Math 33A
Linear Algebra and Applications

Practice Midterm 1

Instructions: You have 24 hours to complete this exam. There are 7 questions, worth a total of 100 points. This test is closed book and closed notes. No calculator is allowed. This document is the template where you need to provide your answers. Please print or download this document, complete it in the space provided, show your work in the space provided, clearly box your final answer, and upload a pdf version of this document with your solutions. Do not upload a different document, and do not upload loose paper sheets. Do not forget to write your name, section (if you do not know your section, please write the name of your TA), and UID in the space below. Failure to comply with any of these instructions may have repercussions in your final grade.

Name: ______________________________

ID number: ___________________________

Section: _____________________________

Question	Points	Score
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
Total:	100	

Problem 1. *10pts.*

Determine whether the following statements are true or false.

(a) If the system $A\vec{x} = \vec{b}$ has a unique solution, then A must be a square matrix.

(b) The system $A\vec{x} = \vec{b}$ is inconsistent if and only if $\mathrm{rref}(A)$ contains a row of zeros.

(c) If $A^2 = A$ for an invertible $n \times n$ matrix A, then A must be I_n.

(d) If matrix A commutes with matrix B, and B commutes with matrix C, then A must commute with C.

(e) If vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$ are linearly independent, then vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}$ are linearly independent.

Problem 2. *15pts.*

Consider the linear system

$$\begin{aligned} x - y - z &= 2 \\ x + 2y + z &= 3 \\ x + y + (k^2 - 5)z &= k \end{aligned}$$

where k is an arbitrary constant. For which values of k does this system have a unique solution? For which value(s) of k does the system have infinitely many solutions? For which value(s) of k is the system inconsistent?

Problem 3. *15pts.*

Determine the values of the constants a, b, c, d for which the vector

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

is a linear combination of the vectors

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 5 \\ 6 \end{bmatrix}.$$

Problem 4. *15pts.*

Find the matrix of the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ given by the reflection about the plane $y = z$.

Problem 5. *15pts.*

Decide whether the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 11 \\ 3 & 7 & 14 & 25 \\ 4 & 11 & 25 & 50 \end{bmatrix}$$

is invertible. If it is, find the inverse.

Problem 6. *15pts.*

Let $\vec{v}$ be a vector in $\mathbb{R}^3$. Describe, geometrically and algebraically, the image and kernel of the transformation T from $\mathbb{R}^3$ to $\mathbb{R}$ given by taking the dot product with $\vec{v}$. In particular, find a basis of the image and a basis of the kernel.

Problem 7. *15pts.*

Consider the basis of $\mathbb{R}^2$ given by $\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ and $\mathfrak{R} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$. Find a matrix P such that $\left[\vec{x}\right]_{\mathfrak{R}} = P\left[\vec{x}\right]_{\mathfrak{B}}$.