Math 31B
Integration and Infinite Series

Practice Midterm 3

Instructions: You have 50 minutes to complete this exam. There are 8 questions, worth a total of 10 points. This test is closed book and closed notes. No calculator is allowed. Please write your solutions on the scantron. Do not forget to write your name, section, and UID in the space below, as well as in your scantron.

Name: ______________________________

ID number: ______________________________

Section: ______________________________

Question	Points	Score
1	2	
2	1	
3	1	
4	2	
5	1	
6	1	
7	1	
8	1	
9	1	
Total:	11	

Problem 1. *2pts.*

11.2.37

Solution: The sums $\sum_{n=3}^{\infty} \frac{1}{n^4}$ and $\sum_{n=0}^{\infty} \frac{n^2}{2^n}$ are not geometric series.

Problem 2. *1pts.*

11.2.55

Solution: We replace each edge by four edges, so the perimeter at the n-th iteration is $4/3$ the perimeter at the $(n-1)$-th iteration, so $P_n = \frac{4}{3}P_{n-1}$. Then $P_n = P_0\left(\frac{4}{3}\right)^n$ so $\lim_{n\to\infty} = \infty$. Each replacement of the edges adds 4 edges, so at the $(n-1)$-th iteration we have $3 \cdot 4^{n-1}$ edges. Moreover each replacement of the edges generates a new triangle, so at the n-th iteration we generate $3 \cdot 4^{n-1}$ triangles. Also each edge at the n-th iteration has has length $1/3$ of the length of the edges at the n-th iteration, so each triangle added at the n-th iteration has area $1/9$ of the area of the triangles added at the $(n-1)$-th iteration. The total new area added is then the number of triangles added at the n-th iteration, which is $T_n = 3 \cdot 4^{n-1}$ the number of edges at the $(n-1)$-th iteration, multiplied by the area of each new triangle added, which is $A_n = \frac{A_{n-1}}{9} = \frac{A_0}{9^n}$ the area of the new triangles added at the $(n-1)$-th iteration divided by 9. Setting A_0 the area of the original triangle, we obtain $A = A_0 + \sum_{n=1}^{\infty} T_n \cdot A_n = \frac{8A_0}{5}$.

Problem 3. *1pts.*

11.3.13

Solution: The integral diverges by the Integral Test.

Problem 4. *2pts.*

11.4.33

Solution: The $(2N-1)$-th partial sum is $S_{2N-1} = \frac{1}{N+1}$ for $2N-1$ odd, $S_N = 0$ for N even. The series converges conditionally to zero.

Problem 5. *1pts.*
11.5.39

Solution: The series converges by the Root Test.

Problem 6. *1pts.*

11.6.11

Solution: The power series has radius of convergence $R = \sqrt{2}$ and interval of convergence $[-\sqrt{2}, +\sqrt{2}]$.

Problem 7. *1pts.*

11.7.37

Solution: The Maclaurin series for $f(x) = \frac{1}{1-x^2}$ is $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(2^{n+1}-1)}{2^{2n+3}}(x-3)^n$, valid for $|x-3| < 2$.

Problem 8. *1pts.*

11.7.45

Solution: The error bound gives $|\ln(1.2) - S_N| \leq a_{N+1} = \frac{1}{(N+1)5^{N+1}}$ so we must choose N such that $\frac{1}{(N+1)5^{N+1}} < 0.0001 = \frac{1}{10000}$. Since $(4+1)5^{4+1} = 5^6 = 15625$ then $N = 4$ is enough.

Problem 9. *1pts.*

BONUS 11.7.21

Solution: The terms through order four of the Maclaurin series for $f(x) = \frac{\sin(x)}{1-x}$ are $x + x^2 + \frac{5x^3}{6} + \frac{5x^4}{6}$.