c13p08=%/. Momentum[q]→ 0
```

$$\text{Out[1106]} = -\frac{i c_1 g \delta^{bc} \bar{\epsilon}^{\alpha\beta\mu} \bar{P}}{\sqrt{m_c^3}}$$

M_cbar8 cbar gluon emission

Singlet channel

1S0

```
In[1107]:= SUNFSimplify[ColorSingletProyector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
M_cbar81s01=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out[1109]} = \frac{c_8 g \bar{P}^\mu \delta^{ab} \sqrt{C_A m_c^3}}{\sqrt{2} C_A m_c^2}$$

1P1

```
SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFInc
%/. {Momentum[p1]-> Momentum[P/2+q], Momentum[p2]-> Momentum[P/2-q]};
Mcbar81p11=FourDivergence[%,FourVector[q,α]]
```

Out[1112]= 0

3S1

`SUNFSimplify[colorsingletprojector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]]`

`Mcbar83s11=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}`

$$\text{Out}[1114]=\frac{c_8 g \bar{P}^{\alpha} \bar{P}^{\mu} \delta^{ab} \sqrt{C_A m_c^3}}{2 \sqrt{2} C_A m_c^3}-\frac{\sqrt{2} c_8 g \delta^{ab} \bar{g}^{\alpha \mu} \sqrt{C_A m_c^3}}{C_A m_c}$$

3PJ

```
SUNFSimplify[ColoringSingleTensor SUNFDelta[SUNFIndex[n], SUNFIndex[i]]*SUNFDelta[SUNFIndex[n], SUNFIndex[i]]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%, FourVector[q, β]];
M_cbar83p11=%/. Momentum[q]→ 0
```

$$\text{Out}[1119]= -\frac{i\,c_8\,g^{\delta ab}\,\sqrt{C_A\,m_c^3}\,\bar{\epsilon}^{\alpha\,\beta\,\mu\,\bar{P}}}{\sqrt{2}\,C_A\,m_c^3}$$

Octet channel

150

```
SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2];
Mcbar81s08=%/. Momentum[p2]→ Momentum[P/2]
```

$$\text{Out}[1122]=\frac{2\,c_8\,g\sqrt{m_c^3}\,\bar{P}^\mu\left(\text{tr}\left(T^c.T^a.T^b\right)\right)}{m_c^2}$$

1P1

```
SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. {Momentum[p1]→ Momentum[P/2+q], Momentum[p2]→ Momentum[P/2-q]};
Mbar81p18=FourDivergence[%,FourVector[q,α]]
```

Out[1125]= 0

3S1

In[1126]:=

```
SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
M_cbar83s18=%/. {Momentum[p1]→ Momentum[P/2], Momentum[p2]→ Momentum[P/2]}
```

Out[1127]=

$$\frac{c_8 g \bar{P}^\alpha \bar{P}^\mu (\text{tr} (T^c . T^a . T^b))}{\sqrt{m_c^3}} - \frac{2 c_8 g m_c^2 \bar{g}^{\alpha \mu} (\text{tr} (T^c . T^a . T^b))}{\sqrt{m_c^3}} - \frac{2 c_8 g \sqrt{m_c^3} \bar{g}^{\alpha \mu} (\text{tr} (T^c . T^a . T^b))}{m_c}$$

3PJ

In[1128]:=

```
SUNFSimplify[ColorOctateProjector SUNFDelta[SUNFIndex[n],SUNFIndex[i]]*SUNFDelta[SUNFIndex[n],SUNFIndex[i]]
%/. Momentum[p1]→ Momentum[P/2+q];
%/. Momentum[p2]→ Momentum[P/2-q];
FourDivergence[%,FourVector[q,β]];
M_cbar83pj8=%/. Momentum[q]→ 0
```

Out[1132]=

$$- \frac{2 i c_8 g \bar{\epsilon}^{\alpha \beta \mu \bar{P}} (\text{tr} (T^c . T^a . T^b))}{\sqrt{m_c^3}}$$