

Math 31B
Integration and Infinite Series

Practice Midterm 2

Instructions: You have 50 minutes to complete this exam. There are 6 questions, worth a total of 100 points. This test is closed book and closed notes. No calculator is allowed. Please write your solutions in the space provided, show all your work legibly, and clearly reference any theorems or results that you use. Do not forget to write your name, section (if you do not know your section, please write the name of your TA), and UID in the space below. **Failure to comply with any of these instructions may have repercussions in your final grade.**

Name: _____

ID number: _____

Section: _____

Question	Points	Score
1	15	
2	17	
3	17	
4	17	
5	17	
6	17	
Total:	100	

Problem 1. *15pts.*

Determine whether the following statements are true or false. If the statement is true, write T in the box provided under the statement. If the statement is false, write F in the box provided under the statement. Do not write “true” or “false”.

- (a) ____ Let S be the solid obtained by rotating the region below a curve $f(x)$. The volume of S is always smaller than the surface area of S .
- (b) ____ Given non-zero polynomials $p(x)$ and $q(x)$, then we can always compute $\int \frac{p(x)}{q(x)} dx$.
- (c) ____ To compute improper integrals we can use limit(s).
- (d) ____ Let $f(x)$ be any function. The n th Taylor polynomial of $f(x)$ is an approximation of $f(x)$ using the first n derivatives of $f(x)$.
- (e) ____ If we have a sequence $\{a_n\}_{n=1}^{\infty}$ such that $\lim_{n \rightarrow \infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ does not converge.

Problem 2. *17pts.*

Find the integral of $f(x) = \frac{10}{x^4 - 2x^3 + 10x^2 - 18x + 9}$.

Problem 3. *17pts.*

- (a) Determine whether $\int_0^1 \frac{dx}{2x^2+5x}$ converges and, if so, evaluate it.
- (b) Determine whether $\int_{-1}^1 \frac{dx}{\sqrt[3]{x}}$ converges and, if so, evaluate it.

Problem 4. *17pts.*

Compute the surface area of revolution about the x -axis of $f(x) = \frac{x^2}{4} - \frac{\ln(x)}{2}$ in $[1, e]$.

Problem 5. *17pts.*

- (a) Let $T_n(x)$ be the Taylor polynomial for $f(x) = \ln(x)$ at $a = 1$. Let $c > 1$. Explain why $|\ln(c) - T_n(c)| \leq \frac{|c-1|^{n+1}}{n+1}$.
- (b) Find the smallest integer value of n such that $|\ln(1.5) - T_n(1.5)| \leq 10^{-2}$.

Problem 6. *17pts.*

- (a) Compute the limit of the sequence with general term $a_n = \sqrt{n+3} - \sqrt{n}$.
- (b) Write $\sum_{n=1}^{\infty} (\sqrt{n+3} - \sqrt{n})$ as a telescopic series and find its sum.