NAME AND UCLA ID:

Task 1: Read Sections 21, 22, 23, and 24.

Exercise 1: Let G be a finite p-group, $p \in \mathbb{Z}^+$ prime. Show that if p^n divides $|G|$ for some $n \in \mathbb{Z}^+$, then G has a normal subgroup of order p^n.

Exercise 2: Let G be a finite group, H and K subgroups of G. Show that $|HK| = |H||K|/|H \cap K|$ (recall that $HK = \{hk | h \in H, k \in K\}$).

Exercise 3: Let G be a finite group, H and K normal subgroups of G having relatively prime order. Show that if H and K are abelian, then HK is an abelian subgroup of G. Show that if H and K are cyclic, then HK is a cyclic subgroup of G.

Exercise 4: Let G be a group having $r \in \mathbb{Z}^+$ distinct subgroups of prime order $p \in \mathbb{Z}^+$. Prove that G contains at least $r(p-1)$ elements of order p. Use this to prove that any group of order 56 has a proper normal subgroup.

Exercise 5: Let G be a group such that $|G|$ is not prime and $|G| \leq 60$. Prove that G has a proper normal subgroup.